Dynamic Shear Modulus of Crushable Sand

잘 부서지는 모래의 동적전단탄성계수

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요 지

동적인 문제를 해석하는 데 있어서 최대전단탄성계수의 결정은 어떤 변형률에서의 전단응력을 추정하는데 필연적이라 할 수 있다. 그동안 실리카질 모래에 관한 모델이 제시되어 왔지만 그러한 모델을 부서지는 모래에 직접적으로 적용하는 데는 모래의 부서짐(crushability) 때문에 어려움이 있다. 본 연구에서는 부서지는 모래의 미소변형률에서의 동적인 거동이 평가되었으며 실험한 모래의 최대전단탄성계수에 관한 모델이 제시되었다. 부서지는 모래의 전단탄성계수는 느슨한 모래의 경우는 실리카질 모래와 유사한 값을 보이나 밀도가 증가하면 입자간의 접촉면적이 커져서 비틂력(torsional force)으로 인해 모래가 부서져 전단저항이 작아지는 것으로 보인다. 그리고 계수치(modulus number)가 Been과 Jefferies(1985)의 상태정수(state parameter)로 표현되었다.

Abstract

In the analysis of dynamic problem, determination of maximun shear modulus is essential for the estimation of shear stress at any strain level. Although many models for silica sands were presented, the direct accomodation of those models to crushable sand would be difficult because of crushability during torsion. In this research dynamic behaviour of crushable sand at small strain is evaluated and model for the maximum shear modulus of tested sand is presented. The shear modulus of loose crushable sand shows similar results to silica sand. However, as the density of crushable sand increases the shear modulus decreases because of crushability by increasing surface contact area. And modulus number is expressed in terms of state parameter by Been and Jefferies (1985).

1. Introduction

In order to analyze the dynamic response of the ground or of the soil – structure system, dynamic properties such as damping capacity and shear moduli should be evaluated.

During last two decades many studies have been undertaken to determine shear moduli

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and damping for cohesionless and cohesive soils. Maximum shear mdoulus is essential for the estimation of shear stress at any strain level. Many researchers presented models for expression of maximum shear modulus of silica sands (non − crushable) as a function of void ratio and effective confining stress. However the models cannot express that of curshable sand because of the different behavior from non − crushable sands during torsion. Therefore it is necessary to know behavioral characteristics of crushable sand during torsion.

Properties of sand cannot be expressed in terms of relative density alone; a description of stress level must be included. Since Schofield and Wroth (1968) introduced the 'critical state' concept for an ideal cohesive and cohesionless soil, it has been used to accomodate for real test results. Recently Been Jefferies (1985) proposed state paramter, which offers some potential for the interpretation of many types of drained test of cohesionless soils.

2. Test Equipment and Type of Soil Tested

2.1 Resonant Column Test

Whatever a method is used to analyze the response of soils to dynamic problems, knowl-

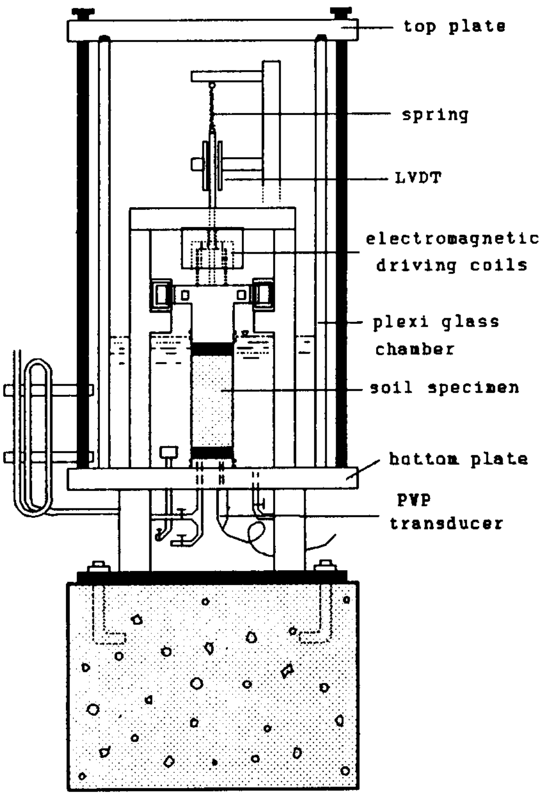


Fig. Resonant column test apparatus.

Tabel 1 Index properties of the sands.

Name of sand	Mol sand	Antwerpian sand
Median grain size, mm D_{50}	0.195	0.23
Uniformity coefficient D60 / D10	1.60	1.48
Degree of curvature $Cc = (D30)^2 / D10\ D60$	1.02	1.19
Effective grain size, mm D10	0.13	0.13
Moh's hardness : quartz glauconite	7.0	7.0 ≈2.0
Specific gravity, Gs	2.65	2.807
Roundness : quartz particles glauconite particles	subangular −	subangular rounded
Maximum void ratio, e_{max}	0.918	1.018
Minimum void ratio, e_{min}	0.585	0.665
Max. dry density, kN / m³ $\gamma_{d,max}$	16.387	16.529
Min. dry density, kN / m³ $\Gamma_{d,min}$	13.553	13.608

edge of the stress − strain relationships for the soil at small deformation level is required. In this research the resonant column test apparatus (called Drnevich apparatus) was implemented (Fig 1). This device goes out from soild cylindrical specimens fixed at the base with excitation forces applied to the top of the specimen.

The apparatus has the capability of applying both longitudinal and torsional excitations. The Young's modulus can be obtained from the longitudinal response and the dynamic shear modulus from the torsional response. Only the torsional mode was used in this test series. The mechanical parts of the appartus consist essentially of the electromagnetic vibration excitation, the force transmission system to the base of the specimen, and the confining cell.

2.2 Soil Types Involved in This Research

2.2.1 Mol Sand

Mol sand is composed of quartz minerals. The Mol sand tested in this research is a uniform, fine quartzitic sand with a median grain size of 0.195mm (Fig. 2). The physical characteristics and the curve of grain size distribution of Mol sand are given in Table 1.

2.2.2 Antwerpian Sand

Antwerpian sand is consisted of about 53% of glauconite and 47% of quartz minerals. According to McRae(1972), glauconite(clay mineral of the illite type) is a hydrated silicate of Potassium, Magnesium, Aluminium, and ferro and ferri iron [$KMg(FeAl)(SiO_3)_6{}^{-3}H_2O$]. The glauconite paraticles are much more crushable than quartz. Glauconite is usually green in color, although the shade may vary from dark green to a pale greenish − yellow. The grain size distribution of Antwerpian sand is shown in Fig. 2 and the physical characteristics of clean Antwerpian sand used for this research are listed in Table 1.

2.3 Specimen Preparation and Test Procedure

It has long been established that different methods of reconstituting samples to the same density produce samples with different structures and behavioural characteristics. Therefor it is important to reproduce the same density and soil fabric according to the purpose of the research. According to Dennis(1988), identical specimens prepared using the undercompaction technique were more resistant to volume change when subjected to a confining vacuum than those prepared using equal - energy technique.

In this research specimen prepartion procedure used through all the studies were moist tamping technique using undercompaction method(Ladd, 1972). This method has an advantage to make same density throughout the layers. The approximate initial dimensions of the test specimens are 35.2mm in diameter and 78mm in height.

Clean sand is mixed with distilled water in an order of water content of 10 percent.

Specimen preparation for the resonant column is identical to that for triaixal specimen using undercompaction concept. Conventional triaxial membranes were used. A split mold with vacuum connections is supplied for making specimens with sands. After constructing a specimen, top platen sustem (the plate with magnet, accelerometer, etc.) is installed next. Extra care was necessary to avoid sample disturbance. Torsional drive coils were next put into place. The heavy bar with circular magnet, linear variable transducer (LVDT), and pressure chamber were put in place. Distilled water was allowed to flow into the chamber until it covers the specimen, so leaving an air cushion in the top of the cell. Finally all connections to the control box and the associated equipment were made. The specimen consolidation procedure is identical to the one used in conventional triaxial test except that a LVDT was used to meaure length change.

The frequency of the applied force was adjusted until the first mode resonance frequency was obtained. When an acceleration transducer was used, the resonance frequency produces an ellipse with axes vertical and horizontal. The phase relationship describing resonance can be established by observing the Lissajous figure formed on an x - y oscilloscope with the voltage proportional to the driving current (Drnevich, 1982). The frequency was readjusted to obtain resonance and readings were recorded. This process was continued until the ill - shaped Lissajous figure was appeared.

3. Literature Review

3.1 Maximum Dynamic Shear Modulus

Hardin and Richart (1963) and Hardin (1978), expressed the maximum shear modulus as a function of void ratio and effective mean confining pressure. The shear moduli are taken as various form depending on the stress, void ratio level and sand type.

· dry Ottawa sand, $\sigma' > 96$ kPa

$$G_o \propto \frac{(2 \cdot 17 - e)^2}{1 + e} \quad (0.35 < e < 0.8)$$

· dry Ottawa sand $\sigma' > 96$ kPa

$$G_o \propto \frac{(2 \cdot 13 - e)^2}{1 + e} \quad (0.35 < e < 0.8)$$

· dry quartz

$$G_o \propto \frac{(2 \cdot 973 - e)^2}{1 + e} \quad (0.6 < e < 1.3)$$

· dry fine quartz sand, $\sigma' > 96$ kPa

$$G_o \propto \frac{(1 \cdot 902 - e)^2}{1 + e} \quad (0.7 < e < 1.0)$$

The differences among the types of function are shown in Fig. 3

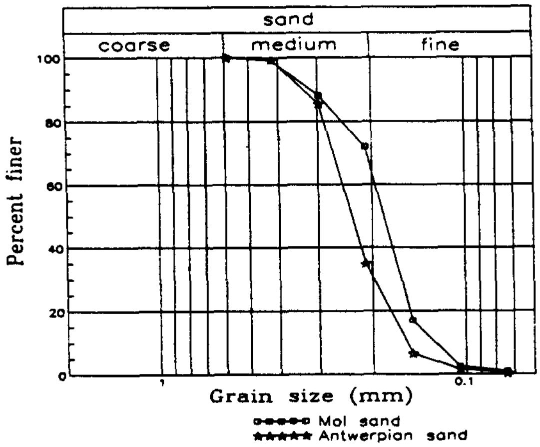


Fig. 2 Grain size distribution of the sands.

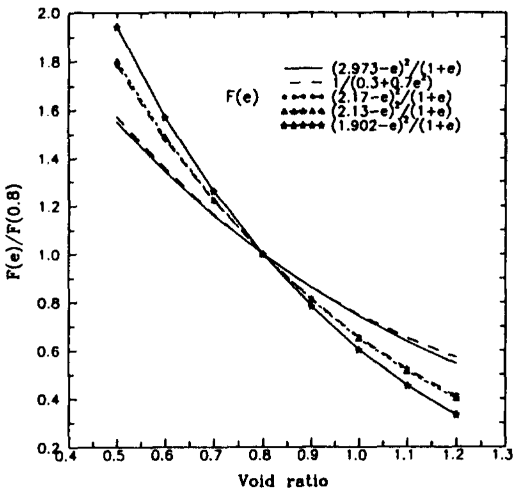


Fig. 3 Comparison of different type of functions in terms of void ratio on mzximum shear modulus.

In 1978 Hardin revised the form of equation formerly proposed by Hardin and Drnevich (1972). to

$$Go = [\ S\ (OCR)^k/(0.3 + 0.7e^2)\ p_a^{1-n}(\sigma_o)^n \qquad (1)$$

where

S = stiffness coefficient (no units)

P_a = atmopsheric pressure, units the same as G

n = power of stress

Eq.(1) applies for all systems of units because it is dimensionally correct and by introducing p_a the parameters is dimensionless. Furthermore Eq.(1) applies also for soils that may have large void ratios.

For clean sands, it has been found that Go is essentially influenced by three main factors : (1) the confining stress ; (2) the strain amplitude ; and (3) the initial void ratio.

Analytical expressions have been presented for the shear modulus of clean sands as (Hardin and Black, 1968) :

for round - grained sands,

$$Go = \frac{6908\ (2.17 - e)^2}{1 + e}\ \sigma_o^{0.5} \qquad (2)$$

and for angular grained sands,

$$Go = \frac{3230\ (2.97 - e)^2}{1 + e}\ \sigma_o'^{0.5} \tag{3}$$

In Eqs. (2) and (3), Go and σ_o' have units of kN/m^2. Both equations were originally established to correspond to shear strains γ fo 10^{-4} or less. Iwasaki and Tatsuoka (1977) have determined experimentally that

$$Go = \frac{900\ (2.17 - e)^2}{1 + e}\ \sigma_o'^{0.38} \tag{4}$$

from tests on clean sands ($0.61 < e < 0.86$ and $0.2 < \sigma_o' < 5kg/cm^2$) at shear strain amplitudes of 10^{-6}

3.2 State Parameter Ψ by Been and Jefferies

State parameter Ψ (Fig. 4), which is defined as the void ratio difference between the initial p′-e state and steady state conditions at the same mean effective stress p′, can be used to describe much of the behavior of granular materials over a wide range of stresses and densities. This parameter is embodied in the concept of critical soil mechanics. Ψ represents an adaptation of the parameter V_λ (specific volume) formerly proposed by Schofield and Wroth (1968). The curve connecting the points when the specimen is in the loosest possible states is often called the virgin compression line, because it is strictly the line connecting equilibrium state. The first stress invariant I_1 is proposed to be a suitable stress measure for incorporation into the state parameter.

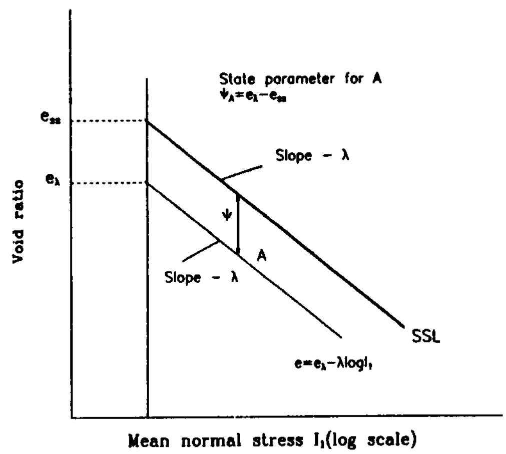


Fig. 4 Definition fo state parameter by Been and Jeffries (1985).

3.2.1 Determination of the Steady State Line

The steady state line (Fig. 4) is defined as the locus of all points in void ratio – stress space at which a soil mass deforms under conditios of constant effective stress, void ratio and velocity. According to Been et al. (1991), for sands there appears to be little distinction between the critical and steady state and the differences appear to lie in the methods of measurement. Critical state generally was determined from drained, strain - rate controlled tests on dilatant samples while the steady state is measured in undrained test, usually on loose samples.

3.2.2 Sand Behavior in terms of State Parameter

Samples with negative state parameter will always have a negative state at phase transformation and samples with high positive state parameter will have a positive state at phase transformation.

For sands with negative parameter, thare is a clean peak in deviator stress which becomes less marked with decreasing negative state parameter, until there is generally no peak for samples with positive state parameter.

The volumetric strain behavior is similarly dependent on state parameter. Strong dilation is apparent for high negative states with little dilation or even contraction observed where state parameter is positive.

4. Test Results and Analysis

4.1 Shear Modulus as a Function of Shear Strain

Fig. 5 for the resonant column (RC) testes in this research shows the effects of strain amplitude, effective mean principal stress on the shear modulus of Antwerpian sand. It can be observed that shear moduli are strain dependent. As widely known and anticipated, the shear modulus increases with increasing confining pressure throughout the range of the shear strain amplitudes studied. As already shown in references by Chung et. al (1984), Georgiannou (1991) and others, the shear modulus variation in the small strain range ($\leqslant 10^{-4}$) is very small. According to Dobry et al. (1980) and Van Impe (1980), below an "elastic threshold ($\leqslant 10^{-5}$)" shear strain the soil behavior is approximately isotropic and linear elastic. As shown in Fig. 5, below a shear strain of about 10^{-5} the shear modulus is almost constant and equlal to maximum shear modulus G_{max}. This "elastic threshold" shear strain is in agreement with the data reported by Hardin and Drnevich (1972), Chung et al. (1984), Georgiannou (1991) and others from RC tests. Those data show that for a value of shear strain of $0.25*10^{-4}$ the difference in shear modulus is within about 2 to 5 percent of the G_{max}. Above the "elastic threshold" shear strain the soil behavior is non - linear elastic until a "plastic threshold" strain ($\approx 5-10*10^{-5}$), the soil behaves as an elastoplastic material.

On the other hand the shear moduli decrease rapidly with increasing shear strain in the range higher than 10^{-4} of shear strain. Below a strain amplitude of approximately 10^{-5} the behavior of the sands tested would be approximately linear elastic.

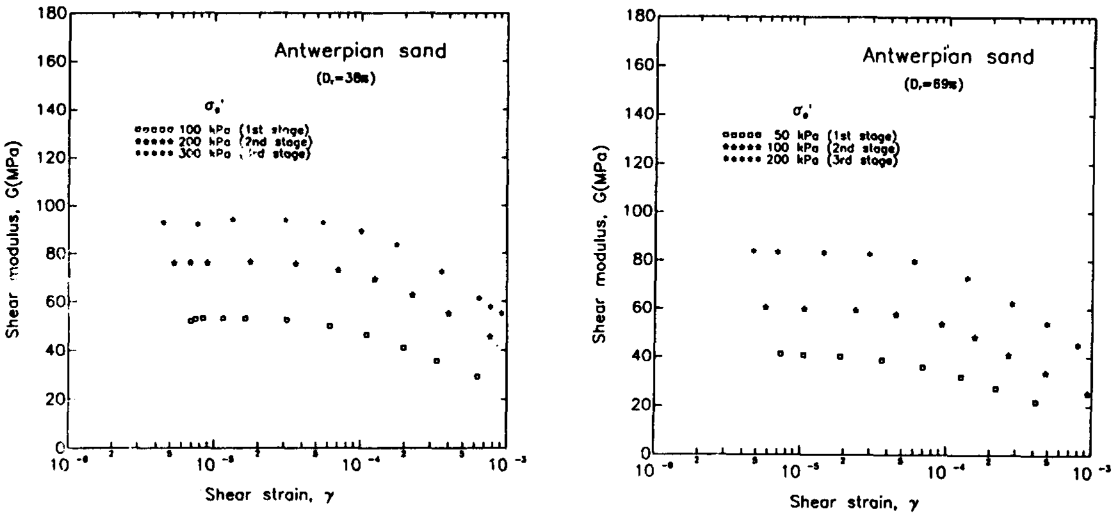


Fig. 5 Shear modulus versus shear strain for Antwerpian sands from resonant column test.

4.2 Estimation of the Maximum Shear Modulus on Sands in Resonant Column Test

In this research 6 series of multistage tests for initially loose, medium, dense sands were carried out in the stress range of 25 kPa to 600 kPa. In general, the maximum shear modulus (defined as shear strain $\leq 10^{-5}$) was determined in the resonance frequency range of 20 to 40 Hz. Once initial shakedown in small amplitude of strain was completed (to evaluate the system response), the shear modulus and damping were determined by applying a torsional excitation force (2.2 to 3.0 mV) of very low magnitude to the specimen that produced average shear strain amplitudes of less than 10^{-5}. The test started at the low confining pressure of 20 − 30 kPa and repeated for intermediate confining pressure up to maximum confining pressure of 600 kPa with considering the change in void ratio by elevating confining pressure. The shear modulus, average shear strain, and damping ratio D of each specimen were calculated from the test results using equations given by Drnevich et al. (1978).

From Fig. 6(a) to 6(f) show the measured shear moduli Go at very small shear strains as a function of confining pressure. For the same sand with different initial densities, the tests were performed and volume changes during tests were taken into account for calculation of void ratio. There are some abnormal values from the regression lines in Fig. 6(e) and Fig. 6(f) which sometimes are discovered at the low confining pressure of 20 to 50 kPa. These deviating test results probably were induced by an improper seating between top platen and the top of the specimen or by bad contact between magnet and magnet plate.

The following type of empirical equation closely approximates the test data presented in Fig. 6.

$$\frac{G_o}{p_a} = [\ S/(0.3 + 0.7e^2)]\ (\frac{\sigma_0}{p_a})^n \qquad (5)$$

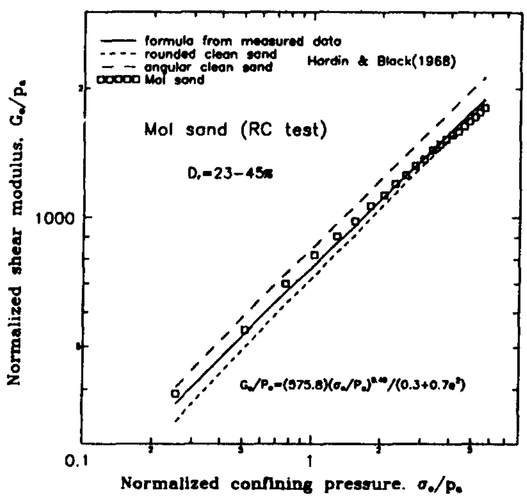
For the sands tested the constants S and n in Eq. (5) are listed in Table 2.

The power of the confining stress varies from 0.45 to 0.49 in Mol sand and from 0.37 to 0.46

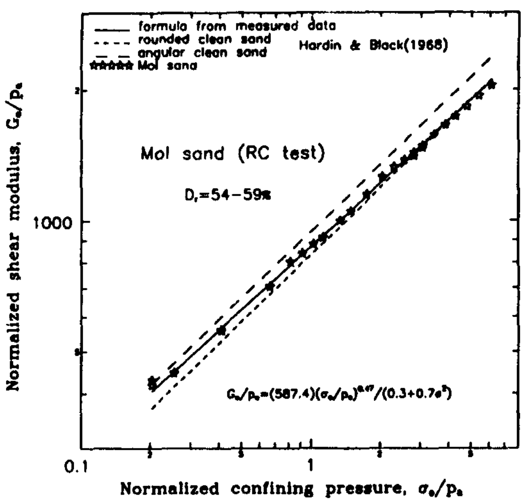
Table 2. The values S and n in Eq. (5) for the sands.

	Mol sand			Antwerpian sand		
Density	S	n	$S^{0.5}$	S	n	$S^{0.5}$
Loose	575.8	0.49	571.8	491.1	0.37	448.8
Medium	587.4	0.47	575.3	479.5	0.43	456.8
Dense	581.9	0.45	562.1	408.0	0.46	396.8

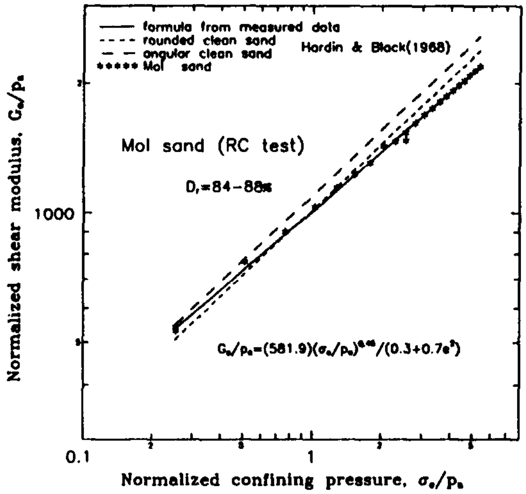
Note : $s^{0.5}$ means the S value in case of n = 0.5 in Eq. (5).



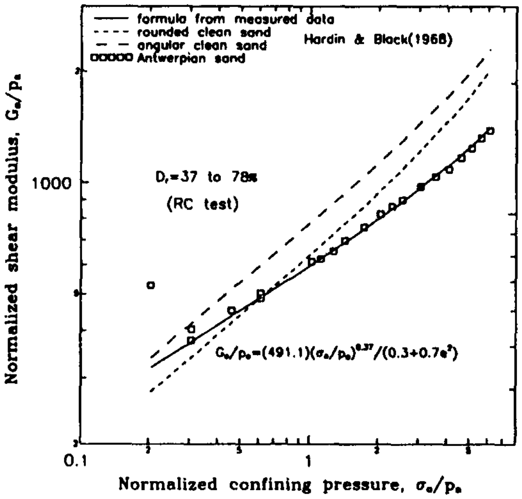
(a) loose Mol sand



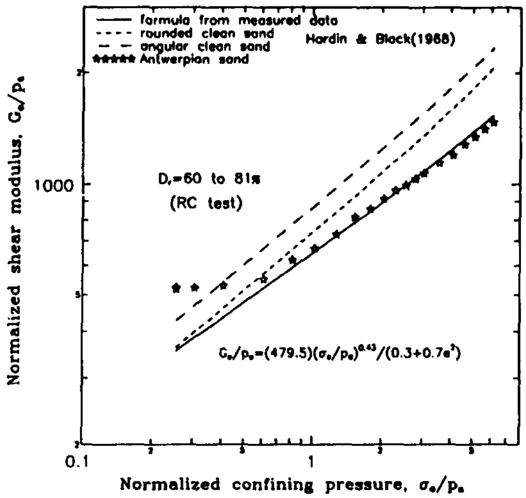
(b) medium Mol sand



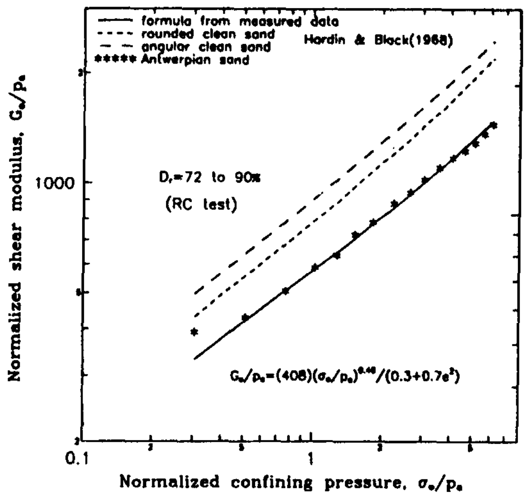
(c) dense Mol sand



(d) loose Antwerpian sand



(e) medium Antwerpian sand



(f) dense Antwerpian Sand

Fig. 6 Maximum shear modulus versus confining pressure for initially loose, medium and dense sands.

in Antwerpian sand. Putting the values of power to be 0.5, the S values in Eq.(5) vary from 551 to 570 for Mol sand approximately. Likewise the shear modulus at small strain range for Antwerpian sand, the S values vary from 410 to 490 approximately. These values are far lower than those of Mol sand and particularly in dense Antwerpian sand. Antwerpian sand, composed of quartz and glauconite, is increasing its silt content at sample preparation by compaction using the undercompaction concept. The low shear modulus probably comes from weak resistance of glauconite to shearing and the crushing will be still increased by torsional vibration in the densely compacted sample where the number of contacts between the quartz and glauconite is higher than in loose Antwerpian sand.

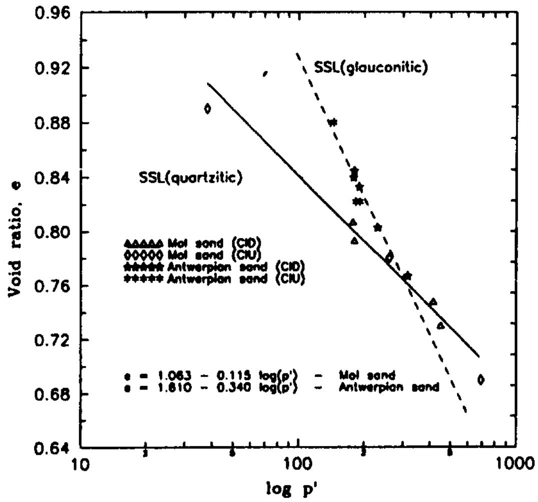


Fig. 7 The steady state lines for the sands tested.

4.3 The Relationship between Modulus Number and State Parameter

As mentioned previously, state parameter is defined as void ratio difference between the initial sand state and steady state at the same mean effective stress. From the comparison of state parameter with maximum shear modulus in Eq. (1), we can find the common parameters, that is, void ratio and mean effective confining stress. Therefore the state parameter can be connected to maximum shear modulus by using modulus number. The steady state line for both sands are shown in Fig. 7.

Definining modulus number(Carriglio, 1989 ; Jamiolkowski, 1991) as

$$M_G = \frac{G_o}{p_a\left(\frac{\sigma'_o}{p_a}\right)^{0.5}} \tag{6}$$

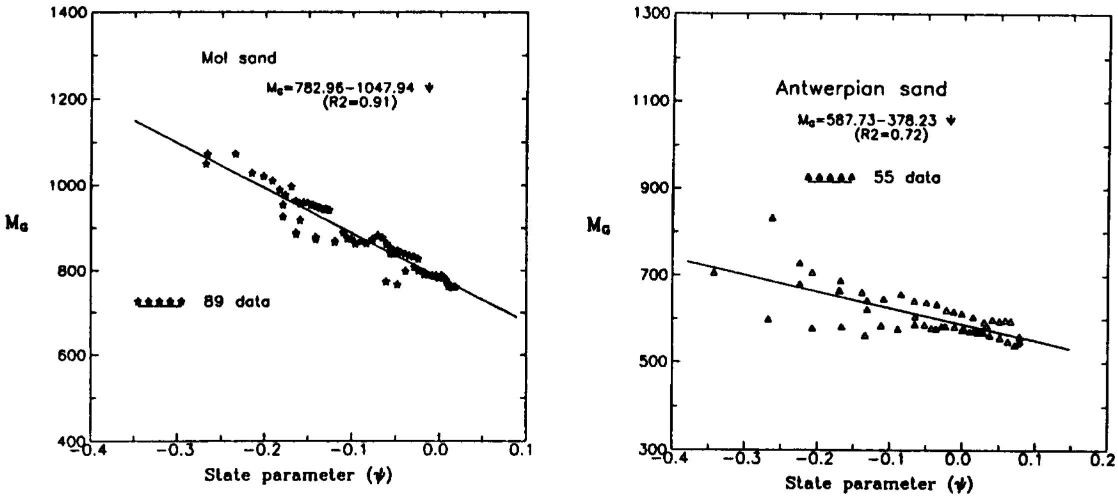
The relationship between state parameter and modulus number can be expressed as (Fig. 8) :

- Mol sand

  $M_G = 782.96 - 1047.94\ \Psi$
- Antwerpian sand

$$M_G = 587.73 - 378.23\,\Psi$$

where Ψ is state parameter defined by Been and Jefferies (1985).



(a) Mol sand

(b) Antwerpian sand

Fig. 8 Modulus number versus state parameter for the tested sand.

5. Conclusions

Based on the beforementioned experimental findings and analyses from RC tests, the following conclusions are drawn.

(1). The maximum shear modulus Go at small strain is closely predicted by Eq.(5) and the constants in Table 2. In particular, Antwerpian sands show lower shear moduli than Mol sand at the same void ratio and confining pressure. Furthermore in dense sand the reduction is prominent. This is, in the author's opinion, attributed to the glauconite mineral which clearly must have weaker shear resistance. In particular the dense Antwerpian sand gives similar shear moduls (Fig. 6) to medium dense Antwerpian sand and far lower shear modulus than quartz sand at similar void ratio. This is probably due to the higher surface contact between quartz and glauconite particles which cause a weaker resistance to torsional shear by crushing of glauconite between quartzs.

(2) The maximum shear modulus determined in RC tests gives good correlation with state parameters defined by Been and Jefferies (1985) as given in Fig. 8.

REFERENCES

1. Been, K. and Jefferies, M. G., "A State Parameter for Sands," Géotechnique, London, England, Vol. 35, No, 2, 1985, pp. 99 – 112.
2. Carriglio, F., "Stress - Strain and Resistance Characteristics for Sands," Doctoral Thesis, Turin Politechnic, 1989, (in Italian).
3. Dennis, N. D., "Influence of Specimen Preparation Techniques and Testing Procedures on Undrained Steady State Shear Strength," ASTM STP 997, Philadephia, 1988, pp. 642 – 654.
4. Dobry, R., Powell, D. J., Yokel, F. Y., and Ladd, R. S., "Liquefaction Potential of Saturated Sand : The Stiffness Method," Proc. of the 7th World Conference on Earthquake Engineering," Istanbul, Turkey, Vol. 3.
5. Drnevich, V. P., "Recent Development in Resonant Column Testing," Richart Commemorative Lectures, Proc. of a session sponsored by ASCE, 1985, pp. 79 – 107.
6. Georgiannou, V. N., Rampello, S., and Silvestri, F., "Static and Dynamic Measurements of Undrained Stiffness on Natural Overconsolidation Clays," Xth ECSMFE, Vol. 2, Firenze, 1991, pp. 91 – 95.
7. Hardin, B. O. and Drnevich, V. P., "Shear Modulus and Damping in Soils : Measurement and Parameter Effects, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM6, June 1972a, pp. 603 – 624.
8. Hardin, B. O. and Drnevich, V. P., "Shear Modulus and Damping in Soils : Design Equations and Curves," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM7, July 1972b, pp. 667 – 692.
9. Ladd, R. S., "Preparing Test Specimens Using Undercompaction," Geotechnical Testing Journal, GTODJ, Vol. 1, No. 1, March 1978, pp. 16 – 23.
10. McRae, S. G., "Glauconite," Earth - Sicience Reviews, Elsevier Co., Amsterdam. Vol. 8, No. 4, Dec. 1972. pp. 397 – 427.
11. Schofield, A. N. and Wroth, C. P., Critical State Soil Mechanics. McGraw - Hill Book Co., Inc., London, 1968.
12. Tatsuoka, F., Iwasaki, T., and Takaki, Y., "Hysteretic Damping of Sands under Cyclic Loading and It's Relation to Shear Modulus," Soils and Foundations, JSSMFE, Vol, 18, No. 2, June 1978, pp. 25 – 40.

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