Application of Nonlinear Goal Programming to Structural Optimization

구조최적화에 관한 비선형 Goal Programming의 응용

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초 록

본 논문은 비선형 goal programming을 이용하여 구조 최적화 문제들의 해를 얻는 방법을 제시한다. 이 방법은 다기준 최적화의 도구로 사용되는데 그 까닭은 goal programming이 목적함수와 제한조건 등을 정의하는데 있어서 발생하는 난점들을 제거해 주며, 또한 설계목적의 우선 순위를 다루는 능력을 갖추고 있기 때문이다.

비선형 goal programming에 대한 구조 최적화의 모델이 제시된다. Hooke-Jeeves의 pattern 탐색, 수정된 Hooke-Jeeves의 Pattern 탐색, 그리고 Powell의 공액방향 탐색방향을 이용한 세 가지의 비선형 goal programming 들이 개발되고 토의된다. 설계 도구로서 이 방법의 유효성을 논증하기 위하여 사례의 해를 모색하며 이를 다른 연구결과와 비교 검토한다.

I. INTRODUCTION

Mathematical programming (MP) is a very important tool in industrial and engineering design. The application of optimization techniques to structural problems has continued to increase and to gain considerable interests since a pioneering work by Schmit [1] who used the mathematical programming model with nonlinear inequality constraints under multiple loading conditions.

The traditional mathematical optimization model is quite suitable for many cases where the problem specification is clear, i.e., the objective function is well defined and the constraints are consistent. However, such is not always the case. There is often no feasible solution due to the conflicting constraints and poor formulation. In the worst situation, there is no simple objective function due to the difficulty in defining the problem. To handle them, a flexible and conceptual optimization technique such as the goal programming technique is needed.

In a goal programming (GP) formulation, the design variables and goals are defined, and

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weights and priority factors are assigned to each one. The algorithm then attempts to minimize the sum of the deviational variables with weights and priority factors, starting with the highest priority goal. The difficulty in defining the optimization problem is eliminated by dealing with an objective function and constraints as goal constraints, which makes goal programming [2-6] an ideal real-life design tool.

In the following the structural optimization model is presented for nonlinear goal programming (NLGP) problems. For structural optimization problems with high nonlinearity, the linear goal programming (LGP) with successive linearization has several drawbacks such as the large memory requirement and non-convergency. To compensate this the nonlinear goal programming techniques are developed. The developed NLGP using the pattern search method by Hooke-Jeeves (NLGP-HJ) is modified to solve structural optimization problems. The NLGP using the modified Hooke-Jeeves method (NLGP-MHJ) which is improved by adding and modifing a line search program is developed. Also the NLGP using the conjugate direction method by Powell (NLGP-PO) is developed. The above NLGP algorithms are used to solve the 10 member truss problems whose objective functions have deviational variables, priority factors and weights. The results obtained according to different goal programming algorithms are compared each other. And their comparisons with Refs. [7,8] are carried out.

2. FORMULATION

2.1 Structural Optimization Model for Nonlinear Goal Programming

The general goal optimization problem is:

$$\text{Minimize} \quad Z=\sum_{i=1}^{I} W_{ki} P_k (d_i^- + d_i^+)$$

$$(k=1, 2, \cdots\cdots, K)$$

$$\text{subject to} \quad gi(x)+d_i^- - d_i^+ = b_i$$

$$(i=1, 2, \cdots\cdots, I)$$

$$\text{and} \quad x^L \le x \le x^U, \ d_i^-, \ d_i^+ \ge 0$$

$$\text{to find} \quad x \in R^N \qquad (1)$$

where vector Z is the objective function of GP to be minimized. The dimension of Z represents the number of preemptive priority levels. The differential weights W_{ki} are mathematical weights which are expressed as cardinal numbers, and are used to differentiate the i^{th} deviational varibles within a single k^{th} priority level. The pre-emptive priority factors P_k represent a ranking system which places the importance of goals in accordance with the following relationship: P_1 (The most important goal) $> P_2 >>> P_k$ (The least important goal). d_i^- and d_i^+ are negative and positive deviational variables that express the possibility of deviation from a right-hand-side value b_i (these variables are conceptually similar to slack variables in LP models). g_i is the goal constraints function we desire to minimize its numerical deviation from a stated right-hand-side value b_i in a selected goal constraint. This goal constraint is expressed as the flexible constraints. x^L and x^U represent the lower and upper bound of design variable, respectively. x is a set of N design variables we seek to determine.

The traditional model of the minimum weight structural optimization problem is:

$$\text{Minimize} \quad M(x)$$

$$\text{subject to} \quad G_i(x) \le 0$$

$$(\text{for } i=1,2,\cdots\cdots,J)$$

and $\quad x^L \leq x \leq x^U$

to find $\quad x \in R^N \qquad (2)$

where M(x) is the objective function, representing the total weight of the structure. G_i (x) represents the J constraint functions including the inequality and equality constraints and it may be a function of stress, displacement, and natural frequency. This constraint is expressed as the hard constraint.

The structural optimization problem, Eq (2), can be rewritten in the multiple objective goal structural optimization model. This is expressed as follows:

$$\text{Minimize} \quad Z=\sum_{i=1}^{I} W_{ki}\, P_k (d_i^- + d_i^+)$$

$$(k=1,2,\cdots\cdots,K)$$

subject to $\quad M(x)/M_a + d_1^- - d_1^+ = 1,$

$$\sigma(x)_{(i-1)}/\sigma_a + d_i^- - d_i^+ = 1$$

$$(\text{for } i=2,3,\cdots\cdots,IK+1)$$

$$u(x)_{(i-IK-1)}/u_a + d_i^- - d_i^+ = 1$$

$$(\text{for } i=IK+2,\ IK+3,\cdots\cdots,$$

$$I-1)$$

$$\Omega(x)/\Omega_a + d_I^- - d_I^+ = 1$$

and $\quad x^L \leq x \leq x^U,\ d_i^-,\ d_i^+ \geq 0,$

to find $\quad x \in R^N \qquad (3)$

M(x) is the total weight of the structure which is a function of the design variables, σ_i (x) is the stress component of the i^{th} member, u_i (x) is the global displacement component of the nodal point, and Ω (x) is the natural frequency. And M_a, σ_a, u_a and Ω_a represent the target weight, allowable stress, allowable displacement and the targeted lowest natural frequency, respectively. I is the total number of goal constraints and IK is the number of members of the truss.

2.2 Nonlinear Goal Programming Using the Hooke-Jeeves Method

The Hooke-Jeeves technique [9,10] is an accelerated pattern search technique. It is based on the assumption that any set of the direction of univariate search which has been successful in improving the value of the objective function is maintained until the improvement stops.

There are two moves. One is the exploratory move which determines the base point. The base point x^m is found as follows:

$$x^m = tn \qquad (4)$$

where

$$\begin{cases} x^{(m-1)} + \delta_1 & \text{if } f(x^{m-1}+\delta_1) < f(x^{m-1}) \\ x^{(m-1)} - \delta_1 & \text{if } f(x^{m-1}-\delta_1) < f(x^{m-1}) \\ x^{(m-1)} & \text{if } f(x^{m-1}) < \min[f(x^{m-1}+\delta_1),\ f(x^{m-1}-\delta_1)] \end{cases}$$

for $j=2$ to n

$$t_j = \begin{cases} t_{j-1}+\delta_j & \text{if } f(t_{j-1}+\delta_j) < f(t_{j-1}) \\ t_{j-1}-\delta_j & \text{if } f(t_{j-1}-\delta_j) < f(t_{j-1}) \\ t_{j-1} & \text{if } f(t_{j-1}) < \min[f(t_{j-1}+\delta_j),\ f(t_{j-1}-\delta_j)] \end{cases}$$

where t_j is temporary head of j^{th} coordinate, step size, and x^{m-1} previous base point. The other is the pattern move that gives the coordinates of pattern move, x_p^{m+1}. The x_p^{m+1} is expressed as follows:

$$x_p^{m+1} = X^{m-1} + a(x^m - x^{m-1}) \qquad (5)$$

where α is a constant and is called an acceleration factor. From x_p^{m+1}, the exploratory move is undertaken again. If there is no improvement, x_p^{m+1} is discarded and x^m is taken as a base point, where an exploratory move is undertaken to find the new pattern move point. Eventually, a situation is reached where this exploratory move fails. Then the step size is reduced by some factor and the exploration resumed. The search is terminated when the step size or the difference between previous and present objective function value becomes sufficiently small. The pattern search method described above is applied to the multi-objective goal programming with the name of NLGP-HJ, in which method is modified and linked to a finite element code to solve structural optimization problems.

2.3 Nonlinear Goal Programming using the Modified Hooke-Jeeves Method

The NLGP-HJ needs many function evaluations and takes a large amount of CPU time to find the optimum solution. This pattern method is improved by adding a line search program [11,12] to find the minimum of an objective function in the pattern move direction. The line search program is used with an iterative application of Eq. (3) which is modified and expressed as:

$$x_p^{m+1} = x^{m-1} + a^*(x^m - x^{m-1}) \qquad (6)$$

where $(x^m - x^{m-1})$ is the same search direction as that of pattern move, and α^* a scalar multiplier determining the amount of change in α for the iteration. The search for α^* proceeds in two phases, called the bounding and interval refinement phases. The former is an initial coarse search that bounds or brackets the optimum and the latter is a finite sequence of interval reductions to reduce the initial search interval to a desired accuracy. The termination criteria is the same as the NLGP-HJ method.

2.4 Nonlinear Goal Programming Using the Powell's Conjugate Direction Method

Powell [13] introduced a method of multivariable optimization. This is one of the most powerful and efficient nongradient based methods availiable today.

This method will minimize a quadratic in the order of magnitude of n steps. The first search is started with a set of n orthogonal and linearly independent search directions, S^i, i = 1 to n, being the coordinate directions. Powell's method performs successive line search along each directions, where each search consists of updating x vector, which is expressed as:

$$x^i = x^{i-1} + a^*_i S^i \qquad (7)$$

where α^*i is a scalar multiplier determining the output of change in x for i^{th} iteration. After performing these successive minimizations, a new search direction is formed between the original starting point and the resulting point of the successive line searches. The first search direction is dropped and the remaining search directions are kept along with the new direction, which is placed last among the direction. The search is continued and the other conjugate directions are found. The search is terminated when the norm of the difference between present and previous design variables is less than some specified small number or when the difference between present and previous objective function value becomes sufficiently small. The Powell's conjugate direction method is applied to the goal programming with the name of NLGP-

PO, in which method is modified and linked to a finite element code to solve structural optimization problems.

3. TEST CASES

3.1 Problem Outline

Fig. 1 shows the geometry and dimensions of the 10 member truss. The design data for the truss elements and load data are given in Table 1. Two loading cases given in Table 1 are considered for the analysis.

For case I, the optimum design is obtained for the problem I-1, whose structure is designed to withstand a single loading conditions subject to weight, stress and displacement goal constraints. For case II, the optimum design is obtained for the problem II-1, whose structure is the same as the problem I-1 except that the natural frequency constraint of 22.Hz is imposed.

Three kinds of NLGP in which the objective function has only the deviational variables are applied to problem I-1 and II-1. The algorithms with priority factors in the objective function are applied to problem II-1, and for this case the problem is designated as II-1p. The NLGP with weights and priority factors is also applied to problem II-1, and the problem is designated as II-1wp. For convenience's sake, the formulation for the problem II-1wp is given in Appendix. A VAX 8650 computer is used to solve the problems.

3.2 Results and Discussion

3.2.1 Objective Function with Deviational Variables

The results for the optimum cross-sectional area of each member, the optimum weight,

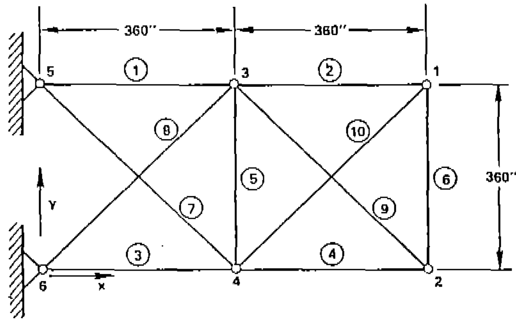

Fig. 1 10 Member Planar Truss

Table 1 Design Data for 10 Member Planar Truss

Modulus of elasticity = 10^4 ksi					
Material density = 0.10 lb/in^3					
Lower limit on cross-sectional areas = 0.10 in^2					
Upper limit on cross-sectional areas = 50 in^2					
Initial value of design variable = 10 in^2					
Stress limit = ±25 ksi					
Number of loading conditions = 1					
Load Data					
			Load component (kips) in Direction		
Load Case No.	Loading Condition	Node	x	y	z
I	1	2	0.0	-100.0	0.0
		4	0.0	-100.0	0.0
II	1	1	0.0	50.0	0.0
		2	0.0	-150.0	0.0
		3	0.0	50.0	0.0
		4	0.0	-150.0	0.0

APPENDIX

Formulation of Problem II-1wp

Minimize $Z = 2P_1(d_1^+ + d_{16}^-) + P_1(d_{13}^- + d_{15}^+) + P_2(d_2^+ + d_3^+ + d_4^+ + d_5^+ + d_6^+ + d_7^+ + d_8^+ + d_9^+ + d_{10}^+ + d_{11}^+ + d_{12}^+ + d_{14}^-)$

subject to

$M/4500 + d_1^- - d_1^+ = 1,$

$\sigma_1/25 + d_2^- - d_2^+ = 1,$

$\sigma_2/25 + d_3^- - d_3^+ = 1,$

$\sigma_3/25 + d_4^- - d_4^+ = 1,$

$\sigma_4/25 + d_5^- - d_5^+ = 1,$

$\sigma_5/25 + d_6^- - d_6^+ = 1,$

$\sigma_6/25 + d_7^- - d_7^+ = 1,$

$\sigma_7/25 + d_8^- - d_8^+ = 1,$

$\sigma_8/25 + d_9^- - d_9^+ = 1,$

$\sigma_9/25 + d_{10}^- - d_{10}^+ = 1,$

$\sigma_{10}/25 + d_{11}^- - d_{11}^+ = 1,$

$((u_y)_1)/2 + d_{12}^- - d_{12}^+ = 1,$

$((u_y)_2)/2 + d_{13}^- - d_{13}^+ = 1,$

$((u_y)_3)/2 + d_{14}^- - d_{14}^+ = 1,$

$((u_y)_4)/2 + d_{15}^- - d_{15}^+ = 1,$

$f/22 + d_{16}^- - d_{16}^+ = 1.$

and $0.1 \leq x_j \leq 50,$ (for $j=1,2,\ldots\ldots,10$)

$d_i^-, d_i^+ \geq 0,$ (for $i=1,2,\ldots\ldots,16$)

to find a design variable, $x \in R^{10}$.

the number of iterations, the number of active constraints, total CPU time, the number of function evaluations, maximum positive deviation and maximum negative deviation are given in Table 2 for problems I-1 and II-1, respectively. The results obtained from the different NLP algorithms [7,8] are also shown in Table 2.

For problem I-1, the target weights are 4500 lb for the NLGP-HJ, 4200 lb for the NLGP-MHJ, and 4210 lb for the NLGP-PO. The optimum weights of 5040.93 lb from the NLGP-HJ, 5051.09 lb from the NLGP-MHJ, and 5051.75 lb from the NLGP-PO are obtained. There are 6 active constraints for the optimum design for the three NLGP algorithms, which are the minimum size constraints on members 2, 5, and 10, the stress constraints on member 5, and the downward displacement constraints at nodes 1 and 2. The NLGP-MHJ takes less total CPU time than the others. The maximum positive deviation occurres

Table 2 Results for Problems I-1 and II-1

	Optimal Cross-Sectional Area in in^2							
	Problem I-1				Problem II-1			
Member Number	NLGP-HJ	NLGP-MHJ	NLGP-PO	Ref. 7	NLGP-HJ	NLGP-MHJ	NLGP-PO	Ref. 8
1	30.5	28.8529	28.7878	30.031	25.1614	24.7132	25.2706	24.8602
2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
3	23.25	24.9725	22.3483	23.274	23.78	24.8796	25.	25.9762
4	15.5	14.1406	16.7847	15.286	13.2175	13.3381	13.0167	13.1350
5	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
6	0.475	0.4888	0.5516	0.5565	1.9888	1.9831	2.0984	1.9727
7	7.4688	7.3284	7.5633	7.4683	13.6644	13.4121	13.5444	13.1924
8	21.0	21.5518	21.9861	21.198	15.8988	15.5336	15.4454	15.3214
9	21.0	21.6864	21.0175	21.618	18.0113	17.5059	17.5334	17.5268
10	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Optimum Weight (lb)	5040.93	5051.09	5051.75	5061.6	4744.41	4714.12	4734.75	4730.29
No. of Iterations	47	35	13	15	39	31	6	9
No. of active constraints	6	6	6	4	6	7	6	
Total CPU Time (sec)	38.07	37.25	64.35		165.67	127.77	78.02	
No. of Function Evaluations	794	779	1352		673	692	422	
Max. Positive Deviation	4.4×10^{-3}	4.4×10^{-3}	4.5×10^{-3}		none	4.7×10^{-3}	2.3×10^{-3}	
Max. Negative Deviation					4×10^{-5}	none	none	
Max. Constraint Violation				2.7×10^{-5}	4×10^{-5}	none	none	

at node 1 of the displacement constraints for three NLGP algorithms. The NLGP-MHJ has fewer function evaluations and lower total CPU time than the others. The optimum weights from the NLGP algorithms are better than that from Ref. [7] due to the flexibility of goal constraints. But it is found that maximum constraints. But it is found that maximum constraint violation from than Ref. [7] is smaller than the maximum positive deviation from the NLGP algorithms as shown in Table 2.

For problem II-1, the target weights are 4300 lb for the NLGP-HJ, 4200 lb for the NLGP-MHJ, and 4150 lb for the NLGP-PO. The optimum weights of 4744.41 lb from the NLGP-HJ, 4714.12 lb from the NLGP-MHJ, and 4734.75 lb from the NLGP-PO are obtained. There are 6 active constraints at the optimum design for the NLGP-HJ, which are the minimum size constraints on members 2 and 10, the stress constraints on members 5 and 6, the downward displacement constraint at node 2, and the natural frequency constraint. There are 7 active constraints for the NLGP-MHJ, which are the same as the NLGP-HJ case, except that the minimum size constraint on member 5 are imposed. The number of active constraints for the NLGP-PO is 6, and they are the same as those of the NLGP-MHJ, except the stress constraint on member 6. The NLGP-PO takes less total CPU time than the others. The maximum positive deviation occurres at node 2 of the displacement constraint for the NLGP-MHJ and NLGP-PO. The maximum negative deviation occurres on the natural frequency constraint for the NLGP-HJ. The NLGP-PO has a smaller number of function evaluations than the others as well as the lowest total CPU time. There is no data available for the problem II-1. The GRGA code based on Ref. [8] is employed to solve the problem. The optimum weights from the NLGP algorithms are found as good as the one from Ref. [8].

3.2.2 Objective Function wigh Priority Factors

For the problem II-1p, priority 1 is given to weight, two Y directional displacements at nodes 2 and 4, and natural frequency constraints and priority 2 is given to the rest of]the constraints. The results for the optimum cross-sectional area of each member, the optimum weight, the number of iterations, the number of active constraints, total CPU time, the number of function evaluations, and maximum positive deviation and maximum negative deviation are given in Table 3.

For problem II-1p, the target weights are 4300 lb for the NLGP-HJ, 4200 lb for the NLGP-MHJ, and 3850 lb for the NLGP-PO. The optimum weights of 4524.49 lb from the NLGP-HJ, 4490.20 lb from the NLGP-MHJ, and 4514.33 lb from the NLGP-PO are obtained. There are 4 active constraints at the optimum design for the NLGP-HJ, which are the minimum size constraints on member 10, the downward displacement constraints at nodes 2 and 4, and the natural frequency constraint. There are 6 active constraints for the NLGP-MHJ and NLGP-PO, which are the same as the NLGP-HJ case, except that the minimum size constraints on members 2 and 5 are imposed. The NLGP-MHJ takes less total CPU time than the others. The maximum positive deviation for priority 1 occurres at node 2 of the displacement constraint for the three algorithms. When the results of problem II-1 are compared with these of problem II-1P, the latter is batter than the former. About 4.7% reduction is obtained from the NLGP algorithms with priority factors.

Table 3 Results for Problems Ⅱ-1p and Ⅱ-1wp

	Optimum Cross-Sectional Area in in^2					
	Problem Ⅱ-1p			Problem Ⅱ-1wp		
Member Number	NLGP-HJ	NLGP-MHJ	NLGP-PO	NLGP-HJ	NLGP-MHJ	NLGP-PO
1	23.5356	25.4331	25.7545	27.8031	23.6067	21.8938
2	0.12	0.1	0.1	0.1	0.1	0.1
3	24.8931	23.8591	21.9696	24.0484	23.9231	25.8496
4	13.3475	13.9055	13.9559	12.025	12.5131	13.7836
5	0.12	0.1	0.1	0.1	0.1	0.1
6	1.16	0.7548	0.6181	1.0984	1.2226	0.7960
7	8.9113	8.9863	9.1645	9.1	9.0230	8.8426
8	15.7525	15.2971	15.8525	14.5	16.6299	16.5805
9	18.7263	18.4498	19.3600	18.6063	18.9234	18.9174
10	0.1	0.1	0.1	0.1	0.1	0.1
Optimum Weight (lb)	4524.49	4490.20	4514.33	4500.18	4487.30	4512.40
No. of Iterations	48	13	8	43	37	12
No. of active constraints	4	6	6	6	6	6
Total CPU Time (sec)	151.38	72.30	201.43	91.69	94.09	103.47
No. of Function Evaluations	809	377	1101	731	765	745
Max. positive Deviation (for Priority 1)	4.6×10^{-4}	5×10^{-3}	8.3×10^{-4}	8.7×10^{-3}	9.6×10^{-3}	6.0×10^{-3}
Max. Negative Deviation (for Frequency) (for Priority 1)	3.2×10^{-4}	none	none	none	none	none

3.2.3 Objective Function with Weights and Priority Factors

For the problem II-1p, we give the same priority for weight, two displacement and the natural frequency constraints. Weight and natural frequency constraints are counted as 2 times more important as the Y directional displacement constraints at nodes 2 and 4 and this problem is designated as the problem II-1wp. The results for the optimum cross-sectional area of each member, the optimum weight, the number of iterations, the number of active constraints, total CPU time, the number of function evaluations, the maximum positive deviation and negative deviation are given in Table 3. For problem II-1wp, the target weights are 4500 lbs for the NLGP-HJ, 4487 lb for the NLGP-MHJ, and 4515 lb for the NLGP-PO. The optimum weights of 4500.18 lb from the NLGP-HJ, 4487.30 lb from the NLGP-MHJ, and 4512.40 lb from the NLGP-PO are obtained. There are 6 active constraints for the optimum design for the three NLGP algorithms, which are the minimum size constraints on member 2,5, and 10, two downward displacement constraints at nodes 2 and 4 and natural frequency constraints. The maximum positive deviation for priority 1 occured at node 2 of the displacement constraint for the three algorithms. When each optimum weight of problem II-1p is compared with that of problem II-1wp, every optimum weight of problem II-1wp is a little bit better than that of problem II-1p in its respective pair.

4. CONCLUSION

To solve nonlinear structural optimization problems, the nonlinear multi-objective goal programming techniques are adopted and the difficulties to distinguish between objective function and constraints are relieved. The techniques employ the traditional nonlinear programming algorithms. Thus the success of one algorithm over the other algorithms to find a solution may be relative to the particular problem as well as the initial problem parameters. As shown in the test cases further weight reduction is achieved due to the flexibility of goal programming. The reduction of structural weight is varied in accordance with different priority levels and weighting factors. Consequently, cautions choice of the priority levels and weighting factors are the essential task to achieve the proper design in the application of nonlinear goal programming.

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