Turbulence Models for the Surface Discharge of Heated Water

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ABSTRACT / In order to predict the dispersion of a thermal discharge with strong turbulent and buoyant effects, the development of a numerical model using turbulence model and its application are significantly increased.

In this study, a three–dimensional steady–state model for the surface discharge heated water into quiescent water body is developed. for the model closure of turbulent terms the four–equation turbulence model is used. For economic numerical simulation, the elliptic governing equations are transformed to the partially parabolic equations.

In general, the simulated results by the present model agree well with the experimental results by Pande and Rajaratnam (1977). The model characteristics are presented in comparison with the predicted results from the two–eqauation turbulence model by McGuirk and Rodi (1979).

1. Introduction

1.1 The Purpose of Study

In order to predict the dispersion of a thermal discharge at near field, the development of a numerical model using turbulence model and its application are significantly increased with sophisticated computer development.

A number of studies have been focused on developing the numerical model with $k-\varepsilon$ turbulence model; Barry et al.(1972), Rastogi et al.(1978), McGuirk et al.(1979), Raithby et al.(1980), Haoging et al.(1987), and Raithby et al.(1988). Kim and Chung(1982) compares the model simulation results between one–equation k model and two–equation $k-\varepsilon$ model which are used to solve the governing equations of thermal discharge hehavior. Park and Chung(1983) compares the model characteristics between the $k-\varepsilon$ model and the four–equation turbulence model which solves the transport equations for T'_2 and its dissipation

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rate ε_T in addition to the k$-\varepsilon$ turbulence model for turbulence model closure of the governing equations of 2–dimensional surface discharge of heated water.

The existing k$-\varepsilon$ turbulence models can represent well the turbulence energy transport and its dissipation for flow field, but the 2–equation turbulence model can not obtain the information for time scale ratio $R=1/C_T$ in the equation 15 (Launder,1975a). Therefore, this study considers the additional transport equations for T'_2 and ε_T which represent the transport and the dissipation of thermal energy in temperature field which has characteristics itself apart from that of flow field for more accurate evaluation and prediction of thermal discharge behavior.

1.2 Basic Concepts of Turbulence

Boussinesq's eddy–viscosity concept assumes that the Reynolds tresses($\overline{u_i u_j}$) are propotional to the mean velocity(U_i) gradients. For general flow situations, the concept may be expressed as,

$$-\overline{u_i u_j} = \nu_T \left(\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right) - \frac{2}{3} k \delta_{ij} \tag{1}$$

where k is the turbulent kinetic energy and νt is the turbulent or eddy viscosity which depends strongly on the state of turbulence.

In direct analogy to the turbulent momentum transport, the turbulent heat or mass transport is often assumed to be related to the gradient of the transport quantity,

$$-\overline{u_i \phi} = \Upsilon t \frac{\partial \phi}{\partial X_i} \tag{2}$$

where Υ_t is the turbulent eddy diffusivity of heat or mass. Like the eddy–viscosity, Υ_t is not a fluid property but depends on the state of the turbulence. In fact, the Reynolds analogy between heat or mass transport and momentum transport suggests that Υ_t is closely related to νt :

$$\Upsilon_t = \frac{\nu_t}{\sigma_t}, \tag{3}$$

σ_t is called the turbulent Prandtl (for heat) or Schmidt (for mass) number (Rodi,1984).

2. Formulation of Turbulence Model

2.1 Governing Equations

The time–averaged equations of mass, momentums and heat conservation for an incompressible flow can be expressed as follows :

$$\frac{\partial U_i}{\partial X_i} = 0 \tag{4a}$$

$$\frac{\partial u_i}{\partial t} + U_j \frac{\partial U_i}{\partial X_j} = -\frac{1}{\rho_r}\frac{\partial P}{\partial X_i} + \frac{\partial}{\partial X_j}\left(\nu \frac{\partial U_i}{\partial X_j} - \overline{u_i u_j}\right) + g_i \frac{\rho - \rho_r}{\rho_r} \tag{4b}$$

$$\frac{\partial T}{\partial t} + U_i \frac{\partial T}{\partial X_i} = \frac{\partial}{\partial X_i}\left(\lambda \frac{\partial T}{\partial X_i} - \overline{u_i T'}\right) + S_T \tag{4c}$$

where λ is the eddy diffusivity and the S_T is the source/sink of heat. The above equations are not a complete system of equations because of the Reynolds stresses and the turbulent heat fluxes which are the unknown quantities. Therefore, turbulence models are required for the determination of the turbulence terms.

In most flow regions, the turbulent stresses and fluxes are much larger than their laminar counterparts $\lambda\ \partial U_i/\partial X_j$ and $\lambda\ \partial T/\partial X_i$ which are therefore often negligible. The assumption is that vertical acceleration is small, so that the vertical momentum equation can be reduced to the hydrostatic pressure relation:

$$p = -\int_{\eta}^{z} \rho\ g\, dz \tag{5}$$

where η is the elevation of the water surface. With the hydrostatic pressure relation, the pressure gradients $\partial p/\partial x$ can be expressed as

$$\frac{\partial p}{\partial X_i} = -g\rho \frac{\partial \eta}{\partial X_i} - \frac{\partial}{\partial X_i}\int_{\eta}^{z} \triangle\rho\ g\, dz \tag{6}$$

where $\triangle\rho$ is the difference between local and ambient density. The surface slope term can be eliminated via the assumption that there should be no horizontal motion.

The turbulent stresses and heat fluxes in the longitudinal and lateral directions have been neglected; those in the vertical direction only has to be considered. Therefore, the equations governing the steady–state three–dimensional heated surface jets with significant buoyancy effects may be written as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \tag{7a}$$

$$\frac{\partial U^2}{\partial X} + \frac{\partial UV}{\partial Y} + \frac{\partial UW}{\partial Z} = -\frac{\partial}{\partial X}\int_{-\infty}^{z} \frac{\triangle\rho}{\triangle\rho_r} g\, dz - \frac{\partial \overline{uw}}{\partial Z} \tag{7b}$$

$$\frac{\partial UV}{\partial X} + \frac{\partial V^2}{\partial Y} + \frac{\partial VW}{\partial Z} = -\frac{\partial}{\partial Y}\int_{-\infty}^{z} \frac{\triangle\rho}{\rho_r} g\, dz - \frac{\partial \overline{vw}}{\partial Z} \tag{7c}$$

$$\frac{\partial UT}{\partial X} + \frac{\partial VT}{\partial Y} + \frac{\partial WT}{\partial Z} = -\frac{\partial \overline{wT'}}{\partial Z} \tag{7d}$$

$$\rho = f(T) \tag{7e}$$

where (7a) is the continuity equation, (7b) and (7c) are momentum equations in the x and y directions, respectively (the Boussinesq's approximation has been invoked), (7d) is the thermal energy equation, and (7e) is the equation of state for water.

2.2 Turbulence Model

To obtain a suitable turbulence model, we investigate the influence of buoyancy on the turbulence from the exact transport equations for the turbulent stresses and heat fluxes as starting point for the model development.

The Reynolds stress transport equation can be derived using the relation $[(m_i)u_j + (m_j)u_i]$, where the denotation (m_i) is obtained by subtracting the Reynolds equation from the original Navier–Stokes equation and u_i is the fluctuating velocity. Turbulent heat flux equation can be derived using the relation $T'(m_i) + u_i(\theta)$, where the denotation θ is the energy transport equation for T'. The turbulent stress and heat flux equations with high Reynolds and Peclet numbers can be written in tensor form as follows (Launder, 1975b; Launder et al.,1975):

$$\frac{D\overline{u_iu_j}}{Dt} = \underbrace{-\frac{\partial}{\partial X_l}(\overline{u_iu_ju_l}) - \frac{1}{P}\left(\frac{\partial \overline{u_jp}}{\partial X_i} + \frac{\partial \overline{u_ip}}{\partial X_j}\right)}_{Diff(\overline{u_iu_j})}$$

$$\underbrace{\underbrace{-\left[\overline{u_ju_l}\frac{\partial U_i}{\partial X_l} + \overline{u_iu_l}\frac{\partial U_j}{\partial X_l}\right]}_{Stress}\underbrace{-\beta(g_i\overline{u_jT'} + g_j\overline{u_iT'})}_{Buoyance}}_{P_{ij}:Production}$$

$$+\underbrace{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right)}_{\pi_{ij}:\ Pressure\ Strain}\underbrace{-2\nu\frac{\overline{\partial u_i\partial u_j}}{\partial X_l\partial X_l}}_{\epsilon_{ij}:\ Viscous\ Dissipation} \tag{8a}$$

$$\frac{D\overline{u_iT'}}{DT} = \underbrace{-\frac{\partial}{\partial X_l}\left[\overline{u_iu_lT'} + \frac{1}{\rho}\delta_{il}\overline{pT'}\right]}_{Diff(\overline{u_iT'})}$$

$$\underbrace{\underbrace{-\left[\overline{u_iu_j}\frac{\partial T}{\partial X_j} + \overline{u_jT'}\frac{\partial U_i}{\partial X_j}\right]}_{Mean\ Field\ P_{iT}}\underbrace{-\beta g_i\overline{T'^2}}_{Buoyance}}_{P_{iT}:Production}$$

$$+\underbrace{\frac{p}{\rho}\left(\frac{\partial T'}{\partial X_i}\right)}_{\pi_{iT}:Pressure\ Temp.\ Gradient}\underbrace{-(\lambda+\nu)\frac{\partial u_i}{\partial X_l}\frac{\partial T'}{\partial X_l}}_{\epsilon_{iT}:Destruction\ of\ \overline{u_iT'}} \tag{8b}$$

At high Reynolds numbers, small eddies in turbulence are locally isotropic so that the viscous dissipation term is approximated as follows (Hanjalic and Launder,1972):

$$\epsilon_{ij} = 2\nu\overline{\frac{\partial u_i}{\partial X_l}\frac{\partial u_j}{\partial X_l}} = \frac{2}{3}\epsilon\delta_{ij} \tag{9}$$

where is the total rate of turbulent kinetic energy dissipation. Launder(1975b) assumes that the pressure−strain term has the contributions from the interaction of fluctuating velocities and the interaction between mean−strain and fluctuating velocities.

$$\pi_{ij}=\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial X_j}\frac{\partial u_j}{\partial X_i}\right)} = -C1\frac{\varepsilon}{k}\left(\overline{u_iu_j}-\frac{2}{3}\delta_{ij}k\right)-C2\left(P_{ij}-\frac{2}{3}\delta_{ij}P\right) \tag{10}$$

where P is the stress production of the turbulent kinetic energy k defined in equation (16a). According to the Kolmogorov's assumption of locally isotropic turbulence, the model approximation have to be introduced only for the pressure−temperature−gradient term and for the diffusion term. As in the case of pressure−strain term, the pressure−temperature−gradient term has the contributions from a turbulence part and a mean−strain part (Gibson and Launder,1976).

$$\pi_{iT}=\overline{\frac{p}{\rho}\left(\frac{\partial T'}{\partial X_i}\right.} = -C1_T\frac{\varepsilon}{k}\overline{u_iT'} - C2_T P_{iT,V} \tag{11}$$

Introducing the assumption that turbulence is near in a state of local equilibrium, the convective and diffusive transport terms are neglected in the above equations, which can be reduced to the simplified algebraic expressions (Launder,1975b).

Introducing the Rodi's assumption(1976) that the transpot of $\overline{u_i u_j}$ is proportional to the transport of k under the non−equilibrium state, the difference between the convection and the diffusion of turbulent stresses may be approximated as follows:

$$\frac{D\overline{u_iu_j}}{Dt}-\text{Diff}\,(\overline{u_iu_j})=\frac{\overline{u_iu_j}}{k}\left[\frac{Dk}{Dt}-\text{Diff}(k)\right]=\frac{\overline{u_iu_j}}{k}(P-\varepsilon) \tag{12a}$$

The values of P and ε in the above equation are locally equilibrium under the local equilibrium state.

Gibson and Launder (1976) proposed to approximate the transport terms in the $\overline{u_iT'}$ equation in a way parallel to (12a). Because both velocity and scalar fluctuations contribute to $\overline{u_iT'}$, the equation (8b) may be approximated as follows:

$$\frac{D\overline{u_iT'}}{Dt}-\text{Diff}(\overline{u_iT'})=\frac{\overline{u_iT'}}{2k}(P-\varepsilon)+\frac{\overline{u_iT'}}{2\overline{T'^2}}(P_T-\varepsilon_T) \tag{12b}$$

Introducing the local equilibrium condition to equation (17a), the last term of equation (12b) can be ignored since $P = (\overline{u_iT'}\,\partial T/\partial Y) = \varepsilon_T$. With the above simplified procedures the $\overline{u_iu_j}$ and $\overline{u_iT'}$ terms may be written by the following algebraic expressions.

$$\frac{\overline{u_iu_j}}{k}=\frac{P_{ij}}{\varepsilon}\frac{1-C2}{C1}-\frac{2}{3}\delta_{ij}\frac{1-C1-C2}{C1} \tag{13a}$$

$$\overline{u_i T'} = \frac{k}{C1_T \varepsilon} \left(P_{iT} - C2_T P_{iTV} \right) \tag{13b}$$

The empirical proposal by Launder(1975b) gives the following relation for ε_T term in the transport equation of $\overline{T'^2}$.

$$\varepsilon_T = C_T \overline{T'^2} \varepsilon / k \tag{14}$$

Thus if we introduce the local equilibrium condition to the advection and diffusion terms in the equation (17a), we can obtain the following relation.

$$\overline{T'^2} = - \frac{2k}{C_T \varepsilon} \overline{u_i T'} \frac{\partial T}{\partial X_i} \tag{15}$$

For turbulence closure the use of k and transport equation with equation (15) is called two–equation turbulence model, and the use of two additional transport equations for $\overline{T'^2}$ and ε_T in order to determine $\overline{T'^2}$ is called as the four–equation turbulence model (Park and Chung, 1983).

1) Two–equation Turbulence Model

The differential equations for k and ε with high Reynolds numbers are as follows (Launder and Spalding,1974):

$$\frac{Dk}{Dt} = \frac{\partial}{\partial X_i} \left(\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial X_i} \right) \underbrace{- \overline{u_i u_j} \frac{\partial U_i}{\partial X_j} - \beta g_i \overline{u_i T'}}_{P\,:\,\text{Production}} - \varepsilon \tag{16a}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial X_i} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial X_i} \right) + C1_\varepsilon \frac{\varepsilon}{k} P - C2_\varepsilon \frac{\varepsilon^2}{k} \tag{16b}$$

where $\nu_t = C_\mu k^2 / \varepsilon$.

The modeling for $\overline{T'^2}$ is the equation (15) which can be obtained by assuming that the time scale ratio R is constant of $1/C_T$.

2) Four–equation Turbulence Model

The differential equations for $\overline{T'^2}$ and ε_T are as follows (Park and Chung,1983; Chen and Singh,1986):

$$\frac{\overline{DT'^2}}{Dt} = - \frac{\partial}{\partial X_i} (\overline{u_i T'^2}) - \overline{2 u_i T'} \frac{\partial T}{\partial X_i} - 2 \varepsilon_T \tag{17a}$$

$$\frac{D\varepsilon^{T}}{Dt}=-\frac{\partial}{\partial X_i}(\overline{\varepsilon_T u_i})-2.0\frac{\varepsilon_T^{2}}{T'^{2}}-1.96\overline{U_iT'}\frac{\varepsilon_T}{T'^{2}}\frac{\partial T}{\partial X_j}$$

$$-0.8\frac{\varepsilon\,\varepsilon_T}{k}+Cs\,P\frac{\varepsilon_T}{k} \tag{17b}$$

The diffusion terms in the above equations are approximated by a gradient type expression.

$$-\overline{u_iT'^{2}}=\frac{\nu_t}{\sigma_T}\frac{\partial \overline{T'^{2}}}{\partial X_i}, \tag{18a}$$

$$-\overline{\varepsilon_T u_i}=\frac{\nu_t}{\sigma\,\varepsilon_T}\frac{\partial\,\varepsilon_T}{\partial X_i} \tag{18b}$$

3. Numerical Analysis of the Model

3.1 Finite Difference Equations

The staggered grid system (Fig. 1) shows how the computational points are arrayed in the y–z plane with the stored the variables U, V ,W, P, and T. The boomerang–shaped envelopes enclose the triads of points denoted by N, S, E, W, or P and the points n, s, e, w are the mid–points of lines PN, PS, PE, and PW, respectively. The general procedure for the finite differencing is as follows:

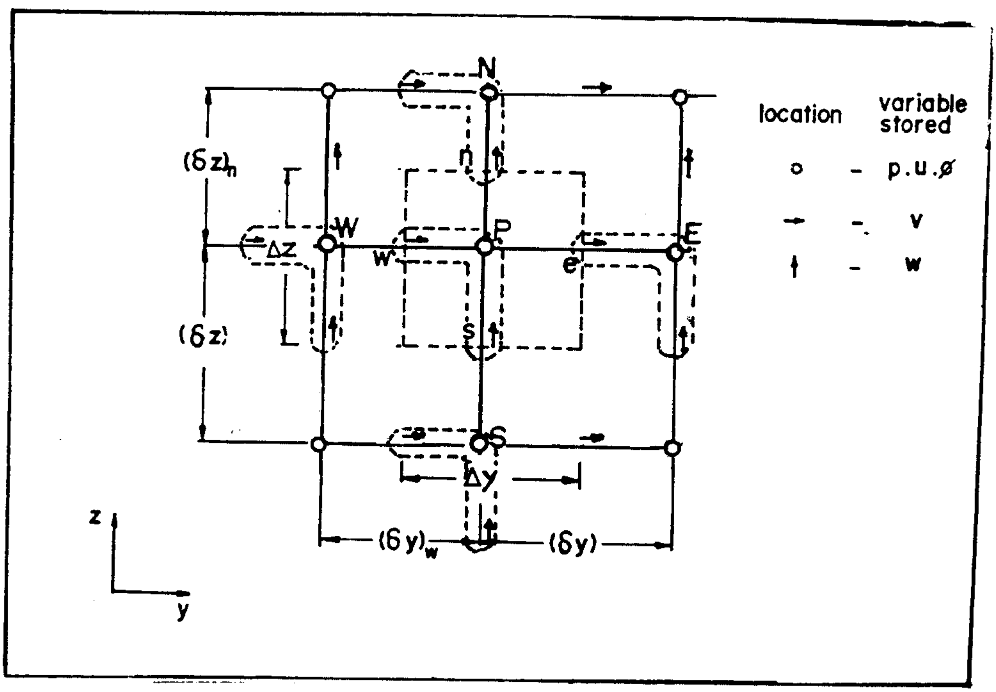

Fig. 1 Staggered Grid System(y–z Face of the Control Volume)

1) In the x direction the dependent variables(U, V, W, $\triangle\rho$, and T) are determined in a stepwise manner.
2) In the y–z plane the values of dependent variables at P(ϕ p) are determined from the values of ϕ p at P and N, S, E, W outside the rectangle surrounding the point P.
3) For the cross–section convection from the x–y and x–z faces of the control volume, the value of ϕ convected is taken to be the arithmetic mean of the ϕ values on either side of that face. The scheme of centered differencing is transformed to that of foreward differencing with the direction of flux, except when this practice is altered by the "high–lateral–flux modification" mentioned below.

According to the above–mentioned decision we can obtain the finite difference equation by integration of continuity equation over the control volume.

$$C^V(V_E - V_P) + C^W(W_N - W_P) = C^U(U_{P,U} - U_{P,D}) \tag{19}$$

where, $C^V = \triangle X\ \triangle Z$, $C^W = \triangle X\ \triangle Y$, $C^U = \triangle Y\ \triangle Z$,

We define the general form for the transport equations of U, V, T, k, ε, $\overline{T'^2}$, and ε_T, which are represented by ϕ, to have the systematic finite difference equation. The S includes the pressure–gradients and turbulent correlation terms appeared in the momentum equations.

$$\frac{\partial U\phi}{\partial X} + \frac{\partial V\phi}{\partial Y} + \frac{\partial W\phi}{\partial Z} = \frac{\partial}{\partial Y}(\Gamma\frac{\partial \phi}{\partial Y}) + \frac{\partial}{\partial Z}(\Gamma\frac{\partial \phi}{\partial Z}) + S \tag{20}$$

where Γ is the transport property such as viscosity. Therefore we transform the equation (20) into a finite difference equation by integrating it over the control volume.

$$F_D\phi_P - F_U\phi_{P,U} + L_{ye}(\phi_E + \phi_P) - L^y_w(\phi_W + \phi_P) - L^z_n(\phi_N + \phi_P) - L^z_s(\phi_S + \phi_P)$$
$$= T^y_e(\phi_E - \phi_P) - T^y_w(\phi_P - \phi_W) - T^z_n(\phi_N - \phi_P) - T^z_s(\phi_p - \phi_S) - S\triangle Y\triangle Z$$

where $F_U = U_{P,U}\ \triangle Y\triangle Z/\triangle X$

$L^y = V_U\ \triangle Z/2$

$L^z = W_U\ \triangle Y/2$

$F_D = F_U - 2L^y_e + 2L^y_w - 2L^z_n + 2L^z_s$

$T^y = \Gamma\triangle Z/\triangle Y$, $T^z = \Gamma\triangle Y/\triangle Z$

In order to transform into the foreward finite differencing with the direction of flux, the terms denoted by L are replaced by Le=[–Le,0], Lw=[–Lw,0], Ln=[–Ln,0], and Ls=[–Ls,0]. Thus the above equation can be rearranged as follows:

$$\phi_P = A_E\phi_E + A_W\phi_W + A_N\phi_N + A_S\phi_S + B \tag{22}$$

where $A_E = A'_E/A'_P$, $A_W = A'_W/A'_P$, $A_N = A'_N/A'_P$, $A_S = A'_S/A'_P$,

$A'_E = T^y_e - L^y_e$, $A'_W = T^y_w + L^y_w$, $A'_N = T^z_n - L^z_n$, $A'_S = T^z_s + L^z_s$,

$A'_P = A'_E + A'_W + A'_N + A'_S + F_U$

$B = (S\triangle Y\triangle Z + U_{P.U}\phi_{P.U}\triangle Y\triangle Z/\triangle X)/A'_P$

When the lateral flow (denoted by symbol L) is large, some of the coefficients A'_E, A'_W, A'_N, A'_S can become negative; this event leads to physically unrealistic results of numerical divergence (Patankar,1980). This cure for this case is a simple one, "high−lateral−flux modification".

3.2 Numerical Solution Procedure

The equations (7d), (16a), (16b), (17a), and (17b) are parabolic and the equations (7b) and (7c) are also parabolic if the pressure gradients arising from density differences are considered as knowns. All these equations can therefore be solved with the general solution procedure for three−dimensional parabolic equation developed by Patankar and Spalding (1972). The solution is obtained by a marching integration in the downstream direction starting from known values of all dependent variables at the discharge cross−section.

3.3 Initial and Boundary Conditions

The cross−section of finite difference grid at any plane may be depicted as shown in Figure 2 on which all the boundary conditions used are shown. In the present case all the variables have been specified as uniform at inlet(zero for the lateral velocities V and W) except the axial velocity U. Because of the boundary layer development at inlet, the one−se-

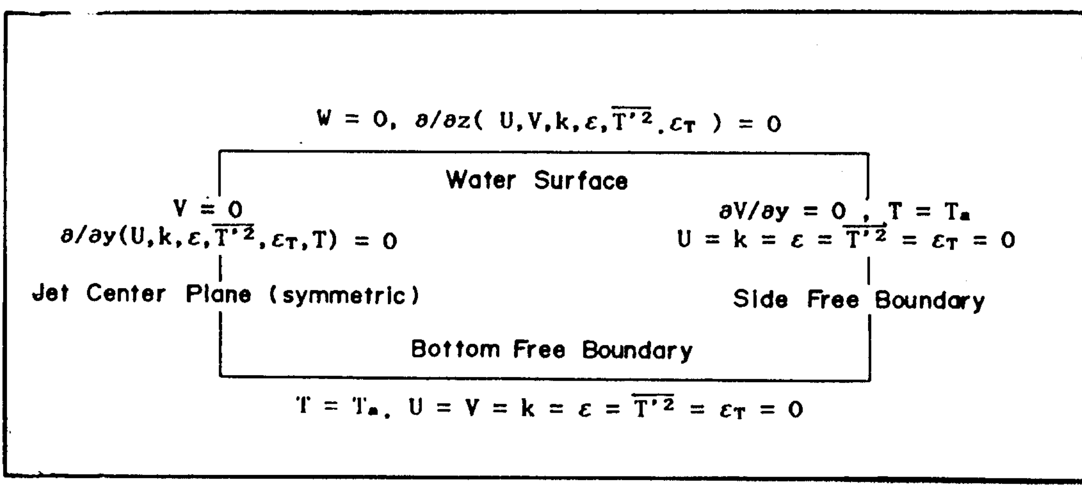

Fig. 2 Boundary Conditions

venth power law distribution is assumed(Cebeci and Smith,1974; Cebeci and Bradshaw,1977). The effects of boundary layer thickness were investigated by McGuirk and Rodi (1979) and were found to be small; here a thickness δ /ho=0.25 is used.

For the turbulence quantities, the values within the boundary layer can be calculated from (Launder and Spalding,1974; Mendoza and Shen, 1987;Celik, Rodi, and Stamou,1987)

$$K_0 = U^2_* \, / \, C_\mu^{1/2} \, , \; \varepsilon_0 = U^3_* \, / \, \kappa l$$

where U_* is the friction velocity which can be calculated from (Cebeci and Smith,1974; Mendoza and Shen,1987)

$$Ue/U_* = Ln[\, Ey \, U_* \, / \, \nu \,]/\kappa , \; E = 9$$

where Ue is an external velocity of boundary layer, l is the length scale proportional to a boundary layer thickness δ , C_μ and κ are the turbulence constants as 0.09 and 0.42, respectively. When the four-equation turbulence model is used, Park and Chung(1983) define the initial turbulence properties as follows:

$$k_0 = 0.005 \, U_0^2, \; \varepsilon_0 = C_\mu k_0^{3/2} \, / \, (0.03H), \; C_\mu = 0.09$$
$$\overline{T'^2_0} = 0.05(\, T_0 - T_a \,)^2, \; \varepsilon_{T0} = \overline{T'^2_0} \, \varepsilon_0/(2k_0R), \; R = 0.8$$

where the subscript 'o' represents the condition at inlet and Ta is an ambient temperature.

4. Numerical Verification and Discussion

4.1 Model Verification

A three-dimensional steady-state model for the surface discharge of heated water with strong turbulent and buoyant effects is developed using the four-equation turbulence model. The simulated results by the present model are compared with the experimental results by Pande and Rajaratnam(1977) and with the simulated results by McGuirk and Rodi(1979), who adopted the two-equation turbulence model for the turbulence closure.

Three experimental situations for model comparison and verification are given in Table 1. The plane and sectional views for flow configuration are shown in Figure 3.

4.1.1 Decay of Centre-line Values and Jet Spreading

Figure 4 compares predicted and measured decay of centre-line surface velocity for Run 1 and 3. The agreement is generally good; in particular the results by present model agree better than those by McGuirk and Rodi (1979). The decay of centre-line surface velocity at

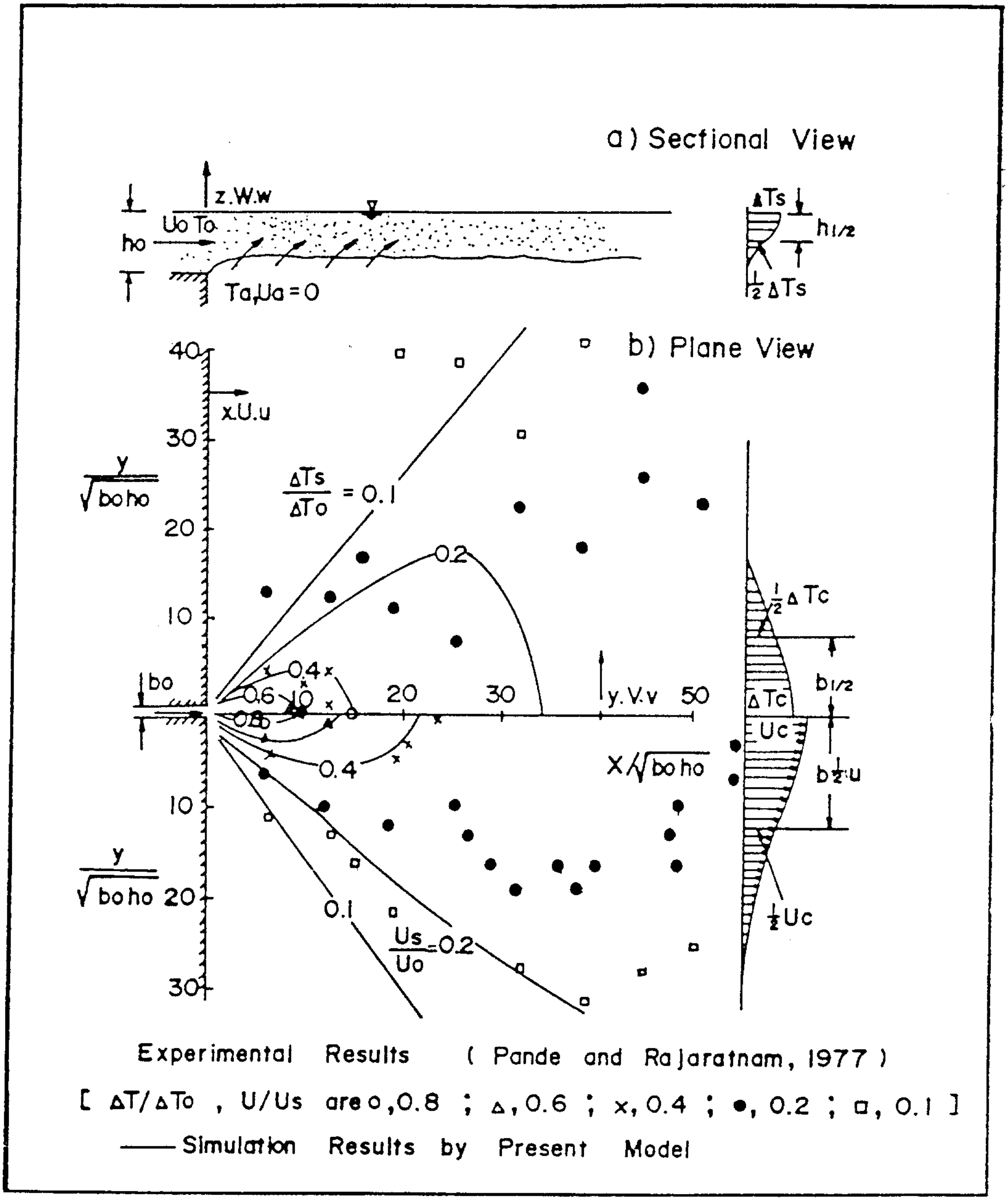

Fig. 3 Flow Configuration and Surface Isotachs and Isotherms(Run # 1)

near outlet is fast due to the high pressure gradients arising from the density difference and is slow further due to the reduction of jet entrainment with ambient flow and finally results in equilibrium state.

The excess temperature decay curves are compared in Figure 5. The general agreement is also satisfactory even if there is local disagreement in the downstream region due to the side wall effects, but the measurements are believed to be less reliable because of the possi-

Table 1 Experimental Situation for Model Simulation(Pande and Rajaratnam,1977)

Run	ho(cm)	bo(cm)	Uo(cm/s)	To(℃)	Ta(℃)	A	Rio	Fo
1	4.62	4.91	9.82	27.5	13.5	0.94	0.15	2.56
2	4.62	4.91	8.03	33.9	13.0	0.94	0.35	1.67
3	4.82	15.2	5.25	30.27	16.25	0.32	1.56	1.32

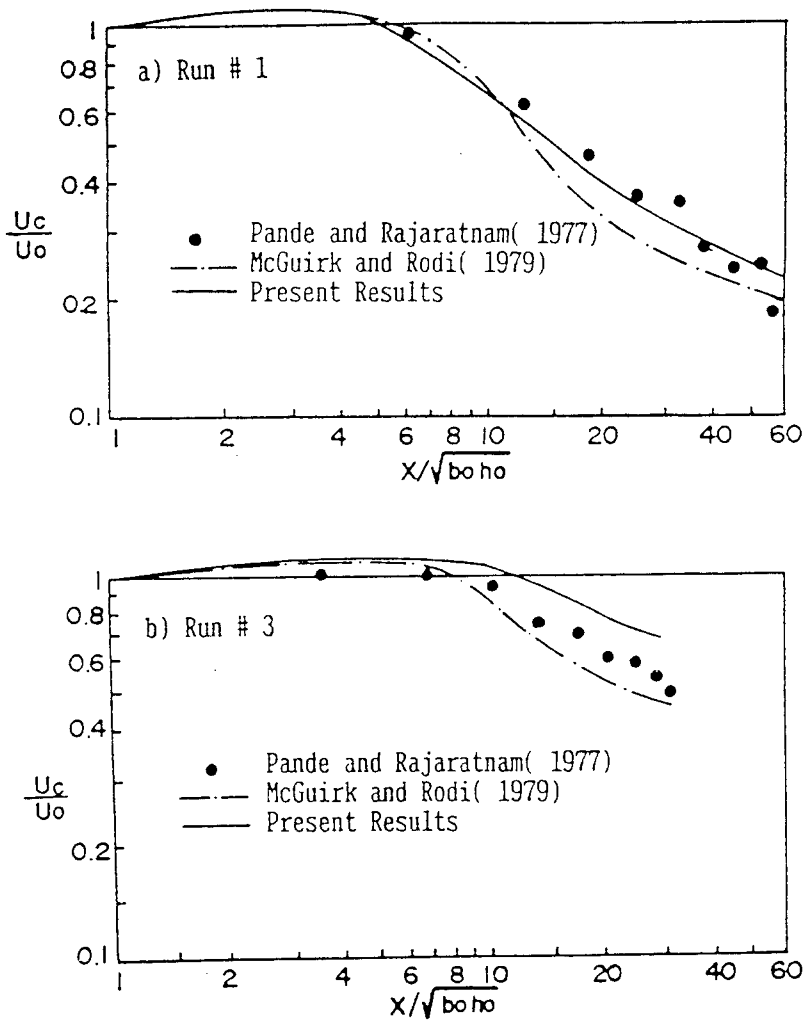

Fig. 4 Surface Centre-line Velocity Decay

ble interference of the side walls of the experimental tank with the very fast spreading jet.The results from the present model agree better than those by McGuirk and Rodi (1979) at the jet entrainment region.

The downstream development of the half width and depth are compared in Figure 6. In the initial region the width is well predicted such like that by McGuirk and Rodi(1979) and in the downstream region the width is somewhat overpredicted, because of side wall effects already—mentioned above.

4.1.2 Profiles of Cross—sectional Values

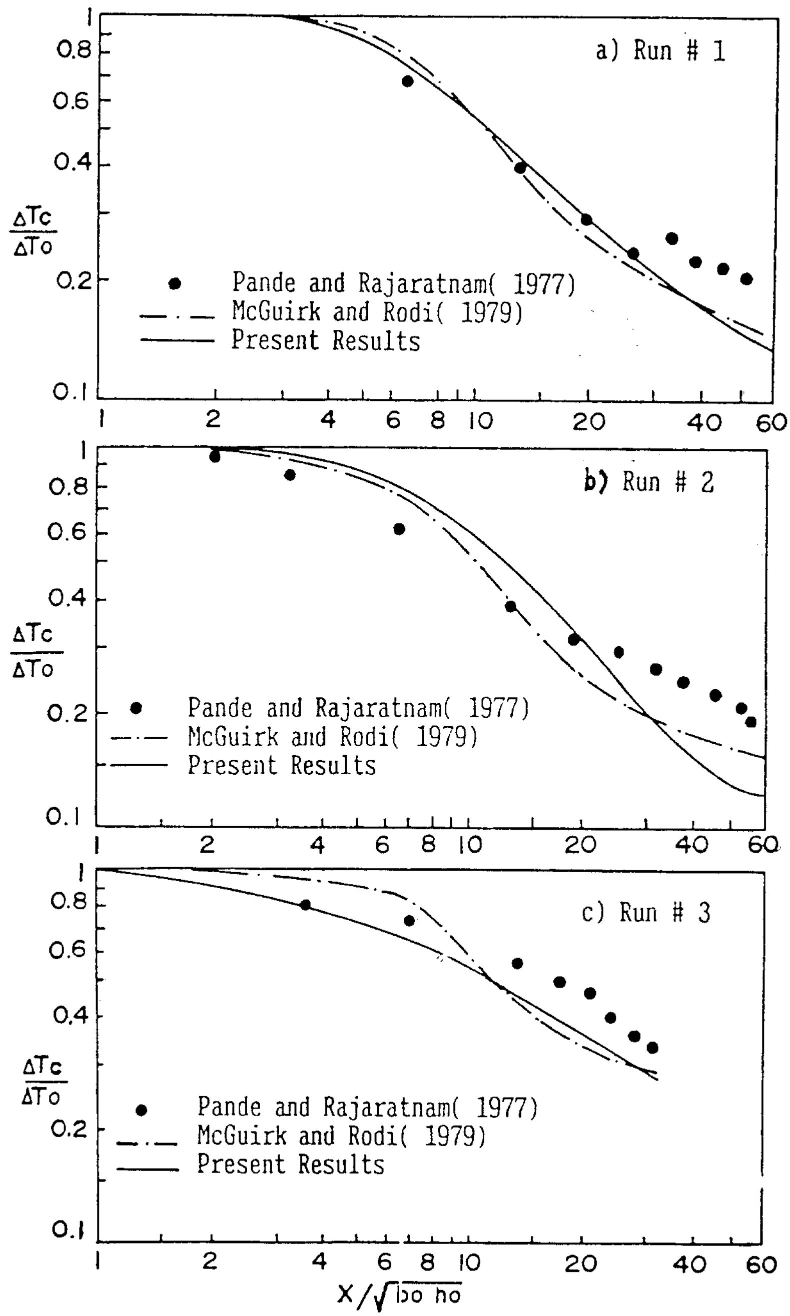

Fig. 5 Surface Centre—line Excess Temperature Decay

Figure 7 compares predicted and measured lateral surface and vertical centre-plane profiles of longitudinal velocity. The agreement can be considered as generally satisfactory except that there is local disagreement at initial region, in which the vertical centre-plane

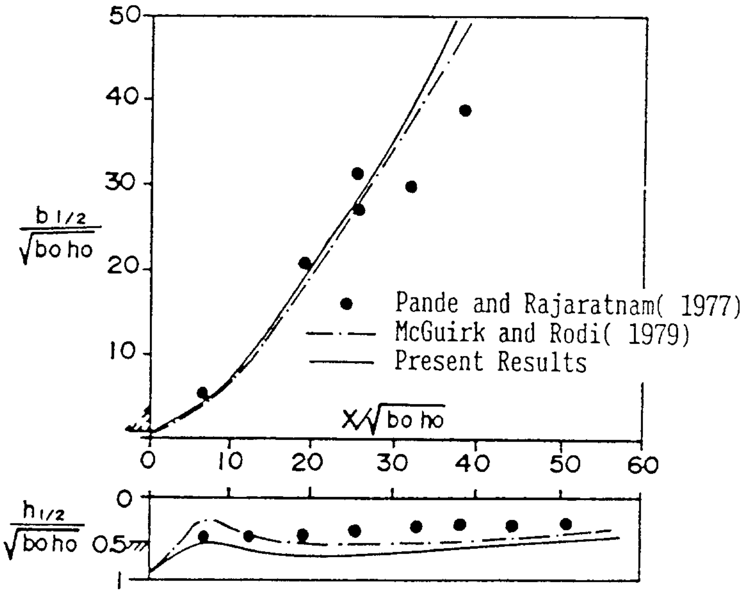

Fig. 6 Development of Half Width and Depth(Run # 1)

profiles are somewhat overpredicted. And the simulation results are satisfactory for the corresponding temperature profiles shown in Figure 8. The vertical centre-line temperature decaying is somewhat better predicted at jet edges than that by McGuirk and Rodi(1979).

The surface isotherms and isotachs for Run 1 is necessary for the prediction of the dispersion of a thermal discharge shown in Figure 3. The agreement can be considered as generally satisfactory even if there is local disagreement at the downstream region.

4.2 Discussion

4.2.1 Profile Shape of Jet

At the outlet the profile is uniform and the half depth is therefore at the bottom of the discharge channel. Two counteracting processes govern the further development of the jet depth; firstly the buoyancy induced upward and lateral motion of the heated water discharged, which reduces the jet depth, and secondly the entrainment of ambient water, which acts to increase the jet depth. In Figure 7 and 8 the lateral and vertical profiles of the jet is not a Gaussian distribution of similarity profiles for velocity and excess temperature by the jet integral and physical model, but nearly follow a straight line in general.

4.2.2 Velocity Vector Plots

Surface and cross-sectional velocity vectors are shown in Figures 9 and 10, respectively. The buoyancy induced upward and lateral motion can best be illustrated by velocity plots as given in Figure 10 for three cross-sections for Run 1. It illustrates clearly the lateral spreading mechanism by means of relatively large lateral velocities at the jet edges. The vertical velocities at the lower jet edges are partly buoyancy induced and partly due to entrainment, these velocities have significant values only near the centre-plane which indi-

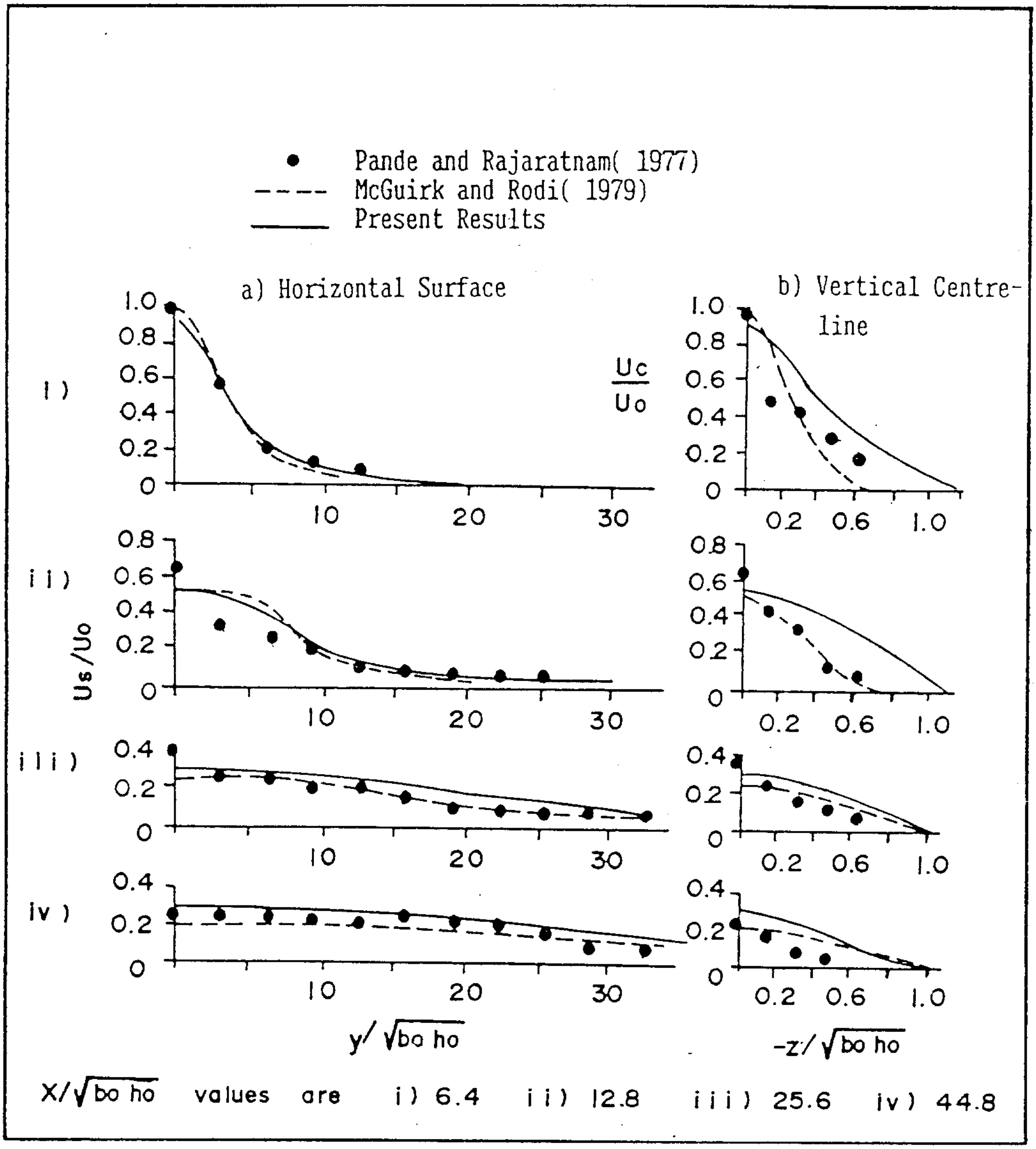

Fig. 7 Velocity Profiles(Run # 1)

cates that entrainment takes place mostly in the inner jet region.

5. Conclusions

1) The predicted results for the verification of the present four-equation turbulence model generally agree well with the experimental results and represents the physical behaviour of thermal discharged jet better.
2) Compared with the two-equation turbulence model results by McGuirk and Rodi, the predicted results at jet entrainment region by the present model are closer to the verification data. The improvement of accuracy at the jet edges is somewhat achieved.

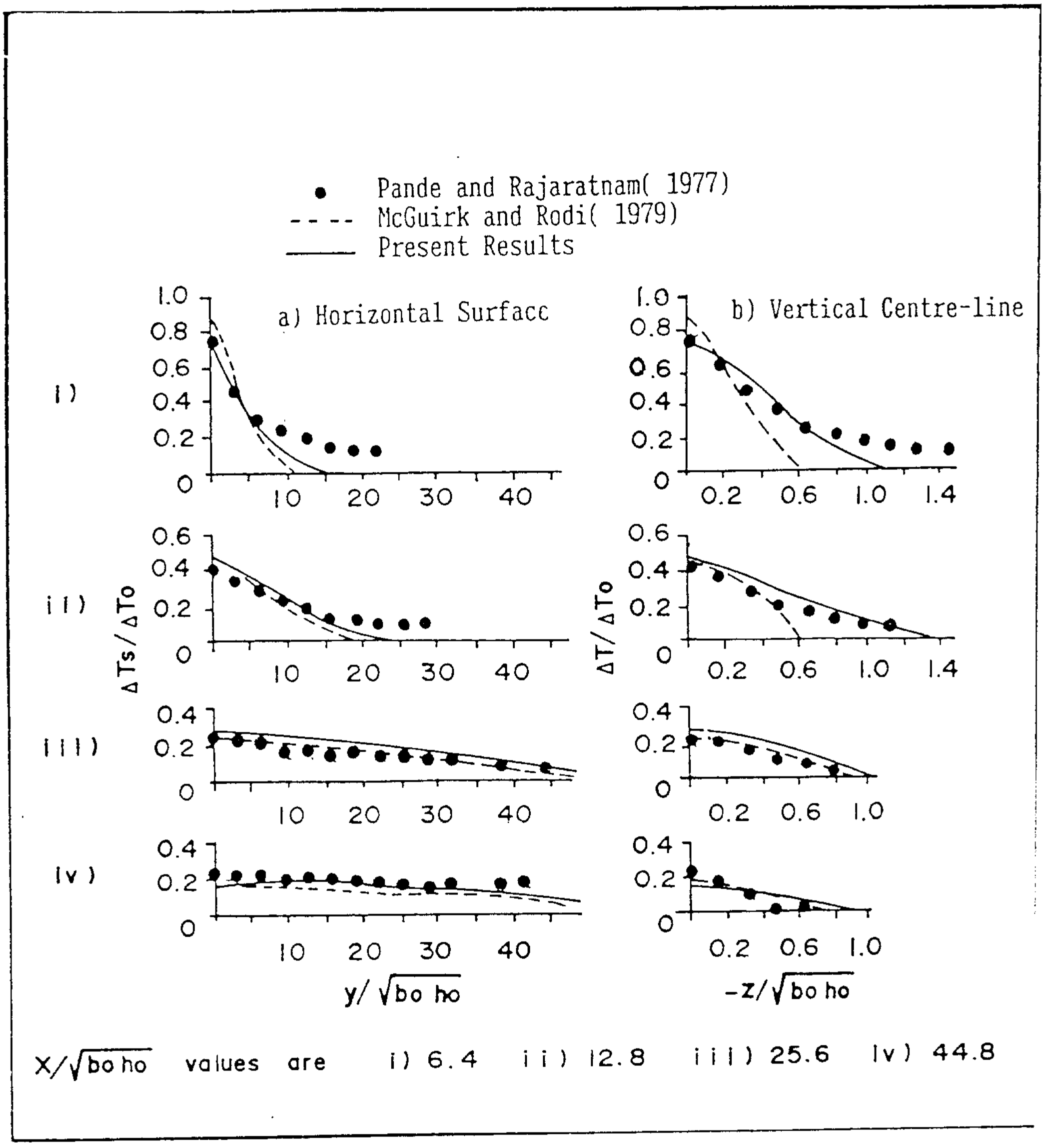

Fig. 8 Excess Temperature Profiles(Run # 1)

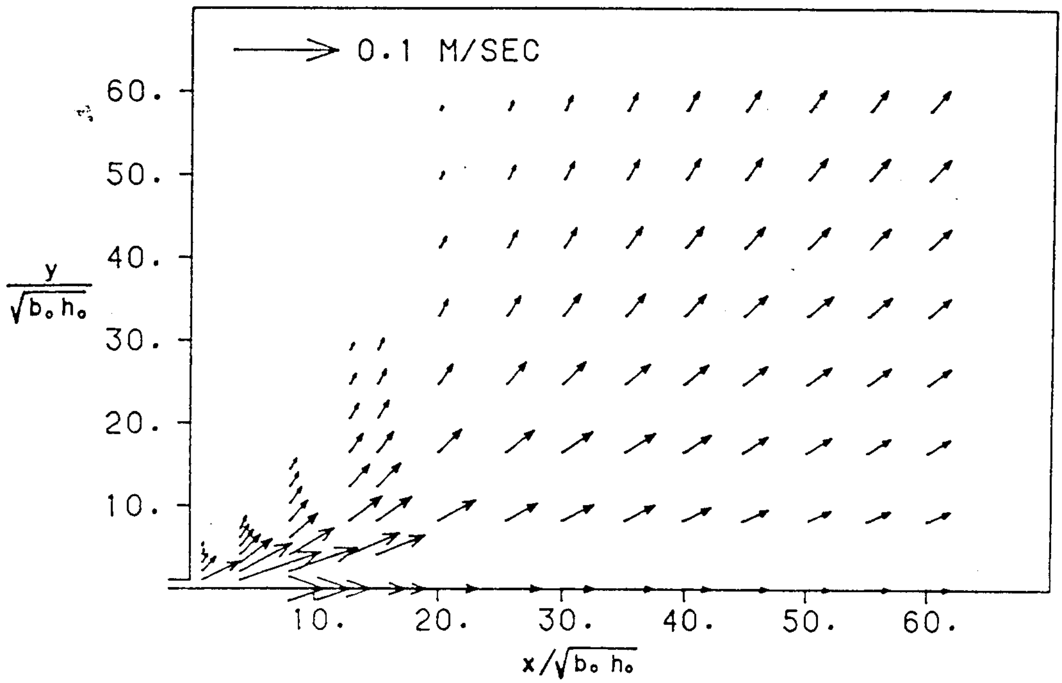

Fig. 9 Predicted Surface Velocity Vector(Run # 1)

3) The limitation of a jet integral model are discussed showing the discrepancy in jet similarity for lateral and vertical values of Gaussian distribution.
4) The present model is an economic simulation model with the simplified governing equations and with the transformation of the elliptic governing equations to the partially parabolic ones.

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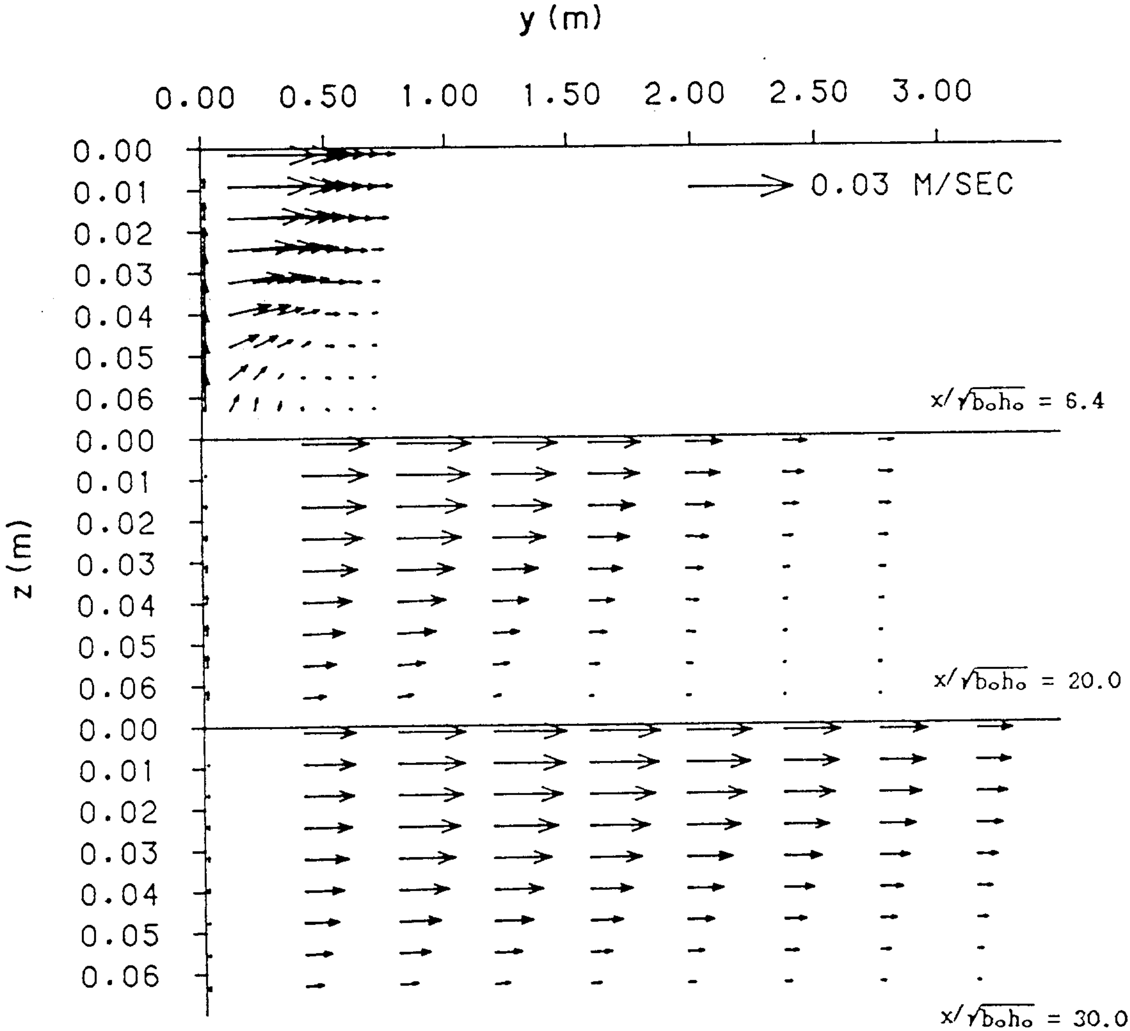

Fig. 10 Predicted Cross-sectional Velocity Vector(Run # 1)

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