論 文
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연속시간 하중최소자승 식별기의 최소고유치 결정

Determination of Minimum Eigenvalue in a Continuous-time Weighted Least Squares Estimator

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Abstract - When using a least squares estimator with exponential forgetting factor to identify continuous-time deterministic system, the problem of determining minimum eigenvalue is described in this paper. It is well known fact that the convergence rate of parameter estimates relies on various factors consisting of the estimator and especially, their properties can be directly affected by all eigenvalues in the parameter error differential equation. Fortunately, there exists only one adjusting eigenvalue in the given estimator and then, the parameter convergence rates depend on this minimum eigenvalue. In this note, a new result to determine the minimum eigenvalue is proposed. Under the assumption that the input has as many spectral lines as the number of parameter estimates, it can be proven that the minimum eigenvalue converges to a constant value, which is a function of the forgetting factor and the parameter estimates number.

Key Words : Weighted Least Squares Algorithm(하중최소자승 앨고리즘), Minimum Eigenvalue(최소고유치), Persistent Excitation(지속여기), Convergence Bounded Value(수렴 유계값)

1. Introduction

There have been many reports treating the estimation of unknown parameters when one wants to obtain an available model. In fact, most adaptive control systems contain the procedure to estimate unknown parameters of controlled system or controller. Hence, parameter and/or system identification may be an important factor in

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designing stable control systems. Various identification algorithms have been discussed in the discrete-time domain and good results have been reported[1~4]. However, we know that most control systems are continuous in nature or sometimes, it may be hard for non-linear physical systems to be discretized correctly. Hence, it is necessary to apply a continuous-time formulation to continuous adaptive control system[5~7].

Many research reports in dealing with exponential stability or parameter convergence property have been given[7~11]. Most of them are developed by defining the minimum or maximum eigenvalue of the coefficient matrix in typical identification algorithms. In particular, paramater convergence has attracted considerable attention in recent years. It is one of the important factors on the design of on-line identification or adaptive control. In general, the rate of parameter estimates depends on several factors like the parameter adaptive gain, initial conditions or input components[5, 7, 10].

Sometimes, it is desirable to know a priori how these varialbes can affect the convegence rate. The exponential convergence is guaranteed under the assumption that the regressor is persistently exciting. Since the regressor consists of the input and its filtered signals, it contains all behaviors of input components. If the input sufficiently rich of any order, the regression vector is sufficient to satisfy the persistent excitation condition, except for an over-parameterization case. Therefore, we can easily guess that the eigenvalue of the updating law may be analyzed by the input spectrum behaviors. Consequently, the convergence rates of parameter estimates can be developed in terms of the eigenvalue.

If system input is sufficiently rich, parameter convergence rates can be quantified with respect to the variation of several variables manipulated by the designer. Of course, the analytic results for them may appear variously for different estimation structures. In this paper, we will discuss parameter convergence properties of only a weighted least squares estimator[12, 13]. One main purpose is to give a new result for the limit of the eigenvalue, under the assumption that the input spectral lines are concentrated on the same points as the number of parameter estimates. In such a case, the minimum eigenvalue becomes as one adjusting factor affecting the parameter convergence rates.

In Section 2, a least squares estimator with exponential forgetting factor[7,12,13] is introduced and we will revise some properties such as persistent excitation, and model error in the estimator.

In Section 3, it can be shown that there exists only one time-varying eigenvalue in the parameter error differential equation, and the parameter convergence rate depends only on this factor. In particular, if the number of input spectral lines is equal to that of parameter estimates, it is proven that the eigenvalue converges to a constant value. It is given as a function of the magnitude of the forgetting factor and the number of parameter estimates, by observing the behavior of the minimum eigenvalue.

Some simulations to verify the given results are also presented.

2. Continuous-time Least Squares Estimator

Consider the case where we wish to perform parameter estimation on the following linear continuous-time system

$$A(s)y(t)=B(s)u(t) \qquad (2.1)$$

where $u(t)\in \boldsymbol{R}^1$ is a quasi-stationary input signal and $u(t)$ and $y(t)\in \boldsymbol{R}^1$ are only available at time t. We suppose that the system is stable and $A(s)$, $B(s)$ are coprime polynomials, given by

$$A(s)=s^n+a_1s^{n-1}+\cdots+a_n$$
$$B(s)=b_0s^m+b_1s^{m-1}+\cdots+b_m$$

Dividing both sides of (2.1) by a known monic polynomial, $A_0(s)$, of oder n, leads to

$$y(t)=\frac{(A_0(s)-A(s))}{A_0(s)}y(t)+\frac{B(s)}{A_0(s)}u(t) \qquad (2.2)$$

where $A_0(s)$ is assumed to be stable, which is given by

$$A_0(s)=s^n+\bar{a}s^{n-1}+\cdots+\bar{a}_n$$

The system described by (2.2) is an ARX model and so we can express the prediction output as a linear regression

$$\hat{y}(t)=\phi^T(t)\hat{\theta}(t) \tag{2.3}$$

where $\phi(t)\in \boldsymbol{R}^p$ and $\hat{\theta}(t)\in \boldsymbol{R}^p$ denote regression vector and parameter estimates vector, respectively, and $\bar{p}$ is the number of parameter estimates. Suppose the upper degrees of $A(s)$ and $B(s)$ are known a priori and the number of unknown parameters can be chosen by the designer to $\bar{p}=n+m+1$. Let $p(=\bar{p})$ define as the number of real parameters for the system model in (2.1). If p can not be given correctly in general identification designs, there exists model error between the property of the real plant and that of an estimated model. In such a case, we can easily guess that the existence condition of a unique solution of parameter estimate vector varies with the model structure, which is arbitrarily chosen by the designer. Since our purpose in this paper is to analyze the parameter convergence rate in an on-line weighted least squares estimator, any property in the presence of unmodeled dynamics will not be treated further.

The nominal parameter vector, $\theta_*\in \boldsymbol{R}^p$ is given as

$$\theta_*^T=[\bar{a}_1-a_1\ \bar{a}_2-a_2 \cdots \bar{a}_n-a_n\ b_0\ b_1 \cdots b_m]$$

In this case, real parameters, $\{a_i, b_i\}$ can be clearly calculated from θ_*. Define the prediction error to estimate unknown parameters in (2.2) as

$$\epsilon(t)=\hat{y}(t)-y(t) \tag{2.4}$$

The most common way to obtain an admissible model which may be matchable with the real system model (2.1) is to adjust a parameter vector $\hat{\theta}(t)$ such that the prediction error is minimized as time goes to infinity. Now, let us define a cost function with exponential forgetting factor as

$$J(t)=\int_0^t e^{-\gamma(t-\tau)}\epsilon^2(\tau)d\tau \tag{2.5}$$

where $\gamma\geq 0$.

If there exists any parameter vector $\hat{\theta}(t)$ to minimize the cost function, then it can be readily shown that[3]

$$R(t)\hat{\theta}(t)=f(t) \tag{2.6}$$

where

$$R(t)=\int_0^t e^{-\gamma(t-\tau)}\phi(\tau)\phi^T(\tau)d\tau \tag{2.7}$$

$$f(t)=\int_0^t e^{-\gamma(t-\tau)}\phi(\tau)y(\tau)d\tau \tag{2.8}$$

Differentiating (2.7) and (2.8) and rearranging them leads to the forms

$$\dot{P}^{-1}(t)=-\gamma P^{-1}(t)+\phi(t)\phi^T(t) \tag{2.9}$$

$$\dot{f}(t)=-\gamma f(t)+\phi(t)y(t) \tag{2.10}$$

where $P(t)=R^{-1}(t)\in \boldsymbol{R}^{p\times p}$ is covariance matrix.

Note that (2.9) can be given under the assumption that $R(t)$ is nonsingular. Therefore, the necessary and sufficient condition such that in the steady state (2.6) has a unique solution, θ_*, is that the covariance matrix $R(t)$ should be positive for all t. It is well known that the regression vector, $\phi(t)$, should be persistently exciting in order that the parameter estimates converge to their nominal values[1~5]. Recall that the regressor is persistently exciting, provided that there exist some positive scalars, α and β such that for some constant T and all t,

$$0<\alpha I\leq \int_t^{t+T}\phi(\tau)\phi^T(\tau)d\tau\leq \beta I<\infty \tag{2.11}$$

Under persistency of excitation, the covariance matrix, $P^{-1}(t)$, in (2.9) remains bounded, as does $P(t)$. Then, the persistent excitation of the regressor results in the exponential stability and so the parameter prediction error converges to zero as time goes to infinity.

Applying (2.9) and (2.10) to (2.6), and using (2.4), we can yield the following updating law

$$\dot{\hat{\theta}}(t)=-P(t)\phi(t)\epsilon(t) \tag{2.12}$$

To avoid computation of the inverse of $P(t)$ for every time, it is desirable to use $P(t)$ directly, rather than using (2.7) and (2.9). By adapting the identity

$$\frac{d}{dt}[P(t)P^{-1}(t)]=\dot{P}(t)P^{-1}(t)+P(t)\dot{P}^{-1}(t)=0$$

we obtain

$$\dot{P}(t)=\gamma P(t)-P(t)\phi(t)\phi^T(t)P(t) \tag{2.13}$$

In using (2.12) and (2.13) for on-line estimation, the initial covariance matrix, $P(0)$, should be positive definite and the initial parameter vector,

$\hat{\theta}(0)$, is arbitrary. There are various least squares algorithms used in identification and adaptive control designs, but we will examine only this formulation to analyze parameter convergence properties.

3. Parameter Convergence Rates

3.1 Eigenvalue of Coefficient Matrix

In fact, the convergence rate of parameter estimates relies on several factors establishing the given estimator. In this section, we shall show some preliminary properties to give a useful quantitative measure to demonstrate the parameter convergence rate when one changes arbitrarily several variables.

Now, define parameter error vector as

$$\tilde{\theta}(t)=\hat{\theta}(t)-\theta_* \tag{3.1}$$

Substituting this into (2.4) leads to

$$\epsilon(t)=\phi^T(t)\tilde{\theta}(t) \tag{3.2}$$

Applying (3.1) and (3.2) to (2.12) gives

$$\dot{\tilde{\theta}}(t)=-P(t)\phi(t)\phi^T(t)\tilde{\theta}(t) \tag{3.3}$$

If $A_0(s)$ and $A(s)$ are stable, then $\phi(t)$ is always bounded for the bounded input. Moreover, as assuming the regressor is persistently exciting, the covariance matrix, $P(t)$, is also bounded for all t. From (3.3) we can easily see that the convergence rate of the parameter error vector depends on all the eigenvalues of coefficient matrix.

Denote the coefficient matrix of $\tilde{\theta}(t)$ as

$$A(t)=P(t)\phi(t)\phi^T(t) \tag{3.4}$$

Note that, for the simplicity of analysis, we will use (3.4) although the coefficient matrix is $-A(t)$ in fact.

Lemma 3.1 Suppose $P(t)$ is invertible. Then $A(t)$ has the same nullspace as $\phi(t)\phi^T(t)$ for all t and there exists the only nonzero eigenvalue given by

$$\lambda(t)=\phi^T(t)P(t)\phi(t) \tag{3.5}$$

Proof : Consider multiplying $A(t)$ on the right by any p-vector, w, orthogonal to $P(t)$. Clearly $A(t)w=0$ and, since the dimension of the space spanned by such w is $p-1$, $A(t)$ has rank 1, i.e., it has at most one nonzero eigenvalue.

Now, premultiplying by $\phi^T(t)$ on both sides of $A(t)$ gives

$$\begin{aligned}\phi^T(t)A(t)&=(\phi^T(t)P(t)\phi(t))\phi^T(t)\\&=\lambda(t)\phi^T(t)\end{aligned}$$

Then, it can be immediately shown that $\lambda(t)$ is the nonzero eigenvalue of $A(t)$.

Note that all eigenvalues of $A(t)$ but the nonzero eigenvalue, $\lambda(t)$, are zeros independently of the variables consisting of $A(t)$ such as the spectrum, magnitudes of the input or the forgetting factor. Thus, we have arrived at an important conclusion that the continuous-time least squares esitimator given by (2.12) and (2.13) has only one non-positive eigenvalue. It can be adjusted by the designer and changes in the range$(-\infty,\ 0]$ for every t. Consequently, it is clear that parameter convergence rate depends only on the time-varying eigenvalue, $-\lambda(t)$, of coefficient matrix. Moreover, it appears as the minimum eigenvalue for (3.3).

3.2 Convergence Bound Analysis

In most general adaptive control systems, it may not be easy that the input components are chosen arbitrarily by the designer in order to ensure the exponential stability of parameter error vector. However, it is possible to do that in identification problems. Here, let the number of distinct spectral lines be q. All signals in the system contain all spectral lines of the system input and then, the time-varying eigenvalue also has the same spectral behaviors. Hence, we can guess that the determination of the convergence bounded value of the eigenvalue may be one way to analyze the parameter convergence rates.

One is then drawn to ask how to analyze the convergence bound of the nonzero eigenvalue. It may look like an impossible problem to calculate the convergence bound of the minimum eigenvalue, for some variables manipulated by the designer. Since the input is given as a function representing several distinct frequency modes, the eigenvalue appears in a very complex form described as a time-varying, non-linear function.

However, we will show a new result determining the convergence bound of the eigenvalue only for $\bar{p}=q$ in this section.

Typically, input is given as a continuous quasi-stationary signal in practical applications. Therefore it can be written as the following almost periodic function, when the spectral lines of the input are located in ω_i, $i=0, \pm1, \cdots, \pm k$ and the amplitude of each spectrum is $\bar{a}_i$,

$$u(t)=\sum_{i=-k}^{k}\bar{a}_i e^{j\omega_i t} \tag{3.6}$$

where $\bar{a}_i=\bar{a}^*_{-i}$, $\omega_i=-\omega_{-i}$ and $\omega_0=0$, and '*' denotes complex conjugate.

For the simplicity of notation, defining a vecotr, $\alpha\in \boldsymbol{C}^q$ and $\beta\in \boldsymbol{R}^q$ as

$$\alpha^T=[\bar{a}_{-k}\ \bar{a}_{-k+1}\cdots\bar{a}_{k-1}\ \bar{a}_k]$$
$$\beta^T=[-\omega_k\ -\omega_{k-1}\cdots\omega_{k-1}\ \omega_k]$$

then, (3.6) can be rewritten

$$u(t)=\alpha^T\varphi(t) \tag{3.7}$$

where

$$\varphi^T(t)=[e^{j\beta_1 t}\ e^{j\beta_2 t}\cdots e^{j\beta_q t}]$$

and $q=2k+1$, and β_i denotes the i-th element of β.

From (2.2) and (2.3), we can directly write the regressor as

$$\phi(t)=H(s)\{u(t)\} \tag{3.8}$$

where $H(s)\in \boldsymbol{C}^q$ is

$$H^T(s)=\frac{1}{A_0(s)}[s^{n-1}G_p(s)\ s^{n-2}G_p(s)\cdots G_p(s)s^{m-1}\cdots 1]$$

and $G_p(s)=B(s)/A(s)$.

Applying (3.7) to (3.8) leads to

$$\phi(t)=N\varphi(t) \tag{3.9}$$

where $N\in \boldsymbol{C}^{q\times q}$ is

$$N=[\alpha_1 H(j\beta_1)\ \alpha_2 H(j\beta_2)\cdots\alpha_q H(j\beta_q)]$$

From (3.9), we see that the regressor can be seperated into a constant matrix and a time-varying vector. The constant matrix is represented in terms of the relations between amplitudes and phases in the system model and algorithm for every spectral line, while the time-varying vector consists of the exponential terms with all complex frequency modes, lineary independent of each other.

Lemma 3.2 Suppose the regressor is persistently exciting. Then, the matrix, N, is invertible.

Proof : Let us assume the contrary to the result and show that this leads to a contradiction. If N is singular, there exists a vector of $\bar{p}$ scalars, $x\in \boldsymbol{C}^p$, at least one of which is nonzero, which satisfies

$$x^T N=0$$

Since the equality does not change even if a nonzero vector, $\varphi(t)$, is multiplied on both sides of the above equation, then

$$x^T\varphi(t)=0 \tag{3.10}$$

Since we assume that the input is supported on $\bar{p}$ distinct spectral lines, then $\varphi(t)$ is persistently exciting.

However, if there exists a nonzero vector x, satisfying (3.10), it means that $\varphi(t)$ does not satisfy persistent excitation This contradicts the above result. Therefore x must be a zero vector and all column or row vectors of N are linearly in dependent. This implies that there exists the inverse of N.

Applying (3.9) into (3.5) and (2.9), then we have the nonzero eigenvalue and a linear transformed covariance matrix described as

$$\lambda(t)=\varphi^T(t)Q(t)\varphi(t) \tag{3.11}$$
$$\dot{Q}^{-1}(t)=-\gamma Q^{-1}(t)+\varphi(t)\varphi^T(t) \tag{3.12}$$

where

$$Q^{-1}(t)=N^{-1}P^{-1}(t)N^{-T}$$

It may not be easy to solve $\lambda(t)$ and $Q(t)$ at each time. However, we may obtain them in the steady-state, by observing the convergence bound of the time-varying eigenvalue. At first, let us determine the steady-state value of $Q^{-1}(t)$. From (3.12),

$$Q_s^{-1}(t)=\frac{1}{s+\gamma}\{\varphi(t)\varphi^T(t)\} \tag{3.13}$$

Substituting $\varphi(t)$ into (3.13) gives

$$Q_s^{-1}(t)=D(t)RD(t) \tag{3.14}$$

where

$$D(t)=\text{diag}[e^{j\beta_1 t}\ e^{j\beta_2 t}\cdots e^{j\beta_q t}]$$

$$R=\begin{bmatrix} \frac{1}{j\beta_{11}+\gamma} & \frac{1}{j\beta_{12}+\gamma} & \cdots & \frac{1}{j\beta_{1q}+\gamma} \\ \frac{1}{j\beta_{21}+\gamma} & \frac{1}{j\beta_{22}+\gamma} & \cdots & \frac{1}{j\beta_{2q}+\gamma} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{j\beta_{q1}+\gamma} & \frac{1}{j\beta_{q2}+\gamma} & \cdots & \frac{1}{j\beta_{qq}+\gamma} \end{bmatrix}$$

where $\beta_{ij}=\beta_i+\beta_j$.

In order to determine the solution of (3.12), $Q(t)$ should be solved a priori. The determinant of $Q^{-1}(t)$ is eqivalent to

$$\det Q^{-1}(t)=\det R$$

because det $D(t)$ is always unity for the practical continuous input signal described by (3.6).

Since all column vectors in R are linearly independent, R is nonsingular, and then substituting (3.14) into (3.11), we can yield.

$$\lambda(t)=U^T R^{-1} U \tag{3.15}$$

where

$$U^T=\varphi^T(t)D^{-1}(t) = [1\ \ 1\cdots 1]$$

From (3.15) it is readily shown that the convergence bound of $\lambda(t)$ in the steady-state becomes a complex function independently of time. Finally, it is given as

$$\lambda=\sum_{i=1}^{p}\sum_{j=1}^{p}\bar{r}_{ij} \tag{3.16}$$

where λ denote $\lim_{t\to\infty}\lambda(t)$, and $\bar{r}_{ij}$ is the ij-th element of R^{-1}.

Here, let u define $K_p\in \boldsymbol{R}^{p\times p}$ as the $\hat{p}$ square counter-identity matrix given by

$$K_p=\begin{bmatrix} 0 & \cdots & 0 & 1 \\ 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \cdots & 0 & 0 \end{bmatrix}$$

We note that $R\in \boldsymbol{C}^{p\times p}$ is a symmetric, counter-Hermitian matrix and it does not depend on time. In particular, we can get

$$R^*=K_p R K_p$$

Since R^{-1} exists and $K_p=K_p^{-1}$, then

$$R^{-*}=K_p R^{-1} K_p$$

Since R^{-1} is also a counter-Hermitian matrix, we can easily show that the convergence bound of $\lambda(t)$ has a real constant value. Although it seems to be impossible to find the convergence bounded value of the minimum eigenvalue, we can obtain the following result for it through some observations.

Theorem 3.1 Suppose that the number of parameter estimates is $\hat{p}$ and the input has $q(=\hat{p})$ distinct spectral lines. If the parameter is estimated by using the least squares identification algorithm given in (2.12) and (2.13), then, there exists a unique convergence bound for $\lambda(t)$, given by

$$\lambda=q\gamma \tag{3.17}$$

where $\lambda=\lim_{t\to\infty}\lambda(t)$.

Proof: In order to simplify matrix operations for $\lambda(t)$ described in (3.15), we begin to analyze from obseving the following relation.

$$\lambda=U^T R^{-1} U = U^T(K_p R)^{-1} U \tag{3.18}$$

Since K_pR results in rotating R by K_p, the sum of all elements in K_pR does not change.

First, let us K_pR denote G. Then, we can write

$$\boldsymbol{G}=\begin{bmatrix} \frac{1}{s_1+s_1^*} & \frac{1}{s_1+s_2^*} & \cdots & \frac{1}{s_1+s_q^*} \\ \frac{1}{s_2+s_1^*} & \frac{1}{s_2+s_2^*} & \cdots & \frac{1}{s_2+s_q^*} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{s_q+s_1^*} & \frac{1}{s_q+s_2^*} & \cdots & \frac{1}{s_q+s_q^*} \end{bmatrix} \tag{3.19}$$

where

$$s_i=\frac{\gamma}{2}+j\beta_{q-i+1} \tag{3.20}$$

Note that all diagonal elements of $\boldsymbol{G}$ are real constants, $1/\gamma$.

In order to diagonalize the matrix $\boldsymbol{G}$, we can partition it into four submatrices, i.e.,

$$\boldsymbol{G}=\begin{bmatrix} G_{11} & G_{12} \\ G^*_{12} & G_{22} \end{bmatrix} \tag{3.21}$$

where G_{11}, G_{12} and G_{22} are $(\hat{p}-1)$, $(\hat{p}-1)\times 1$ and (1×1) matrices, respectively.

Now, let

$$T^*=\begin{bmatrix} I & G_{12}G_{22}^{-1} \\ 0 & 1 \end{bmatrix}$$

then,

$$T=\begin{bmatrix} I & 0 \\ -G_{22}^{-1}G_{12}^* & 1 \end{bmatrix}$$

Hence, using these matrices with respect to $\boldsymbol{G}$, we can obtain

$$T^*\boldsymbol{G}T=\begin{bmatrix} G_{11}-G_{12}G_{22}^{-1}G_{12}^* & 0 \\ 0 & G_{22} \end{bmatrix} =\begin{bmatrix} L & 0 \\ 0 & G_{22} \end{bmatrix} \quad (3.22)$$

where

$$L=G_{11}-G_{12}G_{22}^{-1}G_{12}^*$$

Substituting (3.19) into L, then, the ij-th element of L gives

$$L_{ij}=\frac{s_i-s_q}{s_i+s_q^*}\frac{1}{s_i+s_j^*}\frac{s_j^*-s_q^*}{s_j^*+s_q}$$

Denoting M^* as

$$M^*=\text{diag}\left[\frac{s_1-s_q}{s_1+s_q^*}\ \frac{s_2-s_q}{s_2+s_q^*}\cdots\frac{s_{q-1}-s_q}{s_{q-1}+s_q^*}\right]$$

then, L satisfies the following relation

$$L=M^*G_{11}M \quad (3.23)$$

Using (3.23) with respect to (3.22) gives

$$T^*\boldsymbol{G}T=\begin{bmatrix} M^*G_{11}M & 0 \\ 0 & G_{22}^{-1} \end{bmatrix} \quad (3.24)$$

Consequently, we can easily find the inverse of $T^*\boldsymbol{G}T$ because M and G_{11} are nonsingular That is.

$$(T^*\boldsymbol{G}T)^{-1}=\begin{bmatrix} M^{-1}G_{11}^{-1}M^{-*} & 0 \\ 0 & G_{22}^{-1} \end{bmatrix} \quad (3.25)$$

From (3.18) and (3.25), we have

$$\lambda=U^T\boldsymbol{G}^{-1}U =U^TT(T^*\boldsymbol{G}T)^{-1}T^*U \quad (3.26)$$

Now, observe that

$$U^TT=[1\ \ 1\cdots 1]\begin{bmatrix} I & 0 \\ -G_{22}^{-1}G_{12}^* & 1 \end{bmatrix} =\left[\frac{s_1^*-s_q^*}{s_1^*+s_q}\ \frac{s_2^*-s_q^*}{s_2^*+s_q}\cdots\frac{s_{q-1}^*-s_q^*}{s_{q-1}^*+s_q}\ 1\right] =U^T\begin{bmatrix} M & 0 \\ 0 & 1 \end{bmatrix} \quad (3.27)$$

Applying (3.25) and (3.27) to (3.26), then λ becomes

$$\lambda=U^T\begin{bmatrix} G_{11}^{-1} & 0 \\ 0 & G_{22}^{-1} \end{bmatrix}U =U^T\begin{bmatrix} G_{11}^{-1} & 0 \\ 0 & s_q+s_q^* \end{bmatrix}U \quad (3.28)$$

Continuing this procedure for G_{11} leads to

$$\lambda=U^T\text{diage}[s_1+s_1^*\ \ s_2+s_2^*\cdots s_q+s_q^*]U =\sum_{i=1}^{q}(s_i+s_i^*) \quad (3.29)$$

From (3.20), it is easily verified that, for all i,

$$s_i+s_i^*=\gamma$$

Finally, applying this to (3.29) gives (3.17).

On the other hand, defining

$$\lambda_p(t)=\phi^T(t)\dot{P}(t)\phi(t)$$

then, we can immediately show that from (2.13)

$$\lambda_p(t)=\lambda(t)(\gamma-\lambda(t)) \quad (3.30)$$

Substituting (3.17) into (3.30), in the steady-state $\lambda_p(t)$ becomes

$$\lambda_p=q\gamma^2(1-q)$$

where $\lambda_p=\lim_{t\to\infty}\lambda_p(t)$. Therefore, we can see that $\sigma=-\lambda_p/\lambda$ is

$$\sigma=\gamma(q-1) \quad (3.31)$$

Although the proof of Theorem 3.1 is somewhat long and tedious, it has some interesting features. Clearly, note that the results obtained via (3.17) and (3.31) are very useful when obserring the properties of minimum eigenvalue and further determining the guaranteed rates of exponential convergence. The one immediate observation is that there exists no time-varying variables in (3.17) and (3.31) contrary to one's expectation. This is really a surprising result. It means that $\lambda(t)$ has a constant value in the steady-state.

Assuming the spectral lines of the input are concetrated on p points, the minimum eigenvalue converges to a constant which is only a function of the forgetting factor and the number of parameter estimates. We also note that the convergence bound appears independent of manipulated variables such as amplitude, or frequency components as well as the unknown parameters, except for forgetting factor. If the number of spectrial lines is more than $\bar{p}$, this property does not hold.

4. Simulation Examples

To verify several properties claimed for parameter convergence rates, consider the following 1st order system described as

$$y(t)=\frac{3}{s+2}u(t)$$

For this system, let us choose a stable filter as

$$A_0(s)=s+4$$

and assume the number of parameter estimates to be 2.

To examine the result given in Theorem 3.1, consider the following input with 2 spectral lines,

$$u(t)=a_m\sin\omega t$$

In such a case, the nominal parameter vector is given as $\theta_*^T=[2\ \ 3]$ and all initial parameter estimates are assumed to be zero. Table 4.1 summarizes some data used in simulating the first order plant.

Table 4.1 Some data given in simulations.

symbols	Figure 4.1	Figure 4.3(a)	Figure 4.3(b)
	$a_m=2,\ P(0)=10I$	$\gamma=1,\ P(0)=10I$	$\gamma=1,\ a_m=2$
——	$\gamma=1$	$a_m=1$	$P(0)=I$
----	$\gamma=5$	$a_m=2$	$P(0)=5I$
......	$\gamma=10$	$a_m=3$	$P(0)=10I$
-·-·-	$\gamma=20$	$a_m=5$	$P(0)=20I$

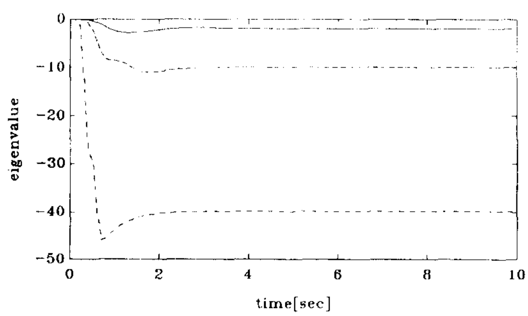


Fig. 4.1 Convergence bound properties of $\lambda(t)$.

Figure 4.1 shows the convergence bound properties of the time-varying eigenvalue when changing the forgetting factor to $\gamma=1$, 5, 10 and 20. Note that from Theorem 3.1, the convergence bounded value can be determined to $\lambda=2\gamma$ for the least squares estimator and then, the minimum eigenvalue of parameter error differential equa-

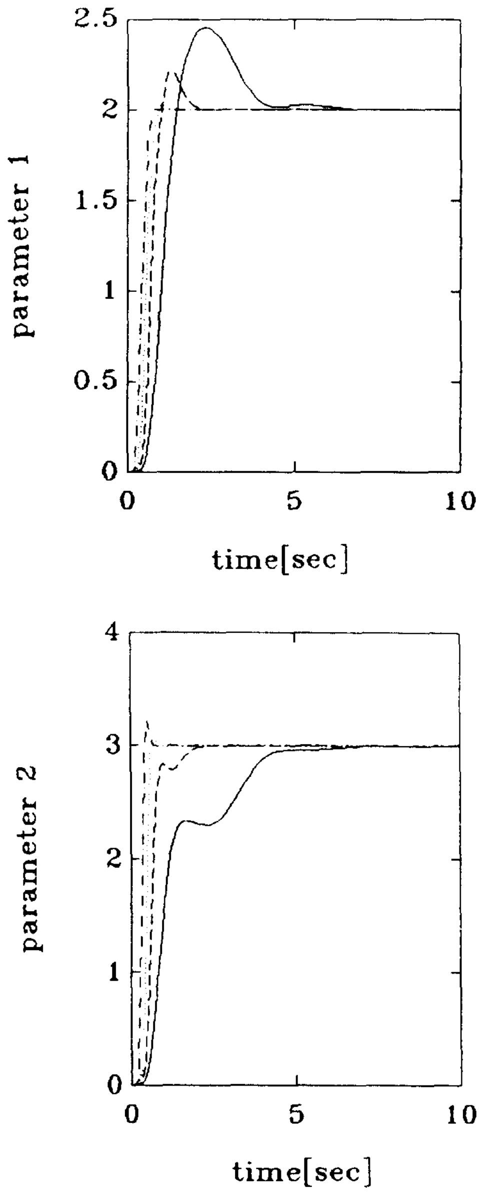
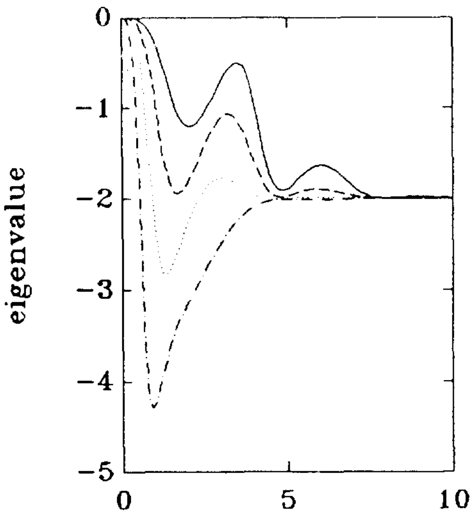
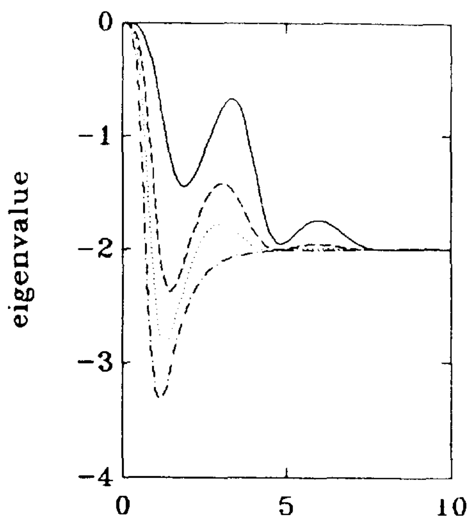


Fig. 4.2 Convergence properties of parameter estimates.



(a) when varying amplitude



(b) when varying P(0)

Fig. 4.3 Convergence bound properties of $\lambda(t)$.

tion becomes -2γ. As we see from Figure 4.1, all eigenvalues converge exactly to -2γ for all γ. Figure 4.2 demonstrates the parameter convergence properties for the given plant. It is clearly seen that the parameter convergence speed directly relies on the minimum eigenvalue.

When input amplitudes and initial covariance matrices are altered, the convergence properties for $-\lambda(t)$ are given in Figure 4.3. From these results, we can immediately see the convergence bounded value of $\lambda(t)$ approaches to -2, independently of a_m and $P(0)$.

5. Conclusions

Parameter convergence properties for a least squares algorithm with exponential forgetting factor are discribed. Assuming input is sufficiently rich, it can be verified that there exists only one time-varying eigenvalue in the coefficient matrix of parameter error differential equation and it can affect the convergence rates of parameter estimates.

In particular, when the system input has as many spectral lines as the number of parameter estimates, the minimum eigenvalue converges to a constant value. This bounded value results in a function of the forgetting factor and the number of parameter estimates. It is independent of the amplitudes and the spectrum of input, the initial covariance matrix, as well as the transfer function model to be identified.

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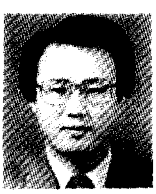

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