投資와 保險需要의 相關關係에 관한 財務經濟學的 研究

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──────── 〈요 약〉 ────────

保險需要者로서의 개인이나 기업은 보험증권과 함께 은행예금과 같은 무위험자산, 혹은 실물자산・자본자산과 같은 위험자산을 보유하고 있는 것이 보통이다. 본 논문은 보험수요자가 보유하는 전체 포트폴리오의 맥락에서 最適保險需要를 분석하는 데 목적이 있다. 이 연구에서 設定한 分析規模에서는 期待效用假說(expected utility hypothesis)에 기초하여 무위험자산과 위험자산에 대한 수요를 보험수요와 동시에 고려하여 保險料의 기회비용이 均衡保險料 개념에 명백히 반영되게 하였다. 이 경우 보험계약은 재난・재해에 대한 危險管理(insurable risk management) 방법의 하나로써 다른 투자기회들과 함께 경쟁관계에서 전체 포트폴리오의 위험을 감소시키는 역할을 담당한다. 본 모형의 분석결과는 기존의 보험경제학설과 다음과 같은 근본적인 상이점을 보이고 있다.

첫째로, 투자자의 효용함수가 一定絶對危險回避(CARA)일 경우, 投資危險(speculative risk)과 財産・災害危險(insurable risk)이 확률적으로 상호의존 관계에 있더라도, 最適保險需要는 다른 투자기회들과 分離(separation)결정될 수 있음을 보였다. 그러나 일반적으로 재산・재해위험이 투자위험과 확률적으로 독립분포되어 있더라도, 보험과 투자간의 상호작용 때문에, 최적보험수요는 다른 투자기회들과 분리결정될 수 없음을 보였다. 이 논문에서는 특별히, 무위험자산 혹은 위험자산에 대한 투자가 財産・災害의 危險(insurable risk)을 헷징(hedging)하는 데 기여하는 고유한 역할을 규명하였고, 또 그 역할을 보험계약에 의해 중복될 수 없는 것임을 보였다.

둘째로, 베르누이 원칙(Bernoulli Principle)을 재검토하여 기존의 베르누이 원칙이 本 분석모형에서는 제한적으로 성립함을 보였다. 이 논문에서는 보다 일반적으로 베르누이 기준이 유지 혹은 위배되는 충분조건을 제시하였고, 그 조건을 전체 포트폴리오 위험에 대한 平均保持擴散(mean preserving spread)의 개념을 도입해 직관적으로 해석하였다.

전통적으로 베르누이 원칙은 보험시장 존재근거에 대해 가장 강력한 이론적 타당성을 부여해 왔으나, 이 논문의 분석결과는 보험수요자의 투자에 대한 기회비용이 보험가격 책정에 반영되지 않으면 보험시장이 붕괴할 수 있음을 시사해 주고 있다.

A Financial Theory of the Demand for Insurance With Simultaneous Investment Opportunities

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ABSTRACT

This paper develops a theory of the demand for insurance. The present model incorporates insurance demand, time value of insurance premium, and demand for riskless and risky assets simultaneously within the expected utility framework. For a special case of CARA, an insurance decision can be made separately from other portfolio decisions. However, in general, the interactions of both decisions cannot be ignored even when insurable and speculative risks are stochastically independent. In particular, the role of risky investment in hedging insurable risk is demonstrated and it is shown that this role cannot be duplicated by an insurance contract. When the investment decision is made simultaneously with the insurance decision, some of the classic theory on insurance should be modified. As an example, the authors characterize the sufficient conditions, under which the Bernoulli criteria (without and with premium loadings) hold or are violated in terms of the net gain of risky investment, the net cost of insurance, and the stochastic relationship between insurable and speculative risks. The authors interpret the results using the Rothschild and Stiglitz's (1970) notion of "increase in riskiness".

I. Introduction

One of the most fundamental theorems in insurance economics is the Bernoulli principle that, with an actuarially fair premium, full coverage is optimal [Arrow (1963, 1974), Smith (1968), Mossin (1968), Ehrlich and Becker (1972), Schlesinger (1981)]. An immediate corollary to this result is that, if the insurance premium includes any positive,

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proportional loading, then only partial coverage of insurance is optimal [Smith (1968), Mossin (1968), Schlesinger (1981)]. However, all of these analyses considered only one source of (completely insurable) risk within a model of timeless uncertain prospects. This approach omits some essential aspects of the individual's insurance purchasing behavior. First, a lack of reality results from isolating the insurance decision from other portfolio decisions [Mayers and Smith (1983)]. A typical individual's opportunity set includes riskless and risky financial/physical assets or investments. The joint interaction between insurable risks and speculative risks may have an impact on the insurance premium and as a consequence on the individual's insurance purchasing behavior. The Bernoulli principle has provided powerful rationale for the existence of insurance market, but the existence of other uninsurable risks, as shown in Doherty and Schlesinger (1983a, 1983b), may cause the Bernoulli principle to be violated.[1] Second, as a result of ignoring other portfolio-decision variables, most of the traditional analyses adopted timeless uncertain prospects. Thus, the time value of money (or opportunity cost of the insurance premium) could not be reflected in the equilibrium premium concept. In practice, there is a significant gap between the time when the premium is collected and the time when insured losses are indemnified if losses are incurred. From the supply side of insurance, it is well known that the premium collected in advance provides funds which can be invested in riskless or risky assets, and therefore the premium should be adjusted to recognize the investment opportunities [Doherty and Garven (1986), Cummins and Harrington (1987), MacMinn and Witt (1987), D'Arcy and Doherty (1988), D'Arcy and Garven (1990)]. Hence, at least in this respect, a limitation of traditional insurance demand theory is its failure to address the relationship between insurance premiums and risk premiums on other marketable assets. According to modern corporate finance, an insurance company can be considered as a financial intermediary between policyholders and the capital market. From a viewpoint of insured, the concept of economic insurance premium should include alternative use of insurance premium. This consideration may allow for different interpretations.

Recently, uninsurable background risks or investment opportunities have been introduced into the traditional risk model of the demand for insurance. In this insurance literature, two approaches can be observed. The first approach employed initial random wealth. Doherty and Schlesinger (1983a, 1983b) and Turnbull (1983) were among the first to examine the theory of optimal insurance purchasing in the presence of uninsurable risk.

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[1]Further discussion of uninsurable background risk can be found in Hirshleifer and Riley (1979), Smith and Buser (1987), Doherty (1984), Kahane and Kroll (1985), Schlesinger and Doherty (1985), Schlenburg (1986) and Briys, Kahane and Kroll (1988).

In partcular, Doherty and Schlesinger (1983b) questioned the existence of "complete" insurance markets and showed that the traditional Bernoulli principle was not always applicable to the "incomplete" insurance markets. Doherty and Schlesinger (1983b) provide perhaps the closest approximation to the problems addressed in this paper. They introduced random initial wealth in the traditional risk model. The rationale for random initial wealth was that the individual wealth typically included uninsurable risky assets. However, Doherty and Schlesinger (1983b) implicitly assumed that the level of risky assets were uncontrollable, or that the insurance decision would be made after the individual's buying strategies for risky assets were completed. This is not a general case. Since investment can serve as an alternative instrument for controlling insurable risk, interaction between investment and insurance decisions cannot be ignored in determining optimal level of insurance. Endogenizing investment decisions in the model will provide a more realistic look at the tradeoff between investment and insurance.

More Recently, when investigating aspects of demand for insurance, investment opportunities were explicitly considered in Doherty (1981, 1984), Mayers and Smith (1983), Kahane and Kroll (1985), and Smith and Buser (1987). All of these studies were based on modern portfolio theory. In particular, Mayers and Smith (1983) analyzed the individual's demand for insurance as a special case of general portfolio hedging activity. In their model, the demand for insurance contracts was determined simultaneously with the demand for other assets in the portfolio. They argued that the demand for insurance contracts was generally not a separable portfolio decision because of significant interdependence between insurance claims and investment returns.

In prior research, it has been frequently convenient to characterize return and risk by the mean and variance of the distribution of final wealth. Although the portfolio approach has provided significant insights about the choice of insurance coverage, it suffers from limitations of the mean-variance criterion. In portfolio theory, it is standard to impose restrictions on utility functions or joint distributions of returns such that all individuals choose mean-variance efficient portfolios. It is well known that, if utility functions are quadratic or if the probability distributions are approximately normal, the mean and variance provide the basis for an acceptable approximation of expected utility. However, the use of quadratic utility functions are regarded as somewhat unreasonable since they exhibit increasing absolute risk aversion (Arrow (1965)). The requirement of normality assumption may even be a more serious obstacle to examining an individual's insurance buying strategies in a mean-variance framework. For the case of the individual

rather than the firm, the exposures to property-liability risks are generally not well diversified. Moreover, property-liability risks are multiplicative type of risks involving the risk associated with the loss frequency distribution and the risk associated with the loss severity distribution. It may not be correct to describe this risk in terms only of a continuous density function which ignorses the non-zero probability of no loss [Smith (1968), Gould (1969)]. Possible skewness of loss distributions may present serious problems [Fieldstein (1969), Schlesinger and Doherty (1985)].

The purpose of this paper is to extend the traditional one source of insurable risk model of the demand for insurance by simultaneously considering the demand for riskless and risky assets within an expected utility framework. Although property-liability risks are crucial in determining the optimal amount of insurance coverage, the riskless interest rate and the random return of the risky assets held in the portfolio are also at the heart of the decision making problem. The model of this paper can be considered as a synthesis and generalization of previous works. That is, on the one hand the present model extends that of Mayers and Smith (1983) in that it assumes entire class of risk averse utility functions and distribution functions by employing expected utility hypotheses. On the other hand, it also generalizes that of Doherty and Schlesinger (1983b) in that it endogenizes riskless and risky investment decisions simultaneously as well as the insurance decision within the context of an individual's whole portfolio, and so explicitly recognizes the opportunity cost of the insurance premium. Moreover, following the insurable risk model of Smith (1968), Gould(1969), or Schlesinger (1981), our model explicitly considers a non-zero probability mass for the case of no loss. The results of our model include those of previous works as special cases, and are substantially different from them in several cases. In particular, we derive the following results.

(1)   It is shown that the individual will invest in a risky asset up to the point where expected marginal rate of return for the risky asset equals the riskfree rate plus the marginal risk premium. However, unlike the results under the traditional investment portfolio model, strict risk aversion is not sufficient to yield a positive marginal risk premium in the presence of an idiosyncratic property-liability risk. The sign and size of marginal risk premium depends on the stochastic relationship between the insurable and speculative risks. Similarly, the individual will choose the level of insurance coverage such that his marginal premium equals the present value of the expected marginal indemnification minus the marginal risk premium for his insurance contract, where the negative marginal risk premium implies a positive

loading. However, unlike the result of the traditional one source of risk model, if investment and insurable risks work as natural hedges each other, the risk averse individual may require a negative loading for his insurance contract.

(2)   We examine the interactions between the investment and optimal insurance decisions in more detail. We show that for the given stochastic conditions, it will never be optimal for the individual exposed to property-liability risks to share his risks only with an insurer without investing in risky assets. The conditions specified on the stochastic relationship between insurable and speculative risks include all independence and positive interdependence cases, and also characterize the extent of the negative interdependence. In these cases, a risky asset can play a positive role in hedging insurable risk, and this role cannot be duplicated by insurance contract alone. That is, we show that the individual with optimal insurance coverage without holding a risky asset can always obtain a final wealth distribution with less dispersion by positively investing in a risky asset, while his mean final wealth remains constant. Therefore his expected utility is improved.

(3)   Demand for insurance is generally not a separable portfolio decision. However, we show that if the utility functions exhibit constant absolute risk aversion (CARA), the insurance decision can be completely separated from other portfolio decisions, even if insurable and speculative risks are stochastically interdependent. This is important because, given separation, most aspects of the optimal amount of insurance under traditional one source of risk model remain valid.

(4)   The Bernoulli principle is reexamined. We show that the Bernoulli principle holds if insurable and speculative risks are uncorrelated or negatively interdependent. This result is an extension of the one obtained by Doherty and Schlesinger (1983), in that the normality assumption is dropped for the negatively interdependent cases. We also demonstrate that if positive interdependence exists the Bernoulli principle will be violated. We provide an intuitive interpretation for these results using the notion of mean preserving spread. That is, for the case of a positive interdependence, since insurable and speculative risks work as natural hedges, by moving to partial insurance from full coverage the individual's total final wealth distribution always loses the weights in both tails, which makes it less "risky" in the Rothschild-Stiglitz (1970).

(5)   Under the more realistic assumption of a positive proportional loading in the premium, we show that the possibility for partial or full coverage as an optimal choice depends on the net expected gain on the risky investment, the loading factor, and the extent of interdependence between the speculative and insurable risks. The results of this model are totally new to the exising insurance-economics literature and may have significant contribution to establishing economic insurance premium principles. We specify sufficient conditions for partial insurance coverage, which include non-negative interdependence and characterize the extent of negative interdependence. We also characterize the extent of negative interdependence which suffices for full insurance coverage. Even though these results are sharply contrasted with the traditional Bernoulli theorem, we provide an intuitive interpretation with the concept of a mean preserving spread.

The paper is organized as follows. In Section II, we develop the model. We also derive and examine the conditions for the optimal investment and insurance plan. In Section III, we demonstrate the role of a risky investment in hedging insurable risk. A separation condition for investment and insurance decisions is displayed in Section IV. In Section V and VI, we reexamine the Bernoulli principle and the possibility of full coverage with proportional loading, respectively. Section VII concludes the paper.

II. The Optimal Investment and Insurance Decisions

We employ a simple two-date model or a *now* and *then* model. An individual makes simultaneous insurance and investment decisions *now*, at time 0, and all uncertainties are resolved *then*, at time 1. The objective of the individual is assumed to be the maximization of expected utility in the von Neumann-Morgenstern sense. At time 0, the individual has to allocate his initial wealth, W, for investment or insurance purposes. Let L denote the random loss amount such that $E[L]<\infty$ where $E[.]$ is the expectation operator. First, by assuming a coinsurance contract,[2] the individual can buy insurance coverage b (0

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[2]Coinsurance and deductibles are the most prevalent insurance contracts observable in the real world. A coinsurance contract approach rather than deductibles is adopted in the present model because of its superiority over deductibles in risk sharing arrangements. As noticed by Borch (1983), deductibles do not seem very relevant in a theory of risk bearing, which assumes that insurance companies are risk neutral. Deductibles should be seen as a practical device for avoiding the expenses involved in checking and paying compensation for negligible losses. Furthermore, unlike coinsurance case, the second-order condition for the optimal deductible is not easily satisfied in maximizing expected utility (see Mossin(1968) and Schlesinger (1981)).

≤ b ≤ 1) for a premium p(b) against random loss L, where b denotes the coinsurance coefficient (p'(b) > 0). Next, suppose that the dollar amount, a, is invested in a risky asset (or a portfolio of risky assets), then the remainder of initial wealth, W-a-p(b), can be used for investment in a riskless asset. We assume that a or W-a-p(b) may be negative since short sales in risky and riskless assets are allowed in the model. However, b will be restricted to 0≤b≤1, since insurance is assumed to be a contract of indemnity.

At time 1, the individual realizes the cash flows from his investment decisions, and the insured losses are paid by a default-free insurance company if the losses are incurred. The dollar amount, a, in risky asset will produce final stochastic return, aR, where R denotes one plus the random rate of return such that $E[R]<\infty$, and W-a-p(b) in the riskless asset will produce final fixed return, $[W-a-p(b)]R_f$, where R_f denotes one plus the riskfree rate of return. Let q denote the probability of loss, then the individual's final wealth, Y, will be given by random amount Y_0 if no losses occur, where

$$Y = Y_0 = [W-a-p(b)]R_f + aR; \text{ with probability } 1-q,$$

or Y_1 if losses do occur, where

$$Y = Y_1 = [W-a-p(b)]R_f + aR - (1-b)L; \text{ with probability } q.$$

More simply, final wealth, Y, can be written by

$$Y = [W-a-p(b)]R_f + aR - h (1-b)L, \tag{2.1}$$

where h is a random variable which has the value of zero with the probability 1-q, or one with the probability q. That is, the random variable, h, denotes the risk of loss frequency, while the random variable, L, reflects the loss severity given that a loss has occurred and is the only insurable risk in the model. Thus, the individual's total loss distribution (or pure premium distribution) can be obtained by combining the loss frequency distribution with the loss severity distribution. The random loss amount, L, has a conditional probability distribution in the sense that it shows the potential size of loss given that a loss has occurred (that is, h=1). By combining or convoluting the loss frequency and loss severity distributions, this makes the model considerably more realistic. In our model, the random variable associated with insurable risk (L) and the random variable associated with speculative risk (R) is assumed to have a joint continuous probability distribution, f(R,L).

We will limit our attention to the situation that random variable concerning the risk of loss frequency, h, is independently distributed from the other random variables concerning insurable or speculative risks (R and L).[3]

Let the individual's expected utility of the final wealth be U(a,b) where:

$$U(a,b) = E[u(Y)]$$

$$= (1-q) \int_{(L=0,\infty)} \int_{(R=0,\infty)} u(Y_0) \, f(R,L) \, dRdL$$

$$+ q \int_{(L=0,\infty)} \int_{(R=0,\infty)} u(Y_1) \, f(R,L) \, dRdL,$$

or equivalently,

$$U(a,b) = (1-q)E[u(Y_0)] + qE[u(Y_1)].$$

We assume that $u'>0$, $u''<0$, then U is concave due to the concavity of u. The first order conditions are given by:

$$D_1U(a,b) = \frac{\partial}{\partial a} E[u(Y)]$$

$$= (1-q) \int \int u'(Y_0)(R-R_f) \, f(R,L) \, dRdL + q \int \int u'(Y_1)(R-R_f) \, f(R,L) \, dRdL$$

$$= (1-q)E[u'(Y_0)(R-R_f)] + qE[u'(Y_1)(R-R_f)] = 0, \qquad (2.2)$$

$$D_2U(a,b) = \frac{\partial}{\partial b} E[u(Y)]$$

$$= - (1-q) \int \int u'(Y_0)p'(b)R_f \, f(R,L) \, dRdL$$

$$- q \int \int u'(Y_1)(p'(b)R_f-L) \, f(R,L) \, dRdL$$

$$= -(1-q)p'(b)R_fE[u'(Y_0)] + qE[u'(Y_1)(-p'(b)R_f+L)] = 0, \qquad (2.3)$$

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[3]This assumption relies on Witt (1973b). He showed that mean loss frequencies and mean loss severities for classes of risks were not correlated by analyzing some empirical classification data for automobile liability insurance.

where $D_i U(a,b)$ denote the partial derivative of U with respect to ith argument. The second-order condition for an interior maximum requires that

$$[x\ y]\ H \begin{bmatrix} x \\ y \end{bmatrix} < 0, \text{ for any } [x\ y] \neq [0\ 0],$$

where H is the Hessian such as

$$H = \begin{bmatrix} D_{11}U & D_{12}U \\ D_{21}U & D_{22}U \end{bmatrix},$$

where $D_{11}U = \dfrac{\partial^2}{\partial a^2} E[u(Y)]$, $D_{12}U = \dfrac{\partial^2}{\partial a \partial b} E[u(Y)]$, and so on. Throughout this paper, we will assume these second order conditions are satisfied so that some optimal values, a^* and b^* exist where $-\infty < a^* < \infty$, and $-\infty < b^* < \infty$. However, due to the property of insurance as a *contract of indemnity*, the feasible choice of insurance, b^{**}, will be restricted to $0 \leq b^{**} \leq 1$.

The first order condition (2.2) with respect to a can be rearranged as follows:

$$D_1 U(a,b) = E(R-R_f) \{(1-q)E[u'(Y_0)] + qE[u'(Y_1)]\}$$

$$+ (1-q)Cov[u'(Y_0);R] + qCov[u'(Y_1);R] = 0,$$

or equivalently

$$E(R) = R_f - \frac{(1-q)Cov[u'(Y_0);R] + q\ Cov[u'(Y_1);R]}{(1-q)E[u'(Y_0)] + qE[u'(Y_1)]} . \tag{2.4}$$

where Cov[.] denotes the covariance operator. For a risk neutral individual, $u'(Y_0)$ or $u'(Y_1)$ would be a constant so that the second term on the RHS of (2.4) should be reduced to zero. However we assumed a strictly risk averse individual ($u''<0$). In this case, the second term on the RHS of (2.4) characterizes the effect of risk aversion on the individual's risky asset choice. A more convenient interpretation can be provided using the concept of risk premium. Let π denote the risk premium which is implicitly defined by the relation $u(E(Y)-\pi)=E[u(Y)]$, then π is a function of both decision variables, a and b. First-order condition for maximizing utility of the certainty equivalent of final wealth can be derived by taking partial derivative of $u[E(Y)-\pi(a,b)]$ with respect to a as follows:

$$\frac{\partial}{\partial a} u[E(Y)-\pi] = u'(E(Y)-\pi)[E(R-R_f)-\pi_a] = 0, \qquad (2.5)$$

where π_a is the marginal risk premium, i.e., the rate of change of π with respect to a (= $\partial \pi/\partial a$). It is evident from (2.4) and (2.5) that

$$\pi_a = \frac{(1-q)Cov[u'(Y_0);R] + qCov[u'(Y_1);R]}{(1-q)E[u'(Y_0)] + qE[u'(Y_1)]}. \qquad (2.6)$$

Equation (2.6) shows that the risk averse individual will require a risk premium for his risky investment. However, risk aversion (u"<0) is not sufficient to sign the marginal risk premium in (2.6). The sign and size of the marginal risk premium depend on the stochastic interdependence between insurable and investment risks.

At this stage, we will derive one important assumption which will be maintained throughout the paper. For the given insurance coverage (0≤b≤1), the individual will buy a positive amount of risky assets if $D_1U(0,b) > 0$. When a = 0, $Y_0 = [W-p(b)]R_f$, $Y_1 = [W-p(b)]R_f-(1-b)L$, and the optimal condition (2.2) may be reduced to

$$D_1U(0,b) = E(R-R_f) \{(1-q)E[u'(Y_0)] + qE[u'(Y_1)]\}$$

$$+ q \, Cov[u'(Y_1);R]. \qquad (2.7)$$

The sign of $Cov[u'(Y_1);R]$ cannot be determined unambiguously. For a special case where the insurable and investment risks are uncorrelated in the sense that E(L|R)=E(L) for all R, one obtains $D_1U(0,b) > 0$ if and only if $E(R)>R_f$ since $Cov[u'(Y_1);R]=0$. In this case, $E(R)>R_f$ will be a necessary and sufficient condition for purchasing a certain amount of the risky asset. It may be noted that the condition $E(R)>R_f$ is equivalent to the boundary condition for positive investment in risky asset derived by Arrow (1963) where an individual's problem is to make an optimal choice between risky and riskfree assets without considering the property-liability risk. Throughout this paper, the assumption that $E(R)>R_f$ will be maintained. However, it should be emphasized that $E(R)>R_f$ will not quarantee a>0 in the present model because insurable risk is possibly correlated with speculative risks.

The first order condition (2.3) for optimal insurance can be rewritten as

$$p'(b)R_f \{(1-q)E[u'(Y_0)] + qE[u'(Y_1)]\} = qE[u'(Y_1)L].\qquad(2.8)$$

To interpret this condition, note that, for a coinsurance contract p'(b) can be considered as a marginal cost of increasing unit coinsurance coverage, and $(1-q)E[u'(Y_0)]+qE[u'(Y_1)]$ is the marginal expected utility of the final wealth at the end of the period (that is, $\partial(E[u(Y)])/\partial Y = (1-q)E[u'(Y_0)] + qE[u'(Y_1)])$. Hence, the LHS of (2.8) is the unit cost of the insurance premium for additional coverage in terms of expected utility at the end of the period. On the right-hand side of (2.8), since L can be seen as a marginal indemnification of increasing unit coverage for a coinsurance contract, the RHS of (2.8) represents the marginal increase in expected utility for additional unit coverage at the end of the period. Thus, the condition (2.8) has a marginal benefit and cost interpretation of buying insurance.

Alternatively, (2.8) can be expressed as

$$p'(b) = \frac{1}{R_f} \cdot \frac{q\,E[u'(Y_1)L]}{(1-q)E[u'(Y_0)] + qE[u'(Y_1)]}.\qquad(2.9)$$

Equation (2.9) reflects a fundamental property of the insurance premium. In equilibrium, the premium will be the present value of the cash equivalent of the loss indemnification. For a more convenient interpretation of (2.9), the actuarial value and the risk premium can be separated from the equilibrium premium concept in (2.9). By letting π_b denote the marginal risk premium with respect to b (that is, $\pi_b = \partial\pi/\partial b$), equation (2.9) can be rewritten as

$$p'(b) = \frac{1}{R_f}[qE(L) - \pi_b],\qquad(2.10)$$

where

$$\pi_b = -\frac{q(1-q)E(L)(E[u'(Y_1)]-E[u'(Y_0)]) + qCov[u'(Y_1);L]}{(1-q)\,E[u'(Y_0)] + q\,E[u'(Y_1)]}.\qquad(2.11)$$

For a risk neutral individual, $u'(Y_0)$ or $u'(Y_1)$ is constant so the numerator of (2.11) will be equal to zero. The required premium for a risk neutral individual should be equal to the present value of expected loss indemnification. Thus, π_b characterizes the effect of individual's risk aversion on the equilibrium premium concept. The risk averse individual

will select his coinsurance level so that his marginal coinsurance premium, p'(b), equals the present value of his expected marginal losses plus the negative marginal risk premium for his insurance contract. However, the negative marginal risk premium, $-\pi_b$, cannot be signed apriori given risk aversion (u"<0). For a special case where the insurable risk is not correlated with the investment risk (E(L|R)=E(L) for all R), one obtains $-\pi_b > 0$, which allows for positive premium loading in the insurance premium. This will be the standard result in the traditional one source of risk model. However, more generally, in the present model, the sign and size of insurance premium loading that the individual is willing to pay in addition to the present value of the expected loss indemnification depend critically on the stochastic relationship between insurable and investment risks. Thus the investment decision (that is, the sign and size of a) plays an essential role in considering the equilibrium premium. In general, it will be impossible to separate both decisions without further critical restrictions because of the interactions between investment and insurance decisions. In the next section, the importance of simultaneous investment decision in purchasing insurance coverage be demonstrated in more detail.

Before discussing further, at this point it may be helpful to clarify the notion of "fair" premium, which will be frequently used later. By assuming proportional loading in the premium, if the total premium for full insurance coverage is denoted P, (that is, P = p(1)), then p(b) = bP and p'(b) = P. From (2.10) it was shown that P = (1/R$_f$)qE(L) is the required premium for full coverage for a risk neutral individual. From now on by assuming a risk neutral insurer in a monopolistically competitive insurance industry,[4] the insurance premium will be defined as the present value of expected indemnification amount plus the proportional loading, that is,

$$p(b) = bP = (1+\lambda)\frac{1}{R_f}b\,qE(L),$$

or

$$P = p(1) = (1+\lambda)\frac{1}{R_f}qE(L), \text{ where } \lambda \geq 0. \tag{2.12}$$

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[4]See Witt (1973a) for an explanation of why monopolisitc insurer provides a reasonable description of the nature of competition in property-casualty insurance lines.

In the remainder of this paper, following the premium schedule (2.12), the insurance premium will be defined as (actuarially) "fair" if $\lambda=0$, or "unfair" if $\lambda>0$ (that is, the premium is said to be fair if $P = (1/R_f)qE(L)$, or unfair if $P > (1/R_f)qE(L)$).[5]

III. The Role of Investment in Risk Management

Risky assets can play a positive role in hedging insurable risk along with the insurance contract. The next proposition shows that for the given stochastic conditions, it will never be optimal for the individual exposed to property-liability risks to share his risks only with insurer without investing in a risky asset. The conditions specified on the joint distribution function include all independence and positive interdependence cases, and characterize the extent of negative interdependence. The proposition is an application of the Diversification Theorem in MacMinn (1984) to the present insurance model.

Proposition 1.
Assume that $E(R)>R_f$ and $P>(1/R_f)qE(L)$. Then $a^* > 0$, if

$$[PR_f\text{-}qE(L)] \ E(R|L) + [E(R)\text{-}R_f]qL$$

is increasing in L.

Proof: First, it is shown that without holding a risky asset, only partial coverage is optimal. That is, if a=0, then $0 \leq b^\circ < 1$, where b° denote the optimal amount of insurance without risky asset in the individual's portfolio. This follows if we evaluate $D_2U(0,1)$. Note that if $(a,b) = (0,1)$, then $Y_0 = Y_1 = Y = (W\text{-}P)R_f$ and

$$D_2U(0,1) = (1\text{-}q) \ u'(Y) \int\int (\text{-}PR_f) \ f(R,L) \ dRdL + q \ u'(Y) \int\int (\text{-}PR_f\text{+}L) \ f(R,L) \ dRdL$$

$$= u'(Y) \ [\text{-}PR_f\text{+}qE(L)]$$

$$< 0,$$

if and only if $P>(1/R_f)qE(L)$. Next, consider the investment and insurance choice such that $(a,b) = (0,b°)$ where $0 \leq b° < 1$. Note that if $(a,b) = (0,b°)$, then $Y_0 = (W-bP)R_f$ and $Y_1 = (W-b°P)R_f - (1-b°)L$. Let the direction $v = (PR_f-qE(L),E(R)-R_f)$, and let $D_vU(a,b)$ denote the partial derivative of U in the direction v. Now, it is shown that $D_vU(0,b°) > 0$, which suffices to show $a^* > 0$ due to the concavity of $U(a,b)$. To show this, let $f(R,L) = g_1(R|L)f_2(L)$ where f_2 is the marginal density of L and g_1 is the conditional density of R. Then by noting that $u'(Y_1)$ is an increasing function of L under risk aversion $(u''<0)$, one obtains

$D_vU(0,b)$

$= [PR_f-qE(L)]\ \{(1-q) \int\int u'(Y_0)(R-R_f)\ f(R,L)\ dRdL + q \int\int u'(Y_1)(R-R_f)\ f(R,L)\ dRdL\}$

$+ [E(R)-R_f]\ \{(1-q) \int\int u'(Y_0)(-PR_f)\ f(R,L)\ dRdL + q \int\int u(Y_1)(-PR_f+L)\ f(R,L)\ dRdL\}$

$= [PR_f-qE(L)]\{(1-q)u'(Y_0)\int\int\ (R-R_f)f(R,L)dRdL+q\int u'(Y_1)[\int (R-R_f)g_1(R|L)dR]\ f_2(L)dL\}$

$+[E(R)-R_f]\{(1-q)u'(Y_0)(-PR_f)\int\int f(R,L)dRdL+q\int u'(Y_1)(-PR_f+L)f_2(L)dL \int g_1(R|L)dR\}$

$= [PR_f-qE(L)]\ \{(1-q)u'(Y_0)[E(R)-R_f] + qE(u'(Y_1)[E(R|L)-R_f])\}$

$+ [E(R)-R_f]\ \{(1-q)u'(Y_0)(-PR_f) + qE[u'(Y_1)(-PR_f+L)]$

$= -q\ (1-q)\ [E(R)-R_f]\ u'(Y_0)\ E(L) + q\ [PR_f-qE(L)]\ E\{u'(Y_1)[E(R|L)-R_f]\}$

$+ q\ [E(R)-R_f]\ E[u(Y_1)(-PR_f+L)]$

$> -q\ (1-q)\ [E(R)-R_f]\ E[u'(Y_1)L] + q\ [PR_f-qE(L)]\ E\{u'(Y_1)[E(R|L)-R_f]\}$

$+ q\ [E(R)-R_f]\ E[u'(Y_1)(-PR_f+L)]$

$= q\ [E(R)-R_f]\ E[u'(Y_1)(-PR_f+L - (1-q)L)]+ q\ [PR_f-qE(L)]\ E\{u'(Y_1)[E(R|L)-R_f]\}$

$= q\ E(u'(Y_1)\ \{[PR_f-qE(L)][E(R|L)-R_f]-[E(R)-R_f][PR_f-qL]\})$

$= q\ E[u'(Y_1)]\ \{[PR_f-qE(L)][E(R)-R_f]-[E(R)-R_f][PR_f-qE(L)]\}$

$$+ q \, \text{Cov}\{u'(Y_1); [PR_f - qE(L)][E(R|L) - R_f] - [E(R) - R_f][PR_f - qL]\}$$

$$= \quad q \, \text{Cov}\{u'(Y_1); [PR_f - qE(L)]E(R|L) + [E(R) - R_f]qL]\}$$

$$> \quad 0,$$

if $[PR_f - qE(L)]E(R|L) + [E(R) - R_f]qL$ is increasing in L. The first inequality follows by noting that

$$Y_0 > Y_1$$

$$\Leftrightarrow$$

$$-u'(Y_0) > -u'(Y_1)$$

$$\Leftrightarrow$$

$$-u'(Y_0)L > -u'(Y_1)L$$

$$\Leftrightarrow$$

$$- u'(Y_0) \int \int Lf(R,L) \, dRdL > - \int \int u'(Y_1)Lf(R,L) \, dRdL$$

$$\Leftrightarrow$$

$$-u'(Y_0)E(L) > -E[u'(Y_1)L]. \hspace{3cm} \text{Q. E. D.}$$

Proposition 1 is illustrated graphically in Figure 1. The expected utility $U(0,b°)$ where $0 \leq b° < 1$ can be always improved if the investment and insurance choice $(0,b°)$ moves into the direction $v = (PR_f - qE(L), E(R) - R_f)$ for the given stochastic conditions. The proposition holds if R and L are stochastically independent (that is, E(R|L) is constant), or R and L are positively interdependent (that is, E(R|L) is increasing in L).[6] It may be interesting to note that, if positive interdependence exist between R and L, speculative risk can work as a natural hedge to insurable risk, and so the individual is initially willing to add a certain amount of the risky asset to his portfolio. More importantly, the proposition also holds if E(R|L) is decreasing in L (that is, negatively interdependent cases) at a rate less than $q[E(R) - R_f]/[PR_f - qE(L)]$. The conditions are also characterized by expected net gain of risky investment $(E(R) - R_f)$ and net expected cost of insurance contract (that is, premium loadings such that $P - (1/R_f)qE(L)$). Thus, the proposition reflects that the relative

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[6]It should be noted that E(R|L) increasing or decreasing in L implies positive or negative correlation between R and L, respectively, since Cov[E(R|L);L] = Cov(R;L). For a formal proof of this claim, see Brumelle (1974)

costs of insurance and investment compete with each other as a means of improving his expected utility or reducing the dispersion of the final wealth, since insurance contracts can be made costly.
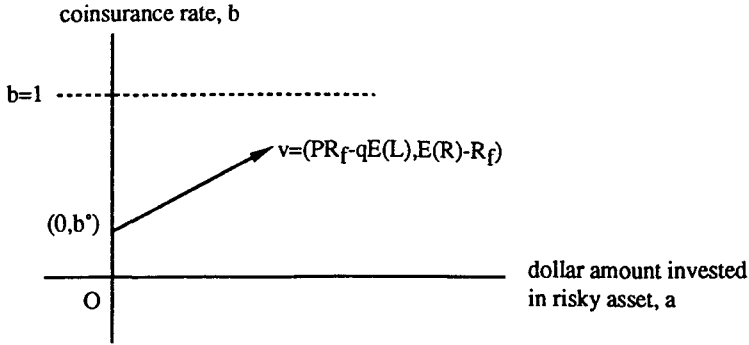

Figure 1. If $[PR_f-qE(L)]E(R|L)+[E(R)-R_f]qL$ is increasing in L, then $D_vU(0,b^\circ)>0$ where $0 \leq b^\circ<1$, which suffices for $a^*>0$ due to the concavity of U (see Proposition 1). This can be interpreted with the dispersion of final wealth distribution. That is, moving in direction v guarantees a reduction in the dispersion of final wealth, $D_vVar_Y(0,b^\circ)<0$, with constant mean final wealth (see Corollary 1).

An intuitive interpretation of Proposition 1 can be given in terms of the dispersion of final wealth. To see this, let $Y(0,b^\circ)$ be random variable denoting individual's final wealth under optimal insurance coverage without holding risky asset such that $Y(0,b^\circ)$ = $(W-bP)R_f-h(1-b^\circ)L$ where $0 \leq b^\circ<1$. Next, let $Y(a,b) = (W-bP)R_f+a(R-R_f)-h(1-b)L$, and this investment and insurance choice of Y(a,b) is assumed to obtain by moving from the investment and insurance choice of $Y(0,b^\circ)$ in the direction $(PR_f-qE(L),E(R)-R_f)$. Note that in this case the individual's expected final wealth remains constant, that is $E[Y(0,b^\circ)]$ = $E[Y(a,b)]$, but higher moments of final wealth distribution are changed. Now, Y(a,b) can be regarded as less risky than $Y(0,b^\circ)$ in the Rothschild-Stiglitz (1970) notion since $E[Y(0,b^\circ)]$ = $E[Y(a,b)]$ and $U(0,b^\circ) < U(a,b)$. These conditions imply that $Y(0,b^\circ)$ has more dispersion than Y(a,b) (see Figure 1). This implication is more formally shown in the following corollary.

Corollary 1.

Assume that $P>(1/R_f)qE(L)$, and $E(R)>R_f$. Then, the individual under optimal insurance coverage $(0 \leq b^\circ<1)$ without holding risky asset can always obtain a final wealth distribution with less dispersion by positively investing in risky asset if

Proposition 1 and Corollary 1 explicitly shows that a risky asset can play a significant role in risk management. The role of investment in our model is similar to "homemade" insurance, which represents the protection provided by marketable assets when the returns of marketable assets are stochastically correlated with insurable risks (Mayers and Smith (1983)). However, our results are somewhat different from homemade insurance in that adding speculative risk may affect the risk of individual's whole portfolio and the insurance coverage even when investment and insurable risks are independently distributed of each other. In the present model, for the independent case, the individual should retain certain positive amount of speculative risk if and only if $E(R)>R_f$ and should adjust the amount of insurance against insurable risk according to the interaction between the two decisions.[7]

IV. Separation

Mayers and Smith (1983) were concerned with the separation question in a mean-variance framework, and found that independence between insurable risk and other risks in the portfolio played a pivotal role for separation. However, as shown in the previous section, the investment decision is generally not irrelevant in determining optimal value of coinsurance rate without a further critical restriction even if insurable and speculative risks are independently distributed. Instead, it will be shown that given CARA the individual's insurance decision is completely separated from other portfolio decisions even when insurable and speculative risks are stochastically interdependent.

Proposition 2.
If the utility function exhibits CARA, then the insurance decision is independent of the investment decision.

Proof: As shown by Pratt (1964), the utility function of a risk averter with constant absolute risk aversion is given by $u(W) = - \exp(-cW)$, where $c > 0$. For the insurance decision to be irrelevant to risky investment decisions, the optimal condition $D_2U(a,b)$ in

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[7]Hong (1992) presented an example which more formally showed how the insurance coverage could change depends on the characterization of risk aversion. Under the independence assumption, if risky investment is introduced to the opportunity set of the individual who initially holds optimal insurance coverage, the individual will always buy certain positive amount of risky asset, and simultaneously, he/she will reduce insurance coverage under DARA, hold it unchanged under CARA or increase it under IARA.

$$[PR_f - qE(L)]\ E(R|L) + [E(R) - R_f]qL$$

is increasing in L.

Proof: Define $Var_Y(a,b)$ to denote the variance of final wealth, Y, then

$$Var_Y(a,b) = a^2 Var(R) - 2aq(1-b)Cov(R;L) + q^2(1-b)^2 Var(L),$$

where Var(.) is the variance operator. Differentiating $Var_Y(a,b)$ in the direction $v = (PR_f - qE(L), E(R) - R_f)$ and evaluating at $(0, b^\circ)$ yields

$$D_v Var_Y(0, b^\circ) = -2q(1-b^\circ)\ \{[PR_f - qE(L)]Cov(R;L) + [E(R) - R_f]qVar(L)\}. \qquad (3.1)$$

The sign of $D_v Var_Y(0, b^\circ)$ will be negative if the sign of the bracketed term on the RHS of (3.1) is positive, and this is the case if $[PR_f - qE(L)]\ E(R|L) + [E(R) - R_f]qL$ is increasing in L, since

$[PR_f - qE(L)]Cov(R;L) + [E(R) - R_f]qVar(L)$

$= [PR_f - qE(L)] \int\int [L - E(L)][(R - E(R)]\ f(R,L)\ dLdR$

$\quad + [E(R) - R_f]q \int\int [L - E(L)]^2\ f(R,L)\ dLdR$

$= [PR_f - qE(L)] \int [(R - E(R)]\ g_1(R|L)\ dR \int [L - E(L)]\ f_2(L)\ dL$

$\quad + [E(R) - R_f]q \int\int [L - E(L)]^2\ f_2(L)\ dL$

$= \int [L - E(L)]\ \{[PR_f - qE(L)][E(R|L) - E(R)] + [E(R) - R_f]q\ [L - E(L)]\}\ f_2(L)\ dL$

$= E[L - E(L)]\ E\{[PR_f - qE(L)][E(R|L) - E(R)] + [E(R) - R_f]\ q\ [L - E(L)]\}$

$\quad + Cov\{L;\ [PR_f - qE(L)][E(R|L) - E(R)] + [E(R) - R_f]q[L - E(L)]\}$

$= Cov\{L;\ [PR_f - qE(L)]E(R|L) + [E(R) - R_f]qL]\}$

$> 0.$ Q.E.D.

(2.3) would have to be identically zero for all possible values of a, and CARA suffices to show this separation. To see this, $D_2U(a,b)$ can be rewritten as

$$D_2U(a,b) = -PR_f\{(1-q)E[u'(Y_0)] + qE[u'(Y_1)]\} + qE[u'(Y_1)L] \qquad (4.1)$$

Let $S=-(1-b)L$, then $Y_1 = Y_0+S$. Now, $E[u'(Y_1)L]$ in the second term on the RHS of (4.1) can be rewritten as follows:

$$E[u'(Y_1)L] = E(L) E[u'(Y_1)] + q \, Cov[u'(Y_1);L]$$

$$= E(L)E[u'(Y_1)] + E\{(u'(Y_1)-E[u'(Y_1)])[L-E(L)]\}$$

$$= \frac{1}{c} E(L) \int c \exp(-cY_0) \, g_1(R|L)dR \int c \exp(-cS) \, f_2(L)dL$$

$$+ \frac{1}{c} \int c \exp(-cY_0)g_1(R|L)dR \int c \exp(-cS)[L-E(L)]f_2(L)dL$$

$$= \frac{1}{c} E[u'(Y_0)|L] \, (E(L)E[u'(S)] + E\{[L-E(L)]u'(S)\}),$$

or equivalently

$$q \, E[u'(Y_1)L] = \frac{1}{c} q \, E[u'(Y_0)|L] \, E[Lu'(S)]. \qquad (4.2)$$

Again, employing similar procedure, the first term on the RHS of (4.1) can be also specified as follows:

$$-PR_f\{(1-q) E[u'(Y_0)] + q E[u'(Y_1)]\} = -PR_f E[u'(Y_0)|L] \{(1-q) + \frac{1}{c} q \, E[u'(S)]\}. \qquad (4.3)$$

Finally, substituting (4.2) and (4.3) into (4.1) gives

$$D_2U(a,b) = E[u'(Y_0)|L] \{-PR_f\frac{1}{c} qE[Lu'(S)] + (1-q) + \frac{1}{c} q \, E[u'(S)]\}$$

$$= -PR_f\frac{1}{c} q \, E[Lu'(-(1-b)L)] + (1-q) + \frac{1}{c} q \, E[u'(-(1-b)L)]$$

$$= 0.$$

Hence, the optimal condition $D_2U(a,b) = 0$ is unaffected by the choice of a, and so separation holds.   Q.E.D.

We claim that this separation condition is especially important in the sense that the optimal insurance decision can be made independently from all other portfolio decisions even when insurable and other uninsurable risks are stochastically correlated with each other when CARA exists. Thus, many of the aspects on the optimal amount of insurance derived under the traditional one source of risk model may possibly remain valid, if CARA is assumed.[8]

V. Bernoulli Principle

In this section, the well-known Bernoulli Principle will be considered. The standard result in the traditional one source of insurable risk model is that all rational risk averse individuals will choose full insurance with fair premium [Arrow (1963, 1974), Smith (1968), Mossin (1968), and Ehrlich and Becker (1972), Schlesinger (1981)]. Recently, Doherty and Schlesinger (1983b) examines the choice of deductible insurance in the presence of uninsurable background risk. They derived some more general results showing that the Bernoulli principle holds if random initial wealth is independent of the insurable loss, or if the two random variables have a bivariate normal distribution with negative correlation. The following proposition is an extension of the results obtained by Doherty and Schlesinger (1983b). The proposition shows that even if the insurable and speculative risks do not have a joint normal distribution but the negative interdependence exists, the Bernoulli principle still holds. In addition, it also demonstrates that if positive interdependence exists then the Bernoulli principle is violated.

Proposition 3.
Assume that $P=(1/R_f)qE(L)$ and $E(R)>R_f$. Then,

    (1) $b = 1$ if $E(L|R)$ is constant for R,

    (2) $b = 1$ if $E(L|R)$ is decreasing in R, or

    (3) $0 \leq b < 1$ if $E(L|R)$ is increasing in R.

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[8]Freifelder (1979) has developed a theoretical insurance ratemaking model based on an assumption of CARA.

Proof: Let a^* denote the optimal value of a when $b = 1$. First, it will be shown that, for the given full insurance ($b=1$), if $E(R)>R_f$ the individual will optimally hold positive amount of risky asset ($a^*>0$) for all possible stochasitc relationship between R and L. It follows if we evaluate $D_1U(0,1)$. If $P = (1/R_f)qE(L)$, then $Y_0 = Y_1 = Y = W-PR_f$ and so

$$D_1U(0,1) = u'(Y) [E(R)-R_f] > 0,$$

which suffices to show that $a^* > 0$. Next, the sign of $D_2U(a^*,1)$ will be evaluated under different stochastic conditions. Let $f(R,L) = g_2(L|R)f_1(R)$ where f_1 is the marginal density of R and g_2 is the conditional density of L. If $(a,b) = (a^*,1)$ and $P = (1/R_f)qE(L)$, then $Y_0 = Y_1 = Y = (W-PR_f)+a^*(R-R_f)$ and

$$D_2U(a^*,1) \quad = (1-q) \int\int u'(Y)(-PR_f) f(R,L) \, dRdL \; + q \int\int u'(Y)(-PR_f+L) f(R,L) \, dRdL$$

$$= \int\int u'(Y)(-PR_f+qL) f(R,L) \, dRdL$$

$$= \int u'(Y) [\int(-PR_f+qL) g_2(L|R)dL] \, f_1(R) \, dR\}$$

$$= \int u'(Y)[-PR_f+qE(L|R)] \, f_1(R) \, dR$$

$$= \{E[u'(Y)] [-PR_f+qE(L)] + Cov[u'(Y);qE(L|R)]\}$$

$$= q \, Cov[u'(Y);E(L|R)].$$

Thus, if $E(L|R)$ is constant, then $D_2U(a^*,1) = 0$, which implies that $(a^*,1)$ is the interior optimal choice. If $E(L|R)$ is decreasing in R, then $D_2U(a^*,1) > 0$. In this case, if we denote b^* to the optimal value of b for maximizing $U(a,b)$ without bound of b (that is, $-\infty < b^* < \infty$), then $1 < b^*$ due to the concavity of U. Therefore, the full coverage insurance portfolio choice $(a^*,1)$ is still optimal, since overinsurance is not permitted in this model. If $E(L|R)$ is increasing in R, then $D_2U(a^*,1) < 0$, which suffices to show that $b^* < 1$ due to the concavity of U. In this case, only partial coverage insurance ($0 \le b < 1$) is optimal and so a^* may no longer be an optimal investment choice.   Q. E. D.

Part (2) of Proposition 3, of course, corresponds to the special case considered by Doherty and Schlesinger (1983b) where a bivariate normal distribution with negative correlation

was employed.[9] This can be easily shown by noting that, if R and L have a joint normal distribution, then

$$E(L|R) = E(L) + \rho \frac{\text{Std}(L)}{\text{Std}(R)} [R-E(R)],$$

where ρ is the correlation coefficient and Std(.) is the standard deviation operator. Proposition 3 thus generalizes the result of Doherty and Schlesinger (1983b).
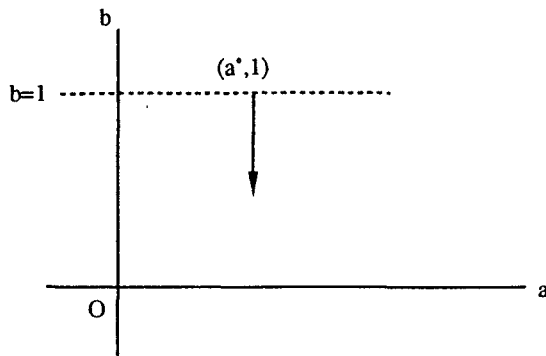

Figure 2. Given fair premium, if E(L|R)is increasing in R, then $-D_2U(a^*,1)>0$ where $a^*>0$. This suffices for $b^*<1$ due to the concavity of U (see Proposition 3). This is because the final wealth distribution loses weights in both tails, that is $-D_2P(Y<y)<0$ and $-D_2P(Y>y)<0$ if evaluated at $(a^*,1)$ without changing the mean final wealth (see Corolllay 3).

If positive interdependence exist between insurable and investment risks, then the insurable risk provides a natural hedge to speculative risk held in the optimal portfolio with full insurance. Thus, the individual is willing to bear a certain amount of insurable risk in the portfolio, which results in partial insurance coverage. As a consequence, the Bernoulli principle no longer holds in this case. This implication can be more formally shown if riskiness of final wealth is defined by the Rothschild and Stiglitz (1970) sense (see Figure 2).

Notice that if the costs of insurance are actuarially fair $(P=(1/R_f)qE(L))$, then for the given risky investment, $a(>0)$, the expected final wealth,

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[9] See Proposition 3 on pp. 561-563 of Doherty and Schlesinger (1983b).

$$E(Y) = WR_f + a[E(R)-R_f] - qE(L),$$

remains constant regardless of all insurance decisions. However, the riskiness of the final wealth distribution will be affected by changing the insurance decision. To see this, let $Y(a^\circ,1)$ be random variable denoting individual's final wealth with full insurance coverage, that is, $Y(a^\circ,1) = (W-P)R_f+a^\circ(R-R_f)$ where $a^\circ>0$. Next, let $Y(a^\circ,b)$ be random variable denoting individual's final wealth with partial insurance coverage, that is, $Y(a^\circ,b) = WR_f + a^\circ(R-R_f) - bPR_f - h(1-b)L$, where $0{\leq}b<1$. Thus, $Y(a^\circ,b)$ is obtained by moving from full coverage to partial coverage insurance without changing risky investment choice. For any $a^\circ>0$, if $E(R|L)$ is increasing in L, $Y(a^\circ,1)$ is more risky than $Y(a^\circ,b)$ in the Rothschild-Stiglitz sense since $E[Y(a^\circ,b)] = E[Y(a^\circ,1)]$ and $U(a^\circ,b)>U(a^\circ,1)$. These conditions are equivalent to $Y(a^\circ,1)$ being a mean preserving spread of $Y(a^\circ,b)$, or $Y(a^\circ,1)$ having more weight in the tails of its distribution than $Y(a^\circ,b)$. Therefore, the proposition shows that if $Cov(R|L)$ is increasing in L, it is always possible to reduce the riskiness of final wealth distribution by moving from full coverage to partial coverage.

We formalize this claim in the following corollary. It may be noted that a reduction in the riskiness of final wealth in the following corollary implies a reduction in variance of final wealth.

Corollary 3
Assume that $P=(1/R_f)qE(L)$, and $E(R)>R_f$. Then, the individual holding full insurance ($b=1$) can always obtain a final wealth distribution with less weight in its tails by reducing the insurance amount to partial coverage ($0{\leq}b<1$) if $E(L|R)$ is increasing in R.

Proof: First, note that if $E(L|R)$ is increasing in R, there exists a fixed number R^* such that $E(L) \lessgtr E(L|R)$ as $R \gtrless R^*$. By noting that the random variable h in (2.1) has the value of zero with the probability 1-q or one with the probability q, $P(Y \leq y)$ can be written as

$$P(Y \leq y) = (1-q) P(Y_0 \leq y) + q P(Y_1 \leq y)$$

$$= (1-q) P[a(R-R_f)-bPR_f \leq y^\circ] + q P[a(R-R_f)-bPR_f -(1-b)L \leq y^\circ],$$

or equivalently,

$$P(Y \leq y) = (1-q) \int_{(L=0,\infty)} \int_{(R=0,k^0(a,b))} f(R,L) \, dRdL$$

$$+ q \int_{(L=0,\infty)} \int_{(R=0,k^1(a,b))} f(R,L) \, dRdL,$$

where

$$y^\circ = y - WR_f,$$

$$k^0(a,b) = \frac{y^\circ + bPR_f}{a} + R_f, \tag{5.1}$$

$$k^1(a,b) = \frac{y^\circ + bPR_f + (1-b)L}{a} + R_f. \tag{5.2}$$

Now it will be shown that the final wealth distribution $P(Y \leq y)$ loses weight in the left tail when the amount of insurance is reduced from full coverage to partial coverage. This follows if we evaluate

$$-\frac{\partial}{\partial b} P(Y \leq y) = -(1-q)\frac{\partial}{\partial b} P(Y_0 \leq y) - q\frac{\partial}{\partial b} P(Y_1 \leq y) < 0, \tag{5.3}$$

at $(a,b) = (a^\circ,1)$ where $a^\circ > 0$. Here, the negative sign on the LHS of (5.3) implies a shift to partial coverage from full coverage. By direct calculation, $\partial P(Y_0 \leq y)/\partial b$ in (5.3) can be written by

$$-\frac{\partial}{\partial b} P(Y_0 \leq y) = -\frac{\partial}{\partial b} \{ \int_{(R=0,k^0(a,b))} f_1(R)[\int_{(L=0,\infty)} g_2(L|R)dL] \, dR \}$$

$$= -D_2 k^0(a,b) \, f_1(k^0(a,b)), \tag{5.4}$$

where $D_2 k^0$ is the partial derivative of $k^0(a,b)$ with respect to second argument, b, that is,

$$D_2 k^0(a,b) = \frac{PR_f}{a}. \tag{5.5}$$

Evaluating the derivatives in (5.4) and (5.5) at $(a,b) = (a^\circ,1)$ yields

$$-\frac{\partial}{\partial b} P(Y_0 \leq y) = -(\frac{PR_f}{a^\circ}) \, f_1(k^0(a^\circ,1)). \tag{5.6}$$

Let y° be such that

$$\frac{y^\circ + PR_f}{a^\circ} + R_f = R < R^*, \tag{5.7}$$

then $k^0(a^\circ, 1) = R$. By substituting (5.7) into (5.6), $-\partial P(Y_0 \leq y)/\partial b$ can be reduced to

$$-\frac{\partial}{\partial b} P(Y_0 \leq y) = -\frac{1}{a^\circ} PR_f f_1(R). \tag{5.8}$$

Again, $-\partial P(Y_1 \leq y)/\partial b$ in (5.3) can be written by

$$-\frac{\partial}{\partial b} P(Y_1 \leq y) = -\frac{\partial}{\partial b} \{\int_{(L=0,\infty)} \int_{(R=0,k^1(a,b))} f(R,L) \, dL dR\}$$

$$= -\int D_2 k^1(a,b) \, f(k^1(a,b),L) \, dL, \tag{5.9}$$

where $D_2 k^1$ is the partial derivative of $k^2(a,b)$ with respect to second argument, b, that is,

$$D_2 k^1(a,b) = \frac{PR_f - L}{a}. \tag{5.10}$$

Evaluating the derivatives in (5.9) and (5.10) at $(a,b) = (a^\circ, 1)$ yields

$$-\frac{\partial}{\partial b} P(Y_1 \leq y) = -\int (\frac{PR_f - L}{a^\circ}) \, f(k^1(a^\circ,1),L) \, dL. \tag{5.11}$$

By noting that $k^1(a^\circ,1) = (y^\circ + PR_f)/a^\circ + R_f = R$, $-\partial P(Y_0 \leq y)/\partial b$ in (5.11) can be rewritten as

$$-\frac{\partial}{\partial b} P(Y_1 \leq y) = -\frac{1}{a^\circ} \int (PR_f - L) \, f(R,L) \, dL$$

$$= -\frac{1}{a^\circ} \int (PR_f - L) \, g_2(L|R) \, dL \, f_1(R)$$

$$= -\frac{1}{a^\circ} [PR_f - E(L|R)] \, f_1(R). \tag{5.12}$$

By substituting (5.8) and (5.12) into (5,3), one obtains

$$-\frac{\partial}{\partial b}P(Y \leq y) = -(1-q)\frac{\partial}{\partial b}P(Y_0 \leq y) - q\frac{\partial}{\partial b}P(Y_1 \leq y)$$

$$= -\frac{1}{a^\circ}f_1(R)\,[PR_f - qE(L|R)]$$

$$= -\frac{1}{a^\circ}f_1(R)\,[E(L) - E(L|R)]$$

$$< 0,$$

since

$$\frac{y^\circ + PR_f}{a^\circ} + R_f = R < R^*$$

yields $E(L) > E(L|R)$.

Following the same manners, one can show that the final wealth probability distribution loses weight in the right tail, that is, $-\partial P(Y \geq y)/\partial b < 0$.   Q.E.D.

The proposition and corollary clearly shows that Bernoulli principle should be reexamined within the context of the insured's whole portfolio. The method of proof for the robustness of the Bernoulli principle (that is, b=1) when $\partial E(L|R)/\partial R \leq 0$ is almost equivalent to that of the preceding proof, so is not duplicated here. In the next section, the Bernoulli criterion is reexamined with more realistic assumption of positive premium loadings.

VI. Full Insurance With a Positive Premium Loading

In the classic theory of the demand for insurance, it would be never optimal for the insured to take full insurance if the insurance premium includes any positive loading factor which is proportionately related to the actuarial value of the policy [Arrow (1964,

1974), Smith (1968), Mossin (1968)]. However, in a real world insurance market it can be frequently observed that the individuals buy full insurance coverage. The results of our model help to explain this phenomenon. In particular, the following proposition specifies sufficient conditions for partial insurance coverage, which includes non-negative interdependence and characterizes the negative interdependence. More importantly, the proposition also characterizes the extent of negative interdependence which is needed for full insurance.

Proposition 4.

Assume that $E(R)>R_f$ and $P>(1/R_f)qE(L)$. Then,

(1) $0 \leq b < 1$, if $-[E(R)-R_f]qE(L|R)-[PR_f-qE(L)]R$ is decreasing in R,

(2) $b=1$, if $-[E(R)-R_f]qE(L|R)-[PR_f-qE(L)]R$ is non-decreasing in R.

Proof: The method of proof is similar to those of Proposition 1 and 3, so it is relegated to the Appendix.

The possibility for partial or full coverage as an optimal choice depends on the net expected gain of the risky investment, the loading factor, and the extent of interdependence between the speculative and insurable risks. Partial coverage ($0 \leq b<1$) with aproportionally loaded premium ($P>(1/R_f)qE(L)$) is consistent with the Bernoulli criterion, and the Bernoulli criterion holds if $E(L|R)$ is either a constant (the independence case) or an increasing function of L (positive interdependence). It also holds if $E(L|R)$ is decreasing with R (negative interdependence) at a rate greater than $[PR_f-qE(L)]/q[E(R)-R_f]$. However, full coverage ($b=1$) with a loaded premium contradicts the Bernoulli criterion. This case results the case if $E(L|R)$ is decreasing with R at a rate smaller than $[PR_f-qE(L)]/q[E(R)-R_f]$. The proposition is graphically illustrated in Figure 3.
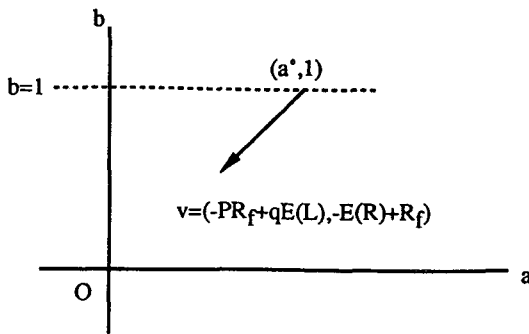
Figure 3. Given an "unfair" or loaded premium, if $-[E(R)-R_f]qE(L|R)-[PR_f-qE(L)]R$ is decreasing in R, then $D_vU(a^*,1)>0$ where $a^*>0$, which suffices for $b^*<1$ due to the concavity of U (see Proposition 4). This can be interpreted by using the notion of a mean preserving spread. Moving in direction v generates a final wealth distribution with less weights in both tails, that is $D_vP(Y<y)<0$ and $D_vP(Y>y)<0$ if evaluated at $(a^*,1)$ while maintaining constant mean wealth (see Corollary 4).

Again, as illustrated in Figure 3, the implication of the proposition can be more intuitively explained by the riskiness of final wealth developed by Rothschild and Stiglitz (1970). The expected final wealth,

$$E(Y) = WR_f + a[E(R)-R_f] - bPR_f - q\,(1-b)E(L),$$

does not change when the portfolio choice moves in the direction $(-PR_f+qE(L),R_f-E(R))$. Let $Y(a^*,1)$ where $a^*>0$ and $Y(a,b)$ be random variables denoting individual's final wealth such that (a,b) is obtained by moving from $(a^*,1)$ in the direction $(-PR_f+qE(L),R_f-E(R))$. Now, if $-(1/R_f)[E(R)-R_f]qE(L|R)-[P-(1/R_f)qE(L)]R$ is decreasing in R, then $Y(a^*,1)$ will be a mean preserving spread of $Y(a,b)$ since $E[Y(a^*,1)] = E[Y(a,b)]$ and $U(a^*,1) < U(a,b)$. Or equivalently $Y(a^*,1)$ has more weight in the tails of its distribution than $Y(a,b)$. The preceding implication for partial insurance is demonstrated directly in the following corollary.

Corollary 4

Assume that $P>(1/R_f)qE(L)$ and $E(R)>R_f$. Then, the individual holding full insurance (b=1) can always obtain a final wealth distribution with less weight in its tails by reducing the insurance amount to partial coverage $(0\leq b<1)$ if

$$-[E(R)-R_f]\,qE(L|R) - [PR_f-qE(L)]R$$

is decreasing in R.

Proof: Since the proof is quite long and complicated, we relegate it to the Appendix.

   Similarly, the reason why full insurance may be optimal under a proportionally loaded premium can be explained in the same lines described above. It has been argued by Borch (1983, 1986) and Schlesinger (1983) that no insurance or full insurance will be the optimal solution to the individual's problem if the premium is given by $P(L) = E(L) + C$, where C is a fixed amount charged to cover the administrative expenses of the insurer. In that case, either full coverage or no coverage at all is purchased depending upon the magnitude of the fixed cost charge. However, as noticed by Witt (1974), the risk charge of an insurance company is usually related to the mean pure premium following the formula (2.12) rather than the variability in the pure premium distribution. This pricing method is consistent with the notion of a risk-neutral monopolistic insurer assumed in this model, since the risk neutral insurance company is interested in only expected loss payment. The traditional results under a proportionally loaded premium seem to be contradicted by observations in real-world insurance markets, where individuals frequently buy full coverage insurance. This incomplete traditional result can be improved within our model. Full insurance proves to be feasible even when the insurance premium includes a proportional loading. The proposition and corollary are not simply a way to hold optimal insurance coverage. Rather, they demonstrate the key role of the risky investment in risk management. Again, the importance of simultaneous insurance and investment decisions should be emphasized because the individual will generally face a trade-off between the net gain of a risky investment $(R-R_f)$ and the net cost of insurance $(P-(1/R_f)qL)$.

VII. Conclusion

   · A model of the demand for insurance under two sources of uncertainty has been presented. The distinguishing characteristics of the present model is that insurance demand, time value of insurance premium, and demand for riskless and risky assets are simultaneously incorporated into the expected utility framework. Based on the view of this paper, the typical individual has two ways to control the risk, risky investment and insurance, and so it may be no longer the case that insurance decision should be governed only by the riskiness of insurable loss. A risky asset play a proper role in hedging insurable

risk. Many of the earlier results obtained from one source of risk model cannot be extended to this model without any strict qualification. The present model provides a more general result to that of Doherty and Schlesinger (1983b), who were concerned with the uncontrollable background risk, and Mayers and Smith (1983), who emphasize the possibility of homemade insurance protection provided by marketable assets which are stochastically correlated with the insurable loss. More generally, we have shown that when investment decision is adjusted simultaneously, nearly all risky assets can play a complementary role to that of insurance in risk management even when insurable and speculative risks are stochastically independent each other, and more importantly, the role of risky asset cannot be duplicated by insurance contract.

Appendix

Proof of Proposition 4: First, it will be shown that under the given full insurance coverage (b=1) the individual always invest positive amount of his wealth in risky assets ($a^*>0$). To see this, note that when $(a,b) = (0,1)$, $Y_0 = Y_1 = Y = W-PR_f$. Evaluating D_1U at $(a,b) = (0,1)$ gives

$$D_1U(0,1) = u'(Y) E(R-R_f) > 0,$$

which implies $a^*>0$ when b = 1. Next, consider the investment and insurance choice such that $(a,b) = (a^*,1)$ where $a^*>0$. Note that when $(a,b) = (a^*,1)$, $Y_0 = Y_1 = Y = W-PR_f + a^*(R-R_f)$. Let the direction $v = (-PR_f+qE(L),R_f-E(R))$, then one obtains $D_vU(a^*,1) > 0$ if $-[E(R)-R_f]qE(L|R)]-[PR_f-qE(L)]R$ is decreasing in R since

$D_vU(a^*,1) =$

$\quad [-PR_f+qE(L)] \{(1-q) \iint u'(Y)(R-R_f) f(R,L) dRdL$

$\quad + q \iint u'(Y)(R-R_f) f(R,L) dRdL\}$

$\quad - [E(R)-R_f] \{(1-q) \iint u'(Y)(-PR_f) f(R,L) dRdL$

$\quad + q \iint u(Y)(-PR_f+L) f(R,L) dRdL\}$

$= \quad [-PR_f+qE(L)]\{\int u'(Y)(R-R_f)(\int f(R,L)dL) dR$

$\quad - [E(R)-R_f] \{(1-q) \int u'(Y)(-PR_f) (\int f(R,L)dL) dR$

$\quad + q \int u(Y)(-PR_f) (\int f(R,L)dL) dR$

$\quad + q \int u(Y) f_1(R) \int L g_2(L|R)dL dR\}$

$= \quad [-PR_f+qE(L)] \int u'(Y)(R-R_f) f_1(R) dR$

$\quad - [E(R)-R_f] \int u'(Y)[-PR_f+qE(L|R)] f_1(R) dR$

$$= -[PR_f - qE(L)] \{E[u'(Y)][E(R)-R_f] + Cov[u'(Y);(R-R_f)]\}$$

$$- [E(R)-R_f] \{E[u'(Y)][-PR_f + qE(L)] + Cov[u'(Y);-PR_f + qE(L|R)]\}$$

$$= Cov\{u'(Y);[E(R)-R_f][PR_f - qE(L|R)] - [PR_f - qE(L)](R-R_f)\}$$

$$> 0.$$

Due to the concavity of U, $D_vU(a^*,1)>0$ suffices to show $0 \le b < 1$. Thus, the proof of the first part is completed. Next, let the direction $v = (PR_f - qE(L), E(R)-R_f)$, then one obtains $D_vU(a^*,1) \ge 0$ if $-[E(R)-R_f]qE(L|R)] - [PR_f - qE(L)]R$ is non-increasing in R since

$$D_vU(a^*,1) = - Cov\{u'(Y);[E(R)-R_f][PR_f - qE(L|R)] - [PR_f - qE(L)](R-R_f)\}$$

$$\ge 0.$$

$D_vU(a^*,1) \ge 0$ where $v = (PR_f - qE(L), E(R)-R_f)$ suffices to show $b=1$, since U is concave and overinsurance ($b>1$) is not feasible. Thus, the proof of the second part is completed.

<div align="right">Q.E.D.</div>

Proof of Corollary 4: First, note that if $-[E(R)-R_f]qE(L|R)] - [PR_f - qE(L)]R$ is decreasing in R, there exists a fixed number R^* such that

$$[E(R)-R_f] [P - \frac{1}{R_f}qE(L|R)] \lessgtr [P - \frac{1}{R_f}qE(L)](R-R_f)$$

as $R \gtrless R^*$. Next, consider the final wealth distribution. $P(Y \le y)$ can be written as

$$P(Y \le y) = (1-q) \int_{(L=0,\infty)}\int_{(R=0,k^0(a,b))} f(R,L) \, dRdL$$

$$+ q \int_{(L=0,\infty)}\int_{(R=0,k^1(a,b))} f(R,L) \, dRdL, \tag{A1}$$

where
$$y^o = y - WR_f,$$

$$k^0(a,b) = \frac{y^\circ + bPR_f}{a} + R_f,$$

$$k^1(a,b) = \frac{y^\circ + bPR_f + (1-b)L}{a} + R_f.$$

Now it will be shown that the final wealth distribution loses weight in the left tail by reducing to partial coverage from full insurance coverage. By direct calculation, the first term on the RHS of (A1) can be rewritten as

$$D_v P(Y_0 \le y)$$

$$= D_v \{ \int_{(R=0,k^0(a,b))} f_1(R) \; [\int_{(L=0,\infty)} g_2(L|R) dL] \; dR \}$$

$$= (-PR_f + qE(L)) \; D_1 k^0(a,b) \; f_1(k^0(a,b))$$

$$+ (R_f - E(R)) \; D_2 k^0(a,b) \; f_1(k^0(a,b)), \tag{A2}$$

where

$$D_1 k^0(a,b) = -\frac{y^\circ + bPR_f}{a^2},$$

$$D_2 k^0(a,b) = \frac{PR_f}{a}.$$

Evaluating the derivative $D_v P(Y_0 \le y)$ in (A2) at $(a,b) = (a^*,1)$ where $a^* > 0$ yields

$$D_v P(Y_0 \le y) = [PR_f - qE(L)] \frac{y^\circ + PR_f}{(a^*)^2} f_1(k^0(a^*,1))$$

$$- [E(R) - R_f] \frac{PR_f}{a^*} f_1(k^0(a^*,1)). \tag{A3}$$

Let y° be such that

$$\frac{y^\circ + PR_f}{a^*} + R_f = R < R^*,$$

then, $k^0(a^*,1) = R$. By noting that

$$\frac{y^\circ + PR_f}{a^\circ} = R - R_f,$$

(A3) can be reduced to

$$D_v P(Y_0 \leq y) = \frac{1}{a^\circ} \{[PR_f - qE(L)](R - R_f) - PR_f[E(R) - R_f]\} f_1(R). \tag{A4}$$

Similarly, the second term on the RHS of (A1) can be rewritten as

$$D_v P(Y_1 \leq y)$$

$$= D_v \{\int_{(L=0,\infty)} \int_{(R=0,k^1(a,b))} f(R,L) \, dLdR\}$$

$$= (-PR_f + qE(L)) \int_{(L=0,\infty)} D_1 k^1(a,b) \, f(k^1(a,b),L) dL$$

$$+ (R_f - E(R)) D_2 k^1(a,b) \, f(k^1(a,b),L) dL, \tag{A5}$$

where

$$D_1 k^1(a,b) = -\frac{y^\circ + bPR_f + (1-b)L}{a^2},$$

$$D_2 k^1(a,b) = \frac{PR_f - L}{a}.$$

Evaluating the derivative in (A5) at $(a,b) = (a^*,1)$ yields

$$D_v P(Y_1 \leq y) = (PR_f - qE(L)) \int (\frac{y^\circ + PR_f}{(a^\circ)^2}) f(k^1(a^*,1),L) \, dL$$

$$+ (E(R) - R_f) \int (\frac{PR_f - L}{a^\circ}) f(k^1(a^*,1),L) \, dL. \tag{A6}$$

By noting that $k^1(a^*,1) = \frac{y^\circ + PR_f}{a^\circ} + R_f = R$, $D_v P(Y_1 \leq y)$ in (A6) can be rewritten as

$D_v P(Y_1 \leq y)$

$$= \frac{1}{a^\circ}(PR_f-qE(L)) \int (R-R_f) f(R,L)\, dL - \frac{1}{a^\circ}(E(R)-R_f) \int (PR_f-L) f(R,L)\, dL$$

$$= \frac{1}{a^\circ}(PR_f-qE(L)) (R-R_f) \int f(R,L)\, dL - \frac{1}{a^\circ}(E(R)-R_f) \int (PR_f-L) g_2(L|R)\, dL\, f_1(R)$$

$$= \frac{1}{a^\circ}\{[PR_f-qE(L)](R-R_f)-[E(R)-R_f][PR_f-E(L|R)]\} f_1(R). \tag{A7}$$

Substituting (A4) and (A7) into (A1) yields

$$D_v P(Y \leq y) = (1-q)\, D_v P(Y_0 \leq y) + q\, D_v P(Y_1 \leq y)$$

$$= \frac{1}{a^\circ} f_1(R)\, [(1-q)\, \{[PR_f-qE(L)](R-R_f) - PR_f[E(R)-R_f]\}$$

$$+ q\, \{[PR_f-qE(L)](R-R_f) - [E(R)-R_f][PR_f-E(L|R)]\}]$$

$$= \frac{1}{a^\circ} f_1(R)\{-[PR_f-E(L|R)][E(R)-R_f] + [PR_f-qE(L)](R-R_f)\}$$

$$< 0,$$

since

$$\frac{y^\circ+PR_f}{a^\circ} + R_f = R < R^*$$

yields

$$[E(R)-R_f]\, [P-\tfrac{1}{R_f}qE(L|R)] > [P-\tfrac{1}{R_f}qE(L)](R-R_f).$$

Following same lines described above, it can be shown that the final wealth probability distribution loses weight in the right tail, that is, $D_v P(Y \geq y) < 0$, where $v = (-PR_f+qE(L), R_f-E(R))$.   Q.E.D.

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