Jour. Korean Inst. Mining Geol.
Vol. 24, No. 1, p. 57-62, 1991.

An Analytical Model Study on the Thermal Stress around the Uplifted Province within the Continental Lithosphere

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ABSTRACT : This study presents results of thermal stress calculations around the uplifted province within the continental lithosphere. A set of thermal stresses for the uplifted provinces are calculated using by assumption of two dimensions along the extension of the strike. The calculations utilize thermoelastic displacement potential function. Thermal stress distribution-faulting conditions inferred from this study are consistent with the suggested surface heat flow and place an important constraint on the thermal state of the uplifted provinces within the continental lithosphere.

INTRODUCTION

Thermal stress distribution within the continental lithosphere affect a variety of physical processes and properties. Seismic properties, rock density and electrical properties are temperature dependent. So are the processes of rock deformation and the stability of mineral phases within the lithosphere.

For these reasons it is important to calculate an estimate of thermal stress around the uplifted province within the lithosphere. For the uppermost parts of the lithosphere, temperature and thermal stress can be either measured directly in drill holes, or extrapolated confidently from surface heat flow and the behavior of thermal properties with depth. The uncertainty of thermal stress estimates become greater with greater depth.

To date the state variable in the thermodynamic equilibrium are temperature, pressure and density. Assuming that a semi-infinite half-space is an uniform elastic medium, we can determine the resultant thermal stresses. The elastic stresses result from sedimentation and erosion and the addition or removal of overburden causes significant deviatoric stresses. Thermal stresses are generated by the temperature change at a given depth.

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The purpose of this study is to obtain the distribution of thermal stress around the uplifted provinces. In doing so I utilize refinements in modeling thermal stresses within the lithosphere. A preferred thermal stresses are derived and presented together with comparisons.

MODELING APPROACH

A simplified model of the anomalous body is shown in Fig. 1. From Fig. 1 z in the downward direction, x in the east and y in the north, represent positive (+) continuation.

k_1 is the thermal conductivity in the continental lithosphere while k_2 is a high conductivity layer with its uplifting influences on the model. This uplift model can be approximated for two dimensions along the extension of the strike.

The temperature field can be considered as a 2-D in the x-z plane because of the elongation in the N-S.

The thermoelastic displacement potential function(Ψ) evidently satisfies Poisson's equation (Timoshenko and Goodier, 1970).

$$\nabla^2\Psi = \frac{1+v}{1-v}\ \alpha T \qquad (1)$$

where v = Poisson's ratio, α = thermal expansion coefficient, and T = temperature.

Solutions of this type of problem are considered in the potential theory. The solution can be written down as the gravitational potential of a distribution of matter density.

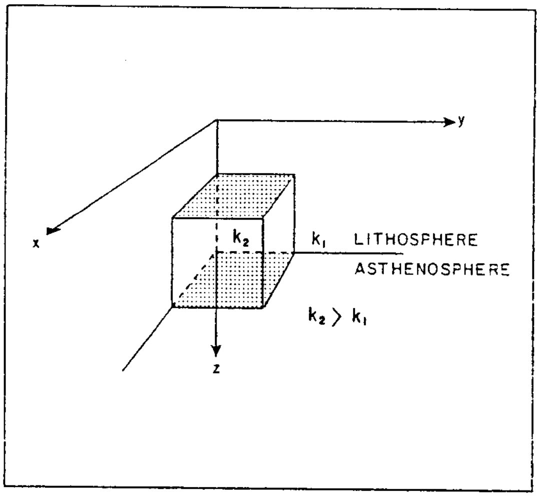

Fig. 1. Characteristic uplift model in the lithosphere.

$$\Psi = -\frac{(1+v)\,\alpha}{4\pi(1-v)} \iiint T(\xi,\eta,\zeta)\frac{1}{r'}\,d\xi\, d\eta\, d\zeta \tag{2}$$

where $T(\xi, \eta, \zeta)$ = temperature at the point ξ, η, ζ at which there is a volume element $d\xi, d\eta, d\zeta$, and r'=distance between this point and the point x, y, z.

The equation (2) gives a complete solution for the thermal stress problem of an infinite solid at temperature zero.

When T is independent of y, and $v = \frac{\partial \Psi}{\partial y} = 0$, we shall have plane strain, with Ψ, $u = \frac{\partial \Psi}{\partial x}$, and $w = \frac{\partial \Psi}{\partial z}$.

The equation (1) becomes

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1+v}{1-v}\,\alpha T \tag{3}$$

A particular solution is given by the logarithmic potential

$$\Psi = -\frac{1}{2\pi}\,\frac{(1+v)}{(1-v)}\,\alpha \iint T(\xi,\zeta) \log r'\, d\xi\, d\zeta \tag{4}$$

where $r' = ((x-\xi)^2 + (z-\zeta)^2)^{\frac{1}{2}}$.

For a thin plate with no variation of T through the thickness, we can assume plane stress.

Then, we have the stress-strain relations

$$P_x = \frac{E}{1-v^2}\left[\frac{\partial u}{\partial x} + v\frac{\partial w}{\partial z} - (1+v)\,\alpha T\right] \tag{5}$$

$$P_z = \frac{E}{1-v^2}\left[\frac{\partial w}{\partial z} + v\frac{\partial u}{\partial x} - (1+v)\,\alpha T\right] \tag{5$'$}$$

$$\tau_{xz} = \frac{E}{2(1+v)}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{5$''$}$$

where E is Young's modulus.

Solving the equation (3) yields $\Psi(x,z)$. The thermal stress can then be determined from the equation (5). Generally it is difficult to solve a heat transfer equation analytically.

The temperature $T(x,z)$ and the thermoelastic displacement potential $\Psi(x,z)$ can usually be obtained by numerical analysis. However, to estimate the order of magnitude of the thermal stress, we can apply equation (4) to seek our solution, using its simplified assumptions in our analysis. Using equation (4), it is obvious that $\Psi(x,z)$ is a linear function of $T(x,z)$.

Assuming the dependence of T on z - direction, the thermal stress which occurs in the uplift of a high conductivity body can be calculated as shown in Fig. 2.

If T is constant, the thermal stress can be estimated. Equaion (4) can be integrated analy-

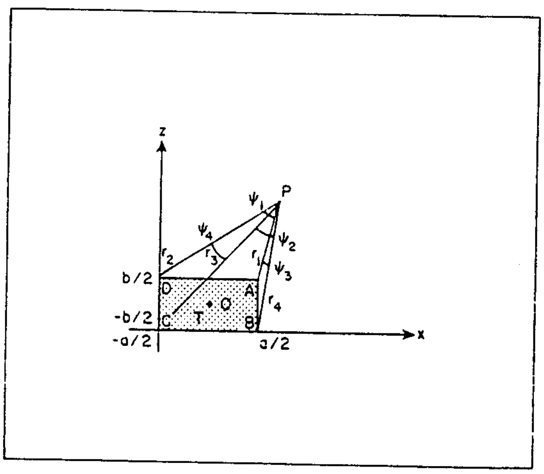

Fig. 2. Basic diagram of the near uplift.

tically by assuming an infinite extension of the surrounding areas. Having made such an assumption, we can apply the model only when considering the stress around the uplift.

If the origin is at the center of the uplift (Fig. 2), we get

$$\Psi(x, z) = -\frac{1}{2\pi}\frac{(1+v)}{(1-v)}\alpha T\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{2}\log\{(x-\xi)^2+(z-\zeta)^2\}\,d\xi\,d\zeta \quad (6)$$

Then, when the following displacements are obtained by differentiation:

$$U(x, z) = -\frac{\partial\Psi}{\partial x} = \frac{1}{2\pi}\frac{1+v}{1-v}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} T\frac{x-\xi}{(x-\xi)^2+(z-\zeta)^2}\,d\xi\,d\zeta \quad (7)$$

$$W(x, z) = -\frac{\partial\Psi}{\partial z}$$

then, the stress components can be found by applying equation (5). Using the integral table, equation (7) will be:

$$U(x,z) = \frac{1}{2\pi}\frac{1+v}{1-v}T$$

$$\{\frac{1}{2}(z-\frac{b}{2})\log\frac{(z-b/2)^2+(x-a/2)^2}{(z-b/2)^2+(x+a/2)^2}+\frac{1}{2}(z+b/2)$$

$$\log\frac{(z+b/2)^2+(x+a/2)^2}{(z+b/2)^2+(x-a/2)^2}+(x-a/2)$$

$$(\tan^{-1}\frac{z-b/2}{x-a/2}-\tan^{-1}\frac{z+b/2}{x-a/2}+(x+a/2)$$

$$(\tan^{-1}\frac{z+b/2}{x+a/2}-\tan^{-1}\frac{z-b/2}{x+a/2})\} \quad (8)$$

with the strain component represented as:

$$e_x = \frac{\partial U}{\partial x}$$

$$= \frac{1}{2\pi}\frac{1+v}{1-v}T(\tan^{-1}\frac{2-b/2}{x-a/2}-\tan^{-1}$$

$$\frac{z+b/2}{x-a/2}+\tan^{-1}\frac{z+b/2}{x+a/2}-\tan^{-1}\frac{z-b/2}{x+a/2})$$

$$e_z = \frac{\partial W}{\partial z} \quad (9)$$

To find both the normal stress P_x and the shear stress (τ_{xz}) for points such as P outside a rectangle ABCD we can apply the following equations:

$$p_x = E\alpha T(\frac{1}{2\pi(1-v)})(\psi_1-\psi_2) = E\alpha T(\frac{1}{2\pi(1-v)})$$

$$(\tan^{-1}\frac{z-b/2}{x-a/2}-\tan^{-1}\frac{z+b/2}{x-a/2}+\tan^{-1}$$

$$\frac{z+b/2}{x+a/2}-\tan^{-1}\frac{z-b/2}{x+a/2}) \quad (10)$$

$$\tau_{xz} = E\alpha T(\frac{1}{4\pi(1-v)})\log\frac{r_1 r_3}{r_2 r_4} \quad (11)$$

where, $r_1 = ((x-a/2)^2+(z-b/2)^2)^{1/2}$
$r_2 = ((x+a/2)^2+(z-b/2)^2)^{1/2}$
$r_3 = ((x+a/2)^2+(z+b/2)^2)^{1/2}$
$r_4 = ((x-a/2)^2+(z+b/2)^2)^{1/2}$

The expression for P_z will be obtained from equation (10)

$$p_x = E\alpha T(\frac{1}{2\pi(1-v)})(\psi_3-\psi_4)$$

THERMAL IMPLICATIONS

For example, if we are considering the Colorado Plateau and we want to discuss the possibility of an uplift which causes normal faultings or earthquakes, thermal expansion in the overlying lithosphere should be considered as a contribution factor in such an uplift.

When the uplift of a high conductivity layer such as that shown in Fig. 1 accompanies high heat flow, it can be assumed that there will be a temperature difference between the uplift part and its surrounding.

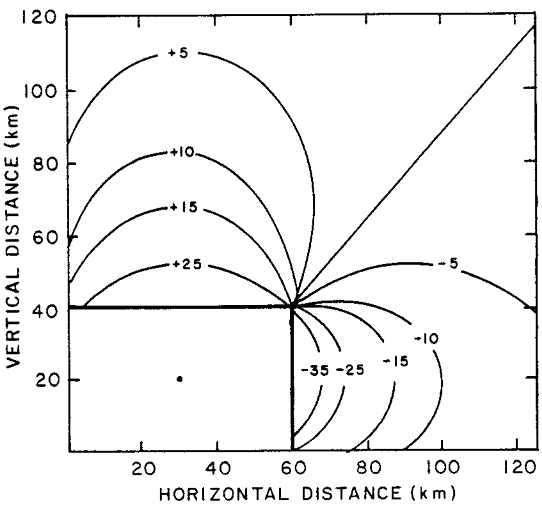

Fig. 3. P_x distribution around an uplift.

The temperature difference is due to thermal stress. According to Bodell and Chapman (1982), the surface heat flow response to lithosphere thinning can only be analysed in terms of a transient thermal problem. When we let a = 60 km, b = 40 km, T = 1,100℃, v = 0.25, E = 1.52×10^{12} dynes / cm², $\alpha = 5.4 \times 10^{-6}$℃$^{-1}$, the stresses ($P_x$, τ_{xz}) around the uplift can be calculated using equations (10) and (11).

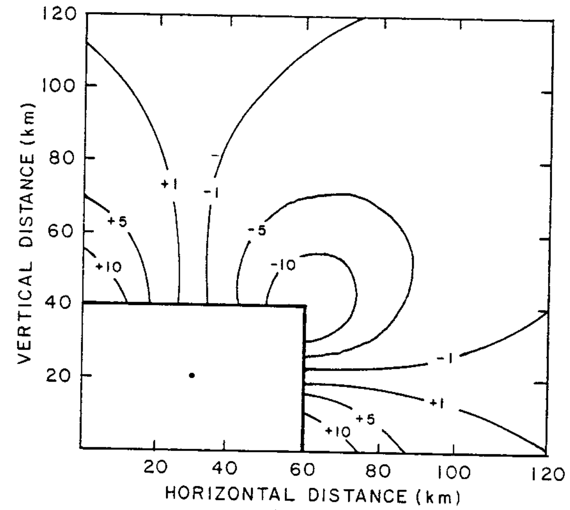

Fig. 4. τ_{xz}distribution around an uplift.

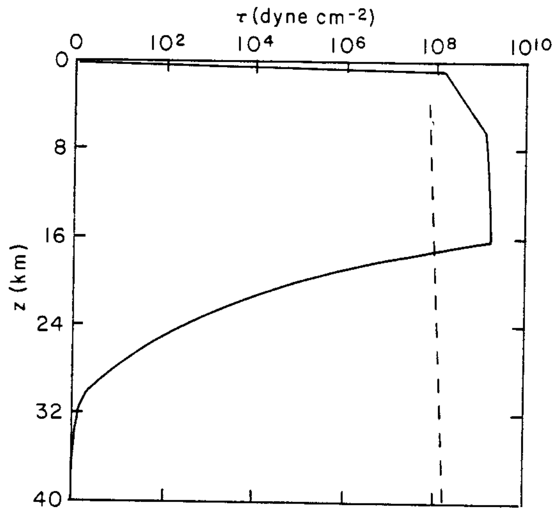

Fig. 5. Shear stress as a function of depth and maximum thermal shear stress (dotted line).

The results of such stresses are shown in Figs. 3 and 4. In both Figs. 3 and 4, we can see that thermal stresses ($P_x = \sim 3 \times 10^9$ dynes / cm², $\tau_{xz} = \sim 1 \times 10^9$ dynes/cm²) around an uplift is a considerable source of faultings or earthquakes and that the tensible stress on the surface of the uplift is at its maximum, therefore, such a situation is most liable to be broken and dangerous.

Next let us consider the distribution of lithospheric stress in two cases. Byerlee (1968) has noted that brittle behavior is a phenomenon of the lithosphere. Since this is so, we can use the equation of Brace and Kohlstedt (1980) to calculate the relationship between shear stress and normal stress:

$$\tau = 0.85\,\sigma_n \qquad \sigma_n < 2\text{kb} \qquad (12)$$

$$\tau = 0.6 \pm 1 + 0.6\,\sigma_n \qquad \sigma_n > 2\text{kb} \qquad (13)$$

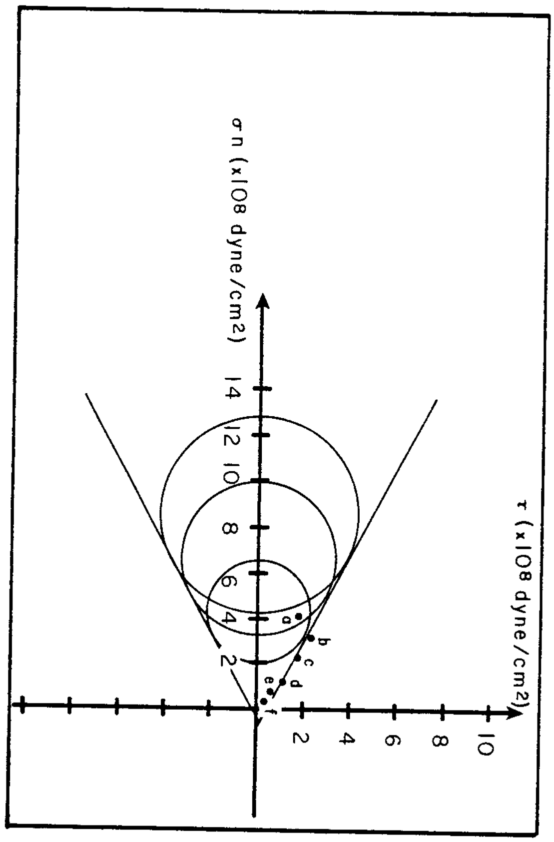

Fig. 6. Mohr's diagram with thermal effects.

Where τ = shear stress, σ_n = normal stress.

Secondly since Weertman (1975), Vetter and Meissner (1979), and Meissner and Strehlau (1982) have noted ductile behavior, we can apply the general creep equation with regard to shear stress τ:

$$\ln \tau = -\frac{1}{3}(C_1 \cdot \frac{Tm}{T} - \ln C_2 + \ln \varepsilon) \quad (14)$$

where C_1, C_2 = Weertman's constants, Tm = melting temperature, T = temperature, and ε = creep rate.

To obtain C_1 and C_2, we can use the results of the peridotite studies of Weertman (1975) and those of Kohlstedt and Goetze (1974). From this application we find that $C_1 = 29$, $C_2 = 4.2 \times 10^{11}$ kb^{-3} s^{-1}. To obtain T(z), we can take the curves for surface heat flow values (q_0 = 80 mWm^{-2}) using the graph of Pollack and Chapman (1977). To obtain ε values, we can use both 10^{-14} s^{-1} and 10^{-17} s^{-1} for the mantle and crustal rheology. Using equations (12), (13) and (14), we can calculate the shear stress in the lithosphere. As shown in Fig. 5, the compressive shear stress can be influenced by the tensile thermal shear stress at depth ranges from 10 km to 30 km.

My consideration of the gravitational loads yields the following equation:

$$\sigma_1 = -\rho gz \quad (15)$$

$$\sigma_3 = -\rho gz \, (v/1-v) \quad (16)$$

where ρ = density, g = gravity, v = Poisson's ratio, and z = depth.

The basic stress equation (Jaeger, 1969) is:

$$\sigma_n = \sigma_3 \cos^2\theta + \sigma_1 \sin^2\theta \quad (17)$$

Using equation (15), (16) and (17), we can calculate normal stress. The configuration that emerges from these calculations is analyzed. The compressive normal stress is subject to a 10% variation in magnitude due to the tensile thermal stress as a function of depth.

When the thermal effect has been calculated for a given depth, we must then add in equation (15) and (16). Plotting both σ_1, σ_3 and τ, I arrive at Mohr's diagram as shown in Fig. 6. Using Mohr's diagram, I then can see the behavior of fracture occurring at a depth range of 5 km to 15 km because of the thermal effect of uplift within the lithosphere.

This result can be compared with the earthquake studies of Utah's Wasatch Front by Arabasz et al. (1980) and those of California's Coso Range by Walter and Weaver (1980). In these comparisons it should be understood that the maximum depths of earthquakes are significantly shallow in high heat flow areas.

CONCLUSIONS

The calculations of the distribution on thermal stress in the lithosphere are in very good agreement with previous knowledge of focal depths and normal faultings.

I can infer from this study that any uplifts within the lithosphere will show a stress distribution similar to Figs. 3 and 4.

Though this result is a preliminary study, it appears to be outlining significant relationships among faultings, heat flows and earthquakes, implying that the thermal process by this model may be an important mechanism for the uplifted provinces.

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Manuscript received 1 April 1990.

대륙 암권내의 융기지역에서 열응력 분포모델에 관한 연구

한 욱

요약: 이 연구는 대륙 암권내에서 융기지역의 열응력 계산결과를 보여주며 주향방향의 연장선상에 2-D 가정하에 열탄성변위 포텐샬 함수를 이용하여 계산이 되었다. 연구로부터 얻은 열응력의 분포와 단층운동조건은 지열류량과 잘 일치하며, 대륙 암권내의 융기지역에서의 지열학적인 상태를 규명하는 데 중요한 역활을 한다.