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Free Vibration of a Thin Circular Cylindrical Shell in Fluid
流體중의 얇은 圓筒셸의 自由振動

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Abstract□Two methods are presented to calculate the natural frequency of an elastic thin circular cylindrical shell vibrating in fluid. Both of them give the natural frequency in analytical expression. One is in a simple form and suitable for higher deformation mode of the shell. Another seems to be exact and be used to a case of the shell partially immersed in fluid. When the shell is fully immersed in fluid, results show: for the lower deformation mode of the shell, the surrounding fluid has remarkable effect upon the natural frequency; for the higher mode, the fluid effect becomes small. When the shell is partially immersed in fluid, it does not occur always that the greatest effect take place at the lowest deformation mode.

要 旨: 流體 중에서 振動하고 있는 얇은 圓筒形 彈性셸의 固有振動数를 計算할 수 있는 두 가지 方法을 제시하였다. 두 方法 모두 固有振動数를 解析的인 形態로 나타내었다. 하나는 簡單한 형태로서 셸의 高次變形모드에 적합하고, 다른 하나는 正確하며 流體중에 部分 潜水된 셸의 경우에 사용된다. 셸이 流體중에 完全 潜水된 경우로서, 低次變形모드에 대해서는 周圍流體가 固有振動数에 상당한 영향을 미침을, 그리고 高次變形모드에 대해서는 그 영향이 작음을 알 수 있었다. 그러나 셸이 流體중에 部分潜水된 경우에는 最低變形모드에서 항상 위 영향이 最大가 되는 것은 아니었다.

1. INTRODUCTION

When we consider a structural vibration, it is necessary to know the natural frequency of the structure at first. In many engineering problems, the structure is vibrating in a fluid. The interaction between vibrating structures and contiguous fluids has a profound influence upon amplitudes and frequencies of the structural vibration. Examples include dams, chimney stacks, heat exchanger tubes, ships and their propeller, off-shore platforms, aircraft, electrical transmission cables, and so on. In many of these cases, the fluid itself is responsible for the vibration. The reaction of the surrounding fluid is such as to alter the natural frequencies of structure from its values in vacuum. It is necessary to consider the fluid effect on the natural frequency of the structure, especially when the fluid is a liquid of high density.

Because a circular cylindrical shell is widely used in various structures, its vibration in fluid were analyzed by many investigators. Breslavskii (1966) calculated the natural frequencies of cylindrical shell filled with liquid, and gave an approximate solution in analytical form. Pallet (1972) applied statistical methods to research the vibration of cylindrical shells. Lomas and Hayek (1977) and Warburton (1978) considered the effect of fluid-loaded on the natural frequencies of plates and shells. There are also a lot of these investigations which used numerical methods to calculate the natural frequencies of shell.

In the following, we present two methods to calculate the natural frequencies of a thin cylindrical shell vibrating in fluid. One is, through a simplification, to solve the coupled equations of shell vi-

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bration and fluid motion. Another is to use the energy relation of the coupled system. By using the Lagrange equation we obtain the natural frequencies. The cylinder shell theory is given by Flügge (1960), which becomes Donnell's approximate equation by simplification. The fluid is assumed to satisfy the potential flow theory. The natural frequency of the shell vibrating in fluid is expressed in a form of combining the natural frequency in vacuum and an influence factor. The influence factor is given in series form, which converges quickly.

2. A FUNDAMENTAL MATHEMATICAL MODEL AND A SOLUTION

We write a cylindrical shell vibration equation after Flügge (1960) as

$$\frac{C}{a^2}[A_F]\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \mu\frac{\partial^2 u}{\partial t^2} \\ \mu\frac{\partial^2 v}{\partial t^2} \\ -\mu\frac{\partial^2 w}{\partial t^2} - P \end{bmatrix}, \qquad (1)$$

where

$$[A_F]=\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

is a symmetrical operator matrix and

$$A_{11}=a^2\frac{\partial^2}{\partial x^2}+(1+K)\frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2},$$

$$A_{12}=A_{21}=\frac{1+\nu}{2}a\frac{\partial^2}{\partial x\partial\theta},$$

$$A_{13}=A_{31}=\nu a\frac{\partial}{\partial x}$$

$$+K\left\{-a^3\frac{\partial^3}{\partial x^3}+a\frac{(1-\nu)}{2}\frac{\partial^3}{\partial x\partial\theta^2}\right\},$$

$$A_{22}=\frac{\partial^2}{\partial\theta^2}+(1+3K)\frac{1-\nu}{2}a^2\frac{\partial^2}{\partial x^2},$$

$$A_{23}=A_{32}=\frac{\partial}{\partial\theta}+K\frac{3-\nu}{2}a^2\frac{\partial^3}{\partial x^2\partial\theta},$$

$$A_{33}=1+K\left(a^4\Delta^2+2\frac{\partial^2}{\partial\theta^2}+1\right),$$

$$C=\frac{E\delta}{1-\nu^2},\quad D=\frac{E\delta^3}{12(1-\nu^2)},\quad K=\frac{D}{Ca^2}=\frac{\delta^2}{12a^2}.$$

In the above equations, a is radius of the middle surface of shell, δ thickness of the shell, E Young's modulus, ν Poisson's ratio, μ mass of the shell per unit area of middle surface, P external compressive force per unit area, and Δ is the Laplacian, which is written in the polar coordinate system

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial x^2}.$$

In the present analysis we put $\frac{\partial}{\partial r}=0$, r=a and use

$$\Delta=\frac{1}{a^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial x^2}.$$

We consider the free vibration of a shell immersed in fluid, with simply supported edges, as illustrated in Fig. 1. The boundary conditions are expressed as follow

$$\left.\begin{aligned} &w=0,\quad \frac{\partial^2 w}{\partial x^2}=0,\quad x=0 \text{ or } x=L; \\ &v=0,\quad \frac{\partial u}{\partial x}=0,\quad x=0 \text{ or } x=L. \end{aligned}\right\} \qquad (2)$$

If the displacement functions are taken in the following forms

$$\left.\begin{aligned} &u=u_l\cos m\theta\cos\frac{n\pi x}{L} && u_l=u_0e^{-i\omega t}, \\ &v=v_l\sin m\theta\sin\frac{n\pi x}{L} \quad \text{where} && v_l=v_0e^{-i\omega t}, \\ &w=w_l\cos m\theta\sin\frac{n\pi x}{L} && w_l=w_0e^{-i\omega t}, \end{aligned}\right\} \qquad (3)$$

the equations of the shell vibration and the boundary conditions are satisfied, in which ω is a circular frequency of vibration, L is the length of the shell, and m is an integer representing deformation mode of the shell, that is, a number of deformation wave in circumferential direction of the shell, and n is also an integer representing a number of half

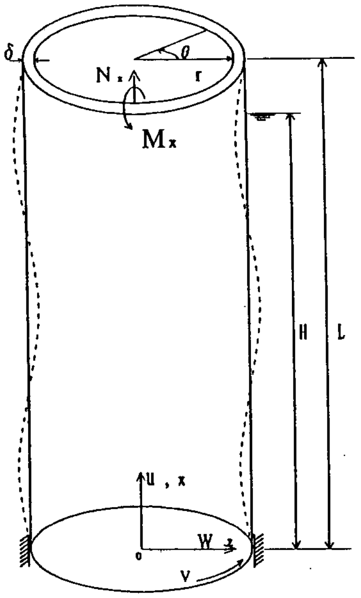


Fig. 1. A simply supported cylindrical shell vibrating in fluid.

wave in axial direction of the shell.

The fluid motion is induced by the shell vibration. We take the fluid surrounding the shell as inviscid and incompressible. The velocity potential of the fluid motion is expressed by Φ, and in the cylindrical coordinate Φ is governed by

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial x^2}=0. \tag{4}$$

The normal component of fluid speed vanishes on the bottom,

$$\frac{\partial\Phi}{\partial x}=0,\ x=0\ ; \tag{5}$$

the free surface condition is expressed as

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial x}=0,\ x=H\ ; \tag{6}$$

the normal velocity component of fluid, on the shell surface, coincides with the velocity of shell

$$\frac{\partial\Phi}{\partial r}=\frac{\partial w}{\partial t},\ r=a,\ 0\leq x\leq H\ ; \tag{7}$$

where H is the fluid depth and g is the gravity acceleration. Wave induced by shell vibration vanishes at infinity

$$\sqrt{\alpha r}\left(\frac{\partial\Phi}{\partial r}-i\alpha\Phi\right)\to 0,\ r\to\infty. \tag{8}$$

which is the Sommerfeld radiation condition in cylindrical polar coordinate (Mei, 1989). The solution of Φ may be taken in the following form

$$\Phi=\Phi_0\cos m\theta\Big[A_0H_m^{(1)}(\alpha r)\cosh\alpha x+\sum_{k=1}^{\infty}A_kK_m(\alpha_k r)\cos\alpha_k x\Big], \tag{9}$$

where $H_m^{(1)}(z)$ is the Hankel function of the first kind, $K_m(z)$ is the modified Bessel function of the second kind, α and α_k are governed by the dispersion relation derived from the free surface boundary condition

$$\omega^2=g\alpha\tanh\alpha H, \tag{10}$$

and

$$\omega^2=-g\alpha_k\tan\alpha_k H. \tag{11}$$

Coefficients A_k are determined by the boundary condition that the normal components of the velocity of the fluid and shell coincide at the interface of fluid and shell. Substituting the velocity potential in the boundary condition of shell surface, we obtain

$$\Phi_0\cos m\theta\Big[A_0H_m'^{(1)}(\alpha a)\cosh\alpha x+\sum_{k=0}^{\infty}A_k\alpha_kK_m'(\alpha_k a)\cos\alpha_k x\Big]=-i\omega e^{-i\omega t}w_0\cos m\theta\sin\frac{n\pi}{L}x. \tag{12}$$

Taking

$$\Phi_0=-i\omega e^{-i\omega t}w_0, \tag{13}$$

and expanding the $\sin\frac{n\pi x}{L}$ in the form of $\cosh\alpha x$ and $\cos\alpha_k x$ series, we get

$$A_0=\frac{2}{F_0\alpha H_m'^{(1)}(\alpha a)}\int_0^H\sin\frac{n\pi}{L}x\cosh\alpha x\,dx$$

$$=\frac{2}{F_0\alpha H_m'^{(1)}(\alpha a)}$$

$$\frac{\alpha \sinh \alpha H \sin \frac{n\pi H}{L} + \frac{n\pi}{L}\left(1 - \cosh \alpha H \cos \frac{n\pi H}{L}\right)}{\alpha^2 + \left(\frac{n\pi}{L}\right)^2}, \tag{14}$$

$$A_k = \frac{2}{F_k \alpha_k K_m'(\alpha_k a)} \int_0^H \sin \frac{n\pi}{L} x \cos \alpha_k x \, dx$$

$$= \frac{2}{F_k \alpha_k K_m'(\alpha_k a)} \frac{(-1)^k \alpha_k \sin \frac{n\pi H}{L} - \frac{n\pi}{L}}{\alpha_k^2 - \left(\frac{n\pi}{L}\right)^2}, \tag{15}$$

where

$$F_0 = \int_0^H \cosh^2 \alpha x \, dx = \frac{H}{2} + \frac{\sinh 2\alpha H}{4\alpha},$$

$$F_k = \int_0^H \cos^2 \alpha_k x \, dx = \frac{H}{2} + \frac{\sin 2\alpha_k H}{4\alpha_k}.$$

As long as we consider small vibration of a shell, the fluid wave amplitude induced by the shell vibration is very small. According to Mei's analysis (Mei *et al.*, 1979; Mei, 1989), in the case, the gravity may be ignored. The free surface boundary condition is simplified to

$$\frac{\partial^2 \Phi}{\partial t^2} = 0, \quad x = H, \tag{16}$$

and we have

$$A_0 = 0, \quad \alpha_k = \left(k - \frac{1}{2}\right)\frac{\pi}{H}, \quad F_k = \frac{H}{2}. \tag{17}$$

When the effect of axial and circumferential inertial force $-\mu\frac{\partial^2 u}{\partial t^2}$ and $-\mu\frac{\partial^2 v}{\partial t^2}$ is small, we can use Donnell's approximation (Hayashi, 1966). We express the vibration equation in forms

$$a\Delta^2 u = -\nu\frac{\partial^3 w}{\partial x^3} + \frac{1}{a^2}\frac{\partial^3 w}{\partial x \partial \theta^2},$$

$$a\Delta^2 v = -\frac{2+\nu}{a}\frac{\partial^3 w}{\partial x^2 \partial \theta} - \frac{1}{a^3}\frac{\partial^3 w}{\partial \theta^3}, \tag{18}$$

$$D\Delta^2 w + \frac{E\delta}{1-\nu^2}\frac{1}{a^2}\left(\frac{\partial v}{\partial \theta} + w + \nu a\frac{\partial u}{\partial x}\right) + \mu\frac{\partial^2 w}{\partial t^2}$$

$$= -P,$$

where P is obtained by the Bernoulli equation

$$P = -\rho\left[\frac{\partial \Phi}{\partial t}\right]_{r=a},$$

ρ being the mass density of fluid. Substituting the displacement functions expressed by Eq. (3) in the above first two equation, we obtain

$$u_0 = \frac{\lambda(\nu\lambda^2 - m^2)}{(\lambda^2 + m^2)^2} w_0, \quad v_0 = \frac{m[(2+\nu)\lambda^2 + m^2]}{(\lambda^2 + m^2)^2} w_0,$$

where $\lambda = \frac{n\pi a}{L}$. When we consider a case that the fluid depth H coincides with the shell length L, we have from the third equation of (18) that

$$\left\{D\frac{1}{a^4}(\lambda^2 + m^2)^2 + D\frac{12(1-\nu^2)}{\delta^2}\frac{\lambda^4}{a^2(\lambda^2 + m^2)^2} - \mu\omega^2\right\}$$

$$\sin \frac{n\pi}{L} x = -\rho\omega^2 \sum_{k=1}^{\infty} A_k K_m(\alpha_k a) \cos \alpha_k x. \tag{19}$$

Expressing

$$\omega_{01}^2 = \frac{D}{a^4 \mu}\left[(\lambda^2 + m^2)^2 + \frac{\eta\lambda^4}{(\lambda^2 + m^2)^2}\right], \tag{20}$$

which is the natural frequency of shell in vacuum (Hayashi, 1966) and $\eta = \frac{12(1-\nu^2)a^2}{\delta^2}$, and expanding

$$\sin \frac{n\pi}{L} x = \sum_{k=1}^{\infty} \frac{2n}{\pi\left(n^2 - \left(k - \frac{1}{2}\right)^2\right)} \cos \alpha_k x, \tag{21}$$

we have that

$$\sum_{k=1}^{\infty}\left[(\omega_{01}^2 - \omega^2)\frac{2n}{\pi\left(n^2 - \left(k - \frac{1}{2}\right)^2\right)} + \omega^2\frac{\rho a}{\mu}\frac{A_k}{a}\right.$$

$$\left. K_m(\alpha_k a)\right] \cos \alpha_k x = 0.$$

The relation must hold along the shell axial direction $0 \leq x \leq L$. We consider a mean value of the above equation by taking the axial deformation mode $\sin \frac{n\pi}{L} x$ as a weight function. We have a natural frequency of the shell in fluid

$$\omega_1^2 = \frac{\omega_{01}^2}{1 + \varepsilon_1}, \tag{22}$$

where ε_1 express the effect of fluid around the shell and is given by

$$\varepsilon_1 = \frac{\rho}{\mu} 4 \frac{a^3}{L^2} \sum_{k=1}^{\infty} \frac{K_m(\alpha_k a)}{-\alpha_k a K_m'(\alpha_k a)} \frac{\lambda^2}{(\alpha_k^2 a^2 - \lambda^2)^2}. \tag{23}$$

3. A SOLUTION OBTAINED FROM THE LAGRANGE EQUATION

In the above calculation we considered a case that the fluid depth H is the same as the shell length L. In order to treat cases in which the shell immerse in the fluid partially, we use the Lagrange equation of motion. The kinetic energy and potential energy of the coupled system (shell and fluid) are expressed by T and U, respectively. The kinetic energy of the shell is given by

$$T_{sh}=\frac{1}{2}\mu\int_0^{2\pi}\int_0^L\left\{\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2\right\}$$

$$a\,dx\,d\theta.$$

Substituting the displacement functions of the shell, we obtain after integration

$$T_{sh}=\frac{\mu\pi aL}{4}(\dot{u}_1{}^2+\dot{v}_1{}^2+\dot{w}_1{}^2). \quad (24)$$

In the case of vortex-free motion of a fluid in a simply connected rigion the kinetic energy of the entire fluid depends on its motion at the boundaries. From the fact and invoking the boundary conditions of Φ mentioned above, we get

$$T_w=\frac{\rho}{2}\int_0^{2\pi}\int_0^H\left(\Phi\frac{\partial\Phi}{\partial r}\right)_{r=a}a\,dx\,d\theta$$

$$+\frac{\rho}{2}\int_0^{2\pi}\int_a^{\infty}\left(\Phi\frac{\partial\Phi}{\partial x}\right)_{x=H}a\,dr\,d\theta,$$

in the last integration the fluid free surface was approximately taken at $x=H$. If the wave amplitude is very small or the gravity is ignored, the last integration may be ignored. Therefore, the total kinetic energy of the system is given by

$$T=T_{sh}+T_w$$

$$=\frac{\pi}{4}a\left\{\mu L(\dot{u}_1{}^2+\dot{v}_1{}^2+\dot{w}_1{}^2)\right.$$

$$\left.+\dot{w}_1{}^2\,\rho H\sum_{k=0}^{\infty}A_k^2\alpha_k K_m'(\alpha_k a)K_m(\alpha_k a)\right\}. \quad (25)$$

According to the Love-Timoshenko theory, the elastic strain energy of a thin cylindrical shell is given as

$$U_{sh}=\int_0^{2\pi}\int_0^L\frac{E\delta}{2(1-\nu^2)}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{a^2}\left(\frac{\partial v}{\partial\theta}+w\right)^2\right.$$

$$+\frac{2\nu}{a}\frac{\partial u}{\partial x}\left(\frac{\partial v}{\partial\theta}+w\right)+\frac{1-\nu}{2}\left(\frac{\partial v}{\partial x}+\frac{1}{a}\frac{\partial u}{\partial\theta}\right)$$

$$+\frac{\delta^2}{12}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\frac{1}{a^4}\left(\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial v}{\partial\theta}\right)^2+\frac{2\nu}{a^2}\frac{\partial^2 w}{\partial x^2}\right.$$

$$\left.\left.\left(\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial v}{\partial\theta}\right)+\frac{2(1-\nu)}{a^2}\left(\frac{\partial^2 w}{\partial x\partial\theta}-\frac{\partial v}{\partial x}\right)^2\right]\right\}$$

$$a\,dx\,d\theta.$$

Substituting the assumed displacement functions into the above expression, we have

$$U_{sh}=\frac{E\delta}{2(1-\nu^2)}\left\{\left(\frac{m}{a}v_1+\frac{1}{a}w_1-\lambda u_1\right)^2+2(1-\nu)\right.$$

$$\left.\left[\lambda u_1\left(\frac{m}{a}v_1+\frac{1}{a}w_1\right)+\frac{1}{4}\left(\frac{m^2}{a^2}u_1^2-2\frac{m\lambda}{a}u_1v_1+\lambda^2v_1{}^2\right)\right]\right\}$$

$$\frac{\pi aL}{2}+\frac{E\delta^3}{24(1-\nu^2)}\left\{\left(\lambda^2w_1+\frac{m^2}{a^2}w_1+\frac{m}{a^2}v_1\right)^2-\right.$$

$$\left.2(1-\nu)\left[\lambda^2w_1\left(\frac{m^2}{a^2}w_1+\frac{m}{a^2}v_1\right)-\left(\frac{\lambda m}{a}w_1+\frac{\lambda}{a}v_1\right)^2\right]\right\}$$

$$\frac{\pi aL}{2}. \quad (26)$$

The potential energy of fluid is expressed as

$$U_w=\int\int_{Sf}\frac{g\rho}{2}[\zeta]_{x=H}^2\,ds, \quad (27)$$

where ζ is the displacement of fluid free surface, i.e., the difference between moving free surface and mean(static) one. For small wave amplitude or ignoring gravity, it can be neglected. The shell strain energy is the total potential energy of the system. Substituting the U and T in the Lagrange equation

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial\dot{Q}}+\frac{\partial U}{\partial Q}=0, \quad (28)$$

we obtain a set of algebra equations in terms of u_0, v_0 and w_0,

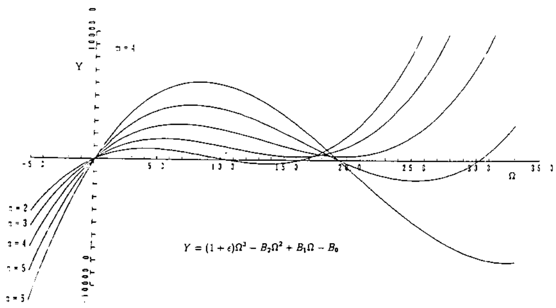


Fig. 2. Curves of eigenequation.

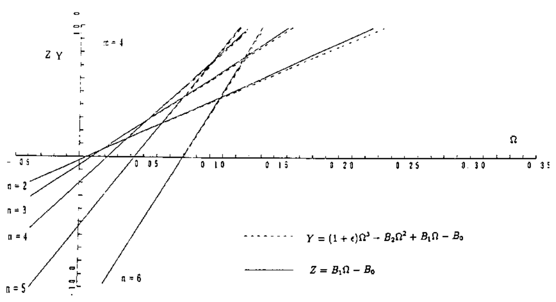


Fig. 3. Approximate solution.

$$\left(\lambda^2+\frac{d}{4}m^2-\frac{a^2\mu}{C}w^2\right)u_0-\frac{1+\nu}{2}\lambda m v_0-\nu\lambda w_0=0,$$

$$-\frac{1+\nu}{2}\lambda m u_0+\left[m^2+\frac{d}{4}\lambda^2+\frac{\delta^2}{12a^2}(m^2+d\lambda^2)-\frac{a^2\mu}{C}\omega^2\right]v_0+\left[m+\frac{\delta^2}{12a^2}\left(\lambda^2 m+m^3+\frac{d}{2}\lambda^2 m\right)\right]w_0=0,$$

$$-\nu\lambda u_0+\left[m+\frac{\delta^2}{12a^2}\left(\lambda^2 m+m^3+\frac{d}{2}\lambda^2 m\right)\right]v_0+\left[1+\frac{\delta^2}{12a^2}(\lambda^2+m^2)^2-\frac{a^2\mu}{C}\omega^2+\omega^2\frac{a^2\rho H}{CL}\sum_{k=1}^{\infty}A_k^2\alpha_k K_m'(\alpha_k a)K_m(\alpha_k a)\right]w_0=0,$$

where $d=2(1-\nu)$. The above set of algebra equation in terms of u_0, v_0 and w_0 must have a nontrivial solution. We put its coefficient determinant zero. This yields a third-degree equation in terms of Ω, which is defined by

$$\Omega=\frac{\mu a^2}{C}\omega^2. \tag{29}$$

By putting

$$\varepsilon=\frac{\rho aH}{\mu L}\sum_{k=1}^{\infty}\left(\frac{A_k}{a}\right)^2(-\alpha_k a)K_m'(\alpha_k a)K_m(\alpha_k a), \tag{30}$$

we have

$$(1+\varepsilon)\Omega^3-B_2\Omega^2+B_1\Omega-B_0=0, \tag{31}$$

where

$$B_2=(1+\varepsilon)\frac{3-\nu}{2}(\lambda^2+m^2)+\kappa^2\left[(\lambda^2+m^2)^2+(1+\varepsilon)(m^2+d\lambda^2)\right],$$

$$B_1=\frac{d}{4}\Big[(1+\varepsilon)(\lambda^2+m^2)^2+(3+2\nu)\lambda^2+m^2\Big]+\kappa^2\Big[\left(1+\frac{d}{4}\right)(\lambda^2+m^2)^4+(1+\varepsilon)\left(\lambda^2+\frac{d}{4}m^2\right)(m^2+d\lambda^2)-2m^2\left(\lambda^2+m^2+\frac{d}{2}\lambda m\right)+m^2+d\lambda^2\Big]=\frac{d}{4}(b_1+\kappa^2 b_1'),$$

$$B_0=\frac{d}{4}(1-\nu^2)\lambda^4+\frac{d}{4}\kappa^2\{(\lambda^2+m^2)^4-2m^2[(2+\nu)\lambda^2+m^2][\lambda^2+m^2+(1-\nu)\lambda m]+[2(1+\nu)\lambda^2+m^2][2(1-\nu)\lambda^2+m^2\}=\frac{d}{4}(b_0+\kappa^2 b_0'),$$

putting $\kappa^2=\frac{\delta^2}{12a^2}$. Considering a thin shell, $\frac{\delta}{a}\ll 1$, we neglected the terms associated with κ^4.

We show the third degree expression Eq. (31) in Fig. 2, for $m=4$, $n=2$, 3, 4, 5, 6. We see three real and positive roots at $m=4$ and $n=2$. As n increases, the bigger two roots become complex conjugate and the smallest one remains real and positive; but for larger n, there still are three real and positive roots. We also show in Fig. 3 the above third degree expression with a first degree expression $B_1\Omega-B_0$. We see that there exists only a small difference between the two expressions as long as Ω takes values between 0 and 1. We have an approximate solution

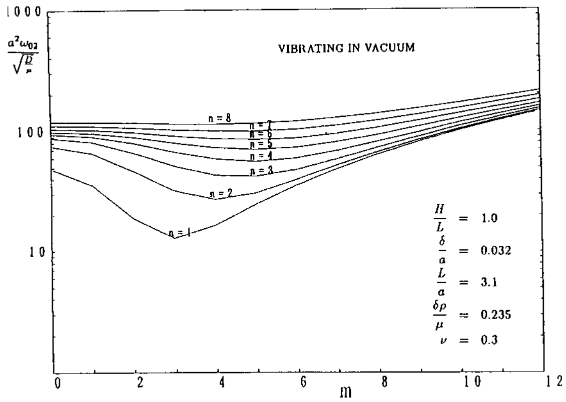


Fig. 4. Natural frequencies of shell from the simplified equation.

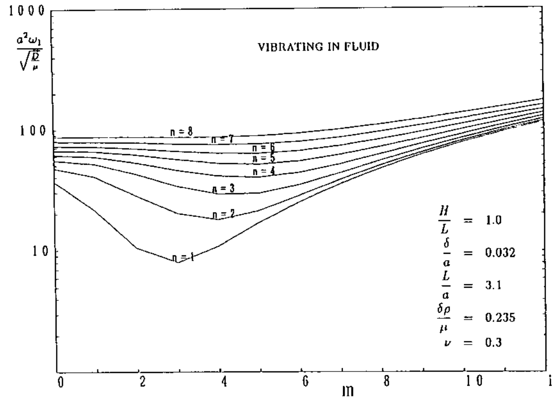


Fig. 5. Natural frequencies of shell vibrating in fluid from the simplified equation.

$$\Omega = \frac{B_0}{B_1}. \tag{32}$$

In order to obtain a solution in simple form, we ignore terms of κ^4 with the higher orders and write

$$\frac{B_0}{B_1} = \frac{(1-\nu^2)\lambda^4 + \kappa^2 R}{(1+\varepsilon)(\lambda^2+m^2)^2 + (3+2\nu)\lambda^2 + m^2},$$

where

$$R = b_0' - b_0 \frac{b_1'}{b_1}.$$

Retaining a term of the highest order of λ and m, we approximate

$$R \doteq b_0' \tag{33}$$

$$= (\lambda^2+m^2)^4 - 2m^2[(2+\nu)\lambda^2+m^2]\,[\lambda^2+m^2+(1-\nu)\lambda m] + [2(1+\nu)\lambda^2+m^2]\,[2(1-\nu)\lambda^2+m^2].$$

We have then

$$\omega_2^2 = \frac{\omega_{02}^2}{1+\varepsilon_2}, \tag{34}$$

in which

$$\omega_{02}^2 = \frac{C}{a^2\mu}\,\frac{(1-\nu^2)\lambda^4+\kappa^2 b_0'}{(\lambda^2+m^2)^2+(3+2\nu)\lambda^2+m^2} \tag{35}$$

$$\doteq \frac{D}{a^4\mu}\,\frac{\eta\lambda^4+b_0'}{(\lambda^2+m^2)^2+(3+2\nu)\lambda^2+m^2},$$

representing the natural frequency vibrating of shell in vacuum, and

$$\varepsilon_2 = \varepsilon\,\frac{(\lambda^2+m^2)^2}{(\lambda^2+m^2)^2+(3+2\nu)\lambda^2+m^2} \tag{36}$$

expressing the fluid effect on the natural frequency of shell.

4. RESULTS AND DISCUSSION

We show results obtained by Donnell's approximation ω_{01} and ω_0 in Fig. 4 and 5; and by Lagrange method ω_{02} and ω_2 in Fig. 6 and 7, respectively. We took modes: $n=1, 2\cdots 8$; $m=0, 1\cdots 12$.

When the shell deforms in the higher mode, i.e., m and n large, values obtained by both method are almost the same; however, in the lower mode the difference is remarkable. This reflects that Donnell's approximation is only suitable for the higher mode deformation (Noff, 1955; Flügge, 1960).

As expected, the natural frequencies in fluid are smaller than those in vacuum. The fluid (added mass) effect are remarkable in the lower mode deformation; and less in the higher mode. Water far from the shell is induced into motion when the shell vibrates in the lower mode; however, only the near field fluid is involved in the higher mode.

In order to study effects of fluid depth, we selected three cases: $\frac{H}{L}=1.0$, 0.5, and 0.3. Results are shown in Fig. 8. We also made a Table 1. It shows the maximum relative difference of the natural frequencies in fluid and vacuum, i.e.

$$\text{diff} = \text{Max}\left\{\frac{\omega_{20}-\omega_2}{\omega_{20}}\right\} = \text{Max}\left\{1-\frac{1}{\sqrt{1+\varepsilon_2}}\right\},$$

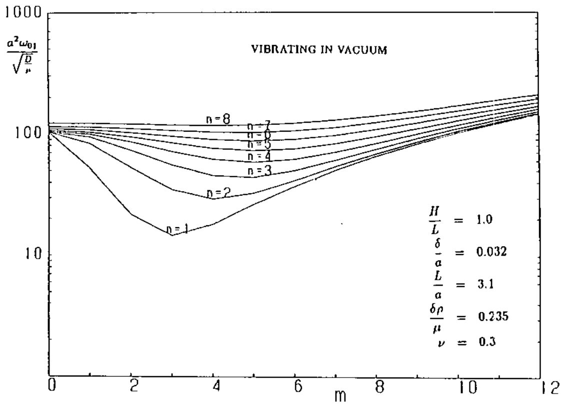


Fig. 6. Natural frequencies of shell vibrating in vacuum.

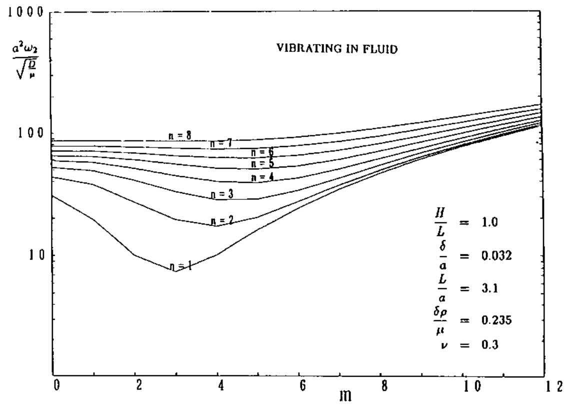


Fig. 7. Natural frequencies of shell vibrating in fluid.

Table 1. Maximum relative difference of ω_2 and ω_{20}

H/L	1.0			0.5			0.3		
n	m	diff	ε_2	m	diff	ε_2	m	diff	ε_2
1	2	0.463	2.484	2	0.258	0.818	2	0.077	0.173
2	1	0.425	2.026	1	0.315	1.133	2	0.174	0.465
3	0	0.407	1.184	0	0.292	0.997	0	0.219	0.638
4	0	0.378	1.583	0	0.271	0.881	0	0.207	0.589
5	0	0.348	1.355	0	0.228	0.678	0	0.179	0.482
6	0	0.321	1.172	0	0.220	0.644	0	0.162	0.425
7	0	0.298	1.027	0	0.195	0.542	0	0.142	0.358
8	0	0.278	0.911	0	0.182	0.496	0	0.120	0.292.

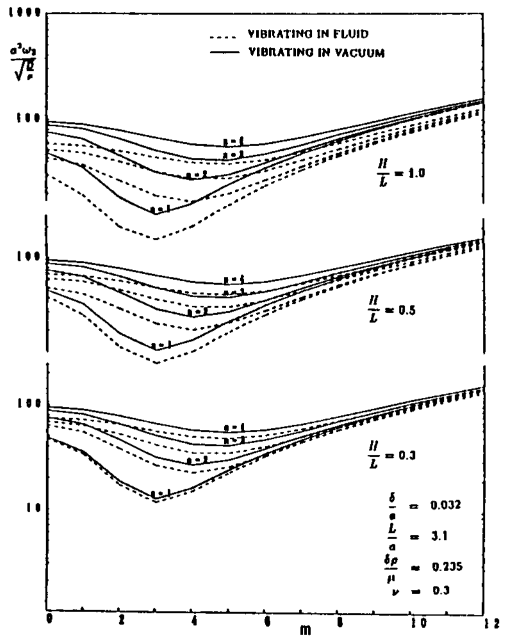
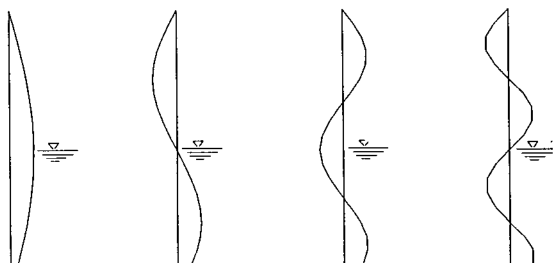


Fig. 8. Effects of fluid depth on the shell natural frequencies of shell vibrating in fluid.



$n=1$ $n=2$ $n=3$ $n=4$

Fig. 9. Axial deformation modes.

with the circumferential mode m at which the above maxima occurs, when we give the axial deformation mode n. As it is conjectured easily, the difference is small as the fluid depth is small. As it is seen from the table, when the shell immerses fully $\frac{H}{L}=1$, the biggest value of the maximum relative difference is obtained at the deformation mode $n=1$ and $m=2$. However, when the shell immerses partially, the bigger one occurs at the different mode. Let us consider a case, for example, the half length of shell is immersed in fluid, $\frac{H}{L}=\frac{1}{2}$. The immersed part has more effect in the second mode ($n=2$) compared to that of the first one ($n=1$); the more interaction between shell and fluid occurs in the second deformation mode, just as illustrated in Fig. 9.

5. CONCLUSION

Approximate analytical expressions of the natural frequency of thin cylindrical shell in fluid have been presented by two methods. Both of them are in forms of the natural frequency in vacuum combined with a factor of the fluid effect.

When the cylindrical shell is vibrating in a fluid, the surrounding fluid has a great effect on the shell natural frequency, particularly in the low-frequency vibration. Generally, the lower the deformation mode of shell is, the more effect the surrounding fluid produces on the natural frequency. But in the case that the shell is partially immersed in fluid, the conclusion is slightly different, at least for these several lower deformation modes.

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