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Reliability Analysis of Floating Offshore Structure
—Fundamental Study of System Reliability Analysis—

by

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부유식 해양구조물의 신뢰성해석
—시스템 신뢰성 해석에 관한 기초연구—

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Abstract

The impact of the system reliability analysis to structural design is described in this paper and various methods for system reliability analysis developed up to the present are reviewed and discussed from the view point of their efficiency.

The paper also includes the detailed formulation procedure of the, so called, extended incremental load method has applied to relatively simple structure to show its usefulness.

요 약

본 논문에서는 구조물의 시스템 신뢰성해석 (Structural System Reliability Analysis)의 설계에 대한 그 필요성과 중요성을 재차 강조하여 기술하였고, 이를 위해 현재까지 개발된 방법들을 효율성 측면에서 비교, 검토하였다. 또한 부유식 해양구조물과 같은 연속계의 구조물 (Continuous Structures)의 설계에 사용되는 강도공식을 직접 이용할 수 있는 소위, 수정된 하중 증분법 (Modified Incremental Load Method)에 대해 그 정식화 과정을 자세히 설명하였으며, 비교적 간단한 구조물에 적용하여 그 유용성을 보여주었다.

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1. Introduction

The development of reliability-based limit state design has been motivated by a desire to quantify preformance of structures and to treat uncertainties in loads, resistance and analysis in a more-rational way[1]. In the context of reliability based structural design, the probability of failure or, alternatively, reliability index (safety index) is the quantitative measure of risk, or safety or serviceability and the basis for achieving uniform performance in probability-based limit state design. This probability has an absolute meaning[2] "the likelihood of occurrence of some pre-defined limit state" : it may be a serviceability limit state, e.g. excessive deflection or rotation, initial yielding, or an ultimate limit state, e.g., partial or total collapse, instability. The reliability is defined as the probability of non-failure and adequately performing the intended function of a structure, when operating under stated environmental conditions during the service life.

In contrast to working stress design, which is a deterministic approach, the reliability-based limit state design is a behaviour-oriented design method which requires specification writers and designers to consider explicitly the structural requirements for function and safety at service and extreme load levels. The probabilistic approach is suggested by the observation that many of the design variables exhibit statistical irregularity[3]. The conceptional framework for reliability-based limit state design has been well established.

The application of the reliability analysis concept to a structure was initiated in the field of aircraft, civil engineering structures and so on, and it is widely applied in the marine, civil, electronic, electrical, aeronautical and nuclear fields. Structural reliability theory has grown rapidly during the last two decades and has evoloved from academic research to practical applications, and becomes a design decision tool based on scientific methods rather than being a scientific theory[4]. The object of its application to design is to give a uniform and consistent reliability within a structural system. Tremendous advances have been made in the area of structural reliability, theoretically and practically, and more recently in the incorporation of probabilistic concepts into codifiable design of marine and civil structures. Since the mid 1970's, the concept of the reliability analysis has been extensively applied to the design of marine structures[4].

It has been recognised for many years that a more complete estimate of the reliability of a structure must include a structural system reliability analysis. Furthermore, it is needed to develop system "partial safety factors" to related component safety to system safety[5]. Possibilities of its application would then exist for[6] :

- 'tuning' new design concepts to calculated reliability levels in existing structures
- making more rational decisions in repair or upgrading situations
- developing improved guidelines for inspection and maintenance purposes.

In the case of statically determinant structures, it is sufficient and reasonable to estimate the reliability of the individual components of a structural system because the failure of any single component will result in the failure of the total structural system. However, in the case of statically indeterminate structures, this is not true because the remaining components will be able to sustain further external load because of load re-distribution after any component fails. In other words, from the system's point of view, the failure of the statically indeterminate structures system always requires that more than one component fails due to much redundancy[7, 8]. Especially for a continuous structure, such as a floating offshore structure or ship's structure, there is considerable statical indeterminancy and hence mathematical redundancy[9-11]. Hence, the concept of system reliability based design is desirable to give more uniform reliability and consistent safety within the overall structure, to achieve the optimal distribution of component strengths within a structure, and to take into account the structural redundancy in the design stage.

During the last decade the necessity for system reliability analysis has been emphasised, and many studies on this subject have been made for marine and civil structures. Some are concerned with sensitivity studies[12-14], some with application to optimal structural design[7, 15-18] to minimise the structural weight and/or the total cost within reliability constraints, some to find the optimum strategy of inspection and maintenance[19,20]. Most of them, however, are concerned with discrete structures such as a truss work or a jacket platform for which the possible failure modes can be idenntified relatively easily. In the case of a continuous structure, such as a TLP, it is not so simple and easy to identify the possible failure modes and to define the redistribution of load effects due to the failure of any component.

Generally system failure is a series system of sub-parallel systems. A sub-parallel system consists of sub-series systems of component failure modes. The important tasks in the structural system reliability generally consists of three parts:

[1] Modelling the structure and defining the random variables

[2] Identification, description and enumeration of the failure modes of the structure

[3] Determination of failure probability or reliability indices of individual modes, and then evaluating the overall system reliability

Among these, the last two are probably more important in the system reliability analysis. Because there is a great number of possible failure modes in a practical structure, it is not practical to include all possible failure modes in evaluating the system reliability of complex structures. The mods which are expected to mainly contribute to the system failure must be taken into account in estimating the system reliability. These are often referred to as the most likely[21] or the most important or the most important or the most significant failure modes[5,22,23], or the stochastically dominant failure modes[24,25]. Hereafter these will be referred to as the most important failure modes.

Identifying the most important failure modes usually takes a great portion of computational time in the system reliability analysis procedure. A major difficulty in the application of system reliability analysis to design is to find an efficient algorithm for identification of the most important failure modes in a complex structure [5,6]. Application of system reliability analysis is a relatively new area. Extensive research has been performed during the last decade and several methods have been developed. Among them approximate methods have been employed for this subject rather than analytical methods. This will be described in Section 3. It is necessary to make a number of simplifications and assumptions not only to the structure itself, but also to the strength and loading properties of the structural elements. These is supposed to be major limitations of the application of structural system reliability theory to practical design procedure. In spite of these limitations it is believed that reliability analysis of structural systems is a useful tool in decision making in offshore engineering[26].

2. Literature Review

2.1 Methods for Deriving Safety Margin

As mentioned before, system reliability has received much interest and its necessity has been well receognised since the mid-1960's. The development of system reliability methods has mainly been concentrated in two parts : How to formulate the limit state equation of the failure mode, and How to evaluate the probability of system failure. Once the limit state is defined the reliability index and/or probability of failure for each mode can be evaluated without much difficulty.

Practical development of the method was initiated by Moses[27] who proposed the incremental load method (ILM) for formulating the limit state equation (system safety margin equation) in which the mean value first-order second moment reliability method (MVFORM) was incorporated to evaluate the system reliability. He applied the method to the system analysis of truss and framework structures. The basic idea behind this method is that a structure is progressively "unzipped" as successive members or components reach their strength capacity until overall failure occurs[22,27]. Later Moses extended the incremental load method to identify the most important failure modes using the deterministic truncating criteria[5]. This method is attractive in that it can allow for the post-ultimate behaviour of a failed component and component strength formulae, can be used during formation of the limit state equation together with the concept of utilised strength at each load increment stage. Recently this method has been incorporated in the cost optimisation studies[16].

After Moses presented the incremental load method, several useful system reliability method have been proposed in the past decade. Moses and Rashedi[28] presented an approach for identifying the important failure modes using the linear programming technique. They presented the results for the structure under multiple loading and having a more realistic post-ultimate behaviour than previously. Gorman[29] presented an automatic generating procedure of failure mode equations based on the rigid-plasticity concept. Murotsu et. al[30] proposed a heuristic procedure of automatically identifying the stochastically dominant failure modes with probabilistic truncating criteria. They extended their proposed method to the system reliability analysis of two dimensional framework structures under combined axial force, bending moment and shear force based on plastic failure criteria[31]. Thoft-Christensen and Sørensen[32] presented the so called "β-unzipping" method for frame structures in which yielding failure was considered. Later Thoft-Christensen[32] extended the method to account for the various failure elements, such as failure due to yielding, buckling, fatigue, punching, etc, by which the system safety index at different failure levels was evaluated. Ditlevsen and Bjerager[34] proposed another approach which was based on the lower and upper bound theorem of plasticity and an optimisation procedure was used in the identifying procedure. This method gave a reasonable upper bound of system failure probability. Melchers and Tang[25,35] extended the incremental load method to truss structures with a more general member behaviour to derive the limit state expression and proposed an iterative approach, the so called "Truncated Enumeration Method (TEM)", to systematically determine the probabilistically most dominant failure modes through an exhaustive searching procedure. Chan and Melcheres[36] applied the method to jacket platforms. Emphasis was placed on semi-brittle components behaviour. More recently Lee and Faulkner[23] have presented "the extended incremental load method (EILM)". This extends the conventional incremental load method by Moses[27] to include structures under multiple loading conditions which has been a major limitation of the incremental load method. Moreover it can more realistically allow for the post-ultimate behaviour of a failed component which can now be characterised by the post-ultimate slope and the residual strength. Also, strength formulae can be used in the limit state equation based on the utilised strengths of components failed at each incremental stage. This method has been successfully used in the reliability analysis of TLP structural systems and for their sensitivity studies[14,37].

As a hybrid approach, Corotis et al[38,39] proposed the load space approach which was a combined form of the incremental load method to evaluate the system resistance and numerical integration techniques to estimate the system failure probability. Later they used this approach in cost optimisation problems of frame structure[40,41]. Edwards et al[6] presented a dual approach based on the Monte-Carlo Importance Sampling procedure and failure mode analysis for offshore jacket platforms using the First-Order Second Moment Method.

All of the above mentioned methods look at the problem in terms of failure events. The complementary approach, or so called the "stable configuration or survival-set approach", was suggested by Bennett[42] and Bennett and Ang [43]. This was proposed as an alternative for system reliability analysis and it was claimed that it could predict upper bounds of system failure probabilities. But computational work is much more expensive than the methods mentioned earlier.

2.2 Methods for Evaluating System Failure Probability

Since the exact evaluation of the system failure probability is difficult, several methods have been proposed to estimate the bounds of the probability. An early simple upper bound of a series system was given by Freudenthal et al [44]. Cornell[45] presented the simple bounds when correlation was assumed to be positive or negative, in which the upper bound was the same as that proposed by Freudenthal[44] when correlation was assumed to be positive. If there are only a few dominant modes these bounds will give reasnable estimates.

For cases that failure modes are correlated with each other, narrow bounds were proposed by Ditlevsen[40]. Vanmarcke[47] and Murotsu et. al[30,31] presented the corresponding upper bounds. More recently Guenard[21] presented the bounding technique with the dominant failure modes and the incomplete interim modes. Ang and Ma[48] used the probabilistic network evaluation technique (PNET) to estimate the upper bound using a grouping technique. Chou et al[49] presented possible alternatives for a grouping technique by using the taxonomic analysis and Tichy and Vorlicek technique.

The concept of an equivalent safety margin of a single failure mode, which applies to a parallel system composing of failed components, was developed by Gollwitzer and Rackwitz[50] using first-order system reliability analysis. This is useful in calculating the probability of system failure of a large system.

These procedures have been successfully used in the system reliability analysis of marine and civil structures. Frangopol[51], Grimmeit and Schueller[52] and Schueller[53] compared the various system reliability analysis methods with regard to their accuracy, capability and efficiency. An overall review of the system reliability methods in formulating a limit state equation and in evaluating the system failure probability can be found in the report by Karamchandani [54] and the paper by Nikolaidis and Kapania [55]. These two well reviewed the current state-of-the-art in system reliability methods and their application to practical structures. The shortcomings and advantages of various methods were discussed with regard to thier practical application and future potentaial.

3. Review of the Methods for Structural System Reliability Analysis

System reliability analysis methods can generally be divided into three categories, i.e., analytical methods, approximate methods and hybrid methods, which are combined forms of analytical methods or analytical and approximate methods. Theoretically, the analtical method may give the exact probability of system failure, but it can be applied to only quite simple idealised problems and therefore not to real structures. Hence approximate methods are usually used for system reliability analysis.

3.1 Analytical Methods

There are several analytical methods for formulating the failure events and evaluating the system reliability. These include Numerical integration, Monte-Carlo simulation, Stratified sampling, Latin hypercube, Importance sampling, Reduced space approach and Response surface based approach. But most of these are limited in application to only idealised simple structures and may be computationally inefficient. The last three methods, however, appear to be the most promising methods in use today[54]. Among them, the importance sampling approach, which is a modified Monte-Carlo simulation, has been used by several researcher. For example, Edwards et al[6] applied the procedure to the system reliability analysis of a jacket platform. The important failure modes are obtained by Monte-Carlo simulation based on an importance sampling approach. The failure probabilities of these important modes are comsidered as decisive for the value of the system failure probability which is then evaluated by Level 2 method. For component reliability analysis for fatigure, Harbitz[56] presented a general procedure based on the importance sampling approach. However, these methods are practically difficult when individually used for practical structural systems. But when combined with approximate methods, as will be discussed in Section 3.3, they can be more easily used in reasonably complex structural systems.

3.2 Approximate Methods

In this method there are generally two types of approach depending on the type of formulation. One is the approach to principly get the failure probability rather than the reliability and the other one is the complementary or vice versa approach. Approximate methods can be categorised into three : Failure Path Approach, Survival-Set Approach and Plasticity-Based Approach.

3.2.1 Failure Path Approach

The failure path approach generates the failure modes which are elements in the system failure event. There are generally two kinds of methods used to derive the failure equation or safety margin equation : Element Replacement Method (ERM) [24,32] and Incremental Load Method (ILM)[5,22,23,27,37].

o Element Replacement Method :

The failure equation can be defined in terms of the forces in components and the strengths of the components. In order to obtain the failure equation for any particular failure mode, the basic idea behind this method is that failed components are removed from the structure and each is replaced by the equivalent load, i.e. the post-failure strength of failed components. The equivalent load replacing the failed component i is $\eta_i R_i$, where η_i is the post-failure strength and the resistance of the component, and R_i is the resistance. For ductile behaviour, $\eta_i = 1.0$, for brittle behaviour, $\eta_i = 0.0$ and for semi-brittle behaviour, $0.0 < \eta_i < 1.0$ [see Fig.1 (a)].

o Incremental Load Method :

This method is based on the mean values of random variables in strength and load. The basic idea behind this method is that the structure collapses progressively in a predefined failure sequence as the load increases, from which a set of these load increments is obtained. These load increments are defined in terms of the strengths of failed components. The total load at a particular component failure stage is the sum of the load increments to that stage and it represents the system resistance which consequently can be expressed in terms of the strengths of failed components. This method is easy to understand and the existing structural analysis program can be used with slight modification. Since, after a component fails, the change of force in a survival component is the sum due to the next load increments and due to the unloading force in the most recently failed component. This procedure can also be applied to the structure with brittle and/or semi-brittle component behaviour.

3.2.2 Stable configuration Approach

This approach[42] is based on looking at the problem in terms of sequences of which survival implies system survival, i.e., the complement of system failure. The survival modes (or configurations) in which a structure may carry an applied load are examined to determine which survival modes are those which the structure is most likely to carry further applied load, i.e., which survival modes have the lowest probability of failure. The probability of system survival (reliability) is obtained from

$$(P_S)_{sys} = P[E(Z_1>0) \cap E(Z_2>0) \cap \ldots \cap E(Z_m>0)] \qquad (1)$$

and the probability of system failure is obtained from Eq.(2) as a complimentary event of the survival event.

$$(P_f)_{sys} = 1-(P_s)_{sys} \qquad (2)$$

The survival modes are found through a heuristic process, in which less important modes are neglected. Neglecting the potential survival modes (stable configurations) will be conservative and an upper bound will be obtained to the probability of system failure.

Bennett and Ang[43] applied this approach to evaluating the reliability of structures whose component behaviours are a non-linear based on the combination of the imposed deformation approach[57]. However, for a given system the number of survival modes is usually much larger than that of the failure modes, and so, in this sense, the magnitude of the problem in the stable configuration approach is larger and its procedure for finding the survival modes is more complicated than the failure path approach. Above all, the evaluation of the probabilities of survival modes (parallel system of element survival events) is difficult and there is no guarantee that the sequences identified are the most important ones.

3.2.3 Plasticity-Based Approach

This approach is based on the two plasticity theory with optimisation theory[34]: Lower-bound theory and Upper-bound theory of plasticity. Although this approach can be applied to a narrower class of structures than the failure path approach and the survival-set approach, i.e. only to the elasto-plastic structure, the advantage over other approximate methods is that it can allow for a more rigorous treatment of the probabilistic aspect of structural systems reliability. Results obtained for a simple structure showed that a lower bound was usually quite close to the true failure probability.

3.3 Hybrid Methods

Among the analytical methods, the reduced space approach and the response surface based approach have been combined with approximate methods to solve practical problems. In this paper this type of approach is termed a hybrid method due to its combined nature of analytical methods and approximate methods. References [38], [39] and [58] used the incremental load method to obtain the system resistance in which nonlinear structural analysis was employed and then the limit state is formed in the load space (or load effect space). The failure probability ofthis deterministic structure is then evaluated by integrating the joint distribution functions of loads over the failure region in the load space. This approach is usually called a load space approach or load effect approach.

In this method, since non-linear structural analysis is included in deriving the system resistance by varying the relative ratio of loadings, it seems to provide one possible way of more realistically taking into account the non-linearity for a reasonably complex structure. However, since the structure analysis takes the main portion of computational time in the system reliability analysis, even a simple structure may require considerable computational time. For this reason this method may be limited in its application to complex and large structures.

3.4 Limitations of the Approximate Methods

The analytical and hybrid methods have the major shortcoming that up to the present it cannot be applied to practical problems. The approximate method is, therefore, usually used to solve practical problems, but it has several limitations. The major limitation is, that first of all, it is applicable only to highly idealised structures and loads. Non-linearity of component (or element) behaviour is usually assumed as a two-state model [Fig.1(a)]. It is not possible to

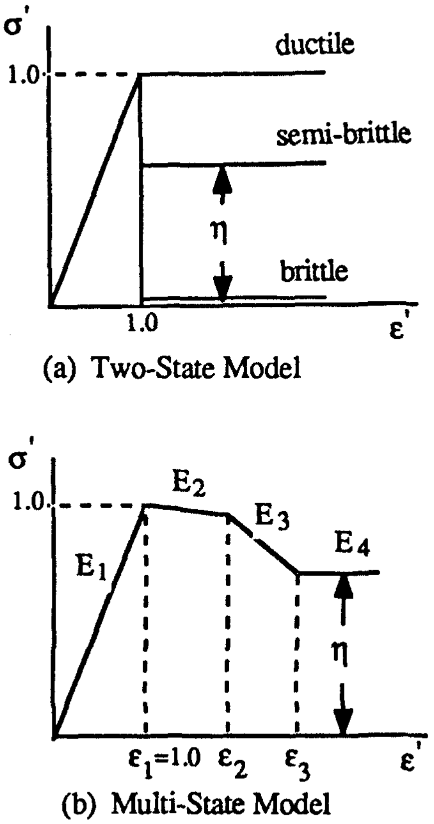


Fig. 1 Typical Piecewise Simplified Models for Nonlinear Component Behaviour

consider the non-linear interaction failure surfaces. Above all, different paths and strain reversal of failed components cannot be allowed for.

3.5 Discussion on the System Reliability Analysis Methods

In spite of their limitations mentioned in the last section approximate methods are usually used to solve the practical problems and the failure path approach is most popular. In this, since the paths which are less important are neglected, the result will be unconservative. Also, it generates the failure modes, not the survival modes as in the stable configuration approach. Considering that the number of survival modes is much larger than that of the failure modes, it is computationally more efficient. In addition this approach can be applied to the structure with ductile, brittle and semi-brittle component behaviour, and is therfore more general than the plasticity-based approach. Because of this

Table 1 Comparison between incremental load method and element replacement method[60]

	ILM	ERM
Behaviour of the failed components	Deformation of failed components can be allowed to follow the post-ultimate behaviour.	Deformation of the failed components are restricted. Failed components are removed and their strength are replaced by the equivalent force acting on the structure.
Merit	Post-ultimate behaviour of failed components can be treated more realistically than the element replacement method.	Applicable to multiple loading case
Limitation	Applicable only to a single loading case	Post-ultimate behaviour of a faild component is idealised as a two-state model : ductile, semi-brittle and brittle.

most work on system reliability analysis has been done by the failure path approach, say using the incremental load method (ILM)[5,14, 16,22,23,25,35,36,59] and the element replacement method (ERM)[7,13,21,24,31,32,39,42,69].

Table 1 shows the comparison betwee the two typical failure path approaches, ILM and ERM [60]. Firat of all, regarding the behaviour of faile components, in the ILM the behaviour of the failed componentcan follow the post－ultimate behaviour and can be allowed for. Since in this method the load factor up to the particular failure stage can be obtained, post-ultimate behaviour of failed components can be considered more realistically by using the multi-state non-linear model[Fig.1(b)]

One major limitation of this method is that it is applicable only to a single loading pattern. In the ERM the failed components are removed from the structure and their strength is replaced by the equivalent forces acting on the structure. Hence, thier deformations are restricted and the post-ultimate behaviour of the failed components may not be considered realistically. But the idealised model of the two-state model [Fig.1(a)], say, ductile, semi-brittle and brittle behaviour, can be used to represent the post-ultimate behaviour in a simple way. Its merit over the ILM is that it is applicable to multiple loading cases. Recently, however, the limitation of the ILM has been removed by Lee [59] and Lee and Faulkner[23] by adopting the concept of a contribution factor which reflects the contribution from the utilised component strength for each loading case in the safety margin equation. This will be detailed in the next section.

4. Formulation of Extended Incremental Load Method

In this section, another method for system reliability analysis, called the extended incremental load method, is introduced, which is an extension to the conventional incremental load method[5,22,27]. The basic idea of the present method is similar to the conventional incremental load method and primarily aims at extending the applicability the multiple loading case, and also to more realistically take into account the post-ultimate behaviour of the failed components using the concept of the load factor.

As described in Section 3.5 [see also Table 1] the applicability of the conventional incremental load method is restricted to a single load pattern. Moses and Rashedi[28] introduced its application to the multi-loading case for the ductile system, but the extension to the multi-loading case is based on incrementing one load and keeping the rest fixed to their final value. However, this is not consistent, i.e. all loads are incremented proportionally till faailure, therefore, the validity of the formulation is not clear.

The procedure which derives the safety margin equation for the multi-loading case is similar to that of the single loading case in the conventional incremental load method. In the present method the contribution factor defined below for each loading is introduced. Let L loading act on a structure in which j components $r_1, r_2,...,r_j$ have failed. The utilisation equation for each loading may be expressed as Eq.(3) similar to the previous single loading case [see Eq.(2.63)]. For the l th loading :

$$\begin{Bmatrix} R_1 \\ R_2 \\ \cdot \\ \\ R_j \end{Bmatrix} = \begin{bmatrix} a_{11}^{(l)} & & & & \\ a_{21}^{(l)} & a_{22}^{(l)} & & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ a_{j1}^{(l)} & a_{j2}^{(l)} & \cdot & \cdot & a_{jj}^{(l)} \end{bmatrix} \begin{Bmatrix} P_1^{(l)} \\ P_2^{(l)} \\ \cdot \\ \cdot \\ P_j^{(l)} \end{Bmatrix} \tag{3}$$

or simply

$$\{R_e\} = [a^{(l)}]\{P^{(l)}\} \tag{4}$$

where $a_{ij}^{(l)}$ is the utilisation ratio for the 1 th load case which generally represents the relationship between component strenght and load increments and may be a stress in some cases or even a more complex expression, such as the interaction formula for combined loading. In this equation the superscript (1) represents the term related to the 1 th loading case. Solving Eq.(4) for the load increment vector, $\{P^{(l)}\}$, summing up each column of $[a^{(l)}]^{-1}$, and normalising the coefficient by the final one gives the resistance coefficients for l th loading case. The resistance coefficient for l th loading case. The resistance coefficient $C_K^{(l)}$ corresponding to resistance P_k of component r_k for 1 th loading case is :

$$C_k^{(l)} = \sum_{i=1}^{i} (a_{ik}^{(l)^{-1}} / C_j^{'(l)}),\ k=1,2,...,j-1 \tag{5.a}$$

and

$$C_j^{(l)} = 1.0 \tag{5.b}$$

The index k(=1,...,j) means the sequence of component failure.

The contribution of resistances of failed components for each loading case can be accounted for by introducting the contribution factor, CF. The contribution factor, $CF^{(l)}$ for the 1 th loading case is here defined as the relative proportion of utilisation of the j th component r_j (the last failed component) for all loading case. Then, resultant resistance coefficients for all loadings are obtained by summing up the resistance coefficients for each loading multiplied by the corresponding contribution factor, i. e., the resultant resistance term is expressed as the sum of contributions of resistance for each loading case to the system resistance.

$$C_k = \sum_{l=1}^{L} C_k^{(l)} . CF^{(l)}\ ,k=1,2,...,j\text{-}1 \tag{6.a}$$

with the contribution factor, $CF^{(l)}$ defined as:

$$CF^{(l)} = \frac{a_{jj}^{(l)}}{\sum_{l'=1}^{L} |a_{jj}^{(l')}|} \tag{6.b}$$

Hence, the resistance term in the equation of safety margin, i.e. system resistance, can be expressed as :

$$R_{sys} = C_1 R_1 + C_2 R_2 + ... + C_j R_j \tag{7.a}$$

where C_j is unity. The loading term can be easily obtained as the sum of the product of the utilisation ratio, $a_{jj}^{(l)}$ and load, $P^{(l)}$.

$$Q = -(a_{jj}^{(1)} P^{(1)} + a_{jj}^{(2)} P^{(2)} + .. + a_{jj}^{(l)} P^{(l)}) \tag{7.b}$$

where Q simply denotes the loading term. With Eqs. (7.a) and (7.b), the equation of safety margin for the m th failure mode becomes :

$$Z_m = R_{sys} - \sum_{k=1}^{j} C_{mk} R_k - \sum_{l=1}^{L} B_{ml} P^{(l)} \tag{8}$$

where C_{mk} and B_{ml} are resistance and loading coefficients for the m th failure mode, respectively, and $B_{ml} = a_{jj}^{(l)}$.

When using the incremental load method, summing up all elements of the inverse of utilisation matrix results in the load factor up to any particular failure. stage. This concept can be extended to compute the load factor at any incremental stage and then to determine the strain of failed components in the mean sense under the multiple loading. In Eq.(3) an element of utilisation matrix is the utilised proportion of a component strength at a particular incremental stage due to unit load, $P^{(l)} = 1.0$. When the mean ualues of the applied loads are substituted the element of utilisation matrx represents the mean utilisation for each loading case. Hence, the total mean utilisation for each loading case. Hence, the total mean utilisation is simply the sum of the mean utilisation for each loading case. Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be mean load factors corresponding to incremental stages. Then, the utilisation equation for the mean load may be expressed as

$$\begin{Bmatrix} R_1 \\ R_2 \\ \cdot \\ \cdot \\ R_j \end{Bmatrix} = \begin{bmatrix} A_{11} & & & & \\ A_{21} & A_{22} & & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ A_{j1} & A_{j2} & \cdot & \cdot & A_{jj} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ {}_2\lambda \\ \cdot \\ \cdot \\ \lambda_j \end{Bmatrix} \quad (9)$$

or simply :

$$\{R_c\} = [A]\{A\} \quad (10)$$

where A_{ki} = is the total mean utilisation given as :

$$A_{ki} = \sum_{l=1}^{L} a_{ki}^{(l)} P^{(l)} \quad (11)$$

and $\underline{P}^{(l)}$ is the mean value of load $P^{(l)}$. By inverting Eq.(10), the vector of the mean load factors for load increments can be obtained.

$$\{\lambda\} = [A]^{-1}\{R_c\} \quad (12)$$

The deformation of already failed component r_i (i=1, j−1) up to the previous stage are determined in the mean value sense. At the j th failure stage (the present failure stage) when component r_j has failed, the mean load factor, λ_i is calculated from Eq.(12). Let $\Delta\epsilon_{ri}^{(l)}$ be the strain increment due to unit value of load $P^{(l)}$ of component r_i, then the mean strain increment of the failed component r_i, then the mean strain increment of the failed component r_i becomes :

$$\sum_{l=1}^{L} \Delta\varepsilon_{r_i}^{(l)} \cdot P^{(l)}, \ i=1,2,...,j-1 \quad (13)$$

Since the load factor of the present stage is λ_j, the mean strain increment of component r_i at the present stage becomes :

$$(\Delta\varepsilon_{ri})_j = \lambda_j \sum_{l=1}^{L} \Delta\varepsilon_{r_i}^{(l)} P^{(l)} \quad (14)$$

Hence, the total strain up to the present stage is

$$(\varepsilon r_i)_j = (\varepsilon r_i)_{j-1} + (\Delta\varepsilon r_i)_j \quad (15)$$

were $(\varepsilon_{r_i})_{j-1}$ is the strain state of component r_i up to the previous incremental stage. Strains of the components are normalised by the strain at the ultimate state, ε_u, i.e. :

$$(\varepsilon_{r_i})' = \frac{(\varepsilon_{r_i})_j}{\varepsilon_u} \quad (16)$$

Referring to the normalised strain state calculated from Eq.(16), the strain state of each failed component up to the present incremental stage can be caculated and at this point the corresponding tangential stiffness (non-dimensional) of components can be obtained from th stress-strain curve. For example, in Fig.1(b) the tangential stiffness of component r_i is

E_2 when $\varepsilon_1 < (\varepsilon_{r_i})_j' < \varepsilon_2$

E_3' when $\varepsilon_2 < (\varepsilon_{r_i})_j' < \varepsilon_3$

E_4' when $\varepsilon_3 < (\varepsilon_{r_i})_j'$

The idea described above may is one possible approach to more realistically consider the post-ultimate behaviour of failed components even for the structure under multiple loading. Since the above procedure of determining the strain sate of failed component is based on the mean load value and strain state of failed component is redicted at each incremental stage, i.e. at the failure stage of each component, when the tangential stiffness abruptly varies, e.g. from E_2' the E_3', of from E_3' to E_4'in Fig.1(b), its effect cannot be precisely taken into account because of the finite size of the strain increments. This is a small limitations of all piecewise linear procedure.

If collapse of a structure occurs when j components $r_1, r_2, \cdots, r_i$ have failed, the total load factor, λ_T is given as the sum of all load factors, i.e.:

$$\lambda_T = \sum_{i=1}^{j} \lambda_i \qquad (17)$$

where elements of λ_T are calculated from Eq.(12). Since when using the interaction equation under the multiple load effects, the elements of utilisation matrix in Eq.(3) and the elements of total mean utilisation matrix in Eq.(9) are nondimensional, λ_T represents the ratio of the load at collapse to the applied load.

5. Procedure of Identifying the most Important Failure Modes

When a structure is treated as a system, its failure should be seen as progressive collapse, or as a mechanism of successive failures of some of its components. Identification of the failure modes is selecting the failure modes and one of the most important parts of the failure path approach[5,7,54]. In a complex structure such as offshore platforms, the number of potential failure modes is usually very large. However, it is not practical to consider all possible failure modes to evaluate the probability of system failure. But, as it has been well recognised, only a few which have high probability of occurrence are important in evaluating the probability of system failure.

At present there are several procedures of identifying the most important failure modes [54]: Monte-Carlo simulation, Utilization ratio-based method, Marginal failure probability-based method, Truncated enumeration method and Branch and Bound method. Details of the procedures can be seen in References 54, 60 and so on.

In finding the most important failure modes one should not overlook the deterministically important modes although they are not always identical with the stochastically most important failure modes[61]. The present identifying procedure is primarily based on the stochastic criteria and aimed at reducing the computational time. The deterministic ones are also considered to identify the most important failure modes. The identifying procedure is composed of two procedures: Searching Procedure to select the most important failure modes and Discarding Procedure to discard the relatively less important modes in which similar deterministic criteria in Reference 5 is employed.

[1] Searching Procedure

○ **Searching Procedure-1**:

At the first stage the failure probabilities of components failure are evaluated using the associated safety margins derived from results of structural analysis at the intace state. They are arranged in decreasing order of probability. As the first candidate component to fail, the component which has the highest probability of failure is chosen. When the component has failed, the stiffness matrix of the associated element is replaced by the reduced one (see Reference 7) and structural analysis is performed. Then the probabilities of failure of the remaining unfailed component are evaluated and arranged in the same way as before. Among these, the mode with the highest probability of failure is chosen (called PATH-A) as the best candidate interim mode. The last component of that path is herein termed as the "focus component". Among possible modes which are at the first failure stage, a searching procedure is performed to check if there is any mode which contains the focus component and which path probability is higher than that of PATH-A. The idea behind this is based on the face that, as the number of failed components increases, the path failure probability usually decreases. If there is a mode of which failure probability is higher than that of PATH-A, the mode having the highest overall failure probability is selected (called PATH-B).

When these procedures are continured, the number of failed components contained in particular modes is different. During the searching procedure the focus component is selected in the modes which have the larger number of failed components, i.e. the modes to which structural failure is more progressed. At the current searching procedure PATH-A is generally the one which has the highest path failure probabilty among the modes having the largest number of failed components, which is denoted herein as N_{max}, i.e., N_{max} is the largest value of the numbers of failed components among all possible interim modes identified up to the current process. The value of N_{max} will be updated as the searching procedure is progressed.

PATH-B is the one which not only has the focus component as the last failed component but also has the highest path failure probability among modes of which the number of failed components are less than N_{max}. If there is one or several modes, the one having the highest probability of failure among them is selected as the best candidate mode and focus is shifted to the mode. When there is no mode like that, PATH-A is selected.

○ **Searching Procedure-2 :**

After passing through Searching Procedure-1, another procedure is introduced to check if there is any mode which is at the lower failure stage than PATH-A, i.e., If which the number of failed components is less than N_{max} has the higher path probability than that of the mode selected before (PATH-A or PATH-B). If there are one or several modes to satisfy these condition, the one having the highest probability of failure among them is selected and focus is shifted to the mode. Otherwise PATH-A (or PATH-B) is selected as the most important interim mode (the best candidate mode) up to the present searching procedure. After then, probability of failure of the next component to fail is calculated for remaining survival components. These procedure will be continued until collapse of structure occurs, which is defined as the occurence of singularity in the structural stiffness matrix, i.e.

$$\text{Det}\,[K] = 0 \qquad (18)$$

where [K] is the total structural stiffness matrix at the current failure stage. Practically, the occurence of the structural collapse may be judged from

$$\frac{\text{Det}\,[K]\text{ of Current Failure State}}{\text{Det}\,[K]\text{ of Intact State}} \le \varepsilon_{det} \qquad (19)$$

where ε_{det} is the prescribed small number.

○ **Searching Procedure-3 :**

In some cases the mode which is not selected as the best candidate interim mode can result in the structural collapse. Hence, one should check if such a mode exists among all the currently possible modes. But this causes a tremendous increase in computational time and a certain restriction may be necessary to save the computational time. Let N_i be the number of failed components of mode i and N_{det} be the specified value to restrict the number of modes for determinat check. Then, the determinatnt check is restricted to the modes such that :

$$\frac{\text{Det }[K_i]}{\text{Det }[K_o]} \leq \varepsilon_{det} \quad \forall N_{max} > N_i \text{ and } (N_{det} < N_i) \tag{20}$$

where Det $[K_i]$ and Det $[K_o]$ are determinatnt of structural stiffness matrix for the i th failure mode and that of the intact structure. When $N_{det} = 1$, then the determinant check is carried out for all interim modes. While $N_{det} = N_{max}$, the determinatn check is ignored.

From the author's experience, in the cases of simple structures as illustrated in the next section, the computational time for the above procedures is not a problem. But in the case of a large structure, the computational time is so great that it might not be practical to pass through all of the avove procedures, especially Procedure-3, In order to reduce computational time the following discarding procedures are introduced.

[2] Discarding Procedure

This procedure is to discard the relatively less important interim modes during the searching procedure from the deterministic and/or probabilistic sense.

○ **Discarding Procedure-1 :**

Because the present method is based on the conventional incremental load method, deterministic discarding procedure can be carried out during formation of the utilisation matrix in such a way that the component having a very small utilisation or a very small change in the utilisation may be regarded as the less important one. Therefore, the associated mode may be discarded[5], i.e., at the first failure stage, when the utilisation of any component is less than a certain given value. The subsequent modes may be discarded or the interim mode of which the ratio of successive utilisation is very small, say, less than ε_{utr}, may be discarded because the mode can be regarded as a deterministically less important one.

○ **Discarding Procedure-2 :**

When there are some interim modes progressed up to the specified failure stage (or level), searching procedure is restricted within the modes which satisfies the following conditions and others are discarded. Let N_i be the number of failed components in mode i and N_{Limit} be an option variable which is used to discard the less important interim modes as below. During the searching procedure, when maximum of N_i is less than N_{Limit} the mode i of which N_i is less than N_{Limit} is discarede based on the assumption that the mode may be less importan than the modes such that $N_{Limit} \leq N_i$. And so, the mode i such that $N_{Limit} \leq N_i$ is considered as a candidate mode in the searching procedure.

The above procedures, Searching Procedures and Discarding Procedures, are to be combined to generate the most important failure modes. Once a mode results in mechanism, the Discarding Procedure-2 is applied and all procedures are to be continued to generate the next most important failure modes. Every time the important failure mode is generated, the bounds of system failure probability (bounds of series system) are evaluated. According to the aboved identifying procedure failure modes are likely to be obtained in decreasing order of failure probability or, alternatively, in increasing order of corresponding reliability indes, i.e.

$$P_{f1} > P_{f2} > \cdots > P_{fm} > \cdots \tag{21.a}$$

or $$\beta_1 < \beta_2 < \beta_m < \cdots \tag{21.b}$$

were P_{f_m} and β_m are the failure probability and the corresponding reliability index of the m th mode. As the number of failure modes identified increases the bounds of probability of the system failure $(P_f)_{sys}$ are expected to increase monotonically. In other words, the corresponding bounds of the system reliability index β_{sys} is expected to monotonically decrease in such a way that :

$$(\beta_{sys,\ lower})_1 > (\beta_{sys,\ lower})_2 > ..$$
$$> (\beta_{sys,\ lower})_m > ... \text{ and } (\beta_{sys,\ upper})_1$$
$$> (\beta_{sys,\ upper})_2 > .. > (\beta_{sys,\ upper})_m > ... \quad (22)$$

Using this concept the searching procedure is terminated if the following criteria are satisfied, or if there is no more modes to be considered :

$$\frac{(\beta_{sys,lower})_{m-1} - (\beta_{sys,lower})_m}{(\beta_{sys,lower})_{m-1}} \leq \varepsilon_{sys}$$

and

$$\frac{(\beta_{sys,upper})_{m-1} - (\beta_{sys,upper})_m}{(\beta_{sys,lower})_{m-1}} \leq \varepsilon_{sys} \quad (23)$$

where

$(\beta_{sys,lower})_m$, $(\beta_{sys,upper})_m$ = lower and upper bound of system reliability index corresponding to bounds of system failure probability up to the present searching stage.

$(\beta_{sys,lower})_{m-1}$, $(\beta_{sys,\ upper})_{m-1}$ = lower and upper bound of system reliability index corresponding to bounds of system failure probability up to the previous searching stage.

ε_{sys} = prescribed small number for convergence checking of the system reliability

The proposed identifying procedure can generate the most important failure from both the probabilistic and deterministic points of view. Theoretically, when $\varepsilon_{det} = 0.0$, $N_{det} = 1$, $\varepsilon_{utr} = 0.0$, $N_{Limit} = 1$, $\varepsilon_{sys} = 0.0$ and $M_{min} >> 1$, all possible failure modes can be identified. But, in fact, since specifying certain values of parameters such as N_{det}, N_{Limit}, etc is inevitable for large and complex structural systems, the identified modes may not be the "truly" most important ones because the identified failure modes certainly depend on the parameters. Consequently, the system reliability must be dependent of the selected values of parameters. In spite of this the identified modes may be the "reasonably" most important ones. The present identifying procedure has a similar nature to the truncated enumeration method (TEM)[25] on one hand, and to the Branch and Bounding technique[21] on the other hand. The present procedure of identifying the important failure modes is detailed in Reference 60 for simple structures.

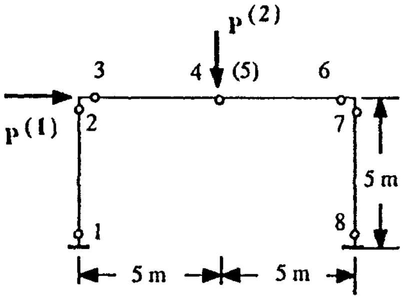
6. Example Application

The plane frame model in Fig.2[7] has 8 possible hinges as components. It is especially sensitive and has been frequently selected for evaluation of the system reliability and sensitivity analysis[12,24,39]. Component failure is assumed to occur when bending moment at a particular hinge reaches the plastic bending moment. For this model the safety margin for component k is given in a simple form as Eq.(24) :

$$Z_K = X_{M_k} - Q_k / R_k \quad (24)$$

where R_k is the plastic bending moment of component k as the stregth and Q_k is the bending moment of component k due to loading as the load effect. X_{M_k} is its modelling error which is assumed to be deterministic having a mean value of 1.0 for all components for the present frame model in Fig.2, and all resistance and loading terms in the safety margin equation (24) are assumed to be normal.

For the parameters controlling the procedure of identifying the most important failure modes, $\varepsilon_{det} = 10^{-1}$, $N_{det} = N_{max}$, $\varepsilon_{utr} = 10^{-3}$, $\varepsilon_{sys} = 10^{-3}$



comp.	A_K (x10^{-3})	I_K (x10^{-5})	R_K
1,2	4.0	3.58	0.075
3,4	4.0	4.77	0.101
5,6	4.0	4.77	0.101
7,8	4.0	3.58	0.075

A_K = cross sectional area (m^2)
I_K = moment of inertia (m^4)
mean yield stress = 276 MPa
$\underline{R}_K$ = Mean strength
(=plastic bending moment, MN)
$\underline{P}^{(1)}$ = 0.02 MN, $\underline{P}^{(2)}$ = 0.04 MN
COV of R_K = 0.05
COV of $P^{(1)}$ and $P^{(2)}$ = 0.3

Fig.2 Frame Model [7]

and M_{min} = 8 are given. The convergence condition is satisfied when 8 important modes have been found. Fig.3. shows the failure states of the identified modes. Actually the number of all possible modes is very much larger. But, as seen in the figure, the reliability index of the 8th mode is 3.29, and so the remaining neglected failure modes are expected to be greater than this value and are unlikely to have much influence on the evaluation of the system reliability.

The results of the frame model are summarised in Table 2. The first three modes seem to be dominatn in evaluating the bounds of β_{sys}. As described in Section 4, the total load factor, λ_T, defined as Eq.(17), may represent the reserve strength. The λ_T's, of the modes in Fig. 3 are also listed in Table 2. From the table it can be seen that within the identified modes the probabilistically most important mode is that for path 4-7-8-2, while the deterministically most important modes are the modes for path 4-8-7-1 and 8-4-7-1 although their path reliability indices (β_{path}) are about 18% greater than that of the mode for path 4-7-8-2 and λ_T's of the modes for path 4-8-7-1 and 8-4-7-1 are about 5% less than that of the mode for path 4-7-8-2. Whereas the mode for path 4-6-8-3 which has the highest β_{path} has λ_T of about 15% greater than that of the mode for path 4-7-8-2. The first three modes result in the same mechanisms, but although their λ_T's are same, they have different levels of β_{path}. This can be also applied for the 5th and the 6th modes and may be due to the different redistribution of load effects according to the different failure sequences. From this point it can said that the deterministically important mode is not identical with the probabilistically important mode, i.e. the mode having the smallest λ_T does not give the lowest β_{path}.

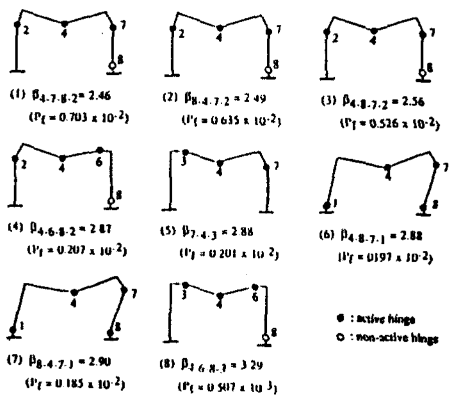


Fig. 3 Important Failure Modes for Plane Frame Model

7. Conclusions

In this study presented is the review of the methods for system reliability analysis developed up to the present from the view point of their efficiency when applying to realistic struc-

Table 2 Summary of Results for Frame Model

Mode No.	failure path	β_{path} [$(Pf)_{path}$]	total load factor, λ_T	$\beta_{sys.lower}$ [$(Pf)_{sys.upper}$]	$\beta_{sys.upper}$ [$(Pf)_{sys.lower}$]
1	4-7-8-2	2.46 [0.703×10^{-2}]	1.76	2.46 [0.703×10^{-2}]	2.46 [0.703×10^{-2}]
2	8-4-7-2	2.49 [0.635×10^{-2}]	1.76	2.22 [0.134×10^{-1}]	2.22 [0.134×10^{-1}]
3	4-8-7-2	2.56 [0.526×10^{-2}]	1.76	2.12 [0.171×10^{-1}]	2.15 [0.157×10^{-1}]
4	4-6-8-2	2.87 [0.207×10^{-2}]	1.89	2.12 [0.171×10^{-1}]	2.15 [0.157×10^{-1}]
5	7-4-3	2.88 [0.201×10^{-2}]	1.89	2.12 [0.171×10^{-1}]	2.15 [0.157×10^{-1}]
6	4-8-7-1	2.88 [0.197×10^{-2}]	1.67	2.10 [0.178×10^{-1}]	2.15 [0.157×10^{-1}]
7	8-4-7-1	2.90 [0.185×10^{-2}]	1.67	2.10 [0.178×10^{-1}]	2.15 [0.157×10^{-1}]
8	4-6-8-3	3.29 [0.507×10^{-3}]	2.02	2.10 [0.178×10^{-1}]	2.15 [0.157×10^{-1}]

tures. Extended incremental load method is introduced as another approximate method. It is an extension of the conventional incremental load method to allow for multiple loading case.

The results for a simple frame model justify the validity and the applicability of the method to a structure under multiple loadings. From the example analysis itis found thatthe deterministically most important failure mode isnot indentical with the probabilistically most important one.

This paper has not shown some results including the post-ultimate behaviour of failed components, in which the post-ultimate behaviour may take the form of multi-stage unloading pattern (as in Fig.1(b)). Nevertheless the extended incremental load method can now allow for such behaviour in evaluating system reliability. The framework is well established by the present author.

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