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Review of the Application of the First-Order Reliability Methods to Safety Assessment of Structures

by

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1차 신뢰성 해석법의 구조적 안전성평가에의 적용에 관한 재고

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Abstract

This paper is concerned with comparison of the first-order reliability methods applied to the assessment of structural safety. For convenience the reliability methods are divided into two categories : the One can explicitly consider the effects of uncertainties in material and geometric variables on those of load effects, say stresses and displacement in the structural analysis procedure and the other one does not. The first method is commonly termed as the stochastic finite element method (SFEM) or probabilistic finite element method (PFEM) and the second method is termed herein as the ordinary reliabilty method to distinct it from the stochastic finite element method in which the structural analysis is carried out just once and the load effects are directly input into the reliability analysis procedure. This is based on the reasonable assumption that the level of uncertainties of load effects is the same as those of load itself.

In this paper the above two different reliability method have been applied to the safety assessment of plane frame structures and compared thier results from the view point of their efficiency and usefulness. As far as results of the present structure models are concerned, it can be said that the ordinary reliability method can give reasonable results when the uncertainties of material and geometric variables are comparatively small, say when less than about 15% and the stochastic finite element method is desired to be applied to the structure in which the COV's are comparatively great, say when greater than about 15%.

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요 약

본 논문은 구조부재의 안전성평가에 적용되는 1차 신뢰성해석법들의 비교를 다룬 것으로서, 구조해석 시 재료와 기하학적 변수들의 불확실성을 고려하는 방법과 이를 고려하지 않는 방법으로 편의상 나누어 비교하여 보았다. 전자를 흔히 확율 유한요소법 (Stochastic Finite Element Method 또는 Probabilistic Finite Element Method)라고 하는데 이와 구별해 주기 위하여 현재 널리 쓰이는 후자의 방법을 통상적인 신뢰성해석법 (Ordinary Reliability Method)이라고 하였다. 이통상적인 방법에서는 구조해석을 단 한번 수행하고 그 결과를 신뢰성 해석과정에 직접 이용한다. 이는 재료및 기하학적 변수들의 불확실성이 응력이나 변위의 불확실성에 영향을 주지않고 따라서 그 불확실성의 정도를 하중의 것과 같다는 가정을 이용한다. 본 논문에서는 위의 두방법을 평면 골조구조물의 안전성평가에 적용하였고 그 결과들을 효율성과 응용성 측면에서 비교하였다. 본 논문의 구조모델에 대한 결과에 따르면 재료 및 기하학적 변수들의 불확실성이 비교적 작을때 (즉, COV가 약15%이하일 때) 통상적인 신뢰성 해석법은 적용해도 만족할 만한 결과를 주고 효율적인데, 확율유한요소법은 그 COV들이 약15%이상인 구조물에 적용하는 것이 바람직하다.

1. Intorduction

During the last decade a general framework for the reliability assessment of structural systems has been well established and it is matured to apply the method to the design of real structures. Some codes for structural design have been already developed based on reliability analysis[1]. The frontier work in this field can be found in the work by Pugsley applied to aircraft structure[2]. Since then many works have been carried out. Some are concerned with the development of the methods for reliability analysis[3-15] and some are concerened with its application to structural reliability analysis and its application to design[16-21]. These two sides have been emphasised in parallel. At these days with regard to the methods for reliability analysis the advanced first-order reliability method (AFORM) is well accepted in assessing structural safety and its based design (especially in the most steel structures). The second-order reliability method, of course may gives a closer solution of reliability to the true solution but not all cases.

This is not the place to go into details of reviewing the state-of the-art in the field of reliability analysis and its application to design. This paper places an emphasis on comparison between the ordinary reliability method and the stochastic finite element method (SFEM). In this the ordinary reliability method is such that in applying the method the structural analysis procedure is carried out just once and its load effects, say stresses and displacement, are directly input in the reliability analysis procedure. The effect of the variation of material and geometric variables on the variation of load effects are, hence, disregarded in the structural analysis and so the level of uncertainties of load effects is assumed to be the same as that of load itself. This approach is commonly employed at present in reliability assessment due to its simplicity. When the variation of material and geometric variables are considerably great the result by this method may be, however, well outside the true solution. This seems to be only one major limitation of this method. To overcome this problem SFEM has been proposed. In this approach the general procedure of the ordinary reliability method is to be, of course, followed and the structural analysis is repeated to get the sensitivities of basic variables. And so theis approach can more explicitly account for the effect of uncertainties in material and geometric variables and may give a closer solution to true solution than the ordinary reliability method. One of major shortcomings is that a great amount of computational efforts is required since the structural analysis should be repeated many times to get the gradient of limit state equation (safety margin) to all basic variables (at every iteration steps when iterative method is used).

In this paper the above two different approaches are compared from the view point of their efficiency in applying to assessment of structural reliability in component level.

2. Formulation of Reliability Methods

2.1 Ordinary Reliability Method : Advanced First-Order Reliability Method

Several methods are available for computing failure probability and reliability index. When the limit state equation has a general form of non-linear equation and basic variables are non-normally distributed, the algorithm proposed by Rackwitz and Fiessles[6] is well accepted which is an extension of Hasofer-Lind[5]. This algorithm is often called advanced first-order second moment reliability method (AFORM). Following briefly describes its formulation procedure.

The limit state equation (for safety margin) is in general of the form :

$$Z = g(y) = (y_1, y_2, \ldots, y_n) \qquad (1)$$

where $y_i(i=1,2,\ldots,n)$ are n reduced variables of basic design variables, $x_i(i=1,2,\ldots,n)$ as basic independent random variables defined as :

$$y_i = (x_i - \underline{x}_i)/\sigma_{X_1} \qquad (2)$$

where $\underline{x}_i$ and σ_{X_1} are mean and standard deviation of variable x_i. Function, g, is some given non-linear function and describes the structural behaviour such that for $g>0$ safe state is defined whereas $g<0$ corresponding to failure, and failure state is given by $g=0$. Expand Eq. (1) in linear Taylor series which should equal zero, if the limit state criterion is fulfilled :

$$g(y_1^* \cdot y_2^*, \ldots, y_n^*) + \sum_{i=1}^{n} gi,(y_i - \overset{*}{\underset{1}{y}}) = 0 \qquad (3)$$

where g,i is partial derivative of function g with respect to y_i evaluated at the unknown point, $y^* = \{y_1^*, y_2^*, \ldots, {}_n^*\}$. The problem is to find the point, y^*, where the distance from origin of the reduced space to the failure surface, β becomes minimal. According to the first order reliability theory the limit state criterion is only fulfilled if the point lies on the failure surface. When such a point is found, the corresponding variable, x^*, can be obtained from :

$$yi^* = \underline{y}i - \alpha_i \beta \sigma_{y_1} \qquad (4)$$

where α_i is referred to as the sensitivity factor as defined in Eq.(6), which represent the relative importance of basic design variables. The point y^* is usually obtained by the following iteration algorthm[6].

In general, at the $(j+1)$th iteration with the assumed point of $y^{(j)}$ obtained at the jth iteration.

Step 1 : Evaluate the partial derivatives to random variable, yi $g,i^{(j)}$ $(=1,n)$ at the current design point,$y^{(j)}$:

$$g_i^{(j)} = \frac{dg(\{y^{(j)}\})}{dy_i} \quad (5)$$

Step 2 : Evaluate the sensitivity factor for all y_i, $\{\alpha^{(j)}\}$, given by :

$$\alpha_i^{(j)} = \frac{g_i \sigma_{X_i}}{\sqrt{\sum_{i=1}^{n} (g_i^{(j)} \cdot \sigma_{X_i})^2}} \quad (6)$$

Step 3 : Calculate the new point for the next iteration :

$$y^{(j+1)} = \left(\{y^{(j)}\}^T \{\alpha^{(j)}\}\right)\alpha^{(j)} - \frac{g(y^{(j)})}{\sqrt{\sum_{i=1}^{n} (g_i^{(j)} \cdot \sigma_{X_i})^2}} \cdot \{\alpha^{(j)}\} \quad (7)$$

Step 4 : Calculate the reliability index :

$$\beta^{(j+1)} = |y^{(j+1)}| \quad (8)$$

Step 5 : Evaluate the design point :

$$x_i^{(j+1)} = \underline{y}_i - \alpha_i^{(j)} \beta^{(j+1)} \sigma_{y_i} \quad (9)$$

Step 6 : Check the convergence :

$$\sqrt{\sum_{i=1}^{n} \left(\frac{y_i^{(j+1)} - y_i^{(j)}}{y_i^{(j)}}\right)^2} \leq \varepsilon$$

and

$$\frac{|\beta^{(j+1)} - \beta^{(j)}|}{\beta^{(j)}} \leq \varepsilon \quad (10)$$

where ε is a prescribed small number as tolerance.

The above procedure will be continued until convergence criteria are satisfied. The failure probability is :

$$P_f = P(g<0) = \Phi(-\beta) \quad (11)$$

For the non-normal variable, its mean and standard deviation in Eq.(2) are replaced by the equivalent mean and standard deviation obtained from the following[5,7] :

$$\Phi\left(\frac{x_i^* - x_i}{\sigma_{X_i}}\right) = F_{X_i}(x_i^*) \quad (12.a)$$

$$\frac{1}{\sigma_{X_i}}\left(\frac{x_i^* - x_i}{\sigma_{X_i}}\right) = f_{X_i}(x_i^*) \quad (12.b)$$

were x_i^* is the approximation point, $F(\cdot)$ and $f(\cdot)$ the distribution function and density function of the non-normal distribution, respectively, and $\Phi(\cdot)$ and $\phi(\cdot)$ the standard normal distribution and density functions, respectively, which has the effect of equating the cumulative probabilities of the probability densities of the actural and approximating normal distributions at the design point x_i^*. The solutions of Eqs.(12.a) and (12.b) are :

$$\sigma_{X_i}^N = \frac{\phi[\Phi^{-1}\{F_{X_i}(x_i^*)\}]}{f_{X_i}(x_i)}$$

$$x_i^N = x_i^* - \sigma_{X_i} \Phi^{-1}\{F_{X_i}(x_i^*)\} \quad (13)$$

x_i^N and $\sigma_{X_i}^N$ are the mean and standard deviation of the equivalent distribution. This approximation may become more and more inaccurate if the original distribution becomes increas ingly skewed.

2.2 Stochastic Finite Element Method

As mentioned in Section 1 the apparent difference of the stochastic finite element method from the ordinary reliability method is that uncertainties in material and geometric variables can be explicityly accounted for. A few algorithms have been proposed[11-15]. Reference 22 well summarise the state-of-the-art in this area. In this paper the algorithm proposed by Kiureghian and Taylor[15] is employed in which an essential point is to find the partial derivatives of limit state equation to random variables. Let divide the random variables into two groups : resistance variables $\{r\}$ and load variables $\{q\}$, that is, random variables are

$$\{x\} = (\{r\},\{q\}) \tag{14}$$

and the limit state equation is expressed in terms of original variables as :

$$g(\{x\}) = g(\{r\},\{q\}) \tag{15}$$

Using the displacement method of structural analysis for a linear system of N degrees of freedom the stiffness equation is given as :

$$[K]\{U\} = \{F\} \tag{16}$$

where [K] is the stiffness matrix of a total structural system. {U} and {F} are nodal displacement and nodal force vectors, respectively. The elements of [K] in general contains such random variables as material and geometric properties and the vector {F} contains geometric properties and the applied loads. It is clear that [K] and {F} are random and hence {U} would be also random. For a linear structural system the load effects {q} would be stress at elements or nodal isplacement. For a component considered now the load effect is obtained form :

$$\{q^{(e)}\} = [B]\{u^{(e)}\} \tag{17}$$

in which superscript (e) is the element which contains the component considered now and matrix [B] is the load effect-nodal displacement reation matrix and of which elements are functions of material and geometric properties.

At the current design point $\{x^*\}$ the limit state equation is $g(\{x^*\})=g(\{r^*\}, \{q^*\})$ where $\{r^*\}$ is explicitly known in terms of $\{x^*\}$ and $\{q^*\}$ is given using Eqs(16) and (17) as :

$$\{q^*\}=[B]^T([K]^{-1}\{F\})^{(e)}_{x=x^*} \tag{18}$$

in which the curled bracket of vectors {x} and $\{x^*\}$ are omitted. Superscript (e) is added to denote that the nodal displacement vector is that of to element (e) with is sorted out using element togpology data (as well known). The partial derivatives of limit state equation (15) to random variables is given by :

$$\{\frac{dg}{dx_i}\}_{x=x^*} = [\{\frac{dg}{dr_k}\}^T\{\frac{dr_k}{dx_i}\} + \{\frac{dg}{dq_k}\}^T\{\frac{dq_k}{dx_i}\}]_{\substack{x=x^*\\ r=r^*\\ q=q^*}} \tag{19}$$

All terms can be easily calculated except the term of $\{dq_k/dx_i\}$. Using Eq.(18) the derivative is given as follow :

$$\{\frac{dq}{dx_i}\}_{x=s^*} = [\{\frac{dB}{dx_i}\}^T\{u^{(e)}\} + [B]^T(\{\frac{dU}{dx_i}\})^{(e)}]_{x=x^*} = [\frac{d[B]^T}{dx_i}\{u^{(e)}\} + [B]^T(\frac{d[K]^{-1}}{dx_i}\{F\} + [K]^{-1}\frac{d\{F\}}{dx_i})^{(e)}]_{x=x^*} \tag{20}$$

It is easily shown that

$$\frac{d[K]^{-1}}{dx_i} = -[K]^{-1}\frac{d[K]}{dx_i}[K]^{-1} \tag{21}$$

Then Eq.(20) becomes

$$\{\frac{dq}{dx_i}\}_{x=x^*} = [\{\frac{dB}{dx_i}\}^T\{u^{(e)}\} + [B]^T [[K]^{-1}(-\frac{d[K]}{dx_i}\{U\} + \frac{d\{F\}}{dx_i})^{(e)}]]_{x=x^*} \tag{22}$$

where {U} is obtained from Eq.(16) at the current design points.

After calculating partial derivative $\{dq_k/dx_i\}$ from Eq.(22) the partial derivative of Eq.(19) is completely calculated. Once obtaining the partial derivatives of limit state equation the iterative procedure described in the previous section can work.

The above formulation of the stochastic finite element method has a merit that the available computer code for the ordinary reliability analysis can be used without much modification.

3. Numerical Examples

3.1 Portal Frame Structure

A simple portal frame model shown in Fig.1 is selected to see the efficiency of the ordinary reliability method and the stochastic finite element method in evaluating the reliability. The model has 4 beam elements and 5 nodes. Both nodes of an element are treated as components. Component failure is assumed to occur when bending moment at a particular element end reaches the plastic bending moment. The limit state equation is simply given by :

$$g(r,q) = r - q \tag{23}$$

and r is the plastic bending moment as strength and q is the applied bending moment. Required data are listed in Table 1. All variables are assumed to be normally distributed and statistically independent. For each component there are 8 variabes. In the stochastic finite element analysis, the structural analysis procedure is repeated many times. After the first iteration the variable of which sensitivity factor given by Eq.(6) is less than ε_α is treated as a deterministic variable from the second iteration to reduce computational time. ε_α is a small number. Doing this is expected not to affect the result. For illustration Table 2 compares the results of Component 7 in Fig.1 when $\varepsilon_\alpha = 0$ and 0.01, respectively and when COV of material and geometric variables, say E, A and I, is 20%. We can see that

Table 1 Data for Portal Frame Model

comp.	$A_k(x10^{-3})$	$I_k(x10^{-5})$	R_k
1,2	4.0	3.58	0.075
3,4	4.0	4.77	0.101
5,6	4.0	4.77	0.101
7,8	4.0	3.58	0.075

A_k = cross sectional area (m^2)

I_k = moment of inertia (m^4)

mean yield stress = 276MPa

R_k = Mean strength (=plastic bending moment, MN)

$P^{(1)} = 0.02$ MN, $P^{(2)} = 0.04$ MN

COV of Rk = 0.05

COV of $P^{(1)}$ and $P^{(2)} = 0.3$

Table 2 Reliability Analysis Result for Component 7 of Portal Frame Model

(1) when $\varepsilon_\alpha = 0$

variable	design point	α
R_7	0.7364E-01	0.2770
E	0.2100E+06	0.2506E-03
A_1	0.400E-02	−0.9939E-04
I_1	0.3677E-04	−0.1038
A_2	0.4001E-02	−0.1306E-02
$P^{(1)}$	0.4631E-04	0.1112
$P^{(2)}$	0.2343E-01	−0.4373
	0.5319E-01	−0.8419

$\beta = 1.306$

$P_f = 0.0958$

No. of iteration = 25

(2) when $\varepsilon_\alpha = 0.01$

variable	design point	α
R_7	0.7364E-01	0.2770
E	0.2100E+06	0.0000
A_1	0.400E02	0.0000
I_1	0.3677E-04	−0.1038
A_2	0.4001E-02	0.0000
I_2	0.4631E-04	0.1112
$P^{(1)}$	0.2343E-01	−0.4373
$P^{(2)}$	0.5319E-01	−0.8419

$\beta = 1.306$

$P_f = 0.0958$

No. of iteration = 5

(Note) A_1, I_1 = A & I of Components 1, 2, 7, 8

A_2, I_2 = A & I of Components 3, 4, 5, 6

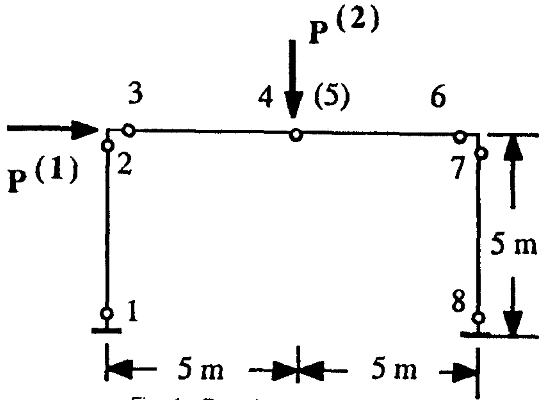


Fig. 1 Portal Frame Model

treating the variables of which sensitivity factor are less than ε_a of $=0.01$ does not affect the result.

For most steel structures the COV of elastic modulus and geometric properties, e.g., E, A and I of the model in Fig.1, is comparatively smaller than that of load effect and usually have the value raging 4 to 8%. To see the effect of variation of such variables on safety level, reliabil ity indices of 7 Components in Fig. 1 (Component 5 is the same as Component 4) are evaluated with varying the COV from 4 to 20%. Table 3 shows results by the ordinary reliability method and the stochastic finite element method When applying the ordinary reliability method the limit state equation is given by :

$$g(r,q) = r-(q^{(1)}+q^{(2)}) \tag{24}$$

in which $q^{(1)}$ and $q^{(2)}$ are bending moments due to load P_1 and P_2, respectively. From Table 3 it would not found appreciable difference of reliability indices when COV of material and geometric properties are less than 10% and even when it is 15%. When the COV is 20% the difference between two methods lies 3 to 8% for this model and the stochastic finite element method gives smaller reliability indices than the ordinary reliability method which can be easily expected.

Table 3 Reliability Indices to Changes in COVs of E, A and I of Portal Frame Model

comp.	by ORM*	by stochastic finite element method			
		COV* = 0.04	0.10	0.15	0.20
1	5.647	5.641	5.612	5.564	5.485
2	4.331	4.313	4.230	4.125	4.000
3	6.093	6.064	5.928	5.763	5.565
4(5)	1.979	1.978	1.968	1.954	1.934
6	3.179	3.175	3.159	3.138	3.113
7	1.322	1.322	1.318	1.313	1.306
8	2.349	2.344	2.321	2.287	2.243

* : ORM = ordinary reliability method
COV* = COV of material and geometric properties

3.2 Building Structure

As a reinforced concrete strucutre (RC structure) model, a 5 story-3 bay building shown in Fig.2 is considered[23]. As it well recognised the variation of material and geometric properties of RC structures are comparatively greater than steel structures. Data for reliability analysis of this model are listed in Table 4. The limit state equation concerns the reliability of element i-j of the model (see Fig.2). A node i is taken here which is under the combined axial force and bending moment. For the purpose of illustration the equation given by Eq.(25) is taken as Reference 23 :

$$g(\{x\}) = 1-\frac{F_2}{A_3R_1} - \frac{F_3}{I_3R_2\left[1-\frac{F_2}{R_3A_3}\right]} \tag{25}$$

where F_2 is the axial force and F_1 is the bending moment at node i. Results by the stochastic finite element analysis illustrated in Table 5 with $\varepsilon_a = 0.01$

As seen in Table 4 distributed loads W_1, W_2

Table 4 Data for Reliability Analysis of 5 Story -4 Bay Building (after Reference 23): unit-Kips, ft

variable	mean	COV	dist.type
W_1	6.00	0.18	log-normal
W_2	7.50	0.18	log-normal
W_3	8.00	0.18	log-normal
P_1	22.5	0.40	extreme type-I
P_2	20.0	0.40	extreme type-I
P_3	16.0	0.40	extreme type-I
E_1	454.0	0.09	normal
E_2	497.0	0.08	normal
I_1	0.94	0.12	normal
I_2	1.33	0.12	normal
I_3	2.47	0.12	normal
I_4	1.25	0.24	normal
I_5	1.63	0.24	normal
I_6	2.69	0.12	normal
A_1	3.36	0.18	normal
A_2	4.00	0.18	normal
A_3	5.44	0.18	normal
A_4	2.72	0.33	normal
A_5	3.13	0.33	normal
A_6	4.01	0.33	normal
R_1	700.0	0.14	log-normal
R_2	500.0	0.10	log-normal
R_3	1400.0	0.11	log-normal

and W_3 have the same probabilistic characteristics except mean values and this also works for the concentrated loads P_1, P_2 and P_3. The load cases can be hence grouped into two cases :

Load case 1 : distributed loads W_1, W_2, and W_3

Load case 2 : concentrated loads P_1, P_2, and P_3

In the ordinary reliability analysis the limit state equation can be expressed referring to Eq.

Table 5 Result of Reliability Analysis for Building by Stochastic Finite Element Analysis (after Reference 23): ($\varepsilon_a=0.01$)

variable	design point	α
W_1	6.35	-0.162
W_2	7.99	-2.00
W_3	8.46	-0.163
P_1	27.9	-0.381
P_2	24.0	-0.116
P_3	17.9	-0.086
E_1	454.0	-0.023
E_2	497.0	0.021
I_1	0.92	-
I_2	1.30	0.038
I_3	1.92	0.270
I_4	1.25	0.267
I_5	2.34	0.114
I_6	2.62	-0.080
A_1	3.24	-
A_2	3.82	-
A_3	3.57	-0.735
A_4	2.52	-
A_5	2.92	-
A_6	3.87	0.012
R_1	562.0	0.283
R_2	432.0	0.187
R_3	1200.0	0.064

$\beta = 2.20$

$P_f = 0.0138$

No. of iteration = 4

(25) as :

$$g(\{x\}) = 1 - \frac{(F_2^{(1)}+F_2^{(2)})}{A_3 R_1}$$

$$\frac{F_3^{(1)}+F_3^{(2)}}{I_3 R_2 \left[\frac{F_2^{(1)}+F_2^{(2)}}{R_3 A_3}\right]} \tag{26}$$

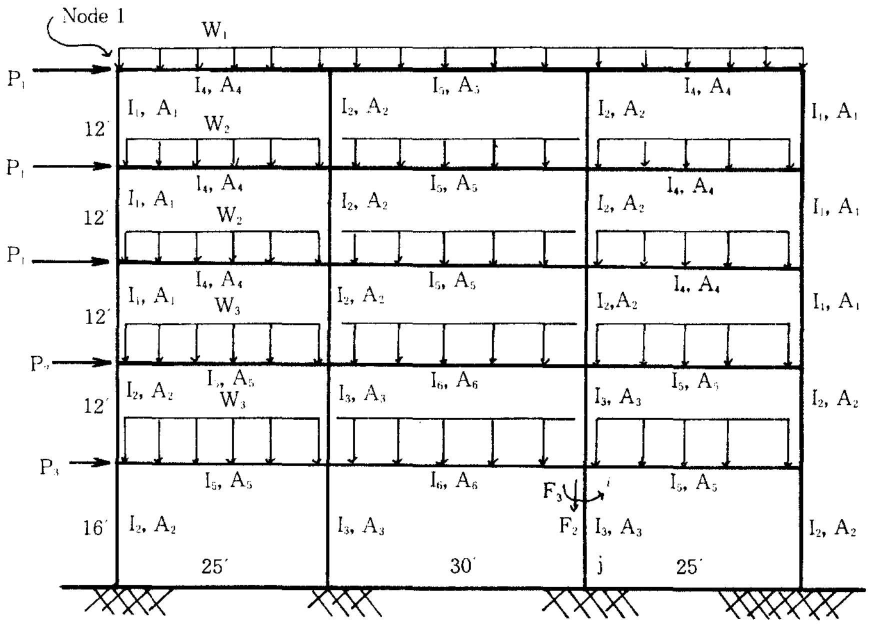


Fig. 2 5 Story-3 Bay Building (after Reference 23)

in which superscript (1) and (2) are refer to the load effects due to Load case 1 and 2, respectively. Table 6 shows the results by the stochastic finite element method and by the ordinary reliability method (AFORM).

Comparing results in Tables 5 and 6 the stochastic finite element method gives 20% smaller reliability index and the ordinary reliability method. This greater difference by the two methods than the protal frame model in Fig.1 may be due to that the larger values of COV's of sectional area and moment of inertia much pull down the reliability in the the stochastic finite element analysis. When COV's of elastic modulus, sectional area and moment of intertia are uniformly given as 10%, the stochastic finite element method gives reliability index of 2.57. This value is 6% smaller than by the ordinary reliabilty method. Table 7 summarises results for building.

Table 6 Data and Result of Reliability Analysis for Buliding by Ordinary Reliability Method (AFORM)

variable	mean	COV	dist. type	design point	α
R_1	700.0	0.14	log-normal	628.4	0.257
R_2	500.0	0.10	log-normal	475.6	0.165
R_3	1400.0	0.11	log-normal	1368.0	0.057
A_3	5.44	0.18	normal	3.78	0.611
I_3	2.47	0.12	normal	2.30	0.213
$F_2^{(1)}$	1035.8	0.14	log-normal	1257.0	−0.428
$F_3^{(1)}$	51.91	0.18	log-normal	52.09	−0.040
$F_2^{(2)}$	1.04	0.40	extreme type-I	0.97	−0.001
$F_3^{(2)}$	204.9	0.40	extreme type-I	339.1	0.548

$\beta = 2.74$
$P_f = 0.303E-02$
No. of iteration = 15

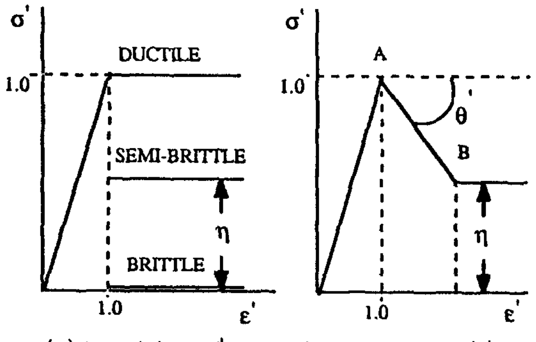
Table 7 Summary of Reliability Analysis for Building

	by ORM*	by stochastic finite element method	
		COV-I[+1]	COV-II[+1]
β	2.74	2.20	2.57
P_f	0.303E-02	0.0138	0.508E-02

ORM* = ordinary reliability method
COV-I[+1] = COV values of E,A and I in Table 4

3.3 Discussion

From results of the last two sections it can be said that the ordinary reliability method can provide the acceptable solution of structural reliability when the variation of material and geometric properties is comparatively small, that is, when they have COV of up to 10% and even up to 15% for a simple structure and with in this range of COV the the stochastic finite element method does not have its merit any more since it is computationally very expensive due to the repeated structural analysis tens of many times to get the derivatives of limit state equation to random variables. And so it is desirable to apply the stochastic finite element method to the cases of structures in which COVs of geometric properties and especially COVs of material properties are comparatively great. RC structures, soil structures, rock structures and so on may be good condidates. Another feasible area of its application may be the system reliability analysis of a structure with considering the post-ultimate behaviour. The post-ultimate behaviour can be characterised by the post-ultimate slope, θ and the residual strength parameter, η as shown in Fig.3 As it is well appreciated the post-ultimate behaviour very much influences the system residual strength and consequently the system reliability. There may be sufficiently great uncertainties in the post-ultimate slope and the residual strenght parameter and hence they should likely be treated as random variables. And then the stochastic finite element method can efficiently evaluate the system reliability. A work is in progress now on by the present author and will be presented at the forthcoming conference[24].



(a) two-state mode (b) three-state model

Fig. 3 Typical Model Post-Ultimate Behaviour

4. Conclusions

The present paper has concerend with comparison of methods for structural reliability analysis: the ordinary reliability method and the stochastic finite element method. Formulation of a kind of the stochastic finite element analysis is illustrated. The two methods have been applied to plane structures. As far as the present numerical results are concerened, when COVs of geometric and material properties are less than 10 to 15%, there is not much difference of reliability indices by the ordinary reliability method and the stochastic finite element method. The later of course always gives the smaller reliability indices. When the COVs are lager than 10 to 15%, the two methods shows appreciable difference.

Based on the present numerical studies, although the ordinary reliability method always gives the higer reliability indices than the stochastic finite element method with can provide the closer solution of reliability to truer one, employing the ordinary reliability method is sufficient in evaluating the structural reliability when the variation of material and geometric properties are comparatively small (say less than 10% and even less than 15%) and hence the stochastic finite element method seems to be computationally less efficient than the ordinary reliability method since the method usually requires the structural analysis repeated many times. It would be, therefore, recommmended that the stochastic finite element method should likely be applied to the structures in which the variation of material and geometric properties are comparatively great, e.g. application to RC, soil and rock structures, and to the reliability analysis of structural system.

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