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Nonlinear Free Surface Flows for an Axisymmetric Submerged Body

by

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Abstract

In this paper the nonlinear free surface flows for an axisymmetric submerged body oscillating beneath the free surface are solved and the forces acting on the body are calculated. A boundary integral method is applied to solve the axisymmetric boundary value problem and the Runge−Kutta 4−th order method is used for the time stepping of the free surface location. The nonlinear forces acting on the axisymmetric body are computed and compared with published results.

요 약

본 논문에서는 축대칭 회전 몰수체가 자유표면하에서 운동하고 있을때 이에 의한 비선형 표면파의 생성과 물체에 작용하는 힘을 계산하였다. 축대칭 포텐시얼 경계치 문제를 해석하기 위하여 경계 적분 방정식을 풀었으며, 시간에 따른 자유표면의 변화를 추적하기 위한 수치적분방법으로 Runge−Kutta 4차 방법을 사용하였다. 이 결과로부터 축대칭 몰수체에 작용하는 힘을 계산하였고, 선형이론과 비선형 이론에 의한 결과를 비교하였다.

1. Introduction

Nonlinear free surface problems have been solved by a semi−Lagrangian method since 1976 (Longuet−Higgins and Cokelet [13]). Longuet−Higgins and Cokelet [13] presented a mixed Eulerian−Lagrangian method for following the time−history of space−periodic irrotational surface waves. The only independent variables at the beginning of each time step were the coordinates and velocity potential of marked particles on the free surface. At each time−step an integral equation was solved for the new normal component of velocity. This me-

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thod was applied to a free, steady wave of finite amplitude, and was found to give excellent agreement with calculations based on Stokes's series. It was then extended to unsteady waves, produced by initially applying an asymmetric distribution of pressure to a symmetric, progressive wave. The results showed the freely running wave then steepened and overturned.

Using a technique similar to that of Longuet－Higgins and Cokelet [13], Faltinsen [7] solved a nonlinear two dimensional free surface problem including a harmonically oscillating body. The body intersected the free surface and was constrained to move in the vertical direction. The numerical calculations were reduced by repressenting the flow far away from the body as a dipole located at the center of the body. A formula to calculate the exact force on the body was presented. It was only necessary to know the velocity potential on the positions of the free surface and the wetted body surface.

Vinje & Brevig [15] presented a numerical method for the time simulation of the nonlinear motions of two dimensional surface－piercing bodies of arbitrary shapes in water of finite depth. Periodicity in space was assumed. At each time step, Cauchy's integral theorem was applied to calculate the complex potential and its time derivative along the boundary. The solution was stepped forward in time by integrating the exact kinematic and dynamic free－surface boundary conditions as well as the equation of motion for the body. They solved the problem of capsizing in beam seas, caused by extreme waves.

Baker, et al. [2] solved two－dimensional nonlinear free surface problems by a dipole (vortex and source) distribution method. The resulting Fredholm integral equation of the second kind was solved by iteration which reduced storage and computing time. Applications to breaking water waves over finite－bottom topography and interacting triads of surface and interfacial waves were given.

Dommermuth & Yue [5] extended the semi－Lagrangian method to vertically axisymmetric free surface flows. Since they solved the finite depth problem, a far field closure was implemented by matching the linearized solution outside a radiation boundary. The intersection line between the body and free surface was treated by extending Lin's [11] method.

In this paper, the nonlinear hydrodynamics of an axisymmetric body beneath the free surface is solved in the time domain. The free surface shape and the forces acting on a sphere oscillating sinusoidally with large amplitude are calculated and compared with published results. The far field flow away from the body is represented by a dipole at the origin of the coordinate system similar to that used by Faltinsen [7]. This is only valid until waves arrive. Waves generated by the numerical error at the truncation boundary are not observed for the numerical calculations given in this paper. When the body motion is unknown, the time derivative of the potential on the body is needed for the time simulation. In two dimensions, Vinje & Brevig[16] derived the integral equation and the boundary conditions for the time derivative of the potential and stream function. However their formulas may not be extended to three－dimensional problems. A three－dimensional form is derived in this work. By using these formulas, the free surface shape and the equations of motion are solved simultaneously. A Runge－Kutta fourth order algorithm is employed in the time stepping scheme.

2. Mathematical Formulation

Consider an ideal fluid below the surface given by $F(\underline{x}, t) = 0$, where $\underline{x}(x, y, z)$ is a right-handed coordinate system with z positive upwards and the origin located at the mean free surface. The fluid is assumed to be inviscid and incompressible and the flow is assumed to be irrotational. The fluid domain is bounded with the following surfaces, the free surface, S_F, the body, S_B, and the surfaces at infinity, S_∞ (Fig. 1). The surfaces, taken as a whole, will be denoted as S. The governing equation and the boundary conditions are as follows(Longuet-Higgins & Cokelet [13] and Dommermuth & Yue [6]) :

Laplace equation :

$$\nabla^2\phi = 0 \qquad \text{in the fluid domain.} \tag{1}$$

Kinematic free surface condition :

$$\frac{D\underline{x}}{Dt} = \nabla\phi \qquad \text{on} \qquad F(\underline{x}, t) = 0. \tag{2}$$

Dynamic free surface condition

$$\frac{D\phi}{Dt} = -gz + \frac{1}{2}\nabla\phi\cdot\nabla\phi \quad \text{on} \quad F(\underline{x}, t) = 0. \tag{3}$$

Body boundary condition :

$$\nabla\phi\cdot\underline{n}(\underline{x}, t) = \underline{V}\cdot\underline{n} \quad \text{on} \quad B(\underline{x}, t) = 0. \tag{4}$$

Radiation condition :

$$\phi \rightarrow 0 \qquad |\underline{x}| \rightarrow \infty, \; t < \infty. \tag{5}$$

where $\frac{D}{Dt}(=\frac{\partial}{\partial t} + \nabla\phi\cdot\nabla)$ is the substantial derivative, $F(\underline{x}, t) = 0$ is the function representing the free surface geometry at time t, $\underline{V}$ includes both translational and rotational velocities, and $B(\underline{x}, t) = 0$, is the function representing the body surface geometry at time t.

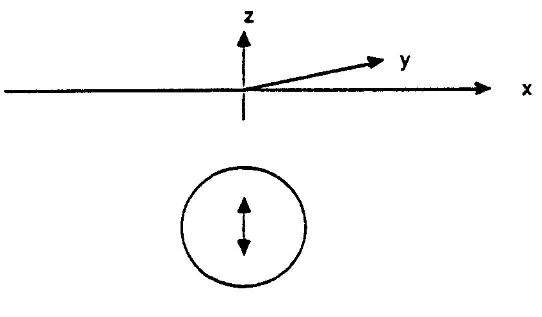

Fig. 1 Coordinate System

The Green function, $G(\underline{x}\,;\,\underline{y})$, satisfies the following equation.

$$\nabla^2 G(\underline{x}\,;\,\underline{y}) = -\delta(\underline{x} - \underline{y}) \tag{6}$$

where $\underline{x}$ is the position vector to the field point, $\underline{y}$ is that to the source point, and $\delta(\underline{x} - \underline{y})$ is the Dirac delta function. Through the application of Green's second identity for the fluid domain, the potential is given as

$$\alpha(\underline{x}, t)\phi(\underline{x}, t) = \iint_S [\frac{\partial\phi}{\partial n} - \phi\frac{\partial}{\partial n}]GdS \tag{7}$$

where α is an included solid angle at $\underline{x}$.

The Green function that satisfies Eq. (6) is

$$G(\underline{x}, \underline{y}) = \frac{1}{R} = \frac{1}{|\underline{x} - \underline{y}|} \tag{8}$$

where $\underline{x}$ is the position vector of a field point and $\underline{y}$ is that of a source point.

The solution of Eq. (3) gives the potential ϕ on the free surface $F(\underline{x}, t) = 0$. Also ϕ_n on the body is known from the body boundary condition, Eq. (4). Consequently, a Fredholm integral equation of the second kind on the body and of the first kind at the free surface may be solved.

For axisymmetric bodies, Eq. (7) can be reduced as follows (Newman, [14]) :

$$\alpha(\underline{x}, t)\phi(\underline{x}, t) = \int_S r'[\frac{\partial\phi}{\partial n'} - \phi\frac{\partial}{\partial n'}]G^a ds \tag{9}$$

where

$$G^a = \int_0^{2\pi}\frac{d\theta}{R} = \frac{4}{\rho_1}K(m)$$

$$\frac{\partial G^a}{\partial n'} = \int_0^{2\pi}\frac{\partial}{\partial n'} = (\frac{1}{R})d\theta$$

$$= \frac{4(z-z')}{\rho_1\rho^2}E(m)n_{z'} + [\frac{4(r-r')}{\rho_1\rho^2}E(m) + \frac{2}{\rho_1 r'}(E(m) - K(m))]n_{r'}$$

$$\rho^2 = (r-r')^2 + (z-z')^2$$

$$\rho_1^2 = (r+r')^2 + (z-z')^2$$

$$m = 1 - \rho^2/\rho_1^2$$

K and E are the complete elliptic integral of the first and the second kind, (r, z) is a field point and (r', z') is a source point in the polar coordinate system.

At the far field, the velocity potential, ϕ, and the wave elevation, η, are small from the radiation condition, Eq. (5). For example,

$$\phi(z=\eta) = \phi(z=0) + \eta \frac{\partial\phi}{\partial z}(z=0) + O(\eta^2)$$

Assuming the behavior of the potential, ϕ, is like that of a dipole at the origin of the coordinate system, it follows that as $r \to \infty$

$$\phi(z=0)=0$$

$$\frac{\partial\phi}{\partial z}(z=0) \sim \frac{1}{r^3}$$

$$\eta = \int_0^t \frac{\partial\phi}{\partial z}(z=0)dt \sim \int_0^t \frac{1}{r^3}dt \qquad (10)$$

$$\phi(z=\eta) \sim \eta \frac{\partial\phi}{\partial z}(z=0) \sim \frac{1}{r^6}$$

This is only valid until waves arrive at the truncation boundary.

The integral over s_F in Eq. (9) can be divided into two integrals :

$$\int_{sF} r'[\frac{\partial\phi}{\partial n'} - \phi\frac{\partial}{\partial n'}]G^a ds = \int_{sFo} r'[\frac{\partial\phi}{\partial n'} - \phi\frac{\partial}{\partial n'}]\cdot$$

$$G^a ds + \int_{ro}^{\infty} r'[\frac{\partial\phi}{\partial z'} - \phi\frac{\partial}{\partial z'}]G^a dr' \qquad (11)$$

where r_0 is the radius of the numerical truncation boundary and s_{Fo} is the actual computational free surface for $r \leq r_0$. Substituting Eq. (10) into the second integral of the RHS in Eq. (11), the following equation can be obtained :

$$\int_{ro}^{\infty} r'[\frac{\partial\phi}{\partial z'} - \phi\frac{\partial}{\partial z'}]G^a dr'$$

$$= \int_0^{2\pi} d\theta \int_{ro}^{\infty} [\frac{C_1}{r'^3(r'^2-2rr'\cos\theta+r^2+z^2)^{1/2}} - \frac{C_2}{r'^6(r'^2-2rr'\cos\theta+r^2+z^2)^{3/2}}]r'dr'$$

If we take a large value of r_0, the second integral on the RHS of Eq. (11) must be relatively small to the first one. By using integral tables of Gradsteyn & Ryzhik (1980), the following equations are obtained :

$$\int_{ro}^{\infty} r'[\frac{\partial\phi}{\partial z'} - \phi\frac{\partial}{\partial z'}]G^a dr'$$

$$\cong -r_0^3 \frac{\partial\phi}{\partial z}(r_0)\int_0^{2\pi} d\theta \int_{ro}^{\infty} \frac{dr'}{r'^2(r'^2-2rr'\cos\theta+r^2+z^2)^{1/2}}$$

$$= -r_0^3 \frac{\partial\phi}{\partial z}(r_0)[(\frac{4\rho_1 E(m)}{r_0} - 2\pi)/(r^2+z^2) - \frac{2r}{(r^2+z^2)^{3/2}}(I_1-I_2)] \qquad (12)$$

with

$$I_1 = \int_0^1 ln \frac{\sqrt{r^2+z^2} - r\sqrt{1-u^2}}{\sqrt{r^2+z^2} + r\sqrt{1-u^2}} du$$

and

$$I_2 = \int_0^1 ln \frac{(r^2+z^2) - rr_0\sqrt{1-u^2}}{(r^2+z^2) + rr_0\sqrt{1-u^2}} \frac{+\sqrt{(r^2+z^2)(r_0^2-2rr_0)\sqrt{1-u^2}+r^2+z^2}}{+\sqrt{(r^2+z^2)(r_0^2+2rr_0)\sqrt{1-u^2}+r^2+z^2}} du.$$

The far field radiation condition is satisfied by including Eq. (12) with its unknown, $\frac{\partial\phi}{\partial z}(r_0)$, in the set of simultaneous linear equations for ϕ on the body and $\frac{\partial\phi}{\partial n}$ on the free surface.

3. Numerical Implementation

All the boundaries, s_B and s_{F0}, are discretized into small segments in order to solve the integral equations numerically. ϕ and ϕ_n are approximated by linear functions of the parameter ζ on a given segment. In particular

$$\phi_j(\zeta) = (1-\zeta)\phi_j + \zeta\phi_{j+1} \qquad \text{for } 0 \leq \zeta \leq 1,$$

$$\phi_{nj}(\zeta) = (1-\zeta)\phi_{nj} + \zeta\phi_{nj+1} \qquad \text{for } 0 \leq \zeta \leq 1.$$

The geometry of the body and free surface is represented by a cubic B-spline (Barsky and Greenberg[3]), or

$$r_j(\zeta) = \sum_{s=-2}^{1} b_s(\zeta) V^r_{j+s} \quad \text{and} \quad z_j(\zeta) = \sum_{s=-2}^{1} b_s(\zeta) V^z_{j+s} \qquad (13)$$

where $b_5(\zeta)$ are the uniform cubic B-spline basis functions and V_i are vertices.

The end condition should be imposed to get a complete B-spline approximation. There are several methods to impose end conditions accroding to the geometrical characteristics (Barsky[4]). The derivative of B-spline interpolation at the end is set to get the tangent of the given geometry if the tangent is known. If the tangent is not known, the derivative at the end is set to be the slope between two vertices at the end obtained by using B-spline algorithm.

To evaluate the integrals over the segments the four point Gaussian Quadrature formula was used (Ferziger [9]), Abramowitz & Stegun [1]). In Eq. (9) G^a_n is not singular but G^a is logarithmically singular as the field point approaches the source point. The logarithmic function is integrable and can be integrated by numerical quadrature. But since an accurate integration of the logarithmic function requires a higher order quadrature formula, the method following Ferziger [9] and Dommermuth & Yue [5] can be used. The integral can be factored into the sum of the logarithmic singular part up to $\zeta\log\zeta$ which is integrable analytically and the non-singular part which requires numerical quadrature(Ferziger[9]).

4. Calculation of the Time Derivative of Potential

For the time simulation, $\frac{\partial\phi}{\partial t}$ needs to be known to calculate the forces and moments acting on the body. For two-dimensional problems, Vinje & Brevig(1982) derived an integral equation and boundary condition for $\frac{\partial\phi}{\partial t}$ by using the ϕ and ψ formulation. However their results can not be extended to the three-dimensional case. Since $\frac{\partial}{\partial n}(\frac{\partial\phi}{\partial t})$ can not be calculated by using the given motion, a boundary value problem for $\frac{\partial\phi}{dt}$, the time derivative of the potential in body fixed coordinates, is derived as follows :

$$\frac{\delta}{dt}(\frac{\partial\phi}{\partial t}) = \underline{n}\cdot\frac{\delta}{dt}\nabla\phi + \nabla\phi\cdot\frac{\delta\underline{n}}{dt}$$

$$=\underline{n}\cdot[\frac{\partial}{dt}\nabla\phi+(\underline{V}\cdot\nabla)\nabla\phi]+\nabla\phi\cdot(\underline{\omega}\mathrm{x}\underline{n})$$

$$=\underline{n}\cdot[\nabla\frac{\partial\phi}{dt}+\nabla(\underline{V}\cdot\nabla\phi)+\underline{\omega}\mathrm{x}\nabla\phi]$$

$$+\nabla\phi\cdot(\underline{\omega}\mathrm{x}\underline{n}) = \frac{\partial}{\partial n}(\frac{\delta\phi}{dt}) \qquad (14)$$

which can be expressed as

$$\frac{\partial}{\partial n}(\frac{\delta\phi}{dt})=\underline{n}\cdot(\frac{\partial\underline{V}_T}{\partial t}+\dot{\underline{\omega}}\mathrm{x}\underline{r}-\underline{\omega}\mathrm{x}\underline{V}_T) \quad (15)$$

Following the nomenclature of Vinje & Brevig (1982), the operator $\frac{\delta}{\partial t}$ is $(\frac{\partial}{\partial t}+\underline{V}\cdot\nabla)$, $\underline{V}=\underline{V}_T+\underline{\omega}\mathrm{x}\underline{r}$, $\underline{V}_T$ is the translational velocity of the center of mass of the body, $\underline{r}$ is the position vector to the boundary from the center of mass of the body, and $\underline{\omega}$ is the angular velocity vector of the body. Eq. (15) is useful in that most quantities of interest are expressed in the body coordinate system rather than a fixed, inertial one.

For axisymmetric motions, the rotational velocity, $\underline{\omega}$, and rotational acceleration, $\dot{\underline{\omega}}$ are zero and only the z-directional velocity w and acceleration a_z may not be zero. Thus for axisymmetric flows, Eq. (15) can be reduced as follows :

$$\frac{\partial}{\partial n}(\frac{\delta\phi}{dt}) = n_z a_z \qquad (16)$$

Since $\underline{V}\cdot\nabla\phi$ satisfies Laplace equation, the time derivative of the potential, $\frac{\delta\phi}{dt}$, can be calculated by using Green's theorem. The limiting behavior of $\underline{V}\cdot\nabla\phi$ at $r\rightarrow\infty$ can also be checked, or

$$\underline{V} \cdot \nabla\phi = O(r\frac{\phi}{r}) = O(\phi) \quad \text{as} \quad r \rightarrow \infty \quad (17)$$

Applying Green's theorem for $\frac{\delta\phi}{dt}$ instead of ϕ in Eqs. (1), (6), and (9), the following equation can be obtained :

$$\alpha\frac{\delta\phi}{dt} = \int_S r'[\frac{\partial}{\partial n}(\frac{\delta\phi}{dt}) - (\frac{\delta\phi}{dt})\frac{\partial}{\partial n}]\, G^a ds \quad (18)$$

The time derivative of the potential, $\frac{\delta\phi}{dt}$, can be decomposed as follows :

$$\frac{\delta\phi}{dt} = a_z\frac{\delta\phi_1}{dt} + \frac{\delta\phi_2}{dt} \quad (19)$$

The auxiliary terms, $\frac{\delta\phi_1}{dt}$ and $\frac{\delta\phi_2}{dt}$, are solutions of the following boundary value problems :

$$\frac{\partial}{\partial n}(\frac{\delta\phi_1}{dt}) = n_z \quad \text{and} \quad \frac{\partial}{\partial n}(\frac{\delta\phi_2}{dt}) = 0 \quad \text{on} \quad B(\underline{x}, t) = 0 \quad (20)$$

and

$$\frac{\delta\phi_1}{dt} = 0 \quad \text{and} \quad \frac{\delta\phi_2}{dt} = \underline{V} \cdot \nabla\phi - \frac{1}{2}\nabla\phi \cdot \nabla\phi - gz \quad \text{on} \quad F(\underline{x}, t) = 0 \quad (21)$$

The time derivative of the potential on the free surface, $\frac{\delta\phi_2}{dt}$, is calculated by using solutions of the integral equation, Eq. (9).

5. The Pressures and Forces

Once the time derivative of the potential is known, the pressures are found by applying Bernoulli's equation. Bernoulli's equation is derived for the variables relative to an inertial coordinate system. However, it is convenient for the purpose of solving the boundary value problem to use body fixed coordinates. Under these circumstances, spatial differentiation is invariant with coordinate transformation, but temporal differentiation is not. Bernoulli's equation can be expressed as

$$\frac{p}{\rho} = -\frac{\partial\phi}{\partial t} - \frac{1}{2}\nabla\phi \cdot \nabla\phi - gz$$
$$= -\frac{\delta\phi}{dt} + \underline{V} \cdot \nabla\phi - \frac{1}{2}\nabla\phi \cdot \nabla\phi - gz. \quad (22)$$

The term $\frac{\delta\phi}{dt}$ in the above equation is calculated by Eq. (19). In this paper the fourth order Runge-Kutta scheme is employed for time integration. With the pressure known, the force becomes

$$\underline{F} = m\dot{\underline{V}}$$
$$= \iint_S p\underline{n}dS - mg\underline{k}$$
$$= -\rho\iint \underline{n}(\frac{\delta\phi}{dt} - \underline{V} \cdot \nabla\phi + \frac{1}{2}\nabla\phi \cdot \nabla\phi + gz) \cdot dS - mg\underline{k}. \quad (23)$$

For axisymmetric bodies moving vertically, only the z-directional force is nonzero :

$$F_z = F_1 + F_2 + F_3 + (\rho g\nabla - mg) \quad (24)$$

where ∇ in Eq. (24) is the displaced volume of a sphere,

$$F_1 = -\rho a_z \iint_{S_B} n_z\frac{\delta\phi_1}{dt}dS = -\rho a_z 2\pi\int_{S_B} n_z\frac{\delta\phi_1}{dt}r'\,ds,$$

$$F_2 = -\rho\iint_{S_B} n_z\frac{\delta\phi_2}{dt}dS = -\rho 2\pi\int_{S_B} n_z\frac{\delta\phi_2}{dt}r'\,ds,$$

and

$$F_3 = -\rho\iint_{S_B} n_z(\underline{V} \cdot \nabla\phi - \frac{1}{2}\nabla\phi \cdot \nabla\phi)dS = -\rho 2\pi\int_{S_B} n_z(\underline{V} \cdot \nabla\phi - \frac{1}{2}\nabla\phi \cdot \nabla\phi)r'\,ds. \quad (25)$$

6. Numerical Calculation

To demonstrate the usefulness of the technique shown in the previous section, the force acting on a sphere oscillating beneath the free surface is computed. The motion of a sphere is given by $z = -h + a\cos wt$ for t greater than zero. Initially both potential and wave elevation at the free surface are zero.

The number of elements on the body is 20 and that on the free surface is 80. The truncation boundary is the position from the origin of the coordinate system where waves reaches in four periods of the body motion. So, it depends on the group velocity of wave. Even spacing is used on the body and free surface. The typical time interval is 0.05 times the period of motion for the time simulation of sphere. The tangential velocity and the tangent vector on the free surface at the vertical axis (r=0) are set to zero. These conditions prevent the numerical instability on the free surface at r=0.

The mean depth of immersion for the center of the sphere, h, is h/R=2.0 Two different amplitudes of motion are considered. The time history of the force acting on the oscillating sphere with a small ratio of motion amplitude, a, to radius, R, (a/R=0.1) was calculated and is shown in Fig. 2 in which the time history of various force components F_1, F_2, and F_3 are also shown. When a neutrally buoyant body is considered, the buoyancy force cancels the weight of the body exactly. The forces were nondimensionalized by $\rho gKaR^3$, where K is a wave number, ω^2/g, and g is the gravitational constant. Fig. 3 is similar to Fig. 2 with the exception of an increase in amplitude of motion (a/R=0.5). The primary non-linear component, F_3, has increased significantly. The harmonic distribution of the total force for different values of KR is shown in Fig. 4 and in Table 1. Fig. 6 shows the three dimensional wave profiles at six different times ($t'(t\sqrt{Kg})$ =1.253, 7.829, 10.647, 20.355, 23.174, 26.618).

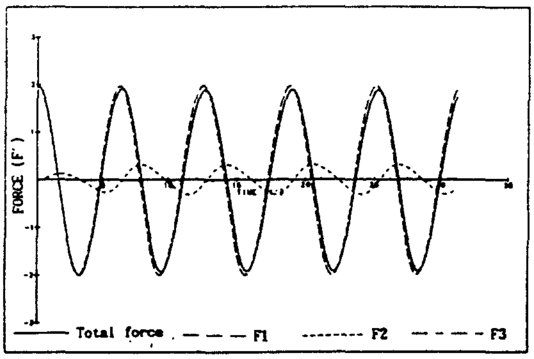

Fig. 2 Time History of the Force Components Acting on the Sphere (a/R=0.1, h/R=2.0, KR=1.0)

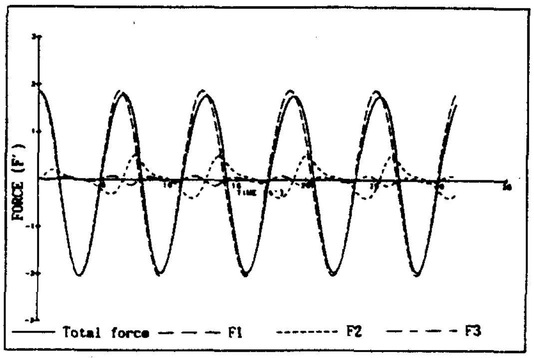

Fig. 3 Time History of the Force Components Acting on the Sphere (a/R=0.5, h/R=2.0, KR=1.0)

For comparison with previously published results, Fig. 5 shows the time history of the force, Eq. (24), and the results given by Ferrant [8]. The body boundary conditions in his paper were satisfied on both the mean position and the exact position of the body. The first body boundary condition, for the purpose of this work, is defined as 'linear' body boundary condition. The second body boundary condition is defined

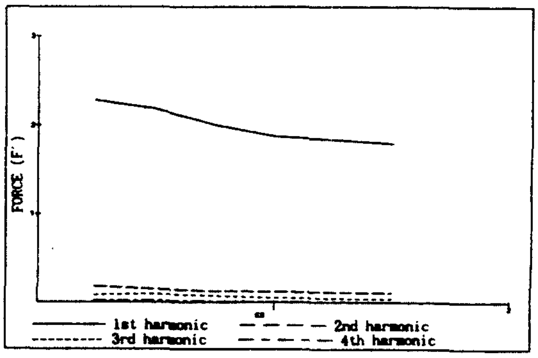

Fig. 4 Harmonic Distribution of the Total Force Acting on the Sphere (a/R=0.5, h/R=2.0, KR=1.0)

as 'nonlinear' body boundary condition, although is not truly nonlinear as long as the body motion is prescribed. Ferrant [8] presented solution for 'linear' and 'nonlinear' body boundary conditions, which are time derivatives of the potential in the body-fixed coordinate, i.e. (F_1+F_2). His free surface condition was still linear. He solved the boundary value problems in the frequency domain, not in the time domain. The fundamental force components for the 'linear' problem, Ferrant's combined linear/'nonlinear' solutions, and the total nonlinear force of this work are presented in Table 2. Ferrant's 'nonlinear' solution (a/R=0.5) shown in Fig. 5 is greater than the 'linear' one. The results (a/R=0.5) given in Fig. 5 show that the full nonlinear solution is less than the 'linear' solution. This means that the force increases due to the 'nonlinear' effect of the body boundary condition but it decreases more due to the nonlinear effect of the free surface condition. We therefore should be careful when we calculate the total force which includes inconsistent nonlinearities.

To investigate the effects on nonlinearity of the motion amplitude and the submerged depth, Fig. 7 (a/R=0.7) shows the results for the increased amplitude and Figs. 8-11 show the no-

Table 1. Harmonic Distributions of the Total Force at a/R=0.5 and h/R=2.0

$F/\rho gKaR^3$

KR	I-TH	COS	SIN
0.25	0	0.8737575E-01	0.0000000E+00
	1	0.2271135E+01	0.1923279E+00
	2	-0.3142179E-02	0.1824121E+00
	3	-0.5602978E-01	0.6227515E-01
	4	-0.1679305E-01	0.1419030E-01
0.5	0	0.8123831E-01	0.0000000E+00
	1	0.2152092E+01	0.3875170E+00
	2	-0.1033199E+00	0.1143350E+00
	3	-0.9440308E-01	0.2080585E-01
	4	-0.1984028E-01	-0.8838424E-02
0.75	0	0.3723103E-01	0.0000000E+00
	1	0.1964790E+01	0.3860812E+00
	2	-0.1026890E+00	0.7736817E-01
	3	-0.7346786E-01	-0.1491389E-01
	4	-0.5125929E-02	-0.1449216E-01
1.0	0	-0.2702199E-01	0.0000000E+00
	1	0.1848863E+01	0.3065330E+00
	2	-0.1106138E+00	0.5816335E-01
	3	-0.5166593E-01	-0.1822874E-01
	4	0.1568122E-02	-0.8852214E-02
1.5	1	-0.8824293E-01	0.0000000E+00
	2	0.1785480E+01	0.1367325E+00
	3	-0.1084778E+00	0.1043831E-01
	4	-0.3629696E-01	-0.1237296E-01
	5	0.1506353E-02	-0.2379999E-02

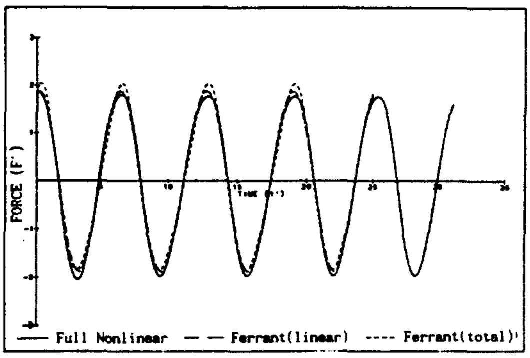

Fig. 5 Comparison of Two Nonlinear Predictions of the Force Time Histories (a/R=0.5, h/R=2.0, KR=1.0)

Table 2. Fundamental Force Components at KR= 1.0, a/R=0.5, and h/R=2.0

$F/\rho gKaR^3$

	Linear	Linear FSBC/ Nonlinear BBC	Nonlinear Eq. (24)
COS	0.185E+01	0.1825292E+01	0.1848863E+01
SIN	0.30 E+00	0.2988316E+00	0.3065330E+00

nlinear solutions for the different submerged depth (h/R=1.8). The harmonic distributions of the total force for the above cases are shown in Table 3. From Figs. 7−11 and Table 3, the nonlinear behavior is strong in low frequency and small submerged depth. But the nonlinearity for a submerged sphere is not dominant in the region where the nondimensional wave number, KR, is greater than 1.0 or the submerged depth, h/R, is greater than 2.0.

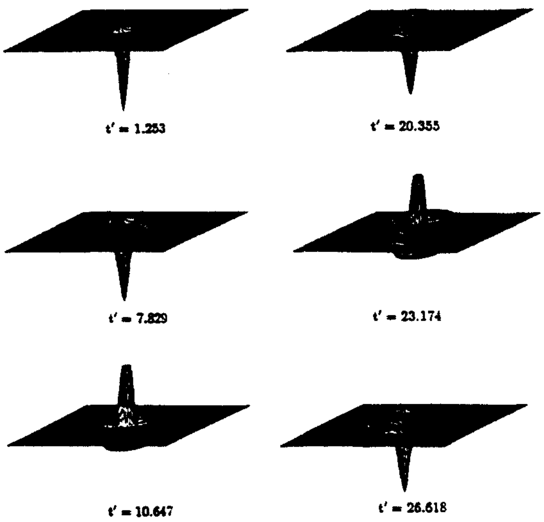

Fig. 6 Wave Shapes in the Time Domain (a/R=0.5, h/R=2.0, KR=1.0)

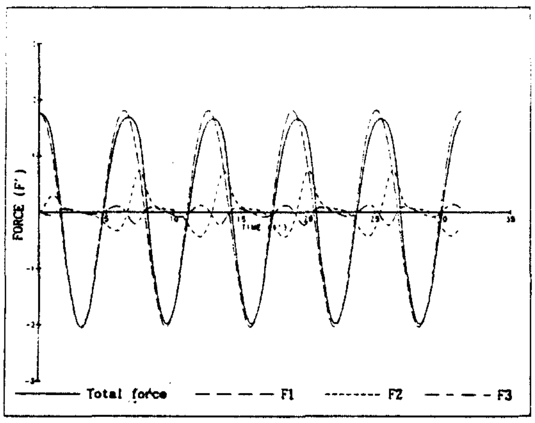

Fig. 7 Time History of the Force Components Acting on the Sphere (a/R=0.7, h/R=2.0, KR=1.0)

7. Conclusion

In this paper the nonlinear hydrodynamics of an axisymmetric body beneath the free surface is solved in the time domain. The free surface shape and forces acting on a sphere oscillating sinusoidally with large amplitude are calculated and compared with other results already pub-

Table 3. Harmonic Distributions of the Total Force.

$F/\rho gKaR^3$

a/R=0.7, h/R=2.0

KR	I-TH	COS	SIN
1.0	0	−0.337773000000E−01	0.000000000000E+00
	1	0.183217500000E+01	0.325255600000E+00
	2	−0.169225800000E+00	0.730429400000E−01
	3	−0.845205700000E−01	−0.484398600000E−01
	4	0.820015900000E−02	−0.317203300000E−01
	5	−0.684934000000E−02	0.184229400000E−02
	6	−0.339760000000E−03	0.798111100000E−02

a/R=0.5, h/R=1.8

KR	I-TH	COS	SIN
0.25	0	0.162845700000E+00	0.000000000000E+00
	1	0.244641100000E+01	0.228842100000E+00
	2	0.920997900000E−01	0.381788400000E+00
	3	−0.117049600000E+00	0.175732900000E+00
	4	−0.778913900000E−01	0.307682700000E−01
0.5	0	0.151077800000E+00	0.000000000000E+00
	1	0.224444300000E+01	0.520554100000E+00
	2	−0.206424300000E+00	0.239120500000E+00
	3	−0.165133700000E+00	0.403128400000E−01
	4	−0.523611500000E−01	−0.447383800000E−01
1.0	0	−0.475435300000E−02	0.000000000000E+00
	1	0.180400700000E+01	0.464940400000E+00
	2	−0.177539500000E+00	0.781713100000E−01
	3	−0.714032200000E−01	−0.564140600000E−01
	4	0.168010400000E−01	−0.230450200000E−01
1.5	0	−0.116472500000E+00	0.000000000000E+00
	1	0.168629800000E+01	0.224889600000E+00
	2	−0.164829600000E+00	0.802789000000E−02
	3	−0.338883300000E−01	−0.301283200000E−01
	4	0.841068800000E−02	−0.211544500000E−02

lished. The far field flow away from the body is represented by a three dimensional dipole at the origin of the coordinate system. This is only valid until waves arrive at the truncation boundary. Waves generated by the numerical error at the truncation boundary are not observed for the numerical calculations given in this paper. The integral equation and boundary conditions to calculate the time derivative of the potential on the body are derived. By using these formulas, the free surface shape and equations of motion are calculated simultaneously. A Runge-Kutta 4-th order scheme is employed in the solution method. Nonlinear effects on the oscillating body submerged in infinite water depth are studied. The magnitude of force increases due to the 'nonlinear' effect of the body boundary condition but decreases more due to the nonlinear effect of the free surface condition. Consequently, care should be exercised when hydrodynamic forces are found by including inconsistent no-

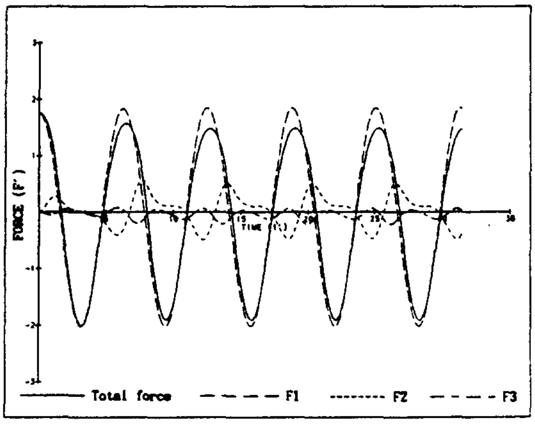

Fig. 8 Time History of the Force Components Acting on the Sphere (a/R=0.5, h/R=1.8, KR=1.5)

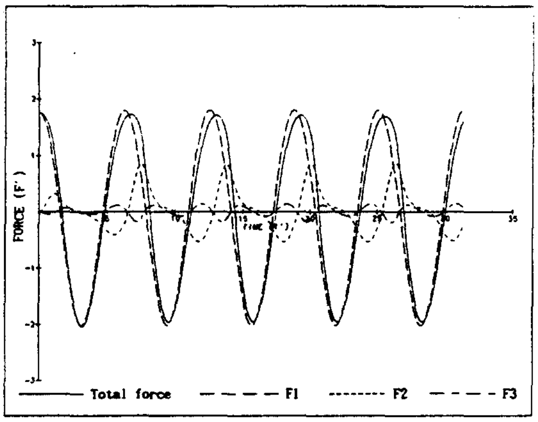

Fig. 9 Time History of the Force Components Acting on the Sphere (a/R=0.5, h/R=1.8, KR=1.0)

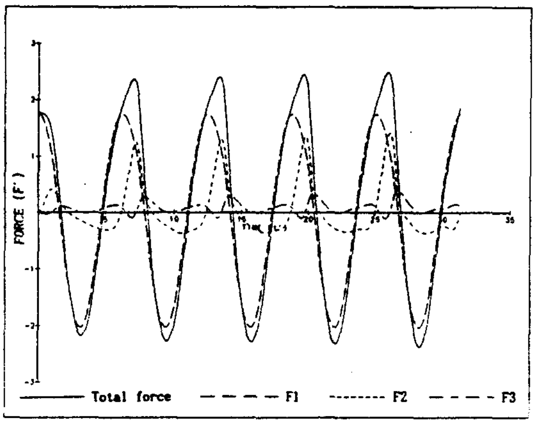

Fig. 10 Time History of the Force Components Acting on the Sphere (a/R=0.5, h/R=1.8, KR=0.5)

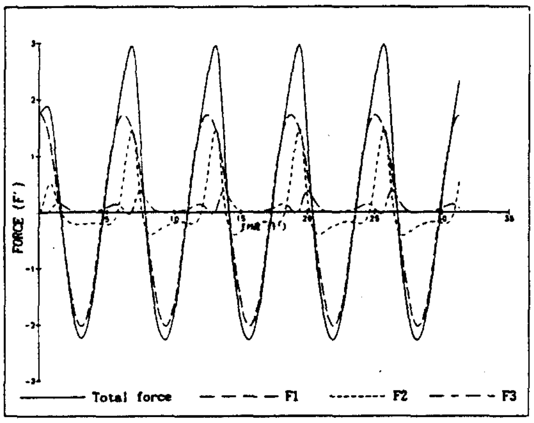

Fig. 11 Time History of the Force Components Acting on the Sphere (a/R=0.5, h/R=1.8, KR=0.25)

nlinearities.

The nonlinear behavior is strong in low frequency and for small submerged depth. But the nonlinearity for a submerged sphere is not dominant in the region where the nondimensional wave number, KR, is greater than 1.0 or the submerged depth, h/R, is greater than 2.0.

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