論 文

Tracking Controller Design Using Delayed Output Feedback For Systems With Stiff Nonlinearities

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심한 비선형성을 갖는 시스템의 시간지연 출력궤환을 이용한 추종제어기의 설계

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ABSTRACT In this paper, a method is presented for designing a tracking and disturbance rejecting controller for a nonlinear control system in which approximate linearization is not applicable due to a stiff nonlinearity. Only the measurable variables are used for the controller synthesis. The system is augmented by a compensator at the output side for the tracking and disturbance rejection. An output delayed feedback controller is designed for the augmented system without nonlinearity. Then the feedback parameters are adjusted by describing function method to overcome the limit cycle due to the nonlinearity.

要 約 본 논문에서는 동작점에서 근사적인 선형화를 할 수없는 심한 비선형성을 갖는 시스템에 대하여 추종 및 외란 제거 제어기의 설계방법을 제안한다. 제어기 합성에는 관측가능한 양만이 사용된다. 추종성과 외란의 제거가 가능하도록 시스템의 출력단에 보상기를 추가하여 비선형성을 제외한 전체 시스템에 시간지연 출력궤환제어를 사용하며, 기술함수방법을 사용하여 출력이 정격내의 값이 되도록 궤환매개변수를 선택하여 비선형성의 난점을 극복한다.

Ⅰ. Introduction

Linear systems are idealized versions of practical nonlinear systems by neglecting nonlinearities when the nonlinear effects are small. In other words, linear systems possess the property of linearity only over a certain range of operation in practice. All physical systems are nonlinear to some degree. Coulomb friction, saturation, hysteresis and limit cycles are common in practical mechanical control systems. But the analysis and design of nonlinear systems are complicated because there are many simplifying properties which are not valid for nonlinear systems.

Of the available methods for analyzing nonlinear systems, the describing function approach which is applicable to any order system is the best one to investigate stiff nonlinear systems(1)(8)(9). This method makes the problem simple by assuming that the input to the nonlinear system is sinusoidal and the only significant frequency component of the output is that component having the same frequency as the input. There are many approaches of design of nonlinear feedback control systems using describing functions. Dither technique, reduction of the system gain, addition of phase-lead network and the introduction of rate feedback have been used to make

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the system stable. But since the describing function analysis is an amplitude-sensitive method, each of the methods has its own difficulties(2)(7)(8). Also the main concern is the elimination of limit cycles.

In this paper, a method is presented for designing an output feedback tracking and disturbance rejecting controller for a nonlinear control system when only partial state variables are available for measurement. To achieve tracking a reference signal and rejecting disturbance signals which are given by differential equations the system is augmented by a compensator at the output side. Removing the stiff nonlinearity from the given system, an output delayed feedback controller is designed for the resulting linear system by minimizing a quadratic performance index.

Using these parameters as initial values, the feedback parameters of the nonlinear closed loop system are adjusted by the describing function mthod to overcome the limit cycle due to the nonlinearity.

Ⅱ. Problem Statement

Consider a single input single output nonlinear system of Fig. 1

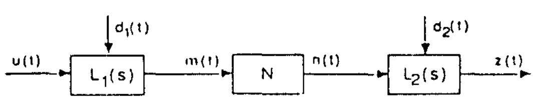

Fig. 1. General nonlinear system.

where $L_1(s)$ and $L_2(s)$ are transfer functions of linear elements, N is the nonlinear part, m(t) and n(t) are the input and output of N, u(t) and z(t) are the input and output of the system, $d_1(t)$ and $d_2(t)$ are disturbances. The nonlinear part N is given by

$$n(t)=a\ m(t)+f(m(t)) \tag{1}$$

where a is a real number and f is a nonlinear bounded function of m(t). The linear parts are represented by

$$\dot{x}_1(t)=A_1\ x_1(t)+B_1\ u(t)+E_1\ d_1(t) \tag{2}$$
$$y_1(t)=C_1\ x_1(t) \tag{3}$$
$$m(t)=D_1\ y_1(t) \tag{4}$$

and

$$\dot{x}_2(t)=A_2\ x_2(t)+B_2\ n(t)+E_2\ d_2(t) \tag{5}$$
$$y_2(t)=C_2\ x_2(t) \tag{6}$$
$$z(t)=D_2\ y_2(t) \tag{7}$$

where $x_1(t)$ and $x_2(t)$ are $n_1\times1$, $n_2\times1$ state variable vectors, $y_1(t)$ and $y_2(t)$ are $m_1\times1$, $m_2\times1$ observable variable vectors, $d_1(t)$ and $d_2(t)$ are $k_1\times1$, $k_2\times1$ disturbance vectors. Let

$$x(t)=\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix} \qquad y(t)=\begin{bmatrix}y_1(t)\\y_2(t)\end{bmatrix} \tag{8}$$
$$d(t)=\begin{bmatrix}d_1(t)\\d_2(t)\end{bmatrix}$$

$$n=n_1+n_2$$
$$m=m_1+m_2$$
$$k=k_1+k_2.$$

Then the whole system can be expressed by

$$\dot{x}(t)=A\ x(t)+B\ u(t)+E\ d(t)+G_1\ f(G_2\ x(t)) \tag{9}$$
$$y(t)=C\ x(t) \tag{10}$$
$$z(t)=D\ y(t) \tag{11}$$

where

$$A=\begin{bmatrix} A_1 & 0 \\ aB_2D_1C_1 & A_2 \end{bmatrix} \quad B=\begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$

$$C=\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \quad D=[0 \quad D_2]$$

$$E=\begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} \quad G_1=\begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$

$$G_2=[D_1C_1 \quad 0].$$

By removing the nonlinearity, that is,

$$G_1 f(G_2 x(t))=0 \tag{12}$$

we can express the linear part of the general nonlinear system by

$$\dot{x}(t)=A x(t)+B u(t)+E d(t) \tag{13}$$

$$y(t)=C x(t) \tag{14}$$

$$z(t)=D y(t) \tag{15}$$

Let $z_r(t)$ be the reference input vector which the output vector $z(t)$ is to follow. It is assumed that the elements of $z_r(t)$ and $d(t)$ satisfy the same differential equation

$$w^{(p)}+a_{p-1} w^{(p-1)}+ \cdots +a_1 \dot{w} +a_0 w=0. \tag{16}$$

Initial conditions are not known a priori and independent of each other for each elements of $z_r(t)$ and $d(t)$.

The problem is to synthesize a feedback controller, using only the observable variables, such that $z(t)$ of the system (9-11) follows $z_r(t)$ without error in steady state for all disturbance signals $d(t)$. For this objective any limit cycle should be removed if the given system has a self-excited oscillation.

Ⅲ. Tracking Controller Design

The design method in this paper to meet the control objective is a two-stage process. For tracking and disturbance regulation a compensator is added to the linear part (13-15). Then quadratic optimal control theory is applied to the augmented linear system with weighting matrices which are chosen so that the system performance requirements are satisfied. Reqirement of all state variables for the above controller is overcome by using state variable reconstruction. Then this tracking controller, which is designed for the linear part using measurable outputs and their delayed values, is applied to the nonlinear augmented system.

In case the response of the system does not have a limit cycle and the transient behavior is not desirable, then the weighting matrices are changed to meet the performance requirements. If the response has a limit cycle the describing function method is used to eliminate it.

1. Augmentation of the system

Let the reference input vector $z_r(t)$ and disturbance vector $d(t)$ be given by the following state equations.

$$\dot{x}_r(t)=A_r x_r(t) \tag{17}$$

$$z_r(t)=C_r x_r(t) \tag{18}$$

$$\dot{x}_d(t)=A_d x_d(t) \tag{19}$$

$$d(t)=C_d x_d(t) \tag{20}$$

Also let M_r and M_d be the minimal polynomials of A_r and A_d, and let M_{rd} be the least common multiple of M_r and M_d. Denote

$$M_{rd}(s)=s^p+a_{p-1} s^{p-1}+ \cdots +a_1 s+a_0 \tag{21}$$

For tracking and disturbance rejection a compensator which is defined as[5]

$$\dot{x}_c(t)=A_c\, x_c(t)+B_c\,(z(t)-z_r(t)) \tag{22}$$

where

$$A_c=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & a_{p-2} & a_{p-1} \end{bmatrix} \tag{23}$$

$$B_c=\begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ 1 \end{bmatrix} \tag{24}$$

is added to the system (13-15) as in fig. 2.

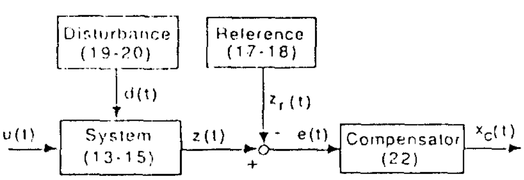

Fig. 2. Augmented system.

Then it is well known[6] that the augmented system (Fig. 2) which is expressed as

$$\dot{x}_a(t)=A_a\, x_a(t)+B_a\, u(t)+B_1\, z_r(t)+B_2\, d(t) \tag{25}$$

$$y_a(t)=C_a\, x_a(t) \tag{26}$$

$$z(t)=D_a\, y_a(t) \tag{27}$$

where

$$x_a(t)=\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} \quad y_a(t)=\begin{bmatrix} y(t) \\ y_c(t) \end{bmatrix}$$

$$A_a=\begin{bmatrix} A & 0 \\ B_cDC & A_c \end{bmatrix} \quad B_a=\begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$B_1=\begin{bmatrix} 0 \\ -B_c \end{bmatrix} \quad B_2=\begin{bmatrix} E \\ 0 \end{bmatrix} \quad C_a=\begin{bmatrix} C & 0 \\ 0 & 1 \end{bmatrix}$$

$$D_a=[D \quad 0]$$

has a stable control law which guarantees asymptotic tracking and disturbance rejection if

$$\text{rank}\begin{bmatrix} \lambda I-A & B \\ DC & 0 \end{bmatrix}=n+1 \tag{28}$$

for all λ's of spectrum of A_c. In other words, as long as the linear part is concerned, there is a stabilizing control

$$u(t)=K_a\, x_a(t) \tag{29}$$

where the constant feedback parameter K_a is given by the cost

$$J=\int_0^\infty [x_a^T(t)\, Q\, x_a(t)+r\, u^2(t)]dt \tag{30}$$

The weighting matrix Q and r are chosen such that the output responses meet the required performance of overshoot, settling time, etc.

2. Elimination of a limit cycle

The next problem is a limit cycle which the output of the nonlinear system (9-11) with the augmentation (22) might have. For this case the describing function method[1-8] is applied to eliminate the limit cycle. Using the

Nyquist diagram or the gain phase plot, the existence of a limit cycle in the nonliear system can be investigated by the locus of the describing function $N(M,\omega)$ of the stiff nonlinearity and the locus of the remaining linear part $L(j\omega)$ on the complex plane. In order to eliminate the limit cycle, the locus of $L(j\omega)$ should be changed to avoid an intersection with tht locus of $-1/N(M,\omega)$ by adjusting the feedback parameters of eq.(29). But the control input of eq.(29) requieres the entire state variable which is a difficult problem in practice. Augmented state variable $x_c(t)$ is observable because it is defined externally. State variable $x(t)$ is reconstructed from the observable vector $y(t)$ and its delayed values.[4]

Ⅳ. Example

Consider a simplified model of a typical second order d.c. motor position control system shown in Fig. 3. The nonlinearity is mainly due to Coulomb firction.

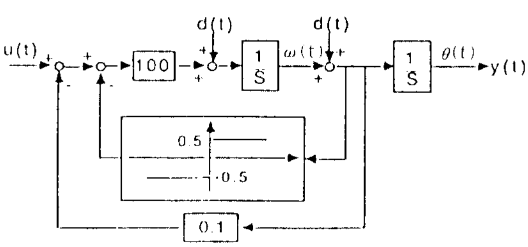

Fig. 3. Nonlinear system with Coulomb firction.

It is desired that the system output variable $\theta(t)$ tracks a ramp reference input. A constraint is imposed that only the output variable is available for measurement. It is obvious that the output feedback alone will not be able to make the system stable. Therefore, the proposed output feedback control is implemented as follows. Let the output variable angular position $\theta(t)$ be $x_1(t)$ and $\omega(t)$ be $x_2(t)$. Then the system equation is given by

$$\dot{x}(t)=\begin{bmatrix}0 & 1\\ 0 & -10\end{bmatrix}x(t)+\begin{bmatrix}0\\ 100\end{bmatrix}u(t)+\begin{bmatrix}1\\ 1\end{bmatrix}d(t)$$
$$\begin{bmatrix}0\\ -100f_n(x_2)\end{bmatrix} \tag{31}$$

where $f_n(\cdot)$ represents Coulomb friction. For simulation let the sampling time $T=0.01$ sec. Then the linear part becomes

$$x(k+1)=\begin{bmatrix}1 & 0.0095\\ 0 & 0.9048\end{bmatrix}x(k)+\begin{bmatrix}0.0048\\ 0.9516\end{bmatrix}u(k)$$
$$+\begin{bmatrix}0.01\\ 0.0095\end{bmatrix}d(k) \tag{32}$$

Suppose the reference input $z_r(t)=r_0+r_1\,t$, disturbance input $d(t)=d_0+d_1\,t$, then the compensator is given by

$$\dot{x}_c(t)=A_c\,x_c(t)+B_c\,e(t)$$
$$=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}x_c(t)+\begin{bmatrix}0\\ 1\end{bmatrix}e(t). \tag{33}$$

But the corresponding sampled version is expressed[3] as

$$x_c(k+1)=\begin{bmatrix}1 & 0.01\\ 0 & 1\end{bmatrix}x_c(k)+\begin{bmatrix}0\\ 0.01\end{bmatrix}e(k) \tag{34}$$

Whithout nonlinearity the state equation for the augmented system is given by

$$x_a(k+1)$$
$$=\begin{bmatrix}1 & 0.0095 & 0 & 0\\ 0 & 0.9048 & 0 & 0\\ 0.01 & 0 & 0 & 1\\ 0 & 0 & 1 & 0.01\end{bmatrix}x_a(k)+\begin{bmatrix}0.0048\\ 0.9516\\ 0\\ 0\end{bmatrix}u(k)$$

$$+\begin{bmatrix}0\\0\\0\\-0.01\end{bmatrix} z_r(k)+\begin{bmatrix}0.01\\0.0095\\0\\0\end{bmatrix} d(k) \qquad (35)$$

$$y_a(k)=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} x_a(k) \qquad (36)$$

$$z(k)=[\,1\quad 0\quad 0\,]\; y_a(k). \qquad (37)$$

Subject to the cost of

$$J=\sum_{k=0}^{\infty}[x_a^T(k)\;Q\;x_a(k)+r\;u^2(k)] \qquad (38)$$
$$Q=\mathrm{diag}\,[\,0\quad 0\quad 1000\quad 10\,]$$
$$r=1$$

the feedback gains are obtained as

$$u(k)=K_a\;x_a(k)$$
$$=[-2.259\quad -0.1316\quad -12.07\quad -29.54\,]\; x_a(k). \qquad (39)$$

Since $x_2(k)$ is not available directly it is replaced by[4]

$$x_2(k)=95.08\;y(k)\;-95.08\;y(k-1)+0.4917\;u(k-1) \qquad (40)$$

Then the observable signal $y(k)$, its delayed value and externally added state variables $x_3(k)$ and $x_4(k)$ are used for the control of

$$u(k)=-14.78\;y(k)+12.52\;y(k-1)-0.06473\;u(k-1)-12.07\;x_3(k)-29.54\;x_4(k). \qquad (41)$$

The feedback system is given in Fig. 4.

The output response without nonlinearity to the unit step reference input is shown in Fig. 5. But the response of the nonlinear system with Coulomb friction to the same feedback gains of eq. (41) shows a limit cycle which is not acceptable as shown in Fig. 6.

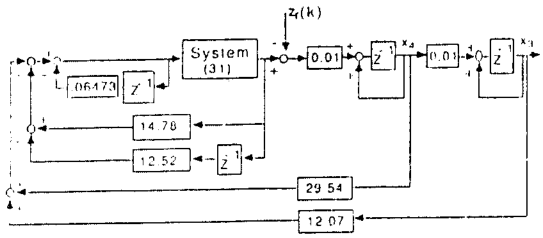

Fig. 4. Output feedback nonlinear system.

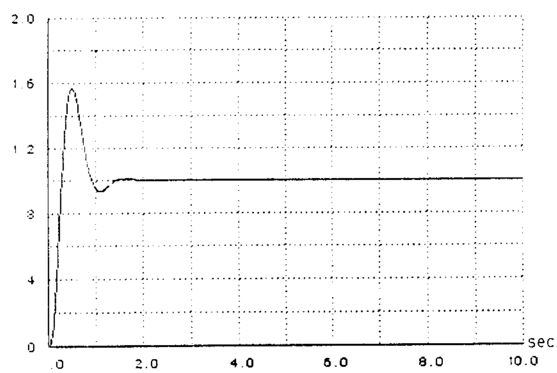

Fig. 5. Output of the linear part for unit step reference.

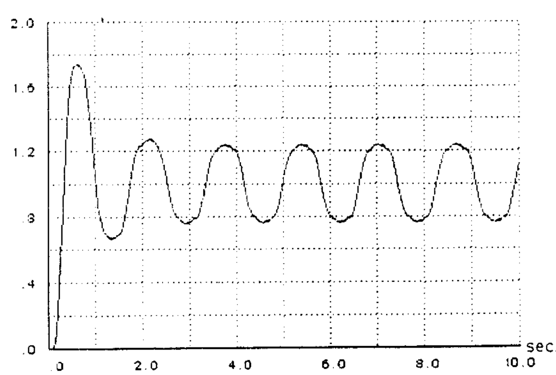

Fig. 6. Output of the nonlinear system for unit step reference.

But the modification of the parameters as

$$u(k)=-18\ y(k)+12.5\ y(k-1) \\ -0.06473\ u(k-1)-25\ x_3(k)-25\ x_4(k) \quad (42)$$

eliminates the limit cycle as shown in Fig. 7.

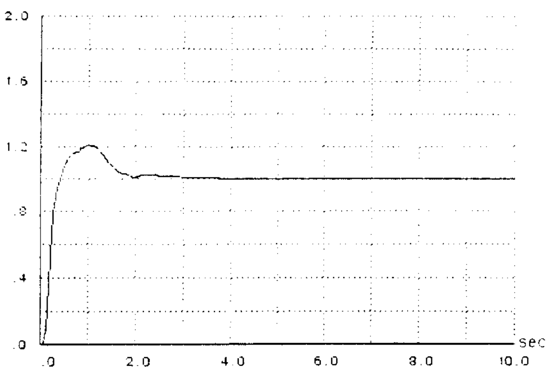

Fig. 7. Output of the nonlinear system with modified parameters for unit step reference.

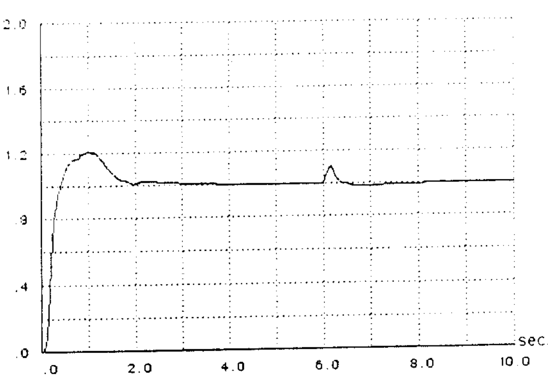

Fig. 8. Output of the nonlinear system with disturbance $d(t)=u(t-6)$.

Disturbance rejection is shown in Fig. 8 when a disturbance of step type is applied at t=6 sec. The system response to the reference input of $z_r(t)=1+t$ is shown in Fig. 9.

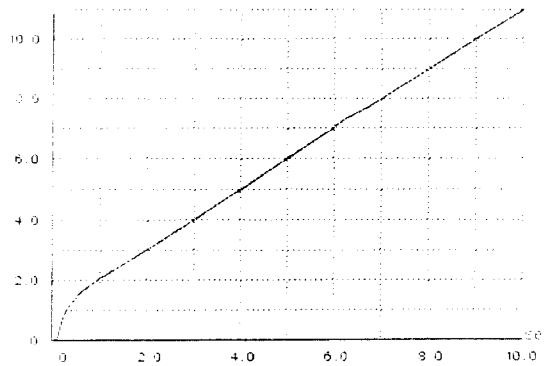

Fig. 9. Output of the nonlinear system for reference $z_r(t)=1+t$.

But the control of eq.(42) is not robust with respect to the output measurement accuracy. In other words, when there is an output measurement accuracy limit, the property of reducing the amount of error in the output signal for a linear system[6] with increased time delays in the control does not hold for a nonlinear system.

V. Conclusion

A design method of tracking and disturbance rejecting controller for a system containing a stiff nonlinearity is presented. It is a two stage process. First, without nonlinearity, a servomechanism problem is formulated and then a feedback controller is designed producing an acceptable output response. The next process is elimination of a limit cycle which might occur due to the nonlinearity for the same controller. This elimination is done by changing the feedback parameters using the describing function plots.

Although this method involves plots on complex plane and numerical analysis, these are inevitable due to the nonlinearity. Except these the proposed method is straightforward and easy to implement.

For ill-conditioned plants which are generally difficult to control, it is also hard to find suitable feedback parameters numerically. Much works are necessary for the case of ill-conditioned plants from the viewpoints of sensitivity problem, robust control, etc.

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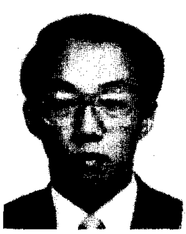

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