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A Goodness－of－Fit Test for the Exponential Distribution with Unknown Parameters

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ABSTRACT

This article is concerned with the goodness－of－fit test for exponentiality when both the scale and location parameters are unknown. A test procedure based on the L_1－norm of discrepancy between the cumulative distribution function and the empirical distribution function is proposed, and the critical values of the test statistic are obtained by Monte Carlo simulations. Also the null distributions of the proposed test statistic are presented for small sample sizes. The power of tests under certain alternative distributions is investigated to compare the proposed test statistic with the well－known EDF test statistics. Our Monte Carlo power studies reveal that the proposed test statistic has good power properties, for moderate－to－large sample sizes, in comparison to other statistics although it is a conservative test.

1. Introduction

Test for exponentiality is a problem of great importance in reliability theory and survival analysis. In practical applications such as Military Standard 781(U.S. Department of Defense 1986), reliability test plans and analyses are wholly based on the assumption that times between failures of a weapon system follow the exponential distribution. Montagne and Singpurwalla(1985) investigate the robustness of the sequential probability ratio test procedures of Military Standard 781, and address some serious problems which may occur when the underlying distribution of times between failures is not exponential. In this context, it would be desirable to make a thorough investigation of exponentiality prior to the analyses or assessments of the reliability testing data.

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There are several types of goodness−of−fit tests which can be applied to the test for exponentiality. Especially, the empirical distribution function(EDF) tests are widely used. However, the well−known EDF tests such as Kolmogorov−Smirov(KS), Kuiper(K), Cramer−von Mises (CM), Watson(W), and Anderson−Darling(AD) test procedures are correctly applicable only when the hypothesized distribution is completely specified. For unspecified distributions cases, certain estimates from the sample data may be used instead of parameters. It is worth noting, however, that even if the unknown parameters are estimated efficiently, the asymptotic distributions of these test statistics may not be the same as they are when parameters are known. Therefore the critical values given by the authors are no longer valid for unknown parameters cases.

Lilliefors(1969) employs Monte Carlo methods to construct the table of critical values for the Kolmogorov−Smirnov test when the mean of an exponential distribution is estimated from the sample. The modifications of the EDF test procedures are made by Spinelli and Stephens(1987). They examine several EDF tests for the exponential distribution with unknown parameters, produce the critical values of these test statistics, and make comparisons of power of tests by Monte Carlo method.

In Section 2 we propose a new test procedure for exponentiality when both the location and scale parameters are unknown, and tables of critical values are constructed by Monte Carlo method. Also some illustrations of the proposed test procedure are provided via example data. Section 3 presents the null distribution of the proposed test statistic for small sample sizes. Section 4 contains Monte Carlo power comparisons of the test statistics under a class of alternative distributions.

2. Proposed Test Procedure

We consider the problem of testing the null hypothesis H_0: a random sample $X_1, X_2, \cdots, X_n$ follows the exponential distribution with unknown origin α and mean β, denoted by $\exp(\alpha, \beta)$,

$$F(x; \alpha, \beta) = 1 - \exp[-(x-\alpha)/\beta],\ x \geq \alpha,\ \beta > 0.$$

In this problem, without loss of generality, α may be assumed to be 0. Many test procedures such as Stephens(1978) and Lee *et al.* (1980) have been developed for this particular case. However, in practical situations, data sets collected from reliability testings are usually not of this type. Stephens(1986) suggests several ways of dealing with unknown parameter cases. We employ an approach in which both α and β are estimated from the sample, and then the probability integral transformation of the ordered sample

$$Z_{(i)} = F(X_{(i)}; \hat{\alpha}, \hat{\beta}) \quad i = 1, 2, \cdots, n$$

where $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$, is carried out, using the estimates in $F(\cdot)$. This transformation converts a given random sample to a sample of uniform values. This procedure is common to the goodness-of-fit tests based on EDF statistics which deal with unknown parameters.

In this article we propose a new test of the above null hypothesis. The test procedure is based on the L_1-norm which measures, in a different way than other commonly used tests, the conformity of the sample data to the exponential distribution. The proposed test is as follows.

(1) Compute the uniformly minimum variance unbiased(UMVU) estimates of α and β, respectively,

$$\hat{\alpha} = [nX_{(1)} - \overline{X}]/(n-1)$$
$$\hat{\beta} = n[\overline{X} - X_{(1)}]/(n-1)$$

wher $X_{(1)}$ is the smallest sample. Note that $\hat{\alpha} + \hat{\beta} = \overline{X}$.

(2) Apply the probability integral transformation to the ordered sample values

$$Z_{(i)} = 1 - \exp[-\{X_{(i)} - \hat{\alpha}\}/\hat{\beta}], \quad i = 1, \cdots, n.$$

Then the test of exponentiality is converted to the test that a random sample is uniformly distributed between 0 and 1.

(3) The test statistic is the L_1-norm of discrepancy between the cumulative distribution function of uniform and the empirical distribution function of the sample values. Test statistic L1 is defined by

$$L1 = \| F(u_i) - F_n(u_i) \|_1, \quad i = 1, \cdots, n,$$

where $\| \cdot \|_1$ denotes the L_1-norm, and $u_i = F^{-1}(i/n)$. More specifically, the definition is

$$L1 = \| i/n - F_n[F^{-1}(i/n)] \|_1, \quad i = 1, \cdots, n-1.$$

The computation of the test statistic L1 is easily performed by the following formula,

$$L1 = \sum_{i=1}^{n-1} | i/n - [\{\text{number of } Z_{(i)}\text{s}\} \leq i/n]/n | .$$

If L1 exceeds the critical value in Table 1, we reject, at a specified significance level, the hypothesis that the observations are from the exponential distribution.

We have obtained the critical values for moderate and large sample sizes by Monte Carlo simulation. It is to be noted that this test statistic is a discrete random variable and hence the test results are conservative in the sense that the probability of a type I error is smaller than the significance level specified. Table 1 gives the critical values for the test statistic L1, for sample sizes $n = 15$ (1) 30 (2) 50 (5) 100 (10) 400, and for significance levels 0.30, 0.25, 0.20, 0.15, 0.10, 0.05, 0.025, 0.01, respectively. These critical values are obtained from 10,000 Monte Carlo runs for each sample size n. The exponential random deviates are generated on a CDC 180/860 computer using the IMSL(International Mathematical and Statistical Library) subroutine

GGEXN. To make approximations of the critical values for sample sizes larger than 400, linear regression models are fitted for each significance levels. The approximation results may highly be reliable since the coefficients of determination of the fitted models are greater than 0.9997 for all cases.

Table 1. Critical Values for Test Statistic L1

Sample size	Significance level							
	0.30	0.25	0.20	0.15	0.10	0.05	0.025	0.01
15	0.867	0.933	1.000	1.067	1.133	1.330	1.467	1.667
16	0.875	0.938	1.000	1.093	1.188	1.375	1.500	1.688
17	0.941	1.000	1.059	1.118	1.235	1.412	1.588	1.824
18	0.944	1.053	1.111	1.167	1.278	1.444	1.611	1.833
19	1.000	1.056	1.150	1.211	1.316	1.526	1.684	1.895
20	1.050	1.100	1.158	1.250	1.350	1.550	1.750	1.950
21	1.095	1.143	1.218	1.286	1.429	1.619	1.762	2.000
22	1.091	1.182	1.227	1.318	1.455	1.682	1.864	2.091
23	1.130	1.208	1.261	1.391	1.478	1.696	1.870	2.130
24	1.167	1.217	1.292	1.417	1.542	1.708	1.917	2.167
25	1.200	1.240	1.320	1.440	1.600	1.800	2.000	2.200
26	1.231	1.269	1.346	1.462	1.615	1.808	2.000	2.269
27	1.259	1.333	1.407	1.516	1.667	1.852	2.074	2.333
28	1.286	1.357	1.429	1.536	1.690	1.929	2.138	2.357
29	1.310	1.379	1.448	1.551	1.714	1.931	2.143	2.448
30	1.333	1.400	1.467	1.600	1.733	1.967	2.200	2.467
32	1.375	1.469	1.531	1.656	1.813	2.031	2.250	2.531
34	1.412	1.500	1.588	1.706	1.853	2.118	2.353	2.618
36	1.472	1.556	1.667	1.778	1.944	2.222	2.472	2.750
38	1.526	1.605	1.711	1.842	2.000	2.237	2.474	2.763
40	1.575	1.650	1.750	1.875	2.050	2.325	2.575	2.925
42	1.595	1.690	1.786	1.929	2.095	2.381	2.643	2.976
44	1.659	1.750	1.841	1.977	2.159	2.455	2.750	3.091
46	1.674	1.761	1.891	2.022	2.196	2.500	2.783	3.167
48	1.729	1.813	1.938	2.083	2.271	2.563	2.833	3.174
50	1.760	1.860	1.980	2.120	2.320	2.640	2.900	3.240
55	1.873	1.964	2.091	2.236	2.455	2.764	3.091	3.455
60	1.950	2.067	2.183	2.350	2.583	2.933	3.250	3.583

Table 1. (Continued)

Sample size	Significance level							
	0.30	0.25	0.20	0.15	0.10	0.05	0.025	0.01
65	2.031	2.154	2.292	2.446	2.692	3.046	3.385	3.800
70	2.129	2.257	2.386	2.557	2.786	3.143	3.529	3.957
75	2.200	2.320	2.467	2.627	2.867	3.267	3.640	4.093
80	2.275	2.400	2.550	2.725	2.975	3.388	3.763	4.163
85	2.329	2.471	2.624	2.812	3.071	3.471	3.835	4.329
90	2.422	2.544	2.700	2.900	3.178	3.600	4.000	4.544
95	2.505	2.642	2.800	2.989	3.253	3.705	4.137	4.642
100	2.570	2.690	2.860	3.040	3.340	3.780	4.190	4.720
110	2.673	2.818	2.991	3.227	3.518	3.999	4.399	4.945
120	2.808	2.967	3.150	3.358	3.667	4.158	4.642	5.199
130	2.931	3.092	3.285	3.515	3.854	4.338	4.869	5.469
140	3.043	3.207	3.393	3.643	3.993	4.536	5.029	5.671
150	3.173	3.340	3.553	3.800	4.133	4.687	5.180	5.807
160	3.256	3.438	3.650	3.913	4.269	4.856	5.394	6.050
170	3.371	3.553	3.776	4.053	4.429	4.994	5.576	6.247
180	3.467	3.650	3.861	4.133	4.517	5.139	5.711	6.472
190	3.568	3.768	3.989	4.258	4.616	5.232	5.816	6.495
200	3.650	3.850	4.085	4.380	4.765	5.415	6.085	6.740
210	3.771	3.986	4.219	4.519	4.929	5.533	6.138	6.914
220	3.827	4.036	4.277	4.577	4.995	5.664	6.327	7.032
230	3.917	4.117	4.369	4.678	5.100	5.783	6.361	7.209
240	4.004	4.208	4.475	4.804	5.213	5.908	6.638	7.400
250	4.108	4.324	4.592	4.924	5.372	6.040	6.752	7.580
260	4.188	4.419	4.681	5.027	5.519	6.250	6.881	7.692
270	4.315	4.552	4.822	5.156	5.611	6.322	7.048	7.963
280	4.336	4.582	4.876	5.200	5.650	6.396	7.168	8.018
290	4.410	4.648	4.928	5.293	5.783	6.500	7.245	8.134
300	4.493	4.747	5.027	5.360	5.883	6.637	7.400	8.303
310	4.568	4.806	5.116	5.432	5.909	6.777	7.529	8.384
320	4.628	4.894	5.175	5.534	6.050	6.809	7.575	8.622
330	4.706	4.967	5.269	5.630	6.142	6.973	7.682	8.630
340	4.788	5.041	5.350	5.712	6.209	7.041	7.800	8.894
350	4.854	5.129	5.437	5.849	6.349	7.174	7.980	9.037

Table 1. (Continued)

Sample size	Significance level							
	0.30	0.25	0.20	0.15	0.10	0.05	0.025	0.01
360	4.958	5.222	5.522	5.939	6.483	7.358	8.175	9.086
370	4.978	5.238	5.570	5.946	6.541	7.462	8.343	9.251
380	5.071	5.329	5.645	6.045	6.603	7.508	8.397	9.476
390	5.131	5.418	5.736	6.138	6.710	7.564	8.533	9.459
400	5.180	5.455	5.798	6.200	6.765	7.740	8.605	9.695
∞	$.2678\sqrt{n}$ $-.1375$	$.2819\sqrt{n}$ $-.1393$	$.2990\sqrt{n}$ $-.1479$	$.3195\sqrt{n}$ $-.1502$	$.3486\sqrt{n}$ $-.1608$	$.3946\sqrt{n}$ $-.1771$	$.4402\sqrt{n}$ $-.2099$	$.4936\sqrt{n}$ $-.2295$

To illustrate the proposed test procedure, we consider the following two examples. From each data set we obtain the UMVU estimates of α and β, and values of test statistics are computed by the following formulas.

$$KS: \max[\max_{1\le i\le n}\{(i/n)-Z_{(i)}\}, \max_{1\le i\le n}\{Z_{(i)}-(i-1)/n\}],$$

$$K: \max_{1\le i\le n}\{(i/n)-Z_{(i)}\}+\max_{1\le i\le n}\{Z_{(i)}-(i-1)/n\},$$

$$CM: \sum_{i=1}^{n}\{Z_{(i)}-(2i-1)/2n\}^2+1/12n,$$

$$W: \sum_{i=1}^{n}\{Z_{(i)}-(2i-1)/2n\}^2+1/12n-n(\bar{Z}-1/2)^2,$$

$$AD: -(1/n)\sum_{i=1}^{n}(2i-1)[ln\{Z_{(i)}\}+ln\{1-Z_{(n+1-i)}\}]-n.$$

Example 1 : Epstein(1960) provides 51 observations from a life testing which employs the type II censoring. He tests the hypothesis that observations are from an exponential distribution with constant parameter, and concludes that the mean life in the first half of the testing is different from the mean life in the second half of the testing. Now, we test exponentiality of the observations regardless of the mean life.

Table 2. Life Times Between Failures of a System

150.0	164.0	179.0	24.0	341.0	7.0	24.0	144.0	187.0
53.0	56.0	72.0	48.0	100.0	167.0	12.0	233.0	204.0
15.0	95.0	7.0	152.0	5.0	2.0	35.0	313.0	49.0
18.0	153.0	185.0	37.0	10.0	32.0	27.0	51.0	9.0
45.0	59.0	40.0	8.0	66.0	2.0	0.0	2.0	42.0
35.0	38.0	66.0	30.0	9.0	11.0			

The computation results are KS=0.1065, K=0.2059, CM=0.1469, W=0.1164, and AD=0.8461. The critical values given by Table 5 in Section 4 show that all of these test statistics are not significant at the 5% level. Now, we test exponentiality of the same data set applying the proposed test statistic, and obtain L1=2.2549 which does not exceed the 5% significant point, 2.640(from Table 1). Hence, we also can not reject the null hypothesis that life times between failures of the system are exponentially distributed.

Example 2 : 32 measurement of modulus of rupture of wood beams are given by Spinelli and Stephens(1987) as follows.

Table 3. Modulus of Rupture of Wood Beams(Ordered)

43.19	49.44	51.55	55.37	56.63	67.27	78.47	86.59
90.63	92.45	94.24	94.35	94.38	98.21	98.39	99.74
100.22	103.48	105.43	105.54	107.13	108.14	108.64	108.94
109.62	110.81	112.75	113.64	116.39	119.46	120.33	131.57

The computed values of test statistics are KS=0.3523, K=0.5487, CM=1.0591, W=0.7776, and AD=5.0580. The critical values given by Table 5 show that all of these test statistics are highly significant at the 5% level. Also we test exponentiality of the data set applying the proposed test statistic, and obtaion L1=4.9688 which is far beyond the 5% significant point, 2.031. Hence, we also reject the null hypothesis that the observations follow the exponential distribution.

3. Distribution of Test Statistic L1

We present the null distribution of the proposed test statistic for small sample sizes, $n=2, 3, 4, 5$. From the characterstic of the proposed statistic, we know that the computation of L1 is not ffected by any shifts of the sample point $Z_{(i)}$ within the k−th interval, $((k-1)/n, k/n]$, $k=1, \cdots, n$. That is, L1 value is not changed unless a sample point within the k−th interval is moved to any other intervals. And the probability of a sample point being located within a specific interval is $1/n$. Consequently, we can obtain the null distribution of the statistic L1 for each smaple size n.

For example, let $n=4$, then we can consider a problem of locating, independently, 4 sample points into 4 intervals of size $1/4$. The total number of events is 35 which can be obtained by the formula ${}_{2n-1}C_n$. All of the events and probabilities corresponding to the events are as follows.

Event	L1(×4)	Prob.(×4^4)	Event	L1(×4)	Prob.(×4^4)
0004	6	1	1030	2	4
0013	5	4	1102	1	12
0022	4	6	1111	0	24
0031	3	4	1120	1	12
0040	4	1	1201	1	12
0103	4	4	1210	2	12
0112	3	12	1300	3	4
0121	2	12	2002	2	6
0130	3	4	2011	1	12
0202	2	6	2020	2	6
0211	1	12	2101	2	12
0220	2	6	2110	3	12
0301	2	4	2200	4	6
0310	3	4	3001	3	4
0400	4	1	3010	4	4
1003	3	4	3100	5	4
1012	2	12	4000	6	1
1021	1	12			

Summarizing the above computational results, we can find the distribution of the test statistic L1 for sample size $n=4$.

x :	0	1/4	1/2	3/4	1	5/4	3/2
$f_{L1}(x)$:	0.09375	0.28125	0.3125	0.1875	0.08594	0.03125	0.00781

Finally, we can construct the null distributions of L1 by utilizing the computer program PARTNN.

Talbe 4. Null Distributions of Test Statistic L1

Sample size	x	$f_{L1}(x)$	x	$f_{L1}(x)$
$n=2$	0	0.50000	1/2	0.50000
$n=3$	0	0.22222	2/3	0.25926
	1/3	0.44444	1	0.07407
$n=4$	0	0.09375	1	0.08594
	1/4	0.28125	5/4	0.03125
	1/2	0.31250	3/2	0.00781
	3/4	0.18750		
$n=5$	0	0.03840	6/5	0.04832
	1/5	0.15360	7/5	0.02304
	2/5	0.24960	8/5	0.00960
	3/5	0.23040	9/5	0.00320
	4/5	0.15360	2	0.00064
	1	0.08960		

4. Power Comparisons

Monte Carlo power studies are conducted to compare various test procedures for exponentiality, KS, K, CM, W, AD, and L1. For the purpose of precise power comparisons, new critical values for KS, K, CM, W, and AD test statistics are obtained from 10,000 Monte Carlo runs which are performed by the same way as for the L1 test. Critical values at the 5% significance level are given in Table 5 for sample sizes $n=30$, 50, 75, 100, 150, and 200.

Table 5. 5% Critical Values for Test Statistics, KS, K, CM, W, and AD

Sample	Test statistics				
size	KS	K	CM	W	AD
30	0.1914	0.2879	0.2058	0.1524	1.1296
50	0.1504	0.2269	0.2149	0.1559	1.2034
75	0.1226	0.1866	0.2155	0.1546	1.2244
100	0.1071	0.1613	0.2146	0.1537	1.2314
150	0.0882	0.1335	0.2196	0.1580	1.2677
200	0.0764	0.1151	0.2196	0.1586	1.2734

For sample sizes $n=30$, 50, 75, 100, 150, and 200, 10000 random samples are generated by simulation from the following distributions : Weibull(2, 3), Weibull(1, 1.5), gamma(0.5, 1), gamma(2, 1), normal(2, 5), log−normal(1, 0.5), half−normal(0, 5), beta(2, 3), beta(2, 2). The IMSL subroutines used to generate the random data are GGWIB for Weibull, GGAMR for gamma, GGNML for normal, GGNLG for log−normal, and GGBTR for beta distribution. These distributions are usually considered as practical alternatives to the exponential distribution in the literature of reliability data analysis such as Lawless(1982).

The proposed test procedure and five other well−known competitors are performed, at the 5% significance level, on each of the random data generated. The power results are the percentage of 10,000 Monte Carlo data sets declared significant by the test statistics at the 5% significance level. For sample sizes less than 30, power comparisons are not presented since the critical values for L1 yield smaller significance probabilities than the specified significance levels.

Table 6. Monte Carlo Power Estimates of Six Test Statistics at the 5% Significance Level under Some Alternatives

Alternative distributions	Test statistics					
	KS	K	CM	W	AD	L1
			Sample size $n=30$			
Exponential[null]	.0499	.0499	.0500	.0499	.0499	.0480
Weibull(2, 3)	.7390	.7388	.8406	.7634	.8317	.8524
Weibull(1, 1.5)	.3787	.3504	.4693	.3813	.4530	.4764
Gamma(0.5, 1)	.5411	.4561	.6139	.5058	.6996	.6033
Gamma(2, 1)	.3051	.2792	.3794	.3086	.3685	.3806
Normal(2, 5)	.9651	.9699	.9862	.9722	.9851	.9880
Log−normal(1, .5)	.4860	.4670	.5666	.5084	.5599	.5656
Half−normal(0, 5)	.2092	.1949	.2734	.2099	.2535	.2817
Beta(2, 3)	.7779	.8132	.8909	.8180	.8903	.9068
Beta(2, 2)	.8960	.9436	.9653	.9302	.9668	.9742
			Sample size $n=50$			
Exponential[null]	.0499	.0499	.0498	.0500	.0499	.0490
Weibull(2, 3)	.9520	.9506	.9826	.9602	.9814	.9847
Weibull(1, 1.5)	.6470	.6011	.7491	.6396	.7469	.7586
Gamma(0.5, 1)	.8000	.7160	.8502	.7592	.9133	.8457
Gamma(2, 1)	.5473	.5001	.6310	.5449	.6323	.6404
Normal(2, 5)	.9992	.9994	.9999	.9996	.9999	1.0000
Log−normal(1, .5)	.7713	.7435	.8248	.7867	.8274	.8227
Half−normal(0, 5)	.3586	.3264	.4419	.3395	.4194	.4555
Beta(2, 3)	.9648	.9778	.9934	.9763	.9940	.9955
Beta(2, 2)	.9955	.9986	.9996	.9980	.9996	.9998
			Sample size $n=75$			
Exponential[null]	.0500	.0498	.0503	.0500	.0499	.0490

Table 6. (Continued)

Alternative distributions	Test statistics					
	KS	K	CM	W	AD	L1
Weibull(2, 3)	.9973	.9971	.9998	.9981	.9998	.9998
Weibull(1, 1.5)	.8628	.8233	.9255	.8581	.9280	.9314
Gamma(0.5, 1)	.9421	.8963	.9621	.9211	.9850	.9598
Gamma(2, 1)	.7825	.7356	.8563	.7869	.8631	.8564
Normal(2, 5)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Log-normal(1, .5)	.9407	.9241	.9626	.9487	.9657	.9589
Half-normal(0, 5)	.5308	.4785	.6478	.5101	.6249	.6623
Beta(2, 3)	.9984	.9995	.9999	.9996	1.0000	1.0000
Beta(2, 2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
			Sample size $n=100$			
Exponential[null]	.0499	.0499	.0500	.0498	.0500	.0495
Weibull(2, 3)	.9999	.9999	1.0000	.9999	1.0000	1.0000
Weibull(1, 1.5)	.9515	.9387	.9831	.9524	.9839	.9847
Gamma(0.5, 1)	.9858	.9692	.9928	.9794	.9982	.9916
Gamma(2, 1)	.9074	.8830	.9494	.9103	.9550	.9487
Normal(2, 5)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Log-normal(1, .5)	.9853	.9825	.9932	.9894	.9940	.9928
Half-normal(0, 5)	.6702	.6385	.7979	.6596	.7773	.8084
Beta(2, 3)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Beta(2, 2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
			Sample size $n=150$			
Exponential[null]	.0500	.0500	.0499	.0500	0.500	.0496
Weibull(2, 3)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Weibull(1, 1.5)	.9956	.9936	.9989	.9952	.9992	.9991
Gamma(0.5, 1)	.9993	.9975	.9997	.9984	.9999	.9996
Gamma(2, 1)	.9873	.9785	.9957	.9868	.9964	.9961
Normal(2, 5)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Log-normal(1, .5)	.9995	.9994	.9997	.9996	.9997	.9996
Half-normal(0, 5)	.8513	.8301	.9353	.8392	.9291	.9440
Beta(2, 3)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Beta(2, 2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 6. (Continued)

Alternative distributions	Test statistics					
	KS	K	CM	W	AD	L1
			Sample size $n=200$			
Exponential[null]	.0498	.0500	.0499	.0500	.0500	.0500
Weibull(2, 3)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Weibull(1, 1.5)	.9995	.9994	.9999	.9995	1.0000	.9999
Gamma(0.5, 1)	1.0000	.9999	1.0000	1.0000	1.0000	1.0000
Gamma(2, 1)	.9980	.9968	.9994	.9986	.9997	.9995
Normal(2, 5)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Log−normal(1, .5)	.9998	.9998	.9999	.9999	.9999	1.0000
Half−normal(0, 5)	.9462	.9410	.9848	.9393	.9835	.9880
Beta(2, 3)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Beta(2, 2)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The results of power comparisons for finite sample sizes reveal that the proposed test is superior or comparable in power properties to other tests except for the cases of gamma(0.5, 1) and log−normal(1, 0.5) distributions. Anderson−Darling test appears to be the most powerful for these two distributions.

5. Concluding Remarks

The proposed goodness−of−fit test based on the L_1−norm always yields conservative test results, particularly for small sample sizes since the test statistic is a discrete random variable. In spite of this property, it has relatively high power for moderate and large sample sizes, compared with several commonly used EDF tests under certain alternative distributions. And it requires no more computation than some other tests. In these respects this test procedure could be efficiently applied, in particular, to the practical situations where the exponentiality of data plays a crucial role in the analyses or assessments.

References

[1] Epstein, B.(1960), "Tests for the Validity of the Assumption that the Underlying Distribution of Life is Exponential: Part II," *Technometrics*, Vol. 2, 167-183.

[2] Lawless, J.F.(1982), *Statistical Models and Methods for Lifetime Data*, New York: John Wiley & Sons.

[3] Lee, S.C.S., Locke, C., and Spurrier, J.D.(1980), "On a Class of Tests of Exponentiality," *Technometrics*, *Vol. 22*, *547-554*.

[4] Lilliefors, H.W.(1969), "On the Kolmogorov−Smirnov Test for the Exponential Distribution with Mean Unknown," *Journal of the American Statistical Association*, Vol. 64, 387-389.

[5] Montagne, E.R. and Singpurwalla, N.D.(1985), "Robustness of Sequential Exponential Life−Testing Procedures," *Journal of the American Statistical Association*, Vol. 80, 715-719.

[6] Spinelli, J.J. and Stephens, M.A.(1987), "Tests for Exponentiality When Origin and Scale Parameters Are Unknown," *Technometrics*, Vol. 29, 471-476.

[7] Stephens, M.A.(1978), "On the W test for Exponentiality with Origin Known," *Technometrics*, Vol. 20, 33-35.

[8] Stephens, M.A.(1986), "Tests for the Exponential Distribution," in *Goodness−of−Fit Techniques*, eds. R.B. D'Agostino and M.A. Stephens, New York: Marcel Dekker, pp.421-459.

[9] U.S. Department of Defense(1986), *Reliability Testing for Engineering Development*, *Qualification*, *and Production*(MIL−STD−781), Washington, DC.

모수가 미지인 상황에서의 지수분포성 적합도 검정방법

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요 약

본 논문은 척도모수와 위치모수가 알려지지 않은 상황에서의 지수분포성 적합도 검정문제를 다루고 있다. 기존의 검정방법들과는 달리 누적분포 함수와 경험분포 함수 사이의 편차의 L_1-norm에 바탕을 둔 새로운 검정방법이 제시되었으며, Monte Carlo 방법에 의하여 검정통계량의 임계치를 구하였다. 그리고 표본의 크기가 작은 경우에 한하여 제시된 검정통계량의 분포가 파악되었다. 한편 이 검정방법의 검정력을 기존의 검정방법들과 비교하기 위하여 응용분야에서 흔히 사용되는 몇가지 분포형태에 대하여 검정력을 측정하였다. 그 결과, 새로운 검정방법이 보수적인 검정임에도 불구하고 다른 검정방법에 비하여 상대적으로 검정력이 우수한 것으로 나타났다.

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