A Damage Model for Reinforced Concrete Members

철근콘크리트 부재의 손상모델

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요 약

반복하중에 의한 철근 콘크리트 부재의 실제 거동에 관한 기본적인 현상들을 면밀히 조사 연구한 후 Low-Cycle 피로(fatigue) 재료인 콘크리트의 손상모델식을 제시하였다. 제안된 모델은 콘크리트가 파괴에 도달할 시의 하중 싸이클수 대신에 부재의 흡수에너지 능력을 주요변수로 택하였으며 본 모델의 정확성은 기 제시된 반복하중에 의한 콘크리트 부재의 해석적인 이력모델[3]을 사용하여 예증하였다.

Abstract

Many different damage models have been proposed for concrete in the past. Most of these are not well suited to predict the residual strength of damaged RC members. This paper reviews some basic facts about concrete damage and uses these to systematically model damage as a low-cycle fatigue phenomenon. Instead of the number of load cycles to failure the energy dissipation capacity of a member is taken as the main variable, which depends on many different factors. The model is capable of simulating reasonably well the strength and stiffness degradation of RC members subjected to strong cyclic loads.

1. Introduction

As concrete is subjected to loading of increasing intensity, it undergoes different phases of damage, from microcracking up to ultimate failure. It is necessary to simulate this process mathematically to accurately predict the residual capacity of damaged RC structures to resist further load. Of particular concern are members that have been subjected to several damaging load cycles of a major earthquake.

A considerable effort has been expanded by many researchers to develop models of concrete damage. In recent study, Chung et al[3] have critically evaluated 17 such models, many of which are either of an empirical nature or were derived originally for metal structures. Most of these models are not well suited to predict the residual strength of damaged concrete members.

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It is the purpose of this paper to review some basic facts about concrete damage and to use this knowledge to systematically construct a new model that is capable of simulating reasonably well the strength and stiffness degradation that accompanies the damage process. Such a model may form the basis for rational seismic risk evaluation of concrete structures, and then give an effective tool to develop an advanced seismic design of RC structures.

2. Damage and Failure of RC Members

The concept of damage is encountered in many different areas of concrete engineering. Defining failure as just an extreme case of damage, it is apparent that the degree of sustained damage is vital factor for both safety and serviceability of a structure. Thus, if a damage index is to be used as a measure of a structure's reliability, it is important that the definition of such a damage index not only be free of any subjective influences, but be based entirely on mechanistic principles. Herein, damage of a RC member shall be defined to signify degree of physical degradation with clearly spelled out consequences regarding the member's capacity to resist further loads.

It is useful to regard the process of damage accumulation under repeated load applications as a fatigue phenomenon. In contrast to metals, concrete is known not to have a fatigue limit, at least not up to 10^7 loading cycles[7]. But our interest here is limited to low-cycle fatigue behavior of concrete. In this case it is expedient to measure the fatigue life not in terms of number of load cycles to failure but rather by the total energy dissipation capacity. In this way it is possible to define the degree of damage as the energy dissipated as a fraction of the total energy dissipation capacity. Failure would correspond to the complete exhaustion of this capacity. It is worth mentioning that the definition of either failure or total energy dissipation capacity are somewhat arbitrary. For example, several authors have suggested to define failure as the point at which the residual strength of a member has dropped below 75% of the initial yield strength[1,6,8]. Fig 1 illustrates the typical response of a reinforced concrete cantilever beam to progressively increasing load cycles[6]. As can be clearly seen, the stiffness of the member decreases gradually, once the yield capacity of the member has been exceeded. It takes a significant further increase in loading until the strength deteriorates as well, i.e. when the force necessary to cause a given tip deflection decreases in subsequent cycles. Fig 1 also demonstrates the difficulty of defining failure, because even beyond the specified strength drop of say 25%, the apparent residual strength may increase with further displacement increase.

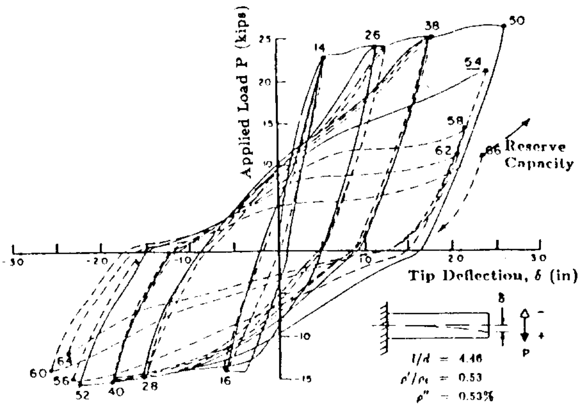

Fig 2 – Typical Inelastic Response of RC Member Tested by Ma et al[6]
(1in = 2.54cm ; 1Kip = 4.448KN)

Each load cylce inflict a certain amount of irreversible damage and can be compared to the passage of some time unit of the life span of the material. If the material is subjected to a history of varying stress levels, it is common practice to utilize Miner's hypothesis,

$$\sum_i \frac{n_i}{N_i} = 1 \tag{1}$$

Where N_i is the number of cycles with stress level S_i leading to failure, and n_i is the number of cycles with stress level S_i actually applied. This Eq(1) assumes that the accumulation of damage is linear and independent of the loading sequence. Neither assumption is supported by experimental evidence. Both the damage accumulation rate and the total energy dissipation capacity are functions of numerous variables : the cross sectional dimensions, concrete material properties, reinforcing steel ratios, confinement steel, shear span ratio, load level and loading sequence. Other influence factors which are more difficult to quantify are the reinforcement detailing and workmanship. All factors which enhance the member's ductility can in general be considered to also increase its energy dissipation capacity. But for the establishment of a clear functional relationship not enough experimental data are available. Until such supporting data are generated, it is necessary to rely on the conceptual model presented herein, calibrated against the few experiments reported in the literature.

3. Previous Damage Models

Numerous models have been proposed in the past to represent damage of structural members or entire structures. Some of these were derived for metal structures. Because of fundamental differences between reinforced concrete and homogeneous materials such as steel, these models are not directly applicable to reinforced concrete. Other models are based on empirical damage definitions[9]. These all but disregard the mechanics of the materials involved when subjected to cyclic load, and therefore do not lend themselves to rational predictions of the strength reserve and response characteristics of a structure with a specified degree of damage.

Several investigators have introduced energy indices which are functions of a few selected parameters[4]. Other notable examples are the damage ratio, $D_R = \frac{K_o}{K_r}$ introduced by Lybas and Sozen[5], where K_o = initial stiffness, and K_r = reduced secant stiffness associated with maximum displacement. Such a simple damage definition obviously ignores many important factors. Banon[2] defined the flexural damage ratio and normalized dissipated eneygy as basic damage state variables which proved to be very useful for reliability computations. Of the more recent damage models, the widely cited model of Park and Ang[8] should be noted,

$$D_e = \frac{\delta_{max}}{\delta_u} + \frac{\beta}{Q_y\delta_u}\int dE \qquad (2)$$

Where δ_{max} = maximum deformation experienced so far, δ_u = ultimate deformation under monotonic loading, $\beta = (-0.357 + 0.73\frac{L}{d} + 0.24N_o + 0.314P_t)\,0.7^{\rho_\omega}$ with $\frac{L}{d}$ = shear span ratio, No = normalized axial force, ρ_ω = confinement ratio, P_t = longitudinal steel ratio, and dE = dissipated energy increment. Although widely quoted in the literature, this damage index exhibits elements of arbitrariness, such as the linear combination of the ductility ratio and the energy ratio. Also, the β lacks a sound physical basis. It is the result of an extensive regression analysis rather than of a rational evaluation of the individual influence factors.

4. A New Damage Model

4.1 Stiffness Degradation

Under load reversals, a RC member experiences a progressive stiffness due to concrete cracking and bond deterioration of the steel-concrete interface primarily in the plastic hinge. The model of Roufaiel and Meyer[10] is used to simulate this behavior. It takes into account the finite size of plastic regions. Fig 2 illustrates the various branches of hysteretic behavior : 1) elastic loading and unloading ; 2)

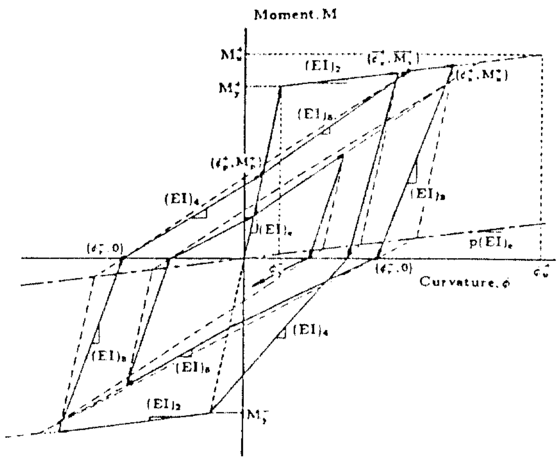

Fig 2 – Typical Hysteretic Moment – Curvature Curve inelastic loading; 3) inelastic unloading; 4) inelastic reloading during closing of cracks; and 5) inelastic reloading after closing of cracks. In a reversed load cycle with high shear, previously opened shear cracks tend to close, leading to an increase in stiffness and a characteristic "pinched" shape of the moment-curvature. Roufaiel and Meyer have modeled this effect by introducing the "crack-closing" moment M'_p, associated with curvature ϕ'_p. If shear stresses are negligible and the hysteresis loops are stable during cyclic loading, no pinching is likely to occur and branches 4 and 5 of Fig 2 will form a single straight line.

4.2 Strength Degradation

In addition to stiffness degradation, RC members experience strength degradation under cyclic loading beyond the yield level, which depends on many variables, such as the degree of confinement and the magnitude of axial force. Atalay and Penzien[1] had noticed some correlation between commencement of strength degradation and the spalling of the concrete cover. But Hwang's experiments[4] showed that strength degradation can start at considerably lower load levels. Even for loads slightly above the yield level, damage and strength degradation can be observed, provided a sufficiently large number of load cycles is applied. It is, therefore, suggested that strength degradation is initiated as soon as the yield load level is exceeded, and the strength degradation accelerates as the critical load level is reached. For this purpose, a strength drop, $\triangle M_i$, in a single load cycle up to curvature ϕ_i is determined as

$$\Delta M_i = \left(\frac{\phi_i - \phi_y}{\phi_f - \phi_y}\right)^{\omega} \times \Delta M_f \qquad (3)$$

where $\triangle M_f = (\phi_f - \phi_y)p(EI)_e + M_y - M_f$ = fictitious moment capacity reduction under monotonic loading up to failure curvature ϕ_f, and ϕ_y = the yield curvature under monotonic loading[3]. For analysis purposes, the strength drop is measured from the second branch of the primary moment-curvature curve, Fig 3. Even though the parameter ω is a function of confinement steel, axial force, and other important factors, studies have shown[3] that a value between 1.5 and 2.0 leads to good correlation with experimental results. With $\triangle M$ denoting the strength drop in the first subsequent load cycle for some curvature ϕ, the residual strength, $m_1(\phi)$, after this load cycle is given by

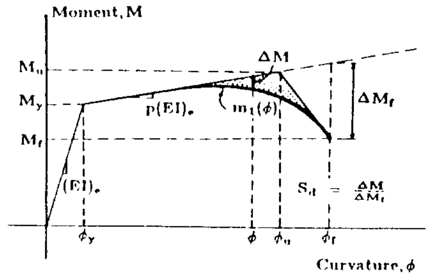

Fig 3 – Strength Deterioration Curve

$$m_1(\phi) = M_y + (\phi - \phi_y)p(EI)_e - \triangle M \qquad (4)$$

In order to incorporate this concept of strength deterioration into the hysteresis model, an imaginary point with coordinates (ϕ_x, M_x), is introduced, at which the load-deformation curve is aimed during reloading, Fig 2. Details are given in Ref[3].

4.3 Definition of Failure

For RC members undergoing cyclic loading, several investigators [1,4,7] have defined failure as the point where the member strength (moment) has dropped below 75% of the initial yield strength (moment). But if the member is subsequently loaded beyond this maximum displacement or curvature, its moment can be observed to increase well above the 75% level[4], even though it has already been assumed to have "failed" [Fig 1]. For this reason it is necessary to relate the failure definition to the actual strength reserve or residual strength, which is a function of the experienced loading history.

First, the failure moment M_f and the corresponding curvature ϕ_f is defined to be the curvature at which the concrete's crushing strain is reached. Given the complete stress-strain curves for steel and concrete and the cross-sectional dimensions, it is relatively straightforward to compute the monotonic moment-curvature curve, by determining the moment M_i associated with any curvature, ϕ_i [3]. A failure moment for some arbitrary curvature level ϕ_i is then defined as

$$M_{fi} = M_f \frac{2\Phi_i}{\Phi_i + 1.0} \qquad (5)$$

where M_{fi} = failure moment for given curvature level, ϕ_i M_f = failure moment for monotonic loading, $\Phi_i = \frac{\phi_i}{\phi_f}$ (curvature ratio), and ϕ_f = failure curvature natio for monotonic loading. According to Fig 4, the failure moment M_{fi} decreases with smaller curvature levels ϕ_i, i.e. larger strength drops from the monotonic loading curve are needed to lead to failure. If the total strength drop down to the failure moment M_{fi} at some curvature ϕ_i is known, the number of cycles for this curvature level needed to cause failure, can be determined.

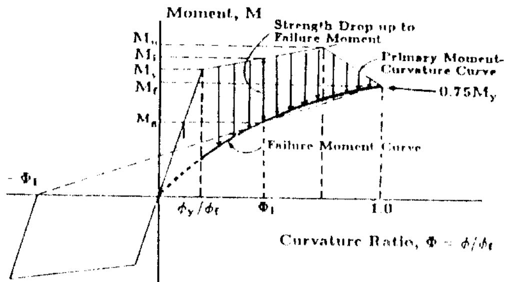

Fig 4 - Definition of Failure

4.4 New Danage Index

Based on the above definition of failure, a new damage index, D_e is proposed as a measure of damage sustained by RC members undergoing inelastic cyclic loading. It combines a modified Miner's hypothesis with damage modifiers, which reflect the effect of the loading history, and it considers the fact that RC members typically respond differently to positive and negative moments :

$$D_e = \sum_i \left(\alpha_i^+ \frac{n_i^+}{N_i^+} + \alpha_i^- \frac{n_i^-}{N_i^-} \right) \qquad (6)$$

where i = indicator of displacement (curvature) level, $N_i = \frac{M_i - M_{fi}}{\triangle M_i}$ = number of cycles to cause failure at curvature level i, n_i = number of cycles actually applied at curvature level i, α_i = damage modifier, and + and − are indicators of loading sense. The loading history effect is captured by including the damage modifier α_i, which for positive moment loment loading is defined as

$$\alpha_i^+ = \frac{\sum K_{ij}^+}{n_i^+ \overline{K_i^+}} \frac{\phi_i^+ + \phi_{i-1}^+}{2\phi_i^+}$$

$$= \frac{M_{i1}^+ - 0.5(n_i^+ - 1)\triangle M_i^+}{M_{i1}^+ - 0.5(N_i^+ - 1)\triangle M_i^+} = \frac{\phi_i^+ + \phi_{i-1}^+}{2\phi_i^+} \qquad (7)$$

where $K_{ij}^+ = \frac{M_{ij}^+}{\phi_i^+}$ is the stiffness during the j-th cycle up to the load level i, $\overline{K_i^+} = \frac{\sum K_{ij}^+}{N_i^+}$ is the average stiffness during N_i^+ cycles up to load level i, and $M_{ij}^+ = M_{i1}^+ - (j-1)\triangle M_i^+$ is the moment reached after j cycles up to load level i, Fig 5. The definition of Eq(7) needs some explanation. The energy that is dissipated during a single cycle up to a given curvature level decreases for successive cycles. That means the damage increment also decrease. In a constant amplitude loading sequence, the first load cycle will cause more damage than the last one. Therefore, the α_i factor decreases as load cycling proceeds, being a function of the stiffness ratio.

The factor $\frac{\phi_i^+ + \phi_{i-1}^-}{2\phi_i^+}$, is necessary to normalize the damage increments in the case of changing load amplitudes. For negative loading, the damage mo-

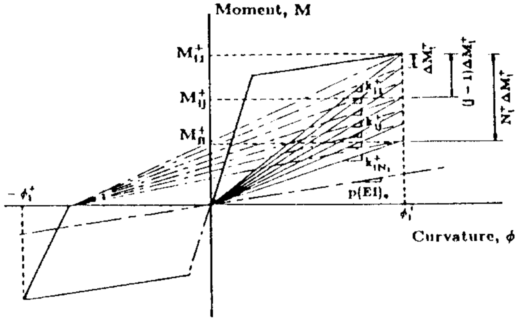

Fig 5 – Strength Drop Due to Cyclic Loading

difer is defined similarly.

To illustrate the accuracy, with which the proposed mathematical model of Eq (6) can simulate hysteretic response of RC members, many experimental results have been reproduced numerically in Ref [3]. The numerical analysis has been performed by all efficient algorithm on the Gaussian elimination method. Since the most of the laboratory experiments to be simulated numerically were displacement-controlled and of guasi-static nature, the effects of inertia and damping were ignored in the analyses. Fig 6 represents an example that is typical for the kind of agreement achieved. Table 1 contains the cumulative damage indices computed for the same specimens tested by Hwang[4] and Ma et al[6]. It is noteworthy that in all but the last case the damage index computed after test termination correlates reasonably well with 1.0, with which corresponds to our definition of failure. In some cases, the testing proceeded well beyond this point, e.g. Specimen S22. This means that testing had continued beyond the point of (artificially defined) failure. Other specimens, most notable B35, appear not to have failed at the time the test was terminated.

5. Conclusions

Any attempts at devising mathematical models to quantify damage in a rational way should set out with a clear and precise definition of damage, because "damage" is a widely used word, that describes all kinds of different phenomena and is prone to subjective interpretation. A new damage model and associated damage index have been developed which are believed to be more rational than previously proposed models and take into account factors such as loading sequence which are usually ignored. An accurate determination of damage is essential for meaningful nonlinear dynamic analysis of concrete structures, because the damage index is closely tied to the strength reserve of a member, after it has undergone large inelastic cycles. To systematically model the damage for reinforced concrete as a low-

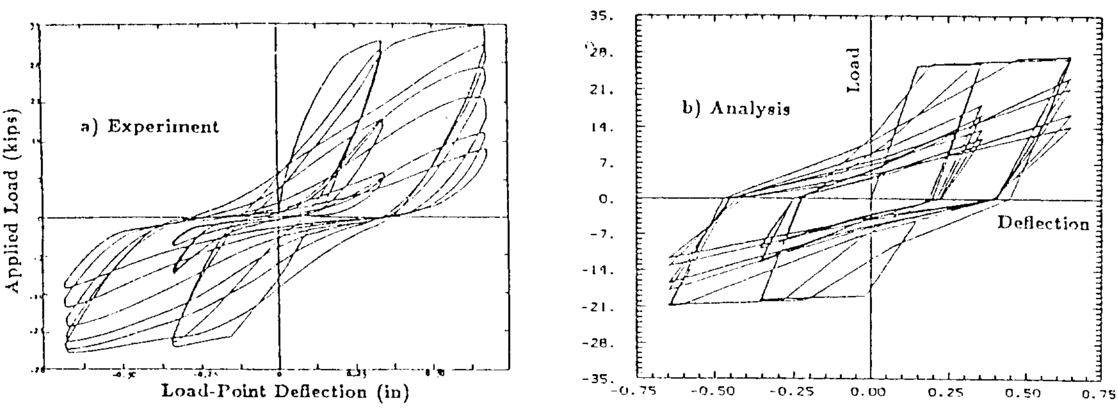

Fig 6 – Experimental and Analytic Load – Deformation Curves for Beam S2 – 3 Tested by Hwang[4]
(1in=2.54cm ; 1Kip=4.448KN) KN)

Table 1 – Numerical Cumulative Damage Indices

No. of Cycles	Specimen									
	S12	S14	S22	S23	S24	S32	S33	S34	R5	B35
1	0.0254	0.0569	0.0797	0.0210	0.0664	0.0722	0.0059	0.0461	0.0041	0.0234
2	0.1250	0.2227	0.2597	0.0685	0.2278	0.2537	0.0225	0.1846	0.0154	0.0697
3	0.2485	0.2644	0.4513	0.1808	0.2863	0.4601	0.0870	0.2234	0.0389	0.1733
4	0.3740	0.2937	0.6422	0.3546	0.3262	0.6545	0.1956	0.2494	0.1043	0.3379
5	0.4871	0.4589	0.8220	0.4089	0.4763	0.8313	0.2156	0.3863	0.1876	0.5816
6	0.5003	0.6439	0.9958	0.4513	0.6453	1.0003	0.2303	0.5445	0.2745	
7	0.7016	0.6794	1.1557	0.5957	0.6959	1.1436	0.3271	0.5740	0.3005	
8	0.8019	0.7040	1.3018	0.7490	0.7296	1.2613	0.4360	0.5993	0.4509	
9	0.9035	0.8326	1.4336	0.7971	0.8552		0.4533	0.6226	0.6354	
10	0.9935	0.9718	1.5552	0.8336	0.9943		0.4661	0.7271	0.8524	
11	1.0721	0.9978		0.9565			0.5491	0.8438	1.0771	
12	1.1395	1.0156		1.0844			0.6392	0.8722	1.3877	
13	1.1960	1.1044					0.6540	0.8910		
14	1.2435	1.1976					0.6647	0.9790		
15	1.2837	1.2142					0.7307			
16	1.3215	1.2252					0.8029			
17	1.3600	1.2837								
18		1.2025								

cycle fatigue phenomenon, the energy dissipation capacity of a member is taken as the main variable. Together with an advanced hysteresis model of frame member behavior, this model is capable of accurately reproducing the load-deformation bahavior of RC members subjected to cyclic load.

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6. References

1) Atalay, M. B. and Penzien, J., "The Seismic Behavior of Critical Regions of Reinforced Concrete Components as Influenced by Moment, Shear and Axial Forces, " Report No. EERC-75-19, Univ. of California at Berkeley, CA, 1975.

2) Banon, H. and Veneziano, D., "Seismic Safety of Reinforced Concrete Members and Structures," Earthq. Engr. and Struct. Dynamics, Vol. 10, 1982.

3) Chung, Y. S., Meyer, C. and Shinozuka, M., "Seismic Damage Assessment of Reinforced Concrete Members," Report No. NCEER-87-22, Nation Center for Earthq. Engr. Research, Buffalo, NY, Oct. 1987.

4) Hwang, T. H., "Effects of Variation in Load History on Cyclic Response of Concrete Flexural Members," Ph.D. Thesis, Dep't of Civil Engr., Univ. of Illinois, Urbana, IL, 1982.

5) Lybas, J. and Sozen, M.A., "Effects of Beam Strength and Stiffness on Dynamic Behavior of Reinforced Concrete Coupled Walls," Civil Engr. Studies, SRS No. 444, Univ. of Illinois, Urbana, IL, Jul. 1977.

6) Ma, S-Y. M., Bertero, V.V. and Popov, E.P., "Experimental and analytical Studies on the Hystere-

tic Behavior of Reinforced Concrete Rectangular and T-Beam," Report No. EERC-76-2, Univ. of California at Berkeley, CA, 1976.

7) Mindess, S. and Young, J.E., "Concrete," Prentice-Hall Inc, Englewood Cliffs, 1981.

8) Park, Y.J., Ang, A.H-S. and Wen, Y.K.,"A Mechanistic Seismic Damage Model for Reinforced Concrete," Journal of Struct. Engr., ASCE, Vol. 111, No. 4, Apr. 1985.

9) Reitherman, R., "Review of Earthquake Damage Estimation Methods," EERI, Earthq. Spectra, Vol. 1, No.4, 1985.

10) Roufaiel, M.S.L. and Meyer, C., "Analytical Modeling of Hysteretic Behavior of R/C Frames," Journal of Struct. Engr., ASCE, Vol. 113, No. 3, Mar. 1987.