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플란지 평행판 복사의 점근 수열 해
(An Asymptotic Series Solution for the Flanged-Waveguide Radiation)

박 타 준* · 엄 효 준*
(Tah J. Park, Hyo J. Eom)

要 約

플란지 평행판 도파관의 복사문제를 고찰한다.
푸리에 변환 기법을 사용하여 복사장을 주파수 영역에 표시한 다음, 근사수열로써 복사장을 구한다. 구해진 복사장을 다른 기법에의해서 구해진 해와 비교하여서, 기존 해의 타당성을 검토한다.

Abstract

The problem of radiation from a flanged parallel-plate waveguide is re-examined. The technique of the Fourier transform is used to represent the radiation fields in the spectral domain. The simultaneous equations for the radiation field coefficients are formulated and solved to give an asymptotic seolution. The asymptotic series solution is compared with other results, thus clarifying the discrepancy among different numerical approaches.

I. Introduction

The subject of radiation from a flanged parallel-plate waveguide has long received a great deal of attention due to its important applications in antenna engineering. Representative theoretical investigations on the flanged waveguide radiation problem include the variational approach[1], the ray optics methods[2], the Fourier transform method[3], the mode-matching technique[4], the Weber-Schafheitlin integral technique[5], and the finite element method[6]. More recently, the technique of the moment method[7] has been used to clarify some numerical discrepancies existing among the different techniques[8].

In this paper, we re-examine the problem of the flanged waveguide radiation by utilizing the Fourier transform the mode-matching technique. We formulate a solution to the flanged-waveguide-radiation problem in an asymptotic series representation which allows us to make a comparison to other solutions, thus clarifying the discrepancy among different numerical approaches. In the next section, we present the expression of the radiation field from the flanged waveguide and investigate its behavior. A brief summary on the thoeretical development is given in Concluding Remarks.

*韓國科學技術院 電氣 및 電子工學科

Ⅱ. Derivation of Radiation Field

Consider a parallel-plate waveguide of width 2a which has an infinite flange as shown in Fig. 1. Here, $exp(-jwt)$ time-harmonic variation is assumed throughout. Assume that a TE_p-mode E_y^i is incident on the aperture at $z=0$ from inside the parallel-plate waveguide(region(Ⅱ), wave number$=k$). Then, the incident and the reflected electric fields inside the waveguide are respectively written as

$$E_y^i(x,z) = \sin a_p(x+a)exp(j\xi_p z)$$
$$E_y^r(x,z) = \sum_{m=1}^{\infty} c_m \sin a_m(x+a)exp(-j\xi_m z)$$
$$where \quad \xi_m = \sqrt{k^2-a_m^2}, \qquad a_m = m\pi/(2a)$$

Note that for odd $p,m=1,3,5,\ldots$, and for even $p,m=2,4,6\ldots$

Outside the waveguide(region(Ⅰ), wave number$=k_0$), we represent the transmitted electric field in the spectral domain ζ such as:

$$E_y^t(x,z) = 1/(2\pi)\int_{-\infty}^{\infty} \tilde{E}_y^t(\zeta)e^{-j\zeta x+jk_1 z}\,d\zeta$$
$$where \quad \tilde{E}_y^t(\zeta) = \int_{-\infty}^{\infty} E_y^t(x,0)e^{j\zeta x}\,dx \qquad (2.1)$$

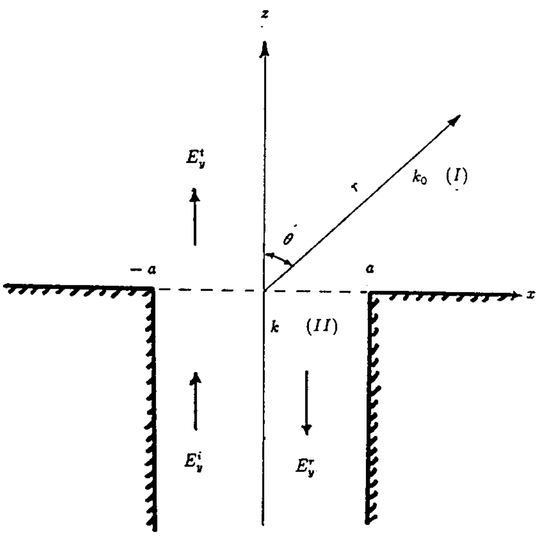

Fig. 1. Ceometry of Waveguide with a Flange

$$k_1 = \sqrt{k_0^2-\zeta^2}$$

Note that $\tilde{E}_y^t(\zeta)$ and $E_y^t(x,0)$ are the Fourier-transformed pair.

Since $H_x(x,z)=-1/(jw\mu)\partial E_y(x,z)/\partial z$, the corresponding x components of the incident, the reflected, and the transmitted H-fields are respectively

$$H_x^i(x,z) = -\frac{\xi_p}{w\mu}\sin a_p(x+a)exp(j\xi_p z)$$
$$H_x^r(x,z) = \sum_{m=1}^{\infty}\frac{\xi_m c_m}{w\mu}\sin a_m(x+a)exp(-j\xi_m z)$$
$$H_x^t(x,z) = \frac{-1}{jw\mu}\frac{\partial}{\partial z}\int_{-\infty}^{\infty}\frac{1}{2\pi}\tilde{E}_y^t(\zeta)e^{-j\zeta x+jk_1 z}\,d\zeta$$

To determine unknown coefficient c_m, it is necessary to match the boundary conditions on tangential E- and H-field continuities. First, the tangential E-field continuity along the x-axis $(-\infty<x<\infty, z=0)$ yields

$$E_y^t(x,0) = E_y^i(x,0)+E_y^r(x,0) \qquad |x|<a$$
$$= 0 \qquad |x|<a$$

Taking the Fourier transform on $E_y^t(x,0)$, we get

$$\tilde{E}_y^t(\zeta) = \int_{-\infty}^{\infty} E_y^t(x,0)e^{j\zeta x}\,dx$$
$$= \int_{-a}^{a}[\sin a_p(x+a)+\sum_{m=1}^{\infty} c_m \sin a_m(x+a)]e^{j\zeta x}\,dx$$
$$= K_p(\zeta)+\sum_{m=1}^{\infty} c_m K_m(\zeta)$$
$$where \quad K_m(\zeta) = \frac{a_m}{(\zeta^2-a_m^2)}[e^{j\zeta a}(-1)^m - e^{-j\zeta a}]$$

Second, the tangential H-field continuity along the aperture, $(-a<x<a, z=0)$, gives

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}[K_p(\zeta)+\sum_{m=1}^{\infty} c_m K_m(\zeta)]k_1 e^{-j\zeta x}\,d\zeta$$
$$= \xi_p \sin a_p(x+a)-\sum_{m=1}^{\infty} c_m\xi_m \sin a_m(x+a)$$

Multipling the above equation by $\sin a_n(x+a)$ and integrate the both sides with respect to x from $-a$ to a, then we obtain

$$\frac{1}{2\pi}[I_{pn}+\sum_{m=1}^{\infty} c_m I_{mn}] = \xi_p a\delta_{np}-\xi_n c_n a \qquad (2.2)$$

where δ_{np} represents Kroneker delta and

$$I_{mn} = \int_{-\infty}^{\infty} \frac{a_m a_n[(-1)^m e^{j\zeta a} - e^{-j\zeta a}][(-1)^n e^{-j\zeta a} - e^{j\zeta a}]k_1}{(\zeta^2 - a_m^2)(\zeta^2 - a_n^2)} d\zeta$$

The evaluation of I_{mn} was first studied in the investigation on scattering from a thick slit in the conducting plane in [9], where it is shown that I_{mn} may be converted into a fast convergent integral using the technique of the contour integration. The analytic contour integral evaluation of I_{mn} may be performed in the complex ζ plane as shown in Fig. 2 to give

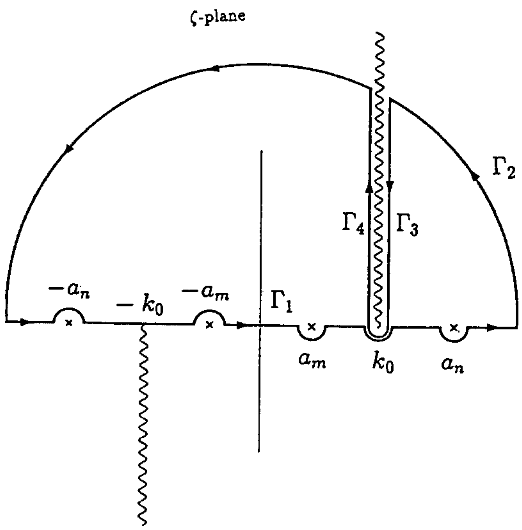

Fig. 2. Diagram of Contour Integration in ζ plane

$$I_{mn} = 2\pi a \eta_m \delta_{mn} - (I_{1mn} + I_{2mn}) \qquad (2.3)$$

where $\eta_m = \sqrt{k_0^2 - a_m^2}$. The first term containing δ_{mn} is a residue contribution at $\zeta = \pm a_m$ whereas I_{1mn}, I_{2mn} arise from the integration along the branch-cut (Γ_3, Γ_4) which is associated with the branch-point at $\zeta = k_0$. The explicit expressions for I_{1mn}, I_{2mn} are given in [10] such as :

$$I_{1mn} = \int_0^\infty \frac{-4j\alpha\beta(-1)^n e^{2jk_0 a} e^{-2k_0 a v}\sqrt{v(-2j+v)}}{[(1+jv)^2 - \alpha^2][(1+jv)^2 - \beta^2]} dv$$

$$I_{2mn} = \int_0^\infty \frac{4j\alpha\beta\sqrt{v(-2j+v)}}{[(1+jv)^2 - \alpha^2][(1+jv)^2 - \beta^2]} dv$$

where $\alpha = a_m / k_0, \quad \beta = a_n / k_0$

Closed forms for I_{1mn}, I_{2mn} are also given in [10] such as :

$$I_{1mn} = -\frac{2\alpha\beta e^{2jk_0 a}(-1)^n}{(\alpha^2 - \beta^2)} \sum_{l=1}^{\infty} S_l[[A(t_1) - A(t_2)]/\alpha - [A(t_3) - A(t_4)]/\beta]$$

$$I_{2mn} = \frac{4j\alpha\beta}{(\alpha^2 - \beta^2)}\left[\frac{\sqrt{1-\alpha^2}}{\alpha} \sin^{-1}\alpha - \frac{\sqrt{1-\beta^2}}{\beta} \sin^{-1}\beta\right]$$

where

$$S_l = \binom{0.5}{l-1}(0.5j)^{l-1.5}$$

$$A(t) = (-1)^l \pi t^{l-0.5} e^{pt} erfc(\sqrt{pt}) + 2^{1-l}\sqrt{\pi} p^{0.5-l} \sum_{r=0}^{l-1} (2l-2r-3)!!(-2pt)^r$$

$$p = 2k_0 a$$

$$t_1 = (\alpha - 1)j, \ t_2 = (-\alpha - 1)j,$$

$$t_3 = (\beta - 1)j, \ t_4 = (-\beta - 1)j$$

$erfc(\ldots)$ denotes the complementary error function.

Note that I_{1mn} is expressed in terms of the asymptotic series of which $l-th$ term has an order of $O(1/(k_0 a)^{l-0.5})$. The series expression for I_{1m} converges for $|2k_0 a/(m\pi)| > 1$. When the operating frequency approaches infinity ($k_0 a \to \infty$), the branch-cut contribution becomes negligible, thus $I_{mn} \to 2\pi a \eta_m \delta_{mn}$.

Substituting I_{mn} given by (2.3) into (2-2), we obtain the simultaneous equations for c_m which may represented in the following series matrix form:

$$C = (U - R)^{-1} Q = Q + RQ + R^2 Q + \ldots \qquad (2.4)$$

where C is the column matrix of elements c_m, U the identity matrix, R the full matrix of elements r_{nm}, and Q the column matrix of elements of q_n. The expressions of r_{nm}, q_n are given as:

$$r_{nm} = \frac{(I_{1mn} + I_{2mn})}{2\pi(\xi_n + \eta_n)a}$$

$$q_n = \frac{(\xi_p - \eta_p)\delta_{np}}{(\xi_n + \eta_n)} + \frac{(I_{1pn} + I_{2pn})}{2\pi a(\xi_n + \eta_n)}$$

The examination of r_{nm} reveals that $r_{nm} \sim O[1/\sqrt{k_0 a}]$ for $k_0 a > 1$, thus c_m may be given as

$$c_m = q_m(1+O[1/\sqrt{k_0 a}])$$

A case of interest is when there is no dielectric discontinuity at the waveguide aperture (*i.e.*, $k=k_0$, $\eta_p=\xi_p$), then

$$q_n = \frac{(I_{1pn}+I_{2pn})}{2\pi a(\xi_n+\eta_n)} \sim O[1/\sqrt{k_0 a}]$$

This means that the reflection coefficient, $|c_m|^2$, is of the order $(1/(k_0 a))$ which is in agreement with the conclusion in [3]. Fig. 3 shows the behavior of c_m versus $2a/\lambda$ for $p=1$ (TE_1 mode). It is seen that $|c_m|<0.1$ when $2a/\lambda>0.7$, thus indicating that total field across the aperture may be approximated with the incident field i.e., $E_y^t(-a<x<a, z=0) \approx E_y^i(-a<x<a, z=0)$. Fig. 4 shows the input impedance Z versus $2a/\lambda$ when $p=1$, where $Z(=R+jX)=[(1+c_1)/(1-c_1)]^*$. The asterisk$(\ldots)^*$ signifies taking the complex conjugation. Note that our computation well agrees with the results in [7], thus reconfirming the numerical accuracy of [7] and [8]. Once the unknown coefficients c_m are determined, it is possible to evaluate the radiation field $E_y^t(x,z)$ which is given by (2.1); for instance, we depict, in Fig. 5, the radiation field pattern near the aperture when $p=1$, $a=0.3\lambda$.

The evaluation of waveguide radiation field in the far-zone is trivial: the farzone transmitted field at distance r from the origin can be evaluated by using the stationary phase approximation in (2.1) where $\theta=\sin^{-1}(x/r)$, $r=\sqrt{x^2+z^2}$.

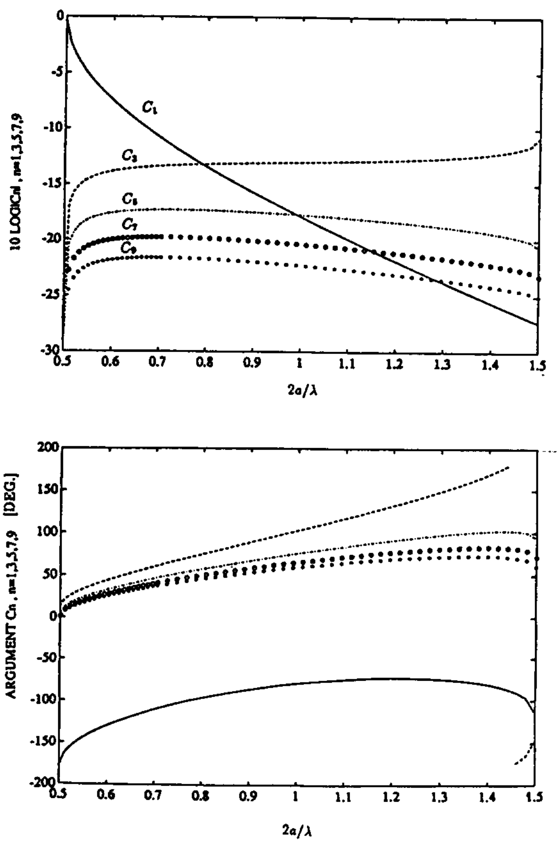

Fig. 3. Behavior of c_m versus $2a/\lambda$ for TE_1 mode ($k_0=k$)

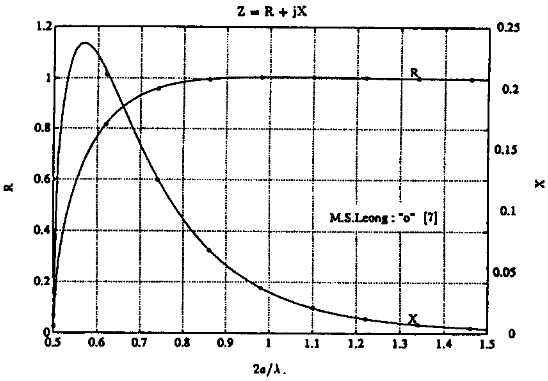

Fig. 4. Input Impedance Z versus $2a/\lambda$ for TE_1 mode ($k_0=k$)

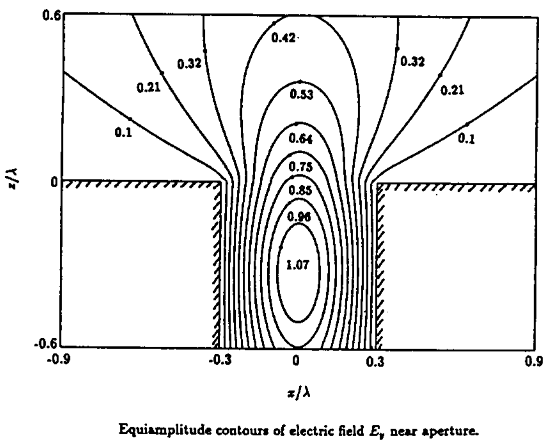

Fig. 5. Equi-amplitude contours of E_y near the aperture for TE_1 mode ($k_0=k$) and $a=0.3\lambda$

$$E_y^t(\theta) = \sqrt{\frac{2k_0}{\pi r}}\, e^{j(k_0 r + 3\pi/4)} \cos\theta \sum_{m=1}^{\infty} (\delta_{mp} + c_m) \frac{a_m \cos(k_0 a \sin\theta)}{((k_0 \sin\theta)^2 - a_m^2)}$$

Ⅲ. Concluding Remarks

Using the Fourier transform and mode-matching approach, we obtain the asymptotic series solution for the radiation from the flanged waveguide. The series solution is compared with other results in order to clarify some ambiguities associated with numerical inaccuracies in the computation of the reflection coefficients. The series solution which is based on (2.4) is exact and very efficient in the numerical computation.

Ⅳ. Acknowledgement

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