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〈연구논문〉

점탄성 용액의 비틀림 흐름에서의 입자의 이동

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Particle Migration in Torsional Flow of a Viscoelastic Solution

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요 약

본 연구에서는 단단한 구형 입자가 적은 변형속도에서 2차 유체로 간주될 수 있는 점탄성 용액의 disc-plate (비틀림)흐름에 놓였을 때의 측면 이동속도에 관하여 분석하였다.

이론적 계산을 통해 비틀림 유체 흐름에서 입자는 항상 중심쪽으로 이동하고 안쪽으로의 이동속도의 크기는 유체의 성질과 흐름의 형태의 함수라는 것을 발견하였으며, 이것은 2차 유체에서 입자는 높은 전단속도 영역에서 낮은 전단속도 영역으로 이동된다는 예측과 일치한다는 것을 알 수 있었다. 그러나 2차 유체 모델로부터의 이 결과는 이전 연구자들에 의해 관찰된 비틀림 유체 흐름에서의 바깥방향으로의 입자의 이동은 설명하지 못하였다.

Abstract—The lateral migration velocity of a rigid single spherical particle is analyzed in this study, when placed in a disc-plate (torsional) flow of a viscoelastic solution which is regarded as a second-order fluid at low deformation rates.

From theoretical calculation, it is found that the particle migrates always toward the center and the magnitude of the inward migration velocity is a function of both fluid properties and flow geometry. The result corresponds with the prediction, i.e. the particle migrates from high shear rate region to low shear rate region in a second-order fluid. However, the result from this second-order fluid model does not show the outward migration of a rigid sphere in torsional flow which has been observed by previous investigators.

Keywords: Particle migration, Second-order fluid model, Torsional flow, Viscoelastic solution

1. Introduction

The motion of a rigid spherical particle in an unbounded Newtonian fluid obeys Faxen's law[1] for low Reynolds numbers, in which the force and torque on the sphere are related to the undisturbed fluid velocity, the translation velocity and the angular velocity of the sphere. If inertia becomes prominent, the particle may undergo lateral migration across streamlines as it translates along a streamline[2]. However, for viscoelastic solutions, lateral migration may occur even in the absence of inertia effects.

Experimental studies on lateral migration of rigid particles in well-defined flow fields of viscoelastic solutions have been also performed extensively by previous investigators[3-11].

In tube flow, it was found that rigid spherical particles which placed near the wall migrated toward the center line and the migration velocity increased with particle size and velocity gradient [3-5]. Karnis *et al.*[3] observed this inward migration in solutions of polyisobutylene in decahydronaphthalene and explained that it may arise from the combined action of normal stresses in the fluid and the variation in the velocity gradient across the particle. In Couette flow, suspended particles form rings coaxial with the cylinder which migrate out of the annulus[4, 6, 7]. It was suggested that since the velocity gradient was greater at the inner cylinder than at the outer, a pressure difference due to the normal stresses would result in a force pushing the spheres toward the outer cylinder. The rate of migration decreases with increasing radial position and decreasing bulk shear rate. In a cone and plate geometry, Highgate[8] reported migration is radially outward.

However, migration in either direction was observed in torsional flow generated from the disc-and-plate geometry[9-11]. When the rigid sphere is located initially inside the critical radius ($r>r_{cr}$), it always migrates radially inward. On the other hand, it migrates radially outward for $r\rangle r_{cr}$. In contrast to the number of experimental papers, any current theory is not available to explain this phenomenon completely.

Some researchers[2,13-18] have conducted the theoretical calculations for second-order fluids which explain some of the experimental observations cited above. Ho and Leal[2] theoretically studied the lateral migration of a neutrally buoyant rigid sphere suspended in a second-order fluid for unidirectional two-dimensional flows and demonstrated the existence of migration induced by normal stresses whenever there is a lateral variation of the shear rate in the undisturbed flow. They assumed any gradients in the deviatoric normal stress components must be balanced by gradients in the pressure so that the net lateral force on any fluid element in the undisturbed flow is precisely zero. Considering the undisturbed bulk flow to be steady, unidirectional and two-dimensional, with a quadratic lateral variation in velocity, they set up the equations of motion of a rigid sphere with a stress tensor for a second-order fluid and also obtained the equations of motion for the undisturbed flow. A disturbance velocity and pressure were defined to be steady and expressed as polynomials with non-Newtonian parameters. Zeroth-order of the polynomial was the equation of motion of a Newtonian fluid and first-order of the polynomial gave the angular and translational velocities of the sphere. Using the reciprocal theorem, they finally obtained the migration velocity of the sphere.

In this study, using the second-order fluid model in the continuum mechanical viewpoint, the particle migration in torsional flow geometry is analyzed.

2. Migration Equation in a Second-Order Fluid

Theoretical analysis of particle migration in a viscoelastic fluid is even more complex than in a Newtonian fluid. This is due to the complexity of the relation of the stress to the deformation of the fluid, and also because of the nonlinear matematical descriptions resulting from the substitution of any trial rheological equation of state. However, the flow of the ambient fluid in this study is steady and so slow that fluid inertia needs not be taken into consideration.

When the corotating rate of strain tensor is expanded in a Taylor series through the n^{th} order in a corotational integral series[12], we obtain an n^{th} order fluid model as follows:

$$\underset{\approx}{\tau}=-\alpha_0\underset{\approx}{\dot{\gamma}}+\alpha_1\frac{\mathcal{D}}{\mathcal{D}t}\underset{\approx}{\dot{\gamma}}-\alpha_2\underset{\approx}{\dot{\gamma}}\cdot\underset{\approx}{\dot{\gamma}}-\alpha_3\frac{\mathcal{D}^2}{\mathcal{D}t^2}\underset{\approx}{\dot{\gamma}}+\cdots \tag{1}$$

where $\underset{\approx}{\dot{\gamma}}=\underset{\sim}{\nabla}\underset{\sim}{v}+(\underset{\sim}{\nabla}\underset{\sim}{v})^T$ is the rate of strain tensor and $\frac{\mathcal{D}}{\mathcal{D}t}$ is the corotational derivative or Jaumann derivative.

Truncating up to second order of eq. (1), we obtain the following second-order fluid model:

$$\underset{\approx}{\tau}=-\eta\dot{\underset{\approx}{\gamma}}+\alpha_1\frac{\mathscr{D}}{\mathscr{D}t}\dot{\underset{\approx}{\gamma}}-\alpha_2\dot{\underset{\approx}{\gamma}}\cdot\dot{\underset{\approx}{\gamma}} \quad (2)$$

Here, η is viscosity and material parameters, α_1 and α_2, are correlated with the material functions as follows:

$$\alpha_1=-\frac{1}{2}\Psi_1,\quad \alpha_2=\frac{1}{2}\Psi_1+\Psi_2 \quad (3)$$

where Ψ_1 is the first normal stress difference coefficient and Ψ_2 is the second normal stress difference coefficient. For this second-order fluid, the viscosity should be constant. It means that the second-order fluid model predicts the shear stress is proportional to the shear rate and that the first normal stress difference (N_1) is proportional to the square of the shear rate as follows:

$$N_1=2\eta\lambda\dot{\gamma}^2 \quad (4)$$

where λ is the relaxation time of polymer solution.

Using this model, Brunn[13-17] and Chan and Leal[18] have developed a rigorous theoretical calculation for the particle migration velocity in a second order fluid. Brunn utilized a perturbation expansion technique for the second order fluid along with the incompressibility assumption to obtain an approximate result. The particle inertia is also neglected, so one obtains the expressions for the translational and rotational velocity of the sphere.

Given the undisturbed fluid velocity $\underset{\sim}{v}$, the governing equations are

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{v}=0$$

$$\underset{\sim}{\nabla}\cdot\underset{\approx}{\tau}=\underset{\sim}{0} \quad (5)$$

where $\underset{\approx}{\tau}$ is the deviatoric stress tensor.

In the limit of small Weissenberg number (We, the ratio of normal stresses to shear stresses), a regular perturbation expansion in powers of We is applied to solve eq. (5). The analysis was carried out as a perturbation expansion around the Newtonian behaviour. This requires the flow to be rheologically slow so that the rheological equation of state was approximated by the equation for a second-order fluid. With the known informations of the Newtonian velocity and pressure fields, the hydrodynamic force and the torque were obatained and the the Brunn finally found the following relationships.

$$\underset{\sim}{\Delta}\equiv\underset{\sim}{v}_p-\underset{\sim}{v}$$

$$=\frac{1}{6}a^2\nabla^2\underset{\sim}{v}$$

$$\underline{-\frac{11}{75}a^2\frac{\alpha_1}{\eta}(1+\frac{14}{11}\beta)\underset{\approx}{E}\cdot\nabla^2\underset{\sim}{v}}$$

$$-\frac{5}{12}a^2\frac{\alpha_1}{\eta}(1+\frac{5}{3}\beta)\underset{\approx}{E}^{(3)}:\underset{\approx}{E}$$

$$+\frac{1}{2}a^2\frac{\alpha_1}{\eta}(1+\frac{1}{9}\beta)\underset{\approx}{\varepsilon}:(\underset{\approx}{E}\cdot\underset{\approx}{\Omega}) \quad (6)$$

where $\underset{\approx}{E}^{(3)}$ is the symmetric irreducible part of $\underset{\sim}{\nabla}\underset{\sim}{\nabla}\underset{\sim}{v}$, $\underset{\approx}{\varepsilon}$ is the alternating unit tensor of rank three, $\underset{\sim}{v}_p$ is the velocity of a rigid sphere, a is the particle radius and $\underset{\approx}{E}$ is defined as a following eq. (15).

The underlined term in eq. (6) differs from that obtained by Chan and Leal [18], who expressed it as follows:

$$-\frac{1}{6}a^2\frac{\alpha_1}{\eta}(1+\beta),$$

$$\text{where } \beta=-2\Psi_2/\Psi_1=1-\frac{\alpha_2}{\alpha_1} \quad (7)$$

And it has been experimentally shown that the ratio of the second to the first normal stress difference is negative and less than 0.2 in magnitude [17]. On the other hand, eq. (6) can rearranged into a following formula:

$$\underset{\sim}{\Delta}=k_1\nabla^2\underset{\sim}{v}+k_2\,\underset{\approx}{E}\cdot\nabla^2\underset{\sim}{v}$$

$$+k_3\underset{\approx}{E}^{(3)}:\underset{\approx}{E}+k_4\underset{\approx}{\varepsilon}:(\underset{\approx}{E}\cdot\underset{\approx}{\Omega}) \quad (8)$$

The scalar paramerers k_i (i=1, 2, 3, 4) depend on only the rheological parameters and the particle radius (a) as follows:

$$k_1=\frac{a^2}{6}$$

$$k_2=-\frac{1}{150}\frac{a^2}{\eta}(11\Psi_1-28\Psi_2)$$

$$k_3 = -\frac{5}{72}\frac{a^2}{\eta}(3\Psi_1 - 10\Psi_2)$$

$$k_4 = -\frac{1}{36}\frac{a^2}{\eta}(9\Psi_1 - 2\Psi_2) \tag{9}$$

To avoid a complicated bookkeeping problem in eq. (8), we first define $\Gamma_{i_1}\cdots_{i_p}$ (an abbreviation for the p^{th} order tensor) as follows:[15]

$$\Gamma_{i_1}\cdots_{i_p} = \frac{\partial^{p-1}}{\partial x_{i_1}\cdots\partial x_{i_{p-1}}} v_{i_p} \Big|_{r=0} \tag{10}$$

Therefore, from eq. (10), the lowest rank tensors which are important to our analysis are expressed as

$$\Gamma_{ij} = \frac{\partial v_j}{\partial x_i}\Big|_{r=0}$$
$$\Gamma_{ij_k} = \frac{\partial^2 v_k}{\partial x_i \partial x_j}\Big|_{r=0} \tag{11}$$

where *i, j, k* are free indices. With this definition, the first term of eq. (6) is

$$(\nabla^2 v)_i = \Gamma_{jji} \tag{12}$$

Here we use Einstein's summation convention, i.e. the repeated indices imply a summation. $E^{(j)}$ and $\Omega^{(j)}$ in eq. (6) are defined as follows[15]:

$$E^{(j)}_{i_1\ldots i_p} \equiv \Delta_{i_1\ldots i_p,\mu_1\ldots\mu_p}\Gamma_{\mu_1\ldots\mu_p}$$

$$\Omega^{(j)}_{i_1\ldots i_p} \equiv -\frac{2p+1}{2p+3}\Delta^{(p+1)}_{i_1\ldots i_p\mu,\ \mu_1\ldots\mu_p\mu}\varepsilon_{\mu\mu'\mu_{p+1}}\Gamma_{\mu_1\ldots\mu_{p+1}}$$

where $\Delta_{\mu_1\ldots\mu_p,\ \nu_1\ldots\nu_p}$ denotes an isotropic tensor of rank 2p which is irreducible in its first p and last p indices, and defined by its projection property[19]. In addition, the explicit forms of the tensor $\Delta_{ij,\ pq}$ and $\Delta_{ij_k,\ pqr}$ are in the following form [14]

$$\Delta_{ij,\ pq} = \frac{1}{2}(\delta_{ij}\delta_{jq} + \delta_{iq}\delta_{jp}) - \frac{1}{3}\delta_{ij}\delta_{pq}$$

$$\Delta_{ijk,\ pqr} = \frac{1}{6}[\delta_{ip}(\delta_{jr}\delta_{kq} + \delta_{jq}\delta_{kr})$$
$$+\delta_{iq}(\delta_{jp}\delta_{kr} + \delta_{jr}\delta_{kp}) + \delta_{ir}(\delta_{jp}\delta_{kq} + \delta_{jq}\delta_{kp})$$
$$-\frac{2}{5}\{\delta_{jk}(\delta_{ip}\delta_{qr} + \delta_{iq}\delta_{pq} + \delta_{ir}\delta_{pq})$$
$$+\delta_{ik}(\delta_{jp}\delta_{qr} + \delta_{jq}\delta_{pr} + \delta_{jr}\delta_{pq})$$
$$+\delta_{ij}(\delta_{kp}\delta_{qr} + \delta_{kq}\delta_{pr} + \delta_{kr}\delta_{pr})\}] \tag{14}$$

where δ_{ij} is the Kronecker delta. Therefore, from eqs. (11), (13), (14) and the equation of continuity, $\Gamma_{jj}=0$, we can obtain the following equation:

$$E_{ij} = \Delta_{ij,\ pq}\Gamma_{pq} = \frac{1}{2}(\Gamma_{ij} + \Gamma_{ji}). \tag{15}$$

Note that E is the rate of deformation tensor and second rank Γ implies velocity gradient tensor. That is, in eq. (15), E is defined as half of the rate of strain tensor in eq. (1).

Furthermore, from the above equations, we also derive

$$E_{ijk}^{(3)} = \frac{1}{6}[2\Gamma_{ijk} + 2\Gamma_{ikj} + 2\Gamma_{kji}$$
$$-\frac{2}{5}(\delta_{jk}(\nabla^2 v)_i + \delta_{ik}(\nabla^2 v)_j + \delta_{ij}(\nabla^2 v)_k)]. \tag{16}$$

Then, using the properties of the alternating unit tensor i.e. $\varepsilon_{iij}=0$ and $\varepsilon_{ijk}=-\varepsilon_{jik}$, we obtain Ω from eq. (13) and

$$\Omega_{ij} = \frac{1}{3}(\varepsilon_{ikm}\Gamma_{jkm} + \varepsilon_{jkm}\Gamma_{ikm}). \tag{17}$$

We now find that eq. (8) can be written in terms of E and Ω in the following forms:

$$(E\cdot\nabla^2 v)_i = E_{ij}\Gamma_{kkj}$$
$$(E^{(3)}:E)_i = E_{ijk}^{(3)}E_{kj}$$
$$(\varepsilon:(E\cdot\Omega))_1 = E_{2i}\Omega_{i3} - E_{3i}\Omega_{i2}$$
$$(\varepsilon:(E\cdot\Omega))_2 = -E_{1i}\Omega_{i3} + E_{3i}\Omega_{i1}$$
$$(\varepsilon:(E\cdot\Omega))_3 = E_{1i}\Omega_{i2} - E_{2i}\Omega_{i1} \tag{18}$$

3. Particle velocity in a Torsional Flow

To calculate a particle migration velocity in torsional flow, we must transform the expression for the bulk flow field to Cartesian coordinates. In the disc and plate flow, since the vertical component of the velocity is zero, the transformed velocity fields are given in the following forms[1]:

$$v_1 = -\frac{x_2 x_3 \omega}{H}, \quad v_2 = \frac{x_1 x_3 \omega}{H} \tag{19}$$

where v_1 and v_2 are the velocity components that are expressed in Cartesian coordinates and x_1, x_2 and x_3 are the Cartesian coordinate components. In addition, H is the height between the disc and plate, and ω is the angular velocity of the disc.

The expressions for v shown in eq. (19) are then employed to calculate $\nabla^2 \underset{\sim}{v}$, $\underset{\approx}{E}$, $\underset{\approx}{E}^{(3)}$ and $\underset{\approx}{\Omega}$, yielding

$$\nabla^2 \underset{\sim}{v} = 0 \tag{20}$$

and

$$\underset{\approx}{E}^{(3)} = 0. \tag{21}$$

Applying eqs. (20) and (21) into eq. (8), we therefore obtain the following simple equation.

$$\underset{\sim}{\Delta} = k_4 \underset{\approx}{\varepsilon} : (\underset{\approx}{E} \cdot \underset{\approx}{\Omega}) \tag{22}$$

Form eq. (22) and the non-zero components of $\underset{\approx}{E}$ and $\underset{\approx}{\Omega}$, we can get

$$\Delta_1 = k_4 \frac{x_1 \omega^2}{H^2}$$

$$\Delta_2 = k_4 \frac{x_2 \omega^2}{H^2} \tag{23}$$

These are the particle migration velocity in Cartesian coordinates in the torsional flow geometry. They can be transformed to cylindrical coordinates as follows:

$$x_1 = r \cos\theta$$
$$x_2 = r \sin\theta$$
$$\Delta r = \Delta_1 \cos\theta + \Delta_2 \sin\theta$$

$$= -\frac{1}{36} \frac{a^2}{\eta} (9\Psi_1 - 2\Psi_2) \frac{\omega^2}{H^2} r \tag{24}$$

Eq. (24) is the final formula of a single spherical particle migration in torsional flow of a viscoelastic solution by means of second-order fluid model. It becomes

$$\Delta r = -\alpha r \tag{25}$$

where $\alpha = \frac{1}{36} \frac{a^2}{\eta} (9\Psi_1 - 2\Psi_2) \frac{\omega^2}{H^2}$

The final result shows that the particle migrates always toward the center and the magnitude of the inward migration velocity is a function of both fluid properties and flow geometry. Fluid properties contain the viscosity and the combination of the first and second normal stress difference coefficients. The first normal stress difference coefficient Ψ_1 in eq. (25) can be easily obtained from the conventional rheological measurements such as slit, cone and plate, and parallel rheometries [20]. For the second normal stress difference coefficient, Ψ_2, it is not nearly as well studied experimentally as Ψ_1, and obtaining reliable values of Ψ_2 is not simple. However, some investigators [21-25] tried to measure the second normal stress difference (N_2) using measurements of the pressure distribution and rim pressure in the cone and plate, and a combination of pressure distribution and total force. N_2 can also be determined from measurement of the surface shape of a polymeric liquid flowing down a semicircular channel, And it is known that the magnitude of Ψ_2 is much smaller than Ψ_1, and that it is negative.

4. Results and Discussion

Based on eq. (25), Fig. 1 shows the inward radial migration velocity of the particle vs. radial position of the particle. The magnitude of the velocity, which depends on α, is a rather arbitrary scale. However, it shows that the migration velocity decreases with decreasing the radial position of the particle. The migration velocity is proportional to square of the radius of the particle, to the square of angular velocity and is inversely proportional to the square of height. In addition, since we choose a neutrally buoyant rigid sphere, vertical migration velocity is neglected in this study.

In recent study, Choi[26] analyzed the radial and vertical migration velocities of a sigle particle (450 μm) in a 0.25% w/w solution of PIB (Mv=9.9

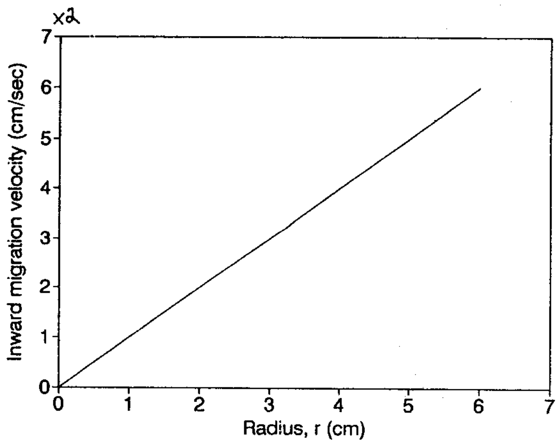

Fig. 1. Inward migration velocity of a spherical particle based on eq. (25) vs. radial position of the particle.

$\times 10^5$) in PB. The radial velocity was found to be increased linearly in absolute magnitude with the radial position. It qualitatively corresponds with the result of eq. (25). In addition, since the density difference between the polymer solution and the particle was narrow and the solution viscosity (81.8 poise) was very high, the vertical migration velocity was very small comparing with the radial migration velocity.

However, it is found that the result from this second-order fluid model does not explain the anomalous migration of a rigid sphere in torsional flow which has been obtained experimentally by previous investigators[9-11]. There might be several reasons to be considered. At first, when the second-order fluid model in eq. (2) is derived from the n-th order fluid model, it should be mentioned that it was truncated up to second order under the assumption of that the shear rate is very small. Therefore, strictly speaking the second-order fluid model can be applied only in the region of very low shear rate. This corresponds with the fact Prieve *et al.* observed the outward particle migration in the region of high shear rate. Therefore eq. (25) is valid only in the low shear rate region. To consider a response of the fluid in the region of high shear rate, third-order fluid model[27] could be applied to this particle migration problem.

On the other hand, one possible description for the occurrence of outward migration in the experiment is that there may be another branch to the solution of the nonlinear descriptive for the particle motion. The solution for inward migration corresponds to a stable fixed point at $r=0$ for $r<r_{cr}$. The other branch of the solution might become stable so that the particle migrates outward for $r>r_{cr}$. For example, eq. (25) is of the form as the simplest differential equation

$$\frac{dr}{dt} = -Dr \qquad (26)$$

where D is a constant.

The solution to the above equation becomes:

$$r(t)=r_i \exp(-Dt) \qquad (27)$$

where r_i is the initial radial position of the particle.

From eq. (27), $r=0$ is the solution for $t=\infty$ when $D>0$. If the other branch of the solution has a negative coefficient in eq. (27), then the stable fixed point becomes $r=\infty$ for $t=\infty$.

In addition, the second reason why the second-order fluid model does not answer about the outward migration is that a great deal of effort must be paid to finding an ideal second-order fluid with constant, hight viscosity and high elasticity at room temperature, as well as transparency at the same time[28-31] to test the theory. Polyisobutylene (PIB) in polybutene (PB) system and PIB in PB and kerosene system are best known second-order fluids for this particle migration experiment in torsional flow. Karis *et al.*[9] observed that a 0.25% w/w solution of a commercial PIB in a PB solvent behaves as a second-order fluid up to a shear rate of 40 sec^{-1}, yet the dynamic shear results from the small amplitude oscillatory shear measurements on the fluid through linear viscoelasticity theory do not indicate second-order behavior. For the second-order fluid, on logarithmic coordinates, N_1 as a function of $\dot{\gamma}$ should have a slope of 2 from eq. (4), and G′ should have a slope of 2 and G″ a slope of 1 in the limit of low frequency[32]. Therefore, it could be said that the non-second order response exists in the certain higher shear rate in the polymer solutions which bave been studied by previous investigators. The factors which contribute to non-second-order response are a large polydispersity, the presence of entanglements and high molecular weight, However, Prieve *et al.*[10] studied the effect of particle size on the inward migration velocity using a 0.1% w/w solution of PIB in PB, the exponent for the form $\Delta_r = \mathcal{O} a^n$ was less than 2.0, which is predicted for inward migration of a sphere. Therefore, for the particle migration theory that incorporates the second-order fluid model to be valid, the fluid should exhibit second-order response.

On the other hand, the possible migration of the polymer molecules themselves in the radial direction can also be considered in torsional flow [33]. Such migration would give rise to a radial gradient of polymer concentration and, hence, to a radial variation in the rheological properties of the fluid. The basis for a theoretical analysis of molecular migration is that a macromolecule in a nonhomogeneous flow field tends to maximize its configurational entropy by migrating toward regions of a lower shear rate. In a region of lower shear rate the molecule has a less oriented configuration than in a region of higher shear rate[34]. In addition, Brunn[35] mentioned the flow induced cross stream migration of the dissolved polymer molecules in torsional flow due to the curvilinearity of the streamlines. From these arguments, polymer concentration decreases with increasing radial position in disc-plate flow. Even though the phenomenon of this polymer migration is not severe, it could give some effect on the motion of a rigid particle.

Explanation of the outward particle migration is not expected from any theory at present. Even though no theory for the outward migration is available, it is useful to speculate on what could give rise to outward migration in torsional flow. To investigate the particle migration in either direction which was observed by some previous investigators in torsional flow, Choi *et al.*[11] correlated this experimental phenomenon with the critical shear rate ($\dot{\gamma}_{cr}$) which was obtained from the reversal in the direction of migration at some shear rate. From the perspective of a polymer molecule in an initially uniform shear field perturbed by the approach of the rigid sphere on a different streamline, the duration of the disturbance was on the order of the reciprocal of the shear rate. If this time was much less than the relaxation time of the molecule, a response characteristic of rigid molecules expected. If the inequality was reversed, the molecule had time to reach a quasi-equilibrium conformation at each instent of the disturbance. A different response characteristic for the flexible molecules was also expected. These two extreme responses might yield opposite directions for lateral migration. From these description, they further correlated the critical shear rate with the relaxation time (λ) of the polymer molecules for different polymer concentrations and different solvent viscosities of PIB in PB. Using the theoretical relaxation times, they speculated that $\dot{\gamma}_{cr}\lambda$ is approximately constant for different solutions.

Since the final equation of this study always gives the inward migration of a spherical particle in torsional flow in the continuum mechanics viewpoint, we are currently trying to analyze this problem from the molecular theoretical viewpoint of dilute polymer solution whether it could predict the migration of a spherical particle in torsional flow in both directions.

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Nomenclature

a	: particle radius
D	: constant in eq. (26)

$\underset{\approx}{E}$	: rate of deformation tensor in eq. (15)
$\underset{\approx}{E}^{(3)}$	: symmetric irreducible part of ∇∇v
H	: height between the disc and plate
k_1, k_2, ⋯	: scalar parameters in eq. (8)
N_1	: first normal stress difference
N_2	: second normal stress difference
r	: radial position
r_i	: initial radial position
r_{cr}	: critical radius
$\underset{\sim}{v}$	: velocity vector
$\underset{\sim}{v}_p$	: particle velocity vector
x_1, x_2, x_3	: Cartesian coordinates
We	: Weissenberg number
α	: coefficient in eq. (25)
α_0, α_1, α_2	: material parameters in eq. (1)
β	: constant in eq. (6)
δ_{ij}	: Kronecker delta
λ	: relaxation time
$\underset{\approx}{\varepsilon}$	: alternating unit tensor of rank three
η	: solution viscosity
$\dot{\gamma}_{cr}$	: critical shear rate
$\underset{\approx}{\dot{\gamma}}=\underset{\sim}{\nabla}\underset{\sim}{v}+(\underset{\sim}{\nabla}\underset{\sim}{v})^T$	
Ψ_1, Ψ_2	: first and second normal stress difference coefficients
$\underset{\approx}{\tau}$	: deviatoric stress tensor
Γ	: defined in eq. (9)
$\underset{\sim}{\Delta}$	: migration velocity
$\Delta_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_p}$	: isotropic tensor of rank 2p
$\underset{\sim}{\nabla}$	: del operator
$\underset{\approx}{\omega}$	: vorticity tensor
$\Omega^{(j)}$	: defined in eq. (12)
$\frac{\mathscr{D}}{\mathscr{D}t}$	: corotational derivative

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