J. Astron. Space Sci., Vol. 8, No. 1, 11~23(1991)

THE DYNAMICAL EVOLUTION OF GLOBULAR CLUSTERS WITH STELLAR MASS LOSS

Chang-Hwan Kim, Mun-Suk Chun
Dept. of Astronomy and Atmospheric Science, Yonsei University, Seoul 120-749

and

Kyoung W. Min
Dept. of Physics, Korea Advanced Institute of Science and Technology, Taejeon 305-701
(Received April 2, 1991 ; Accepted May 27, 1991)

Abstract

The dynamical evolution of globular clusters is studied using the orbit-averaged multi-component Fokker-Planck equation. The original code developed by Cohn(1980) is modified to include the effect of stellar evolutions.

Plummer's model is chosen as the initial density distribution with the initial mass function index α=0.25, 0.65, 1.35, 2.35, and 3.35. The mass loss rate adopted in this work follows that of Fusi-Pecci and Renzini(1976).

The stellar mass loss acts as the energy source, and thus affects the dynamical evolution of globular clusters by slowing down the evolution rate and extending the core collapse time T_{CC}. And the dynamical length scale R_c, R_h is also extended. This represents the expansion of cluster due to the stellar mass loss.

I. Introduction

Globular clusters are the best objects for studying stellar dynamics. Lynden-Bell and Wood (1968) previously mentioned that the self-gravitating finite stellar system had eventually experienced the core collapse and it was called the gravo-thermal catastrophe. This effect mainly due to the negative heat capacity C of the system in the long range force field like gravitation. Therefore the self-gravitating stellar system may have some kind of internal energy source for example stellar mass loss, binary formation, heavy star formation by merger, stellar loss by central black hole.

The first direct numerical integration of the Fokker-Planck equation was presented by Cohn

(1979, 1980) without using Monte Carlo approximation. Further improvements were made by Inagaki and Wiyanto(1984) with two-mass component model in a one-dimensional Fokker-Planck computation. Cohn(1985) also improved his own code over five-mass component model. Extensive numerical calculations of binary heating have performed by Cohn *et al.*(1986), Lee and Ostriker(1986), and Stalter *et al.*(1987). The stellar mass loss effects were studied with simple heat conducting model and hydrodynamical model by Applegate(1986), Angelleti and Giannone(1977, 1979, 1980).

Our present model considers the stellar mass loss is the main energy source and takes three-mass component model with the variation of IMF(Initial Mass Function $\phi(m)=m^{-\alpha}$) index $\alpha=$ 0.65, 1.35, 2.35, 3.35.

II. The Model

The initial density distribution functions of the clusters are multi component Plummer model (Inagaki and Wiyanto, 1984).

$$f_i(E)=\frac{24\sqrt{2}}{7\pi^3}\frac{R_0^2}{G^5M^5}\frac{M_i}{m_i}\left(\frac{m_i}{m_0}\right)^{-1}E^{7/2} \quad \cdots\cdots(1)$$

Where $M(5\times10^{-5}\ M_\odot)$ is the mass of initial cluster, M_i is the mass of each group which is in table 1, and m_i is the mass of each component stars$(m_1, m_2, m_3)=(0.5M_\odot, 1.0M_\odot, 2.0M_\odot)$. m_0 is average mass defined by

$$m_0=\frac{\sum_i n_i m_i}{\sum_i n_i},\quad i=1,3 \quad \cdots\cdots(2)$$

Then the initial density distribution of the cluster is reduced from equation (1)

$$\rho_i(r)=\frac{3M_i}{4\pi R_0^3}\frac{1}{[1+(r/R_0)]^{5/2}} \quad \cdots\cdots(3)$$

and the initial velocity distrition is

$$V_1(r)=\frac{m}{2R_0}\frac{1}{[1+(r/R_0)^2]^{1/2}} \qquad (4)$$

where R_0(2.5pc) is the scale length of Plummer model and taken as the length scale of this study. The unit of energy is $E_0=GM/R_0$, the unit of velocity is $\langle V^2_0\rangle=E_0/2$ and the unit of density is $\rho_0=3M/4\pi R^3_0$.

Table 1. Mass elements and unmber of stars of initial clusters

α	0.65	1.35	2.35	3.35
N_1	2.565×10^5	4.166×10^5	6.468×10^5	8.100×10^5
N_2	1.633×10^5	1.635×10^5	1.268×10^5	7.944×10^4
N_3	1.042×10^5	6.411×10^5	2.490×10^4	7.791×10^3
N	5.240×10^5	6.422×10^5	7.985×10^5	8.970×10^5
$M_1(M_o)$	1.282×10^5	2.083×10^5	3.234×10^5	4.050×10^5
$M_2(M_o)$	1.633×10^5	1.635×10^5	1.268×10^5	7.944×10^5
$M_3(M_o)$	2.084×10^5	1.282×10^5	4.980×10^4	1.558×10^4
$m_o(M_o)$	0.954	0.776	0.626	0.557

The initial density distribution is in Fig. 1. Which is normalized with density $\rho_0=7.64\times10^3M_0/pc^3$ and the radial distribution is in Fig. 2, which is normalized with velocity $\langle V^2_0\rangle^{1/2}=30.5$ km/sec. The initial velocity distribution of all mass group is same and central velocity is 15.3km/sec. Table 1 shows the mass component and the number of stars in initial clusters. Each column is arranged with IMF index α. N_1. N_2, N_3 states the number of stars of the group and N is the total number of stars in the cluster. M_1, M_2, M_3 states the total mass of each group and $\langle m\rangle$ is the average mass of the mass group by eq.(2). The unit of time t_0 is defined as follow

$$t_0=\frac{r_k^{3/2}}{G^{1/2}M^{1/2}}\frac{G^2M^2}{4\pi\Gamma}\frac{1}{N} \qquad (5)$$

where $\Gamma=4\pi G^2m^2\ln\Lambda$, m is the stellar mass and Λ is cut off factor. This time unit t_0 is related to the initial half-mass relaxation time $T_{rh,i}$, which is introduced by Spitzer and Hart(1971), as

$$t_0=T_{rh,i}/32.5 \qquad (6)$$

The time step from the initial evolution to the core collapse is $\Delta t=Tr(0)/40$ where Tr(0) is the

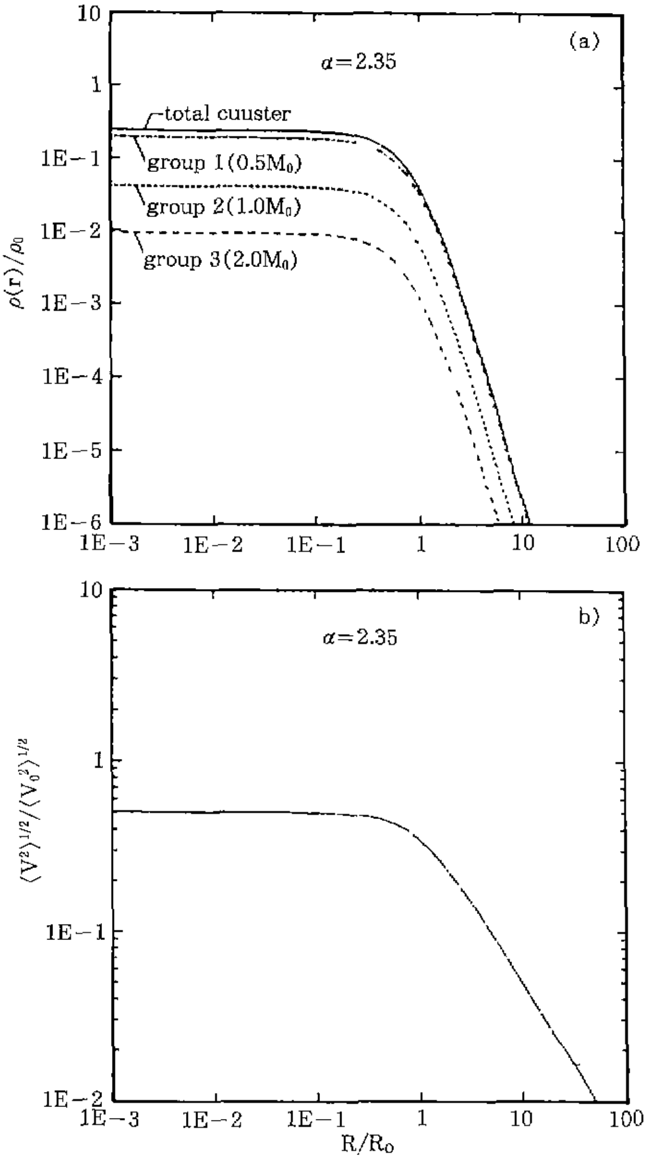

Figure 1. Density and velocity distribution of the initial cluster. a) The density distribution of the initial cluster where ρ_0 is equal to $7.64\times10^3 M_0/pc^3$ and R_0 is 2.5_{pc}. The solid line is the density distribution of the total cluster and the dotted lines are the mass group $m_1 = 0.5M_0$, $m_2 = 1.0M_0$, $m_3 = 2.0M_0$ from top to bottom. b) The velocity distribution of the initial cluster where $\langle V^2\rangle^{1/2}$ is equal to 30.5km/sec and the initial velocity distribution is the same for all mass group. The central velocity is 15.3km/sec.

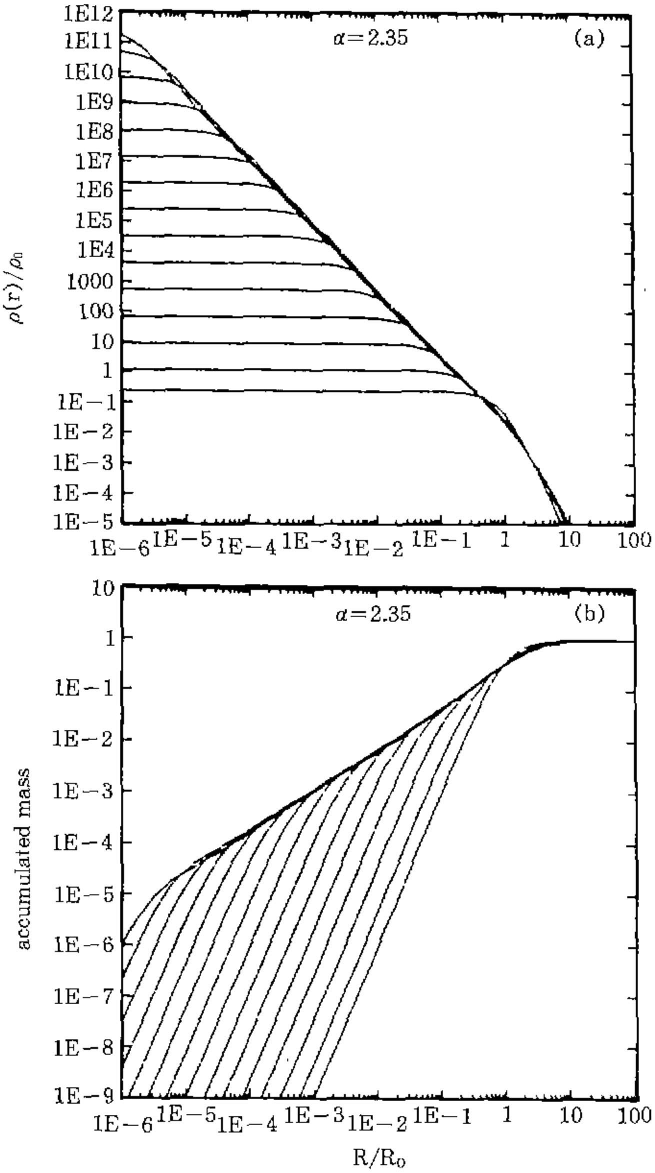

Figure 2. The variation of density and accumulated mass distribution with time. a) The variation of density distribution with time. The abscissa is distance from cluster center and the unit of length is 2.5pc. The ordinate is density and the unit of mass is $7.64\times10^3 M_\odot/pc^3$. The evolution times of each curves are 0.00, 3.96, 6.51, 7.29, 7.48, 7.51, 7.52 $T_{rh,i}$ from bottom to up and the rest is 7.53 $T_{rh,i}$. b) The variation of accumulated mass distribution with time. The abscissa is the same as fig. 2a, and the ordinate is normalized accumulated mass. The evolution time sequence is arranged from right to left and the time is the same as fig. 2a.

central relaxation time.

We adopt the Fusi-Pecci and Renzini's(1976) formula for the mass loss rate of red giant with

$$\dot{M}=3\times10^{-11}\eta_{FPR}(10^3Z)^{-0.04}M^{-1.4}L^{1.92}(M_\odot/yr) \qquad (7)$$

where Z is the metal abundance by mass, M and L are the stellar mass and luminosity in solar units. Then the mass loss rate of $2M_\odot$ and $1M_\odot$ stars are $0.667\times10^{-8}M_\odot/yr$ and $0.347\times10^{-9}M_\odot/yr$ respectively. So the final product of evolution of $2M_\odot$ and $1M_\odot$ stars are $0.9M_\odot$ and $0.5\ M_\odot$ white dwarf stars. The study of stellar evolution shows that the main sequence life time of $2M_\odot$ and $1M_\odot$ stars are 10^9 and 10^{10} yrs respectively. And the dynamic mass loss durations of $2M_\odot$ and $1M_\odot$ stars are 1.65×10^8yrs and 10^9yrs.

The stellar mass loss induces the kinetic energy change of individual star. Hence we must adjust the distribution function f_E with this energy variation. For this we make two assumption. First, the ejection of matter from stars was assumed to take place isotropically so that no dynamical effect on the stars losing mass had to be considered. Second, the ejected matter was supposed to leave the cluster almost instantaneously, and without affecting by direct integration to the mean motion of the stars. The second assumption is reasonable with following observations. No diffused matter in globular clusters detected by the studies of Knapp *et al.*(1973) and Hills and Klein(1973). Cohen(1976) observed larger stellar mass ejection velocities(~45km/sec) than escape velocities which was about 25km/sec at the center of our cluster models.

The total energy of N-body system is

$$E_i=-\sum_j G\frac{m_i m_j}{r_{ij}}+\frac{1}{2}m_i V_i^2 \qquad (8)$$

The energy change produced by Δm_i mass loss of i-th component star is

$$\Delta E_i=-\Delta m_i\sum_{j\neq i}\frac{Gm_j}{r_{ij}}+\frac{1}{2}\Delta m_i v_i^2+\Delta m_i\sum_{j=i}G\frac{(m_i-\Delta m_i)}{r_{ij}} \qquad (9a)$$

$$\Delta E_i=\frac{\Delta m_i}{m_i}\left(-\sum_j\frac{GM_j}{r_{ij}}+\frac{1}{2}v_i^2\right)m_i \qquad (9b)$$

where the order of $\Delta m_i/m_i\sim10^{-8}\sim10^{-9}$ therefore the last term of right hand side of equation

(9b) is approximately zero. Then the energy change is

$$\Delta E_i = \frac{\Delta m_i}{m} E_i \quad \cdots (10)$$

From the analogy of N-body system the distribution function f(E) changes as

$$f_i(E) \rightarrow f_j(E - \Delta E) \quad \cdots (11)$$

where $f_i(E)$ and $f_j(E-\Delta E)$ are the distribution function before and after mass loss. Then we can subtract $\Delta f_j(E)$ from the original distribution function. Therefore the change of the distribution function in each energy mesh with mass loss are

$$f_i^{new}(E) = f_i^{old}(E) - \Delta f_i(E) \quad \cdots (12a)$$

$$f_j^{new}(E - \Delta E) = f_j^{old}(E - \Delta E) + \Delta f_i(E) \quad \cdots (12b)$$

Then we can change the distribution function of eq.(1) whth eq. (12) during mass loss spell.

Ⅲ. Result and Discussion

The following figures are the results of this study. The time variation of density and accumulated mass profiles are in Fig. 2a and 2b. Fig. 2a shows that the central density increase with time and this result due to the mass segregation by dynamical friction. Fig. 2b shows that the mormalized radius shift to right with time by the cluster expansion.

Table 2 presents the time elements of this study. Each column is same to the table 1 and $T_{rh,i}$ is initial half-mass relaxation time defined by eq.(6), $T_r(0)$ is central relaxation time, $T_{cc,n}$ is core collapse time without stellar mass loss, $T_{cc,l}$ is core collapse time with stellar mass loss and $T_{cc,l}/T_{cc,n}$ is the core collapse time ratio with and without stellar mass loss. We calculate the core collapse time T_{cc} from extrapolation of $T_r(o)/T_{rh,i} - t/T_{rh,i}$ diagram. These core collapse times are increased with stellar mass loss. The increment of T_{cc} with IMF index α shows that the median α value cluster have short T_{cc}. Because first the small α value cluster has relatively many heavy mass stars therefore the cluster have experienced heavy heating by larger mass loss. Second the large α value clusters behavior like a single mass component cluster therefore they have longer $T_{rh,i}$ which can be possible to the second stellar mass loss of light component stars.

Table 2. The time elements of the clusters.

α	0.65	1.35	2.35	3.35
$T_{rh,i}$(yr)	8.12×10^{8}	9.84×10^{8}	1.20×10^{9}	1.34×10^{9}
$T_{r(0)}$ (yr)	3.66×10^{8}	4.53×10^{8}	5.50×10^{8}	6.10×10^{8}
$T_{cc,n}$(yr)	2.19×10^{8}	2.75×10^{9}	4.55×10^{9}	8.38×10^{9}
$T_{cc,l}$(yr)	7.90×10^{9}	7.79×10^{9}	9.04×10^{9}	2.05×10^{10}
$T_{cc,l}/T_{cc,n}$	3.61	2.83	1.99	2.44

Figure 3, 4, 5 show the time variation of central parameters of globular clusters. The abscissa is the unit of initial half-mass relaxation time $T_{rh,i}$. The arrow mark is the mass loss point. 1, 2, 3 mark is the evolution track of mass group m_1, m_2, m_3 and tot mark is the evolution track of the total cluster. All figures have converging two curve family where track A signifies the case of no mass loss and track B signifies the case of mass loss.

Figure 3 shows the central density variation with time. The mass loss effect induces the central density decrease of all mass component. The mass loss effect varies with IMF index α. Figure 3a, 3b, 3c have experienced only one mass loss which change the central density increasing rate of mass group m_3, m_3, m_1 to m_2, m_3, m_1. Figure 3d shows second mass loss event then the central density increasing rate is changed from m_3, m_2, m_1 to m_1 m_3. m_2. This effect due to the mass ejecting star eventually arrived lower mass star which expand to the envelope of globular cluster. Figure 4 shows the variation of the central velocity dispersion. The initial velocity dispersion is the same for all mass group. This means that the so called violent relaxation which is generated by the cluster and galaxy formation. Then the kinetic energy of the individual star is proportional to the mass of star. We could acquire the quantity of mass segregation effect by the time evolution of the velocity dispersion. The larger velocity dispersion stellar group is expand to the cluster envelope and the smaller velocity dispersion stellar group is concentrate to the cluster center. This phenomenon is due to the energy equipartition property of the stars in the cluster. Figure 5 shows the evolution of scaled escape energy $X_o=3\phi(0)/v^2(0)$ which is the index of stability of gravitational system. Lynden-Bell and Wood(1968) previously presented that the value of X_0 is larger than 8.5 then the gravothermal instability is formed and the evolution of the stellar system is accelerated to the core collapse.

The evolution of the core is separated from effect of the rest of the cluster with larger X_0 than 8.5. This concept is the same as the Homologous core collapse of Lynden-Bell(1975). The

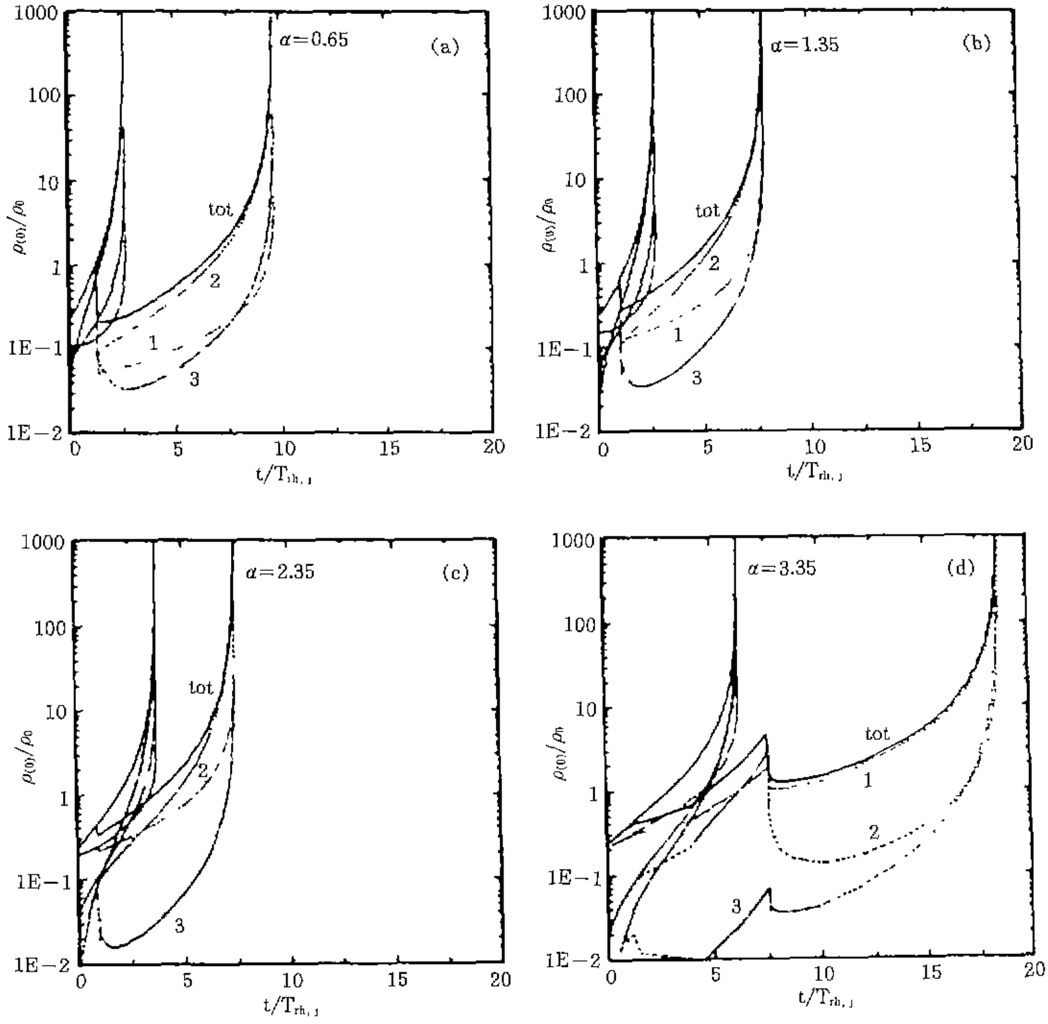

Figure 3. The time evolution of central density. The abscissa is evolution time and the unit of time is $T_{rh,i}$. The ordinate is the central density of the cluster and the unit of mass is 7.64×10^3 M_0. The first converging line family if the evolution of the cluster without mass loss. The second converging line family is the evolution of the cluster with mass loss. a) $\alpha=0.65$, b) $\alpha=1.35$, c) $\alpha=2.35$ and d) $\alpha=3.35$.

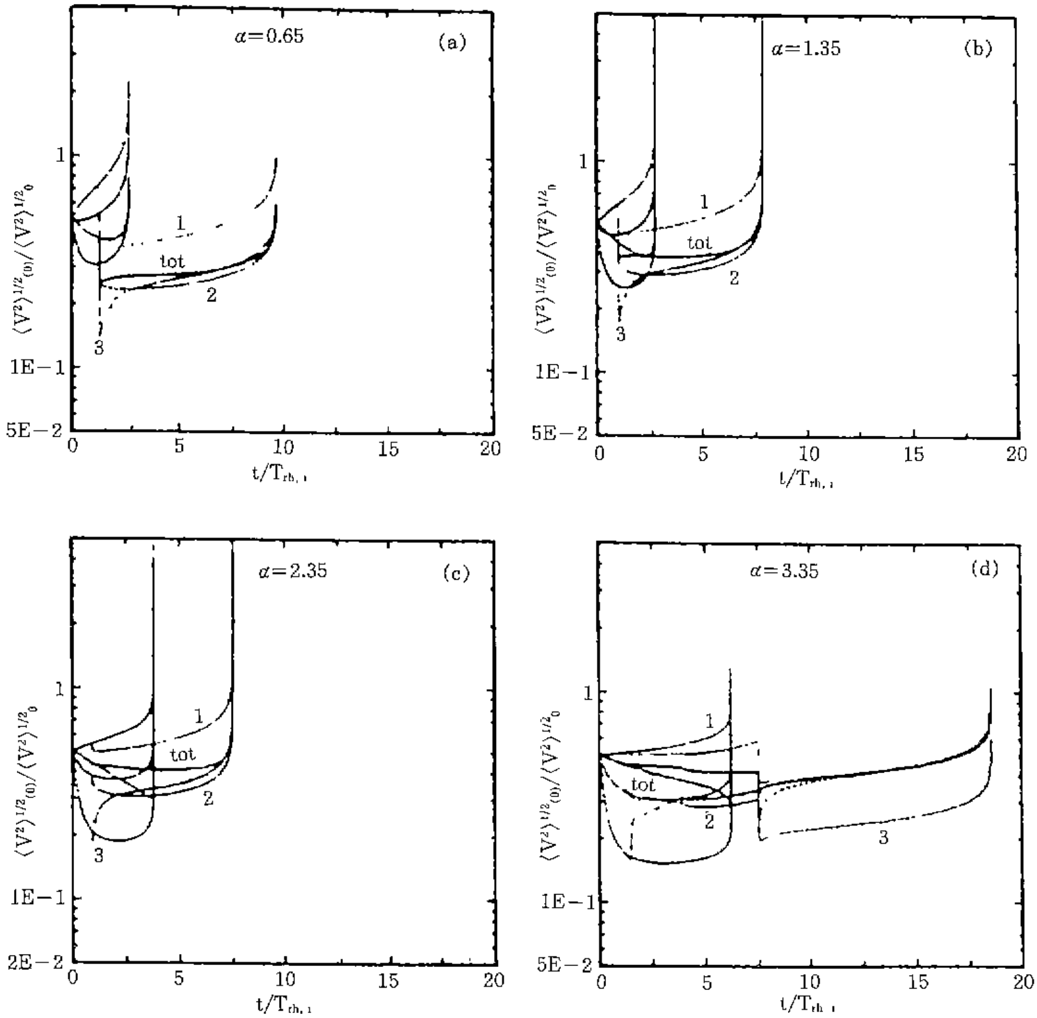

Figure 4. The time evolution of central velocity dispersion. The abscissa is the same as fig. 3, and the ordinate is the central velocity dispersion of the cluster. The unit of the velocity is 30.5km/sec and the rest is the same as fig. 3.

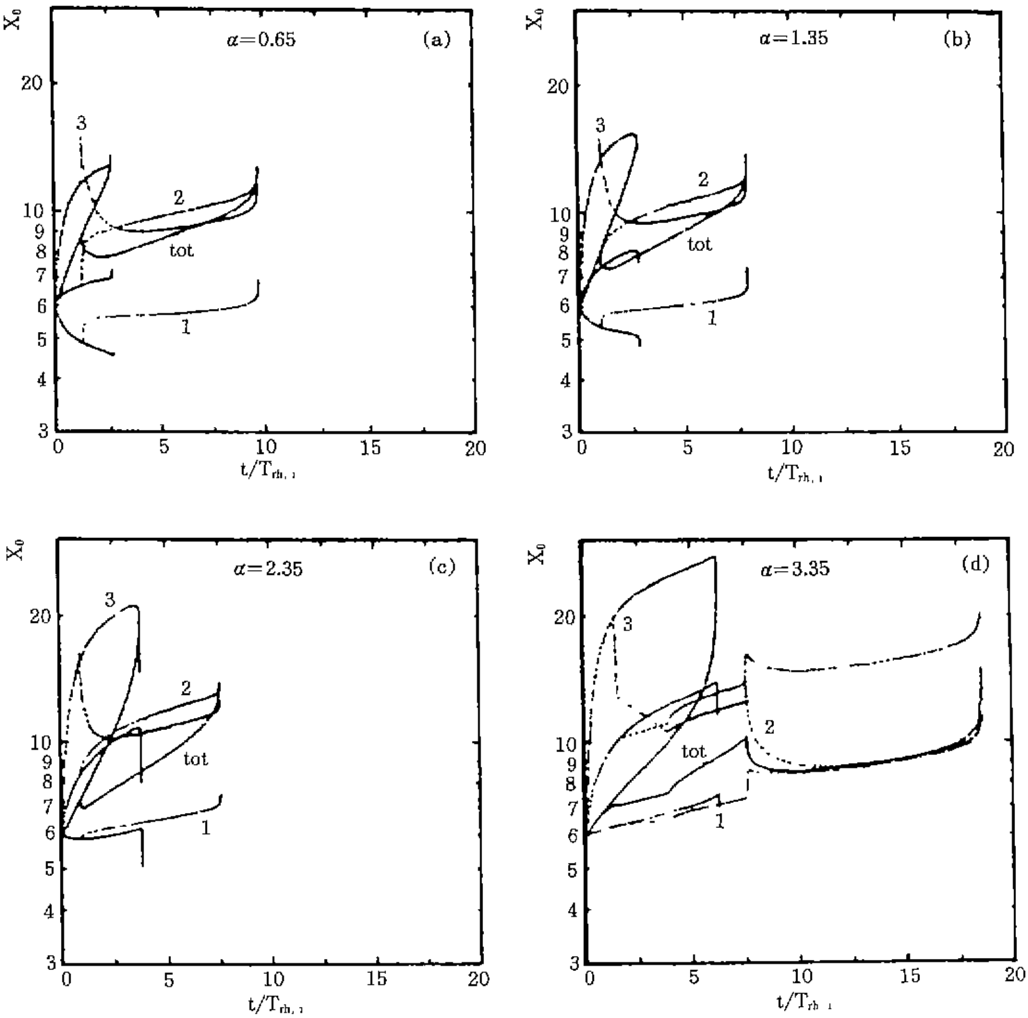

Figure 5. The time evolution of scaled escape energy. The absicssa is the same as fig. 3 and the ordinate is the scaled escape energy $X_0=3\phi(0)/V^2(0)$. The rest is the same as fig. 3.

stellar mass loss changes the central potential $\phi(0)$ and decrease X_0 therefore the core collapse time T_{cc} of the clusters extended with stellar mass loss. In the case of figure 5d the cluster have experienced second stellar mass loss effect and have long duration of lower X_0 state therefore the extension of T_{cc} is larger than other clusters.

Ⅳ. Conclusion

The results of this study have good agreement to the general structure of the globular cluster and the results of fluid dynamical calculation of Angeletti and Giannone(1977). The mass loss effect is dominant to the dynamical evolution of the smaller IMF index cluster and evaporation is dominate to the larger IMF index cluster.

The stellar mass loss effect is energy source of the globular cluster and therefore the stellar mass loss make slowing down the cluster evolution and expanding the cluster envelope. The heating by mass loss can delay the core collapse but not bounce to the core collapse and arrive to the postcollapse evolution. We must consider other energy source for the study of the postcollapse evolution.

References

Angeletti, L. and Giannone, P. 1977, *Astr. Ap.* **58**, 363.

______________. 1979, *Astr. Ap.* **74**, 57.

______________. 1980, *Astr. Ap.* **85**, 113.

Applegate, J. H. 1986, *Ap. J.*(Letters), **301**, 132.

Cohen, J. G. 1976, *Ap. J.* **203**. 132.

Cohn, H. 1979, *Ap. J.* **234**. 1036.

____. 1980, *Ap. J.* **242**, 765.

____. 1985, *Dynamics of Star Clusters,* ed. J. Goodman and P. Hut(D. Reidel: Dordrecht), p. 161.

____. Wise, M. W., Yoon, T. S., Statler, S. T., Ostrike, J. P. and Hut, P. 1986, *The Use of Super Computers in Stellar Dynamics,* ed. P. Hut and S. McMillan(Springer-Verlag: Berlin), p. 206.

Fusi-Pecci, F. and Renzini, A. 1976, *Astr. Ap.* **46**, 447.

Hills, J. G. and Klein, M. J. 1973, *Ap. J.* **186**, 725.

Inagaki, S. and Wiyanto, P. 1984, *Publ. Astron. Soc. Jap.*, **36**, 391.

Knapp, G. R., Rose, W. K. and Kerr, F. J. 1973, *Ap. J.* **186**, 831.

Lee, H.-M. and Ostriker, J. P. 1986, *Ap. J.* **301**. 176.

Lynden-Bell, D. 1975, *Dynamics of Stellar Systems IAU Sympo.* No. **69**, ed. A. Hayli(D. Ridel: Dordrecht), p. 27.

Lynden-Bell, D. and Wood, R. 1968, *M. N. R. A. S.* **138**, 495.

Spitzer, L. and Hart, M. H. 1971, *Ap. J.* **164**, 399.

Statler, T. S., Ostriker, J. P. and Cohn, H. N. 1987, *Ap. J.* **316**, 626.