Dynamic Wave Model for Dendritic River Network

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ABSTRACT / This paper is focused on the development of the RIVNET1 model, which is a dynamic wave model, for flood analysis in dendritic river networks with arbitrary cross-sections. This model adopted the 4-point implicit FDM and utilized a relaxation algorithm in order to solve the governing equations. The double-sweep method was used to reduce the C. P. U. time to solve the matrix system of the model.

This model is applied to analyze flood waves of the Ohio river in the U. S. A and the Keum river in Korea. The results of analysis obtained from this model are compared with those of the DWOPER and observed data.

1. Introduction

The geometrical shapes of channels in any river system can be grouped into three different types; a dendritic type, a loop type, and a complex network. Since most of the rivers in Korea have the dendritic type of tributaries, it is desirable to develop an efficient hydraulic analysis technique for the dendritic river system.

In computing flood waves of a river channel network using the dynamic wave equationd solved by the 4-point implicit scheme, the bottleneck is to solve a sparse matrix due to the internal boundary condition at a junction of several river channels. Since the traditional solution techniques to solve matrix systems do not take advantage of computational efficiency in handling the sparce matrix, it is necessary to develop a more efficient solution procedure.

The representative dynamic wave models for flood wave analysis in a river channel network are BRANCH model from the USGS, DWOPER and FLDWAV model from the NWS, and system 11 model from the DHI.

BRANCH model was developed by Schaffranek at the USGS in 1981. It is applicable to any

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type of river networks. The governing equations include the effect of wind on water surface and the effect of velocity distribution using the momentum correction coefficient. This model can be applied to a river channel which has the gentle gradient subcritical flow. FLDWAV model, which was developed by Fread at the NWS in 1988, use the relaxation algorithm for dendritic type of a river and the network algorithm for complex network type of a river. The relaxation algorithm, which was adopted in the DWOPER model(1976), can handle dendritic-type river channels very efficiently. The network algorithm can handle any type of river channels, which treats the junctions of river channels as internal boundary conditions which are composed of water level and discharge. Both the relaxation algorithm and the network algorithm adopt the weighted 4-point scheme of the dynamic wave equation which transfer nonlinear partial differential equations to nonlinear algebraic equations.

System 11 (MIKE 11 for PC version), which was developed by the DHI, uses the staggered grid scheme as a finite difference method(Abbott-Ionescus , 1967) and uses the double-sweep(DS) method(Richtmyer, R. D, 1957) as a solution algorithm. This model was developed for all types of river channels and have been applied to several delta areas. This model calculates water levels and discharges alternately at each section of a river channel.

Among the above solution techniques, the relaxation algorithm from the DWOPER model was used for the RIVNET1 model, considering topographic features of the rivers in Korea, and the DS method was used to solve the sparce matrix system more efficiently.

2. Dendritic River Network Model

In order to analyze dendritic type of river channel, the dynamic wave equations are differentiated by the Preissmann implicit scheme. The RIVNET1 model, which was designed for dendritic network, use the relaxation algorithm that treats the tributary discharge at confluence as the lateral inflow.

The governing equations of the RIVNET1 model are a complete Saint-Venant equations(1871) considering the effects of lateral inflow and wind stress for the one dimensional unsteady flow, as shown in equation (1) and (2).

$$\frac{\partial Q}{\partial x} + \frac{\partial (A+A_0)}{\partial t} - q_l = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\left(\frac{\partial h}{\partial x} + S_f + S_e\right) + W_f B - q_l V_l = 0 \tag{2}$$

where x is distance in flow direction, t is time, Q is discharge, A is active cross-sectional area, A_0 is inactive(off-channel storage) cross-sectional area, B is wetted top width of cross-section, h is water surface elevation, q_l and V_l are lateral inflow or outflow and its velocity. S_f, S_e are friction slope and local loss slope, and W_f is wind term respectively.

$$S_f = \frac{n^2 g\ Q|Q|}{A^2\ R^{4/3}} \quad (3)$$

$$S_e = \frac{k\ \Delta(Q/A)^2}{2\ g\Delta x} \quad (4)$$

$$W_f = C_w(V_w{}^2 \cos \zeta) \quad (5)$$

where n is Manning's roughness coefficient, R is hydraulic radius, Δx is the computational reach length, Cw is non-dimensional wind coefficient, Vw is wind velocity, ζ is the angle between wind direction and x-axis, K is an expansion(negative) or contraction (positive) coefficient. In order to solve the equations (1) and (2) by the finite difference method, the weighted 4-point implicit scheme (Preissmann, A., 1961) was utilized, which transforms the system of partial differential equations into a couple of nonlinear algebraic equations with the terms of h_j, Q_j, h_{j+1}, and Q_{j+1}.

In order to linearize the nonlinear system of equations, the Taylor series expansion was used. The terms after the first derivative in the series were truncated. Substitution of them into the nonlinear algebraic equations yields equations (6) and (7), which composed of the terms of $\Delta h_j, \Delta Q_j, \Delta h_{j+1}$, and ΔQ_{j+1}, as shown below.

$$A1_j \Delta Q_j + B1_j \Delta h_j + C1_j \Delta Q_j + D1_j \Delta h_j = E1_j \quad (6)$$

$$A2_j \Delta Q_j + B2_j \Delta h_j + C2_j \Delta Q_j + D2_j \Delta h_j = E2_j \quad (7)$$

The values of $A1_j$, $B1_j$, $C1_j$,----, $E2_j$ can be calculated from the known values of $Q^n{}_j, h^n{}_j$ at time level of $n\Delta t$. If the first approximate solutions are estimated at $(n+1)\Delta t$ for N grid points, then the second approximate solutions of the linearized equations (6) and (7) can be derived. Generally this approximation scheme provides sufficiently rapid convergence to the accurate values after few iterations. As the linear equation system has the form of banded matrix, the DS algorithm, which adopts the recurrence equation, is an efficient solution technique in terms of convergence of Δh_j, ΔQ_j, Δh_{j+1}, and ΔQ_{j+1}, calculated from equations (6) and (7). Using the following linear relationship of equations (8) and (9), the solution of equations (6) and (7) can be derived.

$$\Delta Q_j = F_j \Delta h_j + G_j \quad (8)$$

$$\Delta h_j = H_j \Delta Q_{j+1} + I_j \Delta h_{j+1} + J_j \quad (9)$$

where, F_j, G_j, Hj, I_j, and J_j are coefficients of recurrence relations. This procedure is numerically simpler than that of Ammein and Fang's (1970) which is appeared in DWOPER.

As an initial condition , steady-state flow at any cross-section can be determined using the equation (10).

$$Q_{j+1} = Q_j + q_j \Delta x_j \qquad j = 1,2,\ldots\ldots,J-1 \tag{10}$$

where Q_j is upstream flow at time t=0, q_j is average lateral inflow at each Δx. The initial water level, h_j is calculated using equation (11), which is a simplified form of equation (2).

$$(Q^2/A)_{j+1}-(Q^2/A)_j + gA_j(h_{j+1}-h_j+ \Delta x_j\ S_{f,j}) = 0 \tag{11}$$

The calculation process from downstream to upstream and the detailed research can be found in Lee and Han(1986).

The relaxation algorithm(Fread, 1973) is a very efficient method to be applied to dendritic-type rivers which have many tributaries . The merit of the relaxation algorithm is that it forms a banded matrix like those of a simple channel. This algorithm is designed to solve the dynamic equations of mainstream at time of $n\Delta t$ first of all, then solve those of the first tributary . The calculated tributary flow at previous time level $(n-1)\Delta t$ at a confluence is treated as lateral inflow per unit distance of mainstream (Fig. 1). Each tributary flow depends on its upstream boudary condition, lateral inflows along its reach, and the water surface elevaton at the confluence (downstream boundary for the tributary) which is obtained during the simulation of the main channel. Due to the interdependence of the flows in the main channel and its tributaries, the following iterative or relaxation algorithm is used as shown in equation(12).

$$q^* = \alpha\ q + (1-\alpha)\ q^{**} \tag{12}$$

where α is a weighting factor ($0\le\alpha\le1$), q is computed tributary flow at each confluence , q^{**} is a previous estimate of q, q^* is the new estimated value of q for next iteration. It converges when q approach q^{**}, and it usually takes one or two iterations. The optimum value of α can reduce the number of iterations even to the half of the worst case, and the optimum relaxation factor α lies between 0.6 and 0.8.

The acute angle(ω_t) that the tributary makes with the main channel is a specified parameter. This enables the inclusion of the momentum effect of the tributary as used in momentum equation. The velocity of the tributary inflow is given by,

$$V_x = (Q/A)_N \cos \omega_t^t \tag{13}$$

in which V_x is inflow velocity, N denotes the last cross-section of tributary.

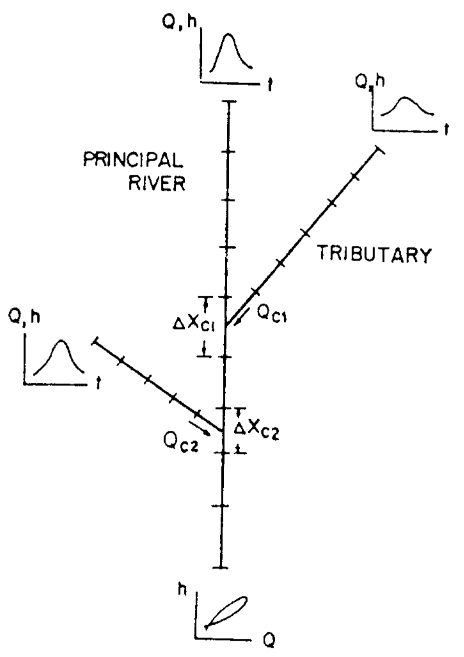

Fig.1 Dendritic river network

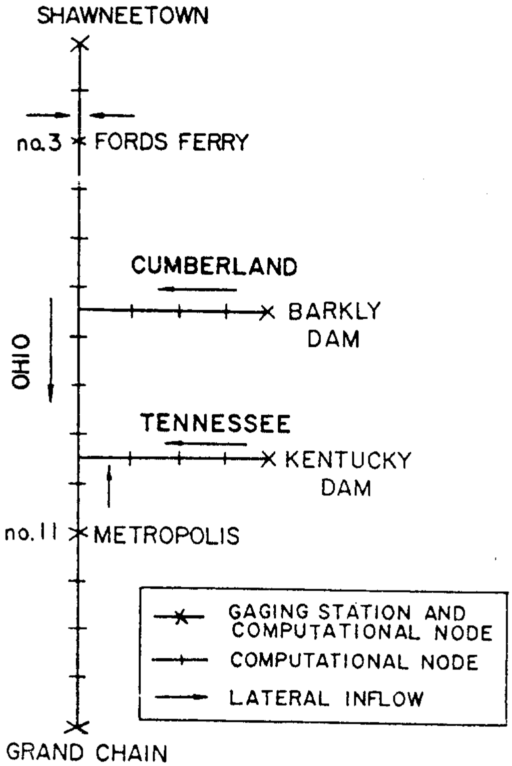

Fig.2 Network of Ohio river

3. Application of Flood Wave Model to River System

Domestic and foreign river channel data were used to estimate the performance of the proposed model RIVNET1. The required data for the application of the model are geometric data of the channel cross-sections, frictional coefficients, hydrographs observed at upstream and downstream boundaries.

In order to investigate application potential of the RIVNET1 model, it was applied to the Ohio river in the U. S. A., and the computed results of the model were compared with those of the DWOPER model and actual measurement data collected by the NWS. This model was also applied to the Keum river in Korea for the analysis of the flood occured in July, 1987.

3.1 Ohio River Channel Network

The Ohio river channel is dendritic-type with two tributaries. The river is 104.3 miles long and its average bed slope is 0.0001. The schematized river channel network for the application of the relaxation algorithm is shown in Fig. 2.

The RIVNET1 model was applied to the flood in 1970, and the initial flow at channel I was 288,000 cfs, that of channel II was 54,800 cfs, and that of channel III was 53,200 cfs. Time interval of calculation(Δt) was set to 24hrs, weighting factor of Preissmann(θ) to 0.6 and weighting factor(α) for calculation of tributary to 0.9. Allowable error limit of flow at confluence was set to 100 cfs. Fig. 3 and Fig. 4 show the comparisons between the simulated water levels and observed ones at no. 3 and no. 11 in channel I respectively. As shown in these figures, both the shapes of the stage hydrographs and the peak values of the simulated water levels are in good agreement with those of the observed data. Since the RIVNET1 model used a constant roughness coefficient throughout the simulation, relatively small variation is observed. If the model after some modifications could accept variable roughness coefficients depending on changes of water level or discharge, it is expected that the simulation would provide more realistic system responses in terms of accuracy of the model results. Two or three iterations were performed to meet the convergence criterion of flow at conlfuence.

The simulated results of the RIVNET1 model are compared with those of the DWOPER model for MOCT(Mississippi-Ohio-Cumberland-Tennessee) river system from the NWS. Fig. 5 and Fig. 6 indicate that the simulated hydrographs from these two models show a perfect agreement at no. 3 and no. 11 respectively. Table. 1 presents a comparison between the RIVNET1 model and the DWOPER model in terms of calulated flows at each station.

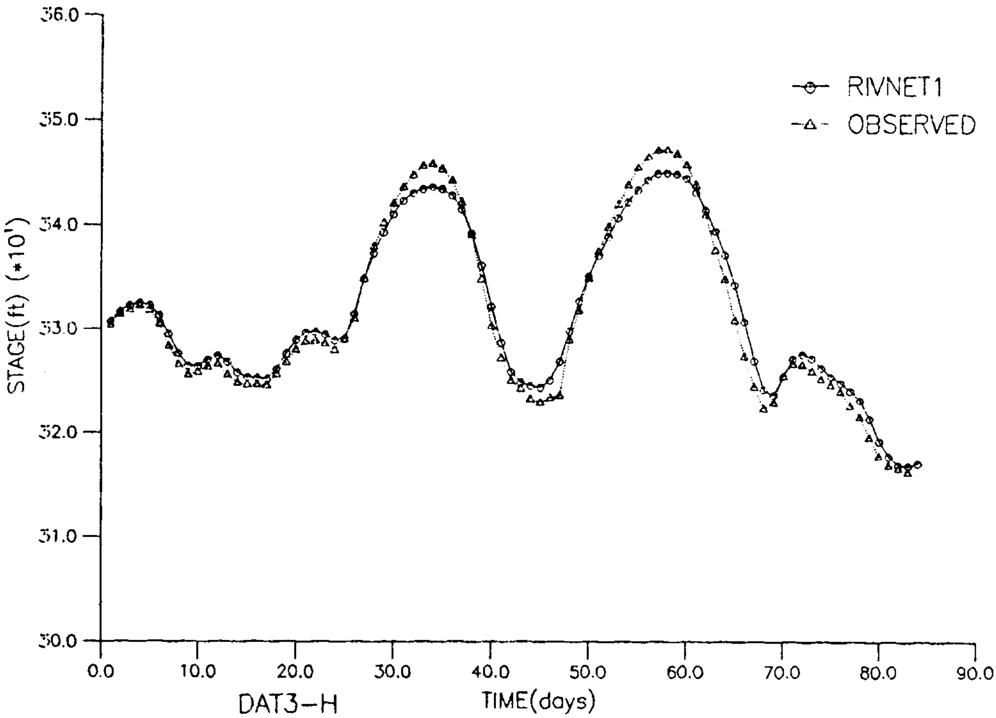

Fig.3 Comparison of water level hydrograph at station no. 3

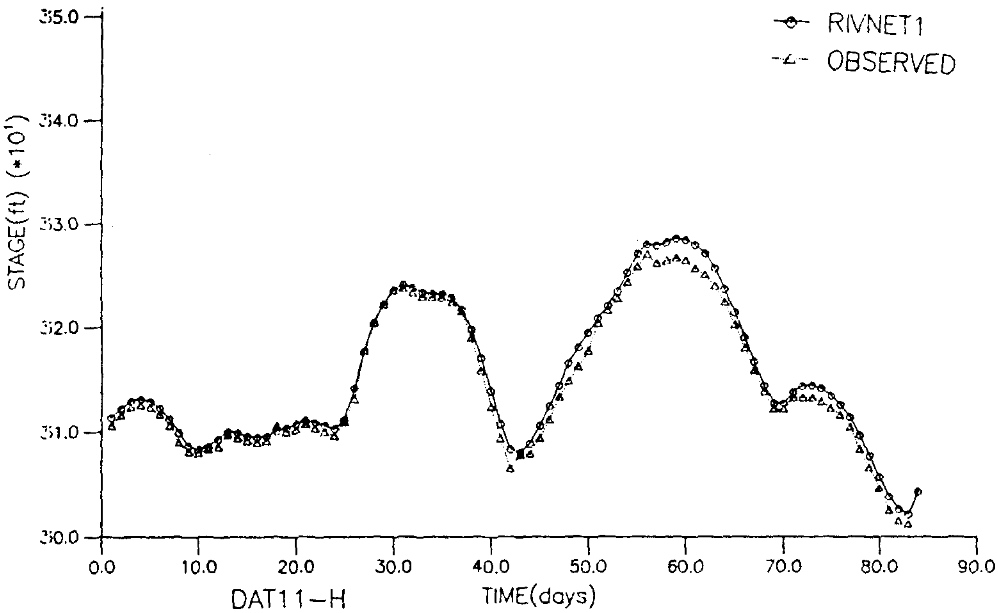

Fig.4 Comparison of water level hydrograph at station no.11

Table 1. Comparison of simulated discharge for RIVNET1 and DWOPER

Sta. No.	t = 504 hrs		t = 1008 hrs	
	RIVNET1	DWOPER	RIVNET1	DWOPER
I – 1	280.00	280.00	205.00	205.00
2	279.96	279.46	208.74	208.72
3	291.25	291.89	219.29	218.77
4	290.49	290.12	223.58	222.78
5	288.56	288.01	231.66	231.02
6	287.56	286.92	235.25	235.05
7	286.95	286.16	237.59	237.66
8	341.39	340.93	265.67	266.23
9	340.89	339.44	268.21	268.54
10	340.57	339.40	270.63	270.21
11	400.01	339.04	306.87	306.33
12	399.79	398.67	308.78	307.35
13	399.54	397.82	310.92	308.35
14	399.04	396.87	316.18	310.52
15	398.75	396.01	322.17	312.84
II – 1	54.90	54.90	30.10	30.01
2	54.86	54.84	28.86	29.43
3	54.80	54.75	28.04	29.05
4	54.72	54.60	27.79	29.05
5	54.61	54.33	28.05	29.40
III – 1	59.00	59.00	32.80	32.80
2	59.08	58.89	33.49	33.09
3	59.10	58.78	34.19	33.44
4	59.09	58.66	34.91	33.82
5	59.91	59.41	36.26	34.81

(Unit: $*10^3$ cfs)

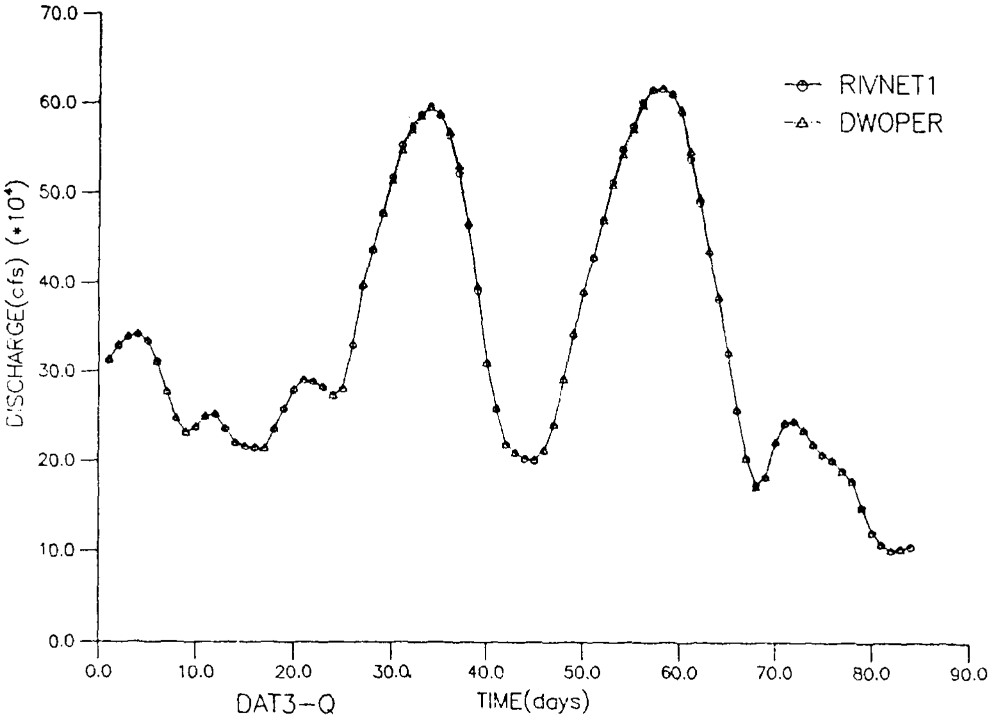

Fig.5 Comparison of flow hydrograph at station no.3

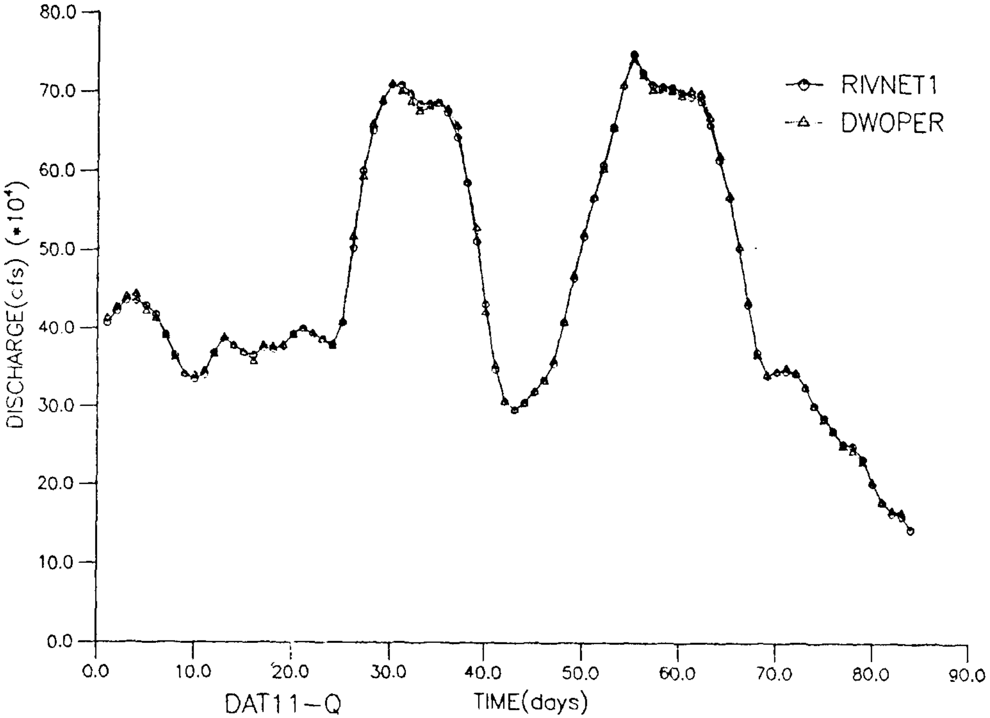

Fig.6 Coparison of flow hydrograph at station no.11

3.2 Keum River Channel Network

Since the Keum river is dendritic type channel, the RIVNETI model was used. The network of Keum river was composed for the middle region of the Keum river basin between Bukang and Kongjoo, which has the first tributary Mihochon-river(Fig. 7). The network consists of channel I and channel II, the lengths of which are 27.5 km and 3.9 km, and average bed slopes of the channels are 0.0002 and 0.0005, respectively.

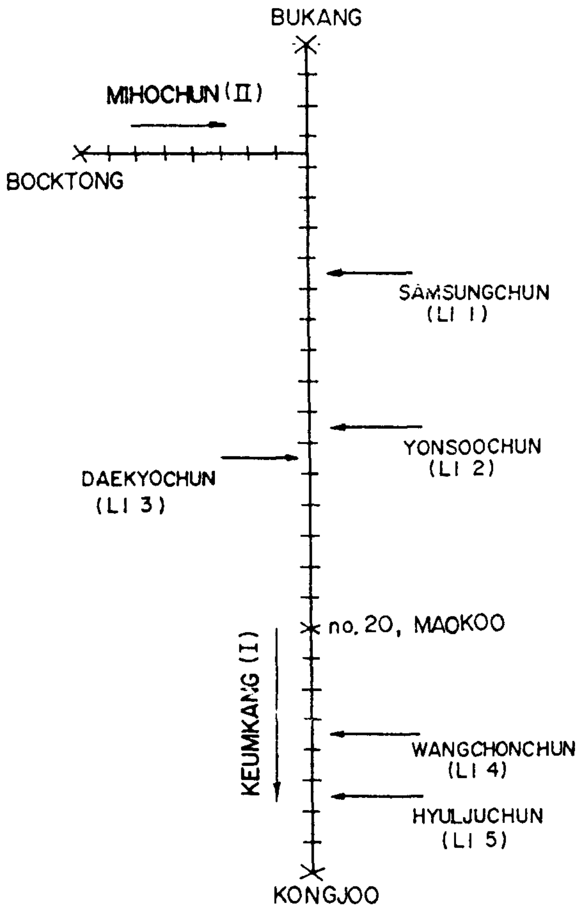

Fig.7 Network of Keum river

Numerical simulation was performed for the flood occurred between July 21-23, 1987. As shown in Fig. 8 and Fig. 9, the stage hydrograph of Bukang station during the flood period was used as upstream boundary condition of channel I, while the stage hydrograph of Kongjoo station was used as downstream boundary condition of channel I. The stage hydrograph of Bocktong station was utilized as upstream boundary condition of channel II (Fig. 8), and average water level of upstream and mainstream at confluence was used as downstream boundary condition of channel Ⅱ. Five lateral inflows of channel Ⅰ were derived from synthetic unit hydrograph by HYMO method(1989, Lee, H. R.). The discharge hydrograph of each lateral inflow is shown in Fig. 10.

Initial flow of channel Ⅰ was 300 CMS, and that of channel Ⅱ was 200 CMS. Average reach lengths of channel Ⅰ and channel Ⅱ were 1.0 km and 0.5 km, respectively. The reach lengths of the grid no. 4 and no. 5, where the confluence point was placed, were reduced significantly to minimize the calculation errors. The time interval was set to 1.0 hr, Preissmann weighting factor(θ) to 0.6, weighting factor(α) to 0.9, for numerical simmulation. Allowable iteration error at confluence was set to 20 CMS.

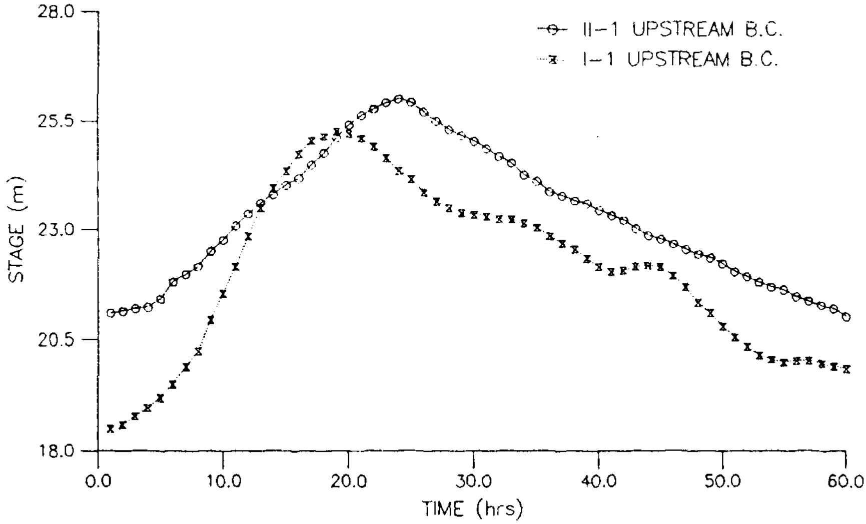

Fig.8 Upstream Boundary conditions for channel Ⅰ and Ⅱ

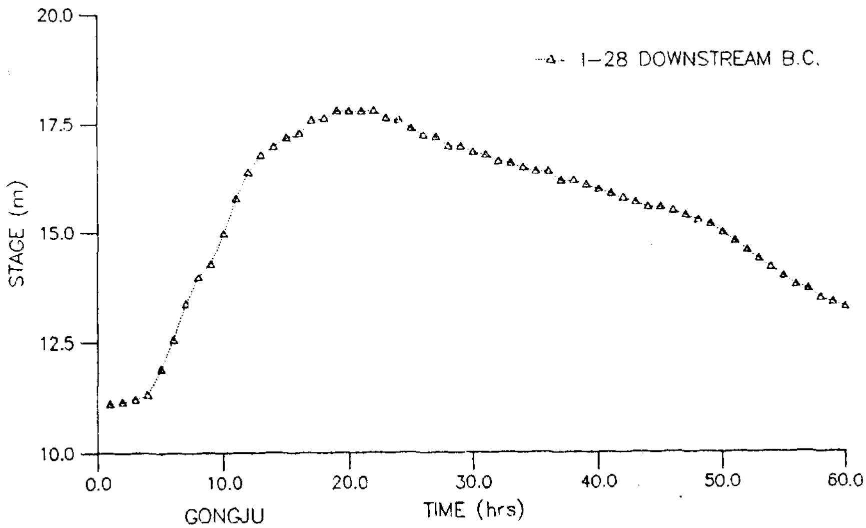

Fig.9 Downstream Boundary condition for channel Ⅰ

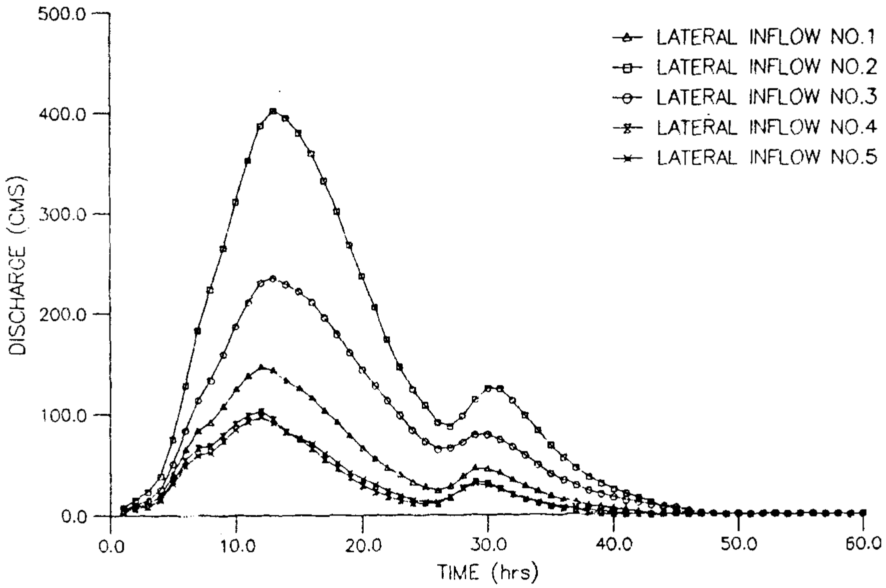

Fig.10 Discharge hydrographs for lateral inflow

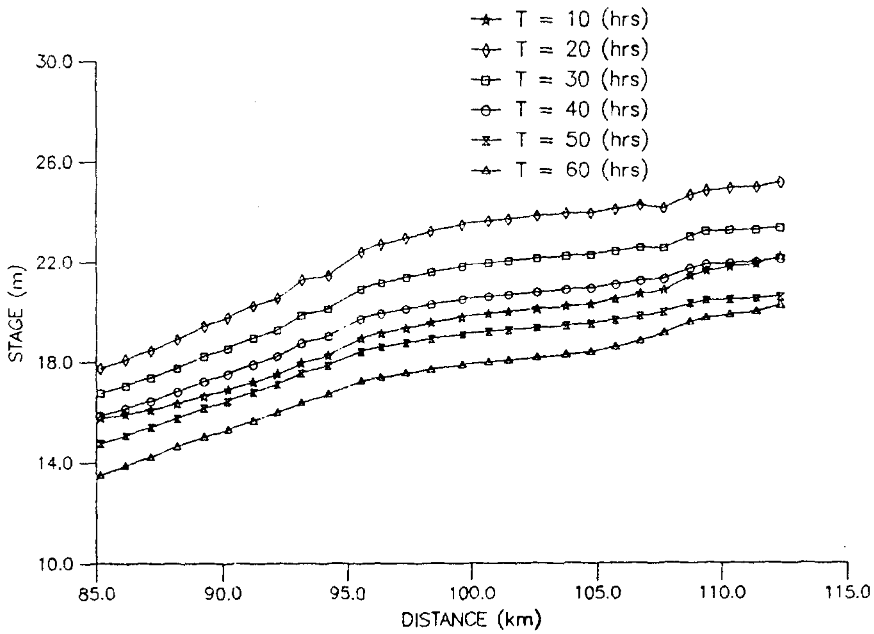

Fig.11 Longitudinal profiles of water level with time

The simulation of flood flow reachs the stable solution within relatively short time, and one or two iteration(s) were performed to meet the convergence criterion of flow at confluence. Fig. 11 shows stage variation with time. Fig. 12 shows the discharge hydrographs of before- and after-confluence of the Mihochun river tributary. This figure indicates wide variation of discharge due to the tributary inflow.

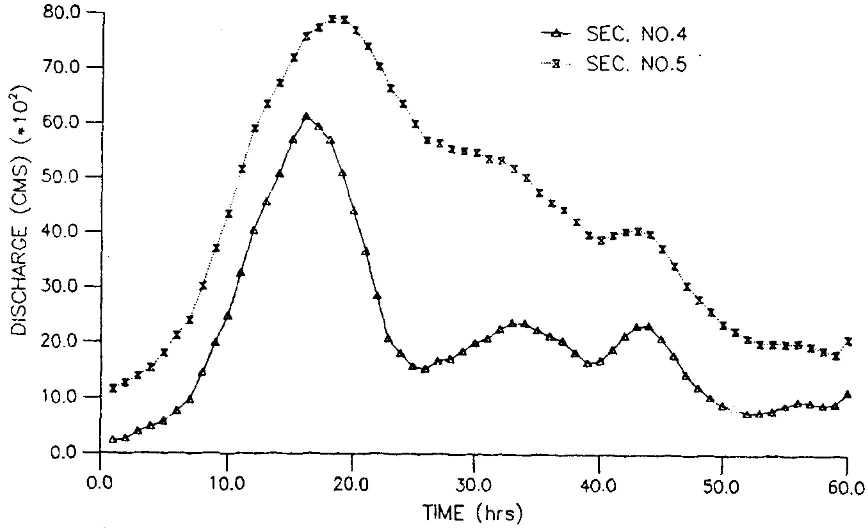

Fig.12 Comparison of discharge hydrographs of before and after confluence

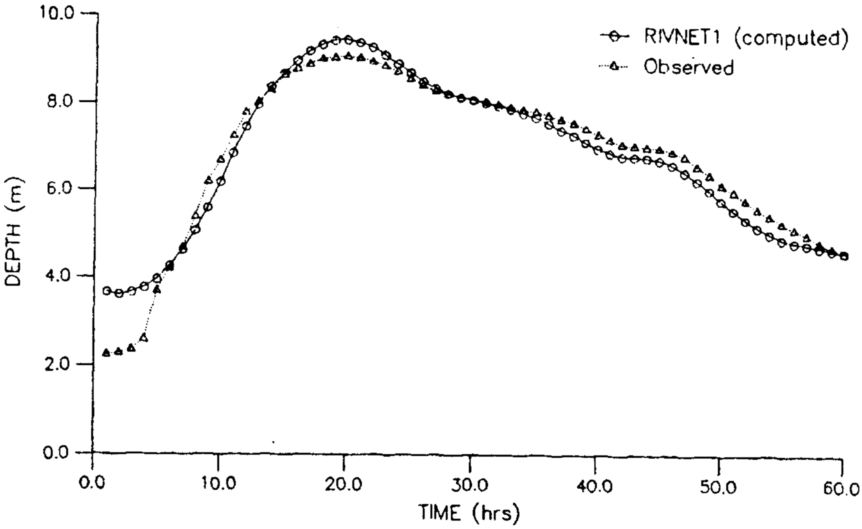

Fig.13 Comparison of simulated and observed stage hydrograph at Maokoo station (no.20)

Since observed data at the Maokoo station(no. 20) was available, the simulated stage hydrograph was compared with data of the Maokoo station as shown in Fig. 13. This figure indicates good agreement between the simulated stage hydrographs and observed ones. The difference between these two hydrographs is due to the use of a constant Manning's roughness coefficient throughout the river channel. If the roughness coefficient is modified at each reach, then the simulated result is expected to approach more closely to the observed data.

From this study, it is proved that the RIVNET1 model is numerically stable and has application potential to analyze dendritic-type river channels. It is very important to collect more observed data and to select a roughness coefficient depending on variation of flow and stage for flood management of catchment area in Korea.

4. Conclusion

This study aimed at developing the RIVNET1 model, which is a dynamic wave model to analyze flood waves in dendritic river networks with arbitrary cross-sections. The Preissmann 4-point implicit FDM is adopted in order to solve the continuity equation and the momentum equation. Fread's relaxation algorithm is utilized for the analysis of characteristics of tributaries and the double sweep method is used to solve the resulting matrix system. This model was applied to the MOCT river network in the U. S. A. (1970 flood). The comparison of the simulated results of RIVNET1 model and those of DWOPER model indicate very good agreement between these two models. This model was also applied to the Keum river in Korea to analyze the 1987 flood, and a good result was obtained from the comparison between the simulated results and the observed data.

Since most of the rivers in Korea have dendritic-type tributaries, the RIVNET1 model might be used as an efficient tool to analyze flood waves in the rivers of Korea.

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