A Channel Flood Routing by the Implicit Dynamic Wave Model

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ABSTRACT/ /US NWS/NETWORK is applied for the analysis of the flood of July 11-15, 1981 through the Goan-Indogyo reach of the Han River. For the flood hydrograph synthesis of the lateral inflows from the major tributaries into the main reach the Clark method is employed. NETWORK coupled with the Clark method of hydrograph synthesis simulated with a fair accuracy the observed flood hydrograph at the downstream boundary of the routing reach. The effect of SCS runoff curve number for tributary flood synthesis is evaluated. The characteristics of the station variations and time variations of the flood discharges in the reach is also analyzed.

1. Introduction

The methods of flood routing through a river channel reach can be classified into two which are known as the hydrologic routing and hydraulic routing(Chow, 1964;Viessman, Knapp and Harbaugh, 1972;Chow, Maidment and Mays, 1988). The hydrologic routing method is an approximate method of solution for river flow based only on the continuity equation of the unsteady flow, and hence accuracy cannot be maintained for the reaches under considerable backwater effects, or for the reaches with abrupt changes in flow regime due to incoming tributary flow. On the other hand, the hydraulic method of routing utilizes the continuity and momentum equations to express the movement of flood wave through the channel and solve the simultaneous nonlinear differential equations numerically for the river stages and discharges at specified stations at specified times in accordance with the initial and boundary conditions for the reach under consideration (Amein and Chu, 1975;Chow, Maidment and Mays, 1988;Weinmann and Laurenson , 1979). For the hydraulic flood routing the data requirement is much heavier and the solution technique is very much complicated,

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but it gives more accurate results at specified sections in the flow domain.

In solving the system of continuity and momentum equations for unsteady flows three different methods are usually employed, i. e. the method of characteristics, explicit, and implicit method. Among these three methods the implicit method is known by far the best method which gives not only more accurate but also stable solution for a flood of considerable duration, although the computational procedure is most complicated and hence the computation time is the longest(Wylie, 1970;Price, 1974;Amein and Chu, 1975).

In applying the implicit method of numerical solution of the unsteady flood flow alternative hydraulic routing models can be produced by using the full continuity equation while eliminating some terms of the momentum equation(Weinmann and Laurenson, 1979;Chow, Maidment and Mays, 1988). The simplest model is the kinematic wave model, which neglects the local acceleration, convective acceleration, and pressure terms in the momentum equation. The diffusion wave model neglects the local and convective acceleration terms but incorporates the pressure term. The dynamic wave model considers all the acceleration and pressure terms in the momentum equation. In other words, the dynamic model deals with the full Saint Venant equations for unsteady flood flow.

In the present study, a significant flood of 1981 for the Goan-Indogyo reach of the Han River was routed by the implicit dynamic wave model, NETWORK, which is a modified version of DWOPER(Dynamic Wave Opertional Model) of the U. S. National Weather Service(Fread, 1978). The major tributary inflows to the routing reach were computed by the Clark method(Clark, 1943), which were input as the lateral inflows at the confluence of the tributaries joining with the routing reach. The computed flood hydrographs at the Indogyo Station, the downstream boundary, are compared with the observed and the applicability of the implicit dynamic model is evaluated to analyze the flood characteristics of the routing reach.

2. Theoretical Background

2.1 Mathematical Basis of the Implicit Dynamic Wave Method

The basis of NETWORK is a finite difference solution of the conservation form of the one-dimensional equations of unsteady flow consisting of conservation of mass and momentum equations. Noting Fig. 1, the complete Saint Venant equations can be expressed as follows. (Fread, 1978):

$$\frac{\partial Q}{\partial x} + \frac{\partial (A+A_0)}{\partial t} - q = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA(\frac{\partial h}{\partial x} + S_f + S_e) - qV_x + W_f B = 0 \tag{2}$$

in which Q is discharge, A is cross-sectional area, A_0 is off-channel cross-sectional area wherein flow velocity is considered negligible, q is lateral inflow or outflow, x is distance along the channel, t is time, g is gravitational acceleration constant, h is water surface elevation (stage), V_x is the velocity of lateral inflow in the x-direction, W_f is the wind term, B is the channel top width, S_f is the friction slope, and S_e is the energy gradient due to channel expansion or contradiction. Neglecting the small magnitude terms of S_e, qV_x, and W_fB and assuming V_x=0, Eq(2) becomes

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA(\frac{\partial h}{\partial x} + S_f) = 0 \tag{3}$$

in which

$$S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} \tag{4}$$

where n is Manning's roughness coefficient, and R is the hydraulic radius. Eq(1) and (3) are nonlinear partial differential equations which may be solved by the numerical methods previously described. Among the numerical methods of solutions NETWORK employs the weighted four-point implicit scheme first used by Preissman (1961) and later by Quinn and Wylie (1973), Chaudry and Contractor (1975), Fread (1973, 1974) and others. This method is accepted as the most advantageous of the various implicit schemes which have been proposed from time to time because it can readily be used with unequal distance steps and its stability-convergence properties can be controlled.

In the weighted four-point scheme, the continuous x-t region in which solutions of h and Q are sought is represented by rectangular net of discrete points, as shown in Fig. 2 at equal or unequal intervals of x and t along the x and t axis, respectively.

The time derivatives are approximated as follows:

$$\frac{\partial \Gamma}{\partial t} = \frac{\Gamma_i^{j+1} + \Gamma_{i+1}^{j+1} - \Gamma_i^j - \Gamma_{i+1}^j}{2\Delta t} \tag{5}$$

in which Γ represents any variable. The spatial derivatives are approximated by a finite difference quotient positioned between two adjacent time lines according to weighting factors θ and $(1-\theta)$, i.e.

$$\frac{\partial \Gamma}{\partial x} = \theta \frac{\Gamma_{i+1}^{j+1} - \Gamma_i^{j+1}}{\Delta x} + (1-\theta)\frac{\Gamma_{i+1}^j - \Gamma_i^j}{\Delta x} \tag{6}$$

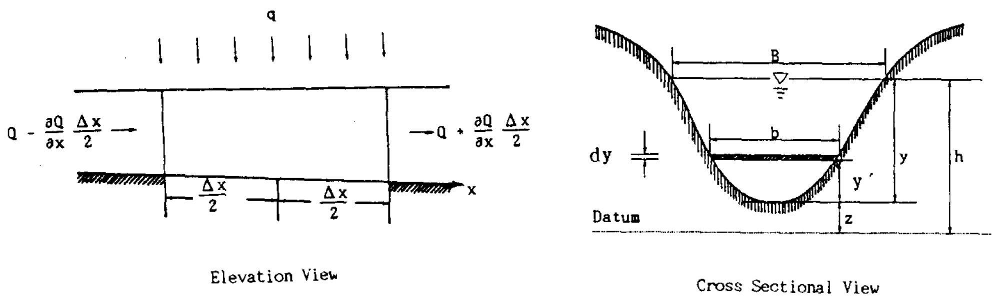

Fig. 1 Flow Element for Saint-Venant Eguations

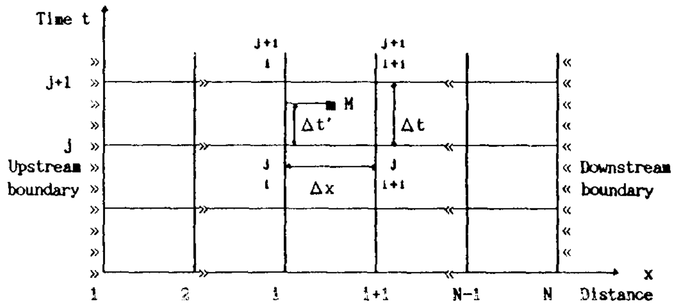

Fig.2 Discrete x-t Solution Domain

and variables other than derivatives are approximated in a similar manner, i. e.

$$\Gamma = \theta \frac{\Gamma_i^{j+1} + \Gamma_{i+1}^{j+1}}{2} + (1-\theta)\frac{\Gamma_i^j + \Gamma_{i+1}^j}{2} \tag{7}$$

The influence of the weighting factors θ on the stability and convergence properties was examined by Fread(1974), who concluded that the accuracy decreases as θ departs from the values of 0.5 and approaches 1.0. This effect becomes more pronounced as the time step size increases. NETWORK allows θ to be an input parameter. A value of 0.55 is often used to minimize loss of accuracy while avoiding weak or pseudo instability as reported by Baltzer and Lai(1968) and Fread(1975) when θ of 0.5 is used.

Substitution of the finite difference quotients defined by Eq(5)-(7) into Eq(1)and (3) for the derivatives and non-derivative terms produces two algebraic equations which are nonlinear with respect to the unknown h and Q at the net points on the j+1 time line as shown in the followings.

$$\begin{aligned} C &= \theta\left(Q_{i+1}^{j+1} - Q_i^{j+1} - q_i^{j+1}\Delta x_i\right) + (1-\theta)\left(Q_{i+1}^j - Q_i^j - q_i^j\Delta x_i\right) \\ &\quad + \frac{\Delta x_i}{2\Delta t_j}\left[(A+A_0)_i^{j+1} + (A+A_0)_{i+1}^{j+1} - (A+A_0)_i^j - (A+A_0)_{i+1}^j\right] \\ M &= \frac{\Delta x_i}{2\Delta t_j}\left(Q_i^{j+1} + Q_{i+1}^{j+1} - Q_i^j - Q_{i+1}^j\right) \\ &\quad + \theta\left[\left(\frac{Q^2}{A}\right)_{i+1}^{j+1} - \left(\frac{Q^2}{A}\right)_i^{j+1} + g\left[\theta\bar{A}_i^{j+1} + (1-\theta)\bar{A}_i^j\right]\left(h_{i+1}^{j+1} - h_i^{j+1} + (\bar{S}_f)_i^{j+1}\Delta x\right)\right] \end{aligned} \tag{8}$$

$$+(1-\theta)\left[\left(\frac{Q^2}{A}\right)_{i+1}^{j}-\left(\frac{Q^2}{A}\right)_{i}^{j}+g\left(\theta\bar{A}_i^{j+1}+(1-\theta)\bar{A}_i^{j}\right)\left(h_{i+1}^{j}-h_i^{j}+(\bar{S}_f)_i^{j}\Delta x\right)\right] \quad (9)$$

All terms associated with the jth time line in Eq(8) and (9) are known from either the initial conditions or previous computations. The initial conditions are values of h and Q at each computational point along the x-axis for the first time line j+1. They are obtained from the previous unsteady flow solution; or they can be estimated since small errors in the initial conditions dampen out within a few time steps.

2.2 Finite Difference Solution of Nonlinear Algebraic Equations

The two nonlinear algebraic equations, Eq(8) and Eq(9), can not be solved explicitly since there are 4 unknowns, h and Q, at points i and (i+1) on the (j+1) time line and only two equations. However, if similar equations are formed for each of the (N-1) Δx reaches between the upstream and downstream boundaries, a total of (2N-2) equations with 2N unknowns result. Here, N denotes the total number of computational points or cross-sections. Then, prescribed boundary conditions, one at the upstream extremity of the routing reach and one at the downstream end, provide the necessary two additional equations required for the system to be determinate. The resulting system of 2N linear equations with 2N unknowns can be solved by a functional iterative procedure, the Newton-Raphson method(Amein and Fang, 1970).

2.3 Boundary Conditions

Boundary conditions must be specified in order to obtain solutions to the dynamic wave equations. In fact, in most unsteady flow problems, the unsteady disturbance is introduced into flow at the boundaries of the river reach. NETWORK can readily accomodate either of the following boundary conditions at the upstream extremities of the river reach:

1) known stage hydrograph, $h_1(t)$, or
2) known discharge hydrograph, $Q_1(t)$

Downstream boundary conditions included as options in NETWORK are:

1) known stage hydrograph, $h_N(t)$
2) known disharge hydrograph, $Q_N(t)$, or
3) A known stage-discharge relationship(Rating Curve)

In the case of the rating curve downstream boundary condition, the rating may be single valued or looped.

2.4 Initial Conditions

NETWORK allows initial conditions to be obtained from the following sources:

1) Estimated stages and discharges at each cross-section
2) Observed stages at each cross-section where a river stage gauge is located. (Intermediate stages are linearly interpolated)
3) Computed stages and discharges which have been saved from a previous unsteady flow simulation
4) Assumed steady flow to obtain discharges and backwater computation to obtain stages.

In each of the above cases, the unsteady flow equations are solved for several time steps using the initial conditions together with boundary conditions which are held constant during the time steps. This allows the errors in the initial conditions to dampen out which results in the initial conditions being more nearly error free when the actual simulation commences and transient boundary conditions are used.

3. Data

3.1 River Stage and Discharge Data

The river reach taken in the present study is the Goan-Indogyo reach of the Han River. The flood stage data during the flood of July 11-15, 1981 were taken at the Goan and Indogyo stations (Han FCO, 1983), and these were converted into discharge hydrogrphs by the rating curves lists in Table 1 (Han FCO, 1986).

Table 1. Stage – Discharge Relations at Upstream and Downstream Boundaries

Discharge \ Station	Goan	Indogyo
Flood	$H > 5.9$ m $Q = 226.5648H^{1.8369}$	$H > 8.10$ m $Q = 150.21H^{2.19}$
Low Flow	$H \leqslant 5.9$ m $Q = 198.244H^{1.91187}$	$H \leqslant 8.10$ m $Q = 354.59(H+0.6)^{1.7078}$
Zero EL	EL. 10.255 m	EL. 1.945 m

Remark; H: River Stage (m), Q: Discharge (m^3/sec)

The flood discharge hydrograph at the upstream boundary, Goan, shown in Fig. 3 was used as the upstream boundary condition for the numerical solution of the dynamic wave equations and, on the other hand, the stage - discharge relation was employed as the downstream boundary condition at the Indogyo station.

3.2 Cross-Section Data

The cross-section data for the routing reach were taken from the river surveying results for the cross-sections and longitudinal profile along the reach (MOC, 1981). The cross sections taken for the present computation are listed in Table 2. In the computation each cross-section is read in as tabular values of the channel width and elevation which together constitute a piece-wise linear relationship. That is, any areas or widths associated with a particular water surface elevation are linearly interpolated from the piece-wise linear-relationships of width to elevation read in.

Dead storage areas (off - channel storage) wherein the flow velocity in the x-direction is considered negligible relative to the velocity in the active area of the cross-section is a feature of NETWORK. To define the dead storage areas the off-channel storage cross-sectional properties are described by a table of widths and elevations for each cross-section read in along with the area associated with the lowest elevation. A table of area-elevation is created within NETWORK and intermediate storage top widths or areas are linearly interpolated.

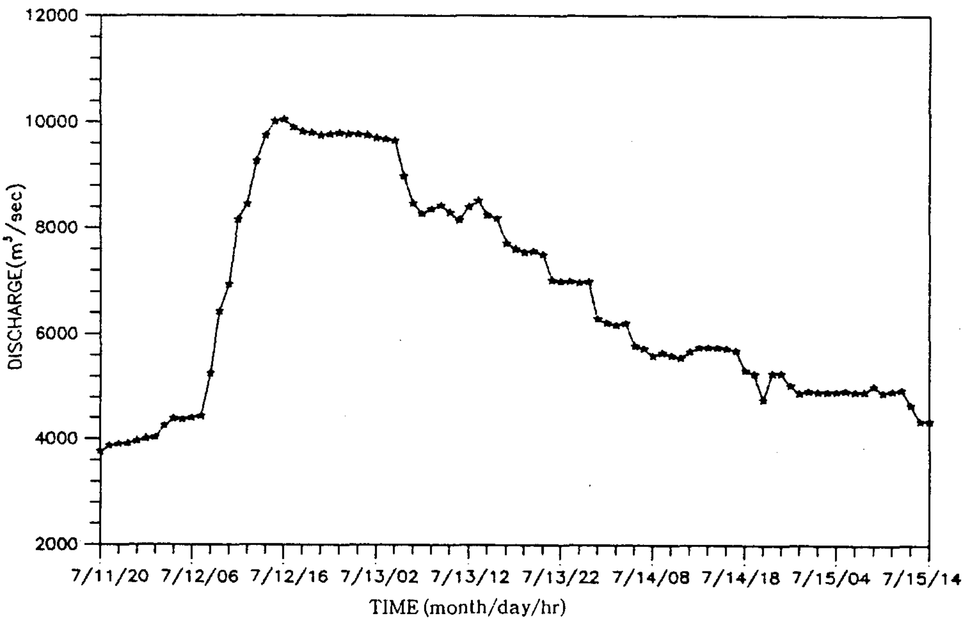

Fig. 3 Flood Hydrograph at Goan Station (Upstream Boundary Condirion for Numerical Computations)

Table 2. Cross-Sections Selected for the Computation

Station	Distance (Km)	Interval (Km)	Lowest EL. (m)	Remarks
1	0.00	0.00	2.17	Goan St.
2	1.03	1.03	-1.90	
3	2.25	1.21	4.50	
4	2.68	0.43	7.00	
5	3.48	0.80	7.80	
6	3.85	0.37	3.50	
7	4.67	0.82	8.20	
8	6.66	1.99	1.20	
9	7.42	0.76	0.75	
10	8.12	0.70	4.00	
11	8.54	0.41	4.50	
12	9.38	0.84	0.70	
13	10.63	1.25	2.15	Confluence with Wangsuk River
14	11.81	1.18	1.20	
15	13.41	1.60	1.90	
16	14.41	1.00	-1.10	
17	15.81	1.40	-1.20	
18	17.91	2.10	-1.40	Kwangjang St.
19	19.81	1.90	4.60	
20	21.81	2.00	0.40	
21	22.81	1.00	-0.50	Confluence with Tan River
22	24.41	1.60	-2.90	
23	25.41	1.00	-1.80	
24	25.71	0.30	-0.30	Dukdo St.
25	26.81	1.10	-3.20	Confluence with Jungrang River
26	27.81	1.00	-2.35	
27	29.81	2.00	-4.30	
28	31.81	2.00	-1.90	
29	32.81	1.00	-4.70	
30	34.41	1.60	-5.80	Indogyo St.

4. Analysis and Results

4.1 General Procedure for the Analysis

The general procedure for the analysis closely followed the mathematical basis and the finite difference solution techniques described previously. As mentioned earlier the upstream boundary condition was taken as the flood discharge hydrograph at Goan station and the downstream boundary as the stage-discharge relation at Indogyo station. To start the routing computation for the reach the initial conditions should be determined first and then the unsteady flow computation follows for which the major tributary inflow hydrographs coming in the routing reach should be determined first and then the unsteady flow computation follows, for which the major tributary inflow hydrographs coming in the routing reach should also be computed to be incorporated into the flood routing through the main channel reach.

4.1.1 Computation of Initial Conditions

Among the options for initial condition allowed in NETWORK the steady flow assumption (option - (4)) was used in the present study. Assuming the steady flow prevails throughout the reach the continuity and momentum equations, Eq(1) and Eq(3), become

$$Q_i = Q_{i-1} + q_{i-1}\Delta x_{i-1} \tag{10}$$

$$(Q^2/A)_i - (Q^2/A)_{i-1} + g\bar{A}(h_i - h_{i-1} + \bar{S}_f \Delta x_{i-1}) = 0 \tag{11}$$

where $i = N, N-1, N-2, \cdots 3, 2, 1$

Based on Eq(10) and Eq(11) the backwater computations (Fread, 1988) were made with the initial recorded flood stage at the downstream boundary and the steady flood discharge for the reach, with Manning's n-values being varied to result the best fit stage compared with the observed flood stage at the upstream boundary, Goan.

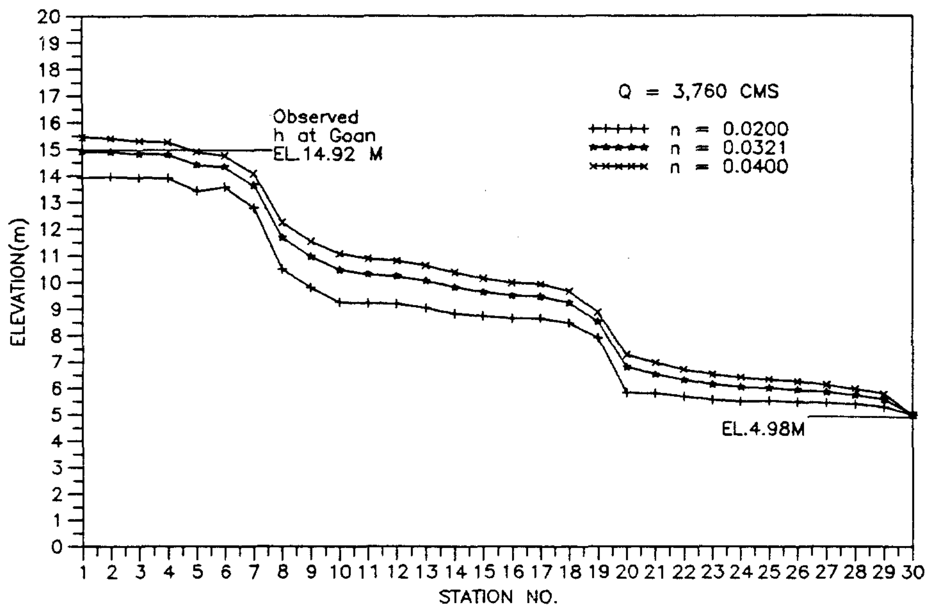

Fig. 4 Initial Conditions for the River Reach

The result of backwater computations with varying n-values for the initial condition of Q= 3, 760 m^3/sec and river stage of h=4. 98 m at Indogyo station is shown in Fig. 4. It can be seen from Fig. 4. that n=0. 031 gives the best fit of the computed stage with the observed values of h=14. 92 m at Goan station.

Hence, the flood discharge of Q=3, 760 m^3/sec and the stage profile obtained with n=0. 0321 as shown in Fig. 4 were taken as the initial conditions for the unsteady flow computations. The values of n was kept constant as 0. 031 throughout the computations.

4. 1. 2 Computation of Lateral Inflows

The lateral inflows from the major tributaries joining with the main reach, Goan-Indogyo, should be considered in the routing computation. Among the tributaries coming in the reach, Wangsuk, Tan, and Jungrang River take more than 90% of the contributing drainage areas between Goan and Indogyo, and their respective watershed characteristics are shown in Table 3. For these tributary rivers sufficient data are not available for hydraulic routing for flood hydrographs at the outlet, and hence a simple hydrologic watershed routing model, Clark method (Clark, 1943), was used to synthesize the tributary flood hydrographs. For the three tributaries the effective rainfall hyetographs were established from the recorded rainfall during the flood of 1981 at Naechon for Wangsuk River, Naksaeng for Tan River and Uijeongbu for Jungrang River by the method of SCS (SCS, 1972). The effective rainfall hyetographs corresponding to the average runoff curve numbers CN=60, 70, 80, were obtained. Each trib-

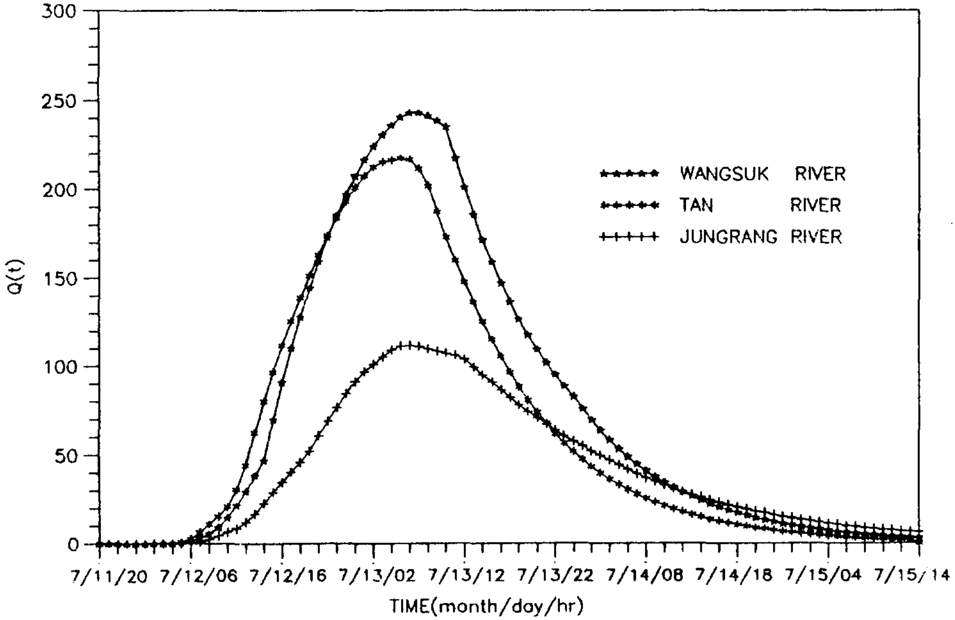

Fig. 5 Lateral Inflow Hydraugraphs form Tributaries (CN=70)

utary watershed was zoned by isovels with due consideration to the time of concentration of the watershed, the watershed storage constant being estimated by an empirical formula(see Table 3).

Table 3. Watershed Characteristics of Tributaries

Tributary	Incoming St.	Area $A(Km^2)$	Length L(m)	Slope S	Storage constant K(hr)
Wangsuk	13	331.2	36.5	0.0055	11.5
Tan	21	351.9	34.5	0.0052	11.3
Jungrang	25	332.5	32.8	0.0024	17.0

The application of effective rainfall hyetographs to each of the time-area histograms for the tributaries resulted the flood hydrographs at the outlets.

Based on the study done by Han River Flood Control Office(1985) the average runoff curve number of CN=70 was selected for the region, and the computed hydrograph, which should be added to the main channel flood flows, are shown in Fig 5. As can be seen in Fig. 5, during the flood of 1981 the Wangsuk and Tan River basin experienced heavier storms than the Jungrang basin. However, the magnitude of peak flows for three basins turned out to be less than 250 m^3/sec which is far smaller than the flood magnitude(see Fig. 6) in the main channel reach. Therefore, it can be said that the rough computation of lateral inflows by Clark method is tolerable in this study.

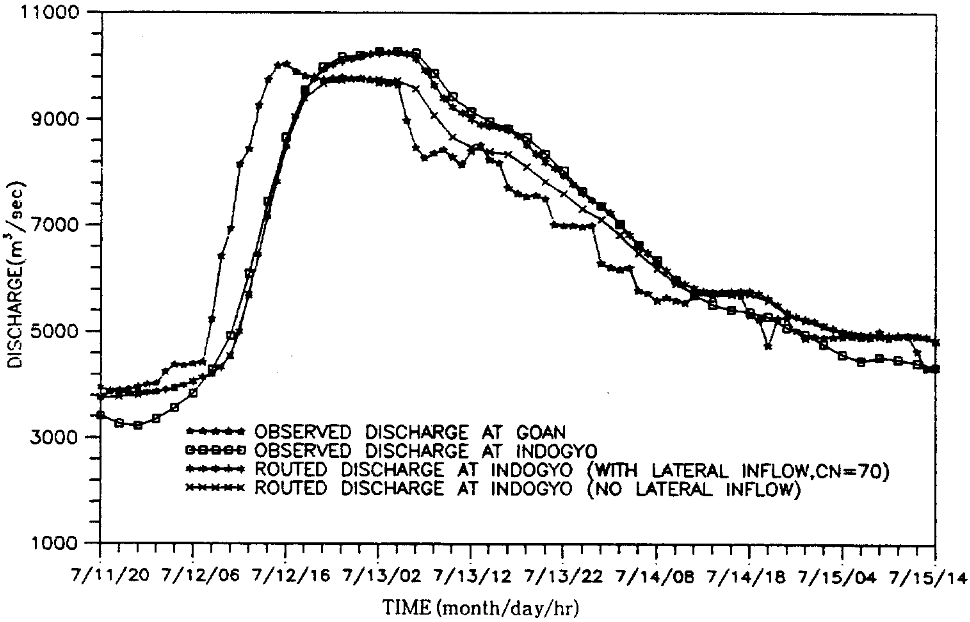

Fig. 6 Comparison of the Computed and Observed Flood Hydrographs at Indogyo station

4.2 Results of Flood Routing Computations

The routing computation for the flood of July 11-15, 1981 was executed for the reach with the upstream boundary condition of flood hydrograph at Goan, the downstream boundary condition of stage-discharge relation at Indogyo, and the initial conditions along the reach computed under the assumption of steady flow at the beginning of flood flows. For the numerical computations of the unsteady flood flow the time increment of $\triangle t$=1hr and the distance increments of $\triangle x$=0.3-2.1 km were used with the Manning's n value fixed at n=0.0321. Since the time increment of $\triangle t$= 1hr selected in the computation is reatively small the influence of time weighting factor θ on the stability and convergence turned out to be not significant. Hence, the value of θ = 0.6 was consitently used throughout the computations as recommended by Fread (1975).

4.2.1 Comparisons of the Computed and Observed Flood Hydrographs at Indogyo Station

The routed flood hydrographs at the downstream boundary, Indogyo, by NWS/NETWORK are compared with the observed hydrograph in Fig. 6. As can be seen in Fig. 6, the routed hydrograph with lateral inflow neglected well compares with the reach inflow hydrograph at Goan; that is, the peak attenuation and the peak time delay demonstrate the usual characteristic of channel flood routing. However, on the falling limb of the hydrograph there appears a significant differences between the observed and routed flood discharges due to the lateral tributary inflow which were neglected in the routing computation.

On the other hand, the routed hydrograph with the lateral inflows corresponding to CN=70 taken into considerartion well simulates the observed hydrograph at Indogyo as can be seen in Fig. 6, in which the peak and the flood volume on the falling limb are shown to increase due to the presence of local inflows. This result clearly demonstrates that NETWORK is highly applicable for flood routing through a river reach with local inflows.

4.2.2 Station Variations of Flood Disicharges at a Specified Time

From the routing results the station-by-station variations of the flood discharges at t=10, 20, 30, 50, 70 hours from the beginning of the flood are plotted in Fig. 7. It can be seen in Fig. 7 that the discharge decreases in the downstream direction at initial stage (t=10, 20 hours) of the flood, but it increases in the downstream direction at later time (t=30, 50, 70 hours). This implies that the flood discharge rises up to t=20-30 hours and then recedes afterward. The effect of lateral inflows from the major tributaries is also shown in Fig. 7

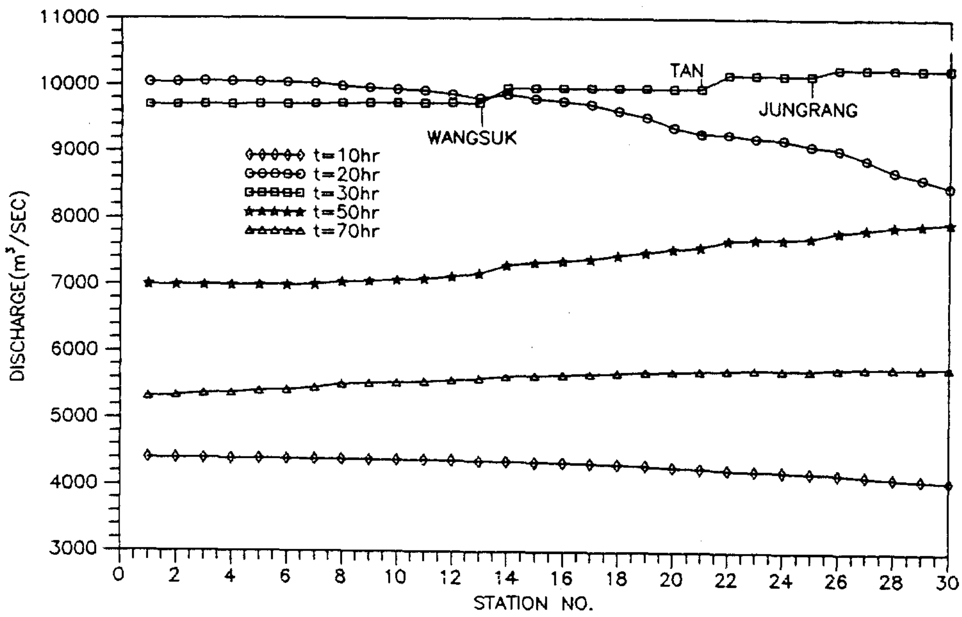

Fig. 7 Station Variations of Flood Discharges at a Specified Time

4.2.3 Time Variations of Flood Discharges at a Station

The time variation of the routed flood hydrographs at the intermediate stations between the upstream and downstream boundaries of the reach is shown in Fig. 8. The peak and flood volume at the Kwangjang station are consistently smaller than those at Dukdo and Indogyo

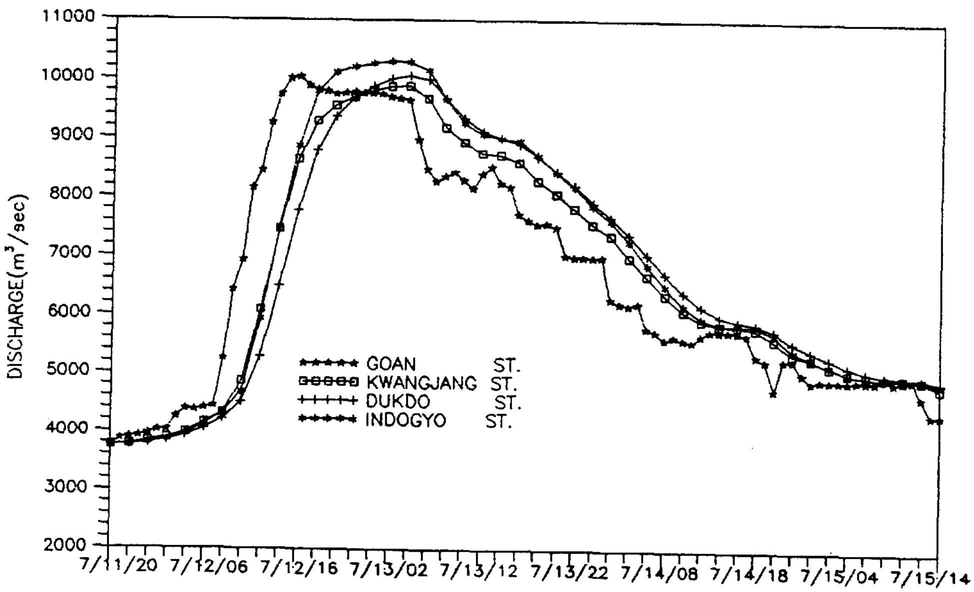

Fig. 8 Time Variation of Flood Discharges at Selected Stations

stations which are located downstream of the Kwangjang station. The overall magnitude of the peak and flood volume at Dukdo shows the same trend comparing with those at the Indogyo station. Fig. 8 well describes the characteristics of peak time delay representing the channel storage effect and flood volume increase due to the tributary inflows.

5. Conclusions

The flood of July 11-15, 1981 through the Goan-Indogyo reach of the Han River was routed by US NWS/NETWORK program partially revised to take into consideration of the tributary inflows coming into the main reach, the tributary inflows having been computed by one of the hydrologic model, the Clark method. The results of computation showed that the NETWORK, a weighted four-point implicit method for the solution of finite difference equations of unsteady flood flow in irregular channels, could simulate with a fair accuracy the observed flood hydrograph at Indogyo station when incorporated with the Clark watershed routing method for the tributary lateral inflows.

The model resulted stable numerical solutions irrespective of the size of reach intervals. The tributary inflows with SCS runoff curve number CN=70 showed the best routing results for the tributary watersheds coming in the reach. This study demonstrated the applicability of NWS/NETWORK for fast and stable routing computations for a river reach with major tributaries coming in it.

However, it appears that more research should be done to more accurately compute the flood from tributary watersheds and to effectively combine them into the implicit dynamic wave model for better routing results at the outlet.

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