論 文
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상태 및 출력에 시간지연이 존재하는 시스템을 위한 칼만필터의 강인성 분석

Robustness Properties of Kalman Filters for Systems with Delays in State and Output

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Abstract - This paper presents robustness properties of Kalman filters for linear time-invariant systems with delays in both the state and the output. The circle condition concerning the return difference matrix is derived. From the circle condition, it can be seen that the Kalman filter guarantees such nondivergence margins as (½,∞) gain margin and ±60° phase margin, which are the same as those for ordinary systems. The results in this paper might be expected to make theoretical background on extending the LQG/LTR method to systems with delay in the output.

1. Introduction

During the last decade, great attention has been devoted to the multivariable robust control design [1]~[6]. Most of theoretical results developed during this period have a remarkable feature that they have extended classical frequency-domain concepts to multivariable systems. In particular, the LQG/LTR method for ordinary systems has received special attention as a robust control design method. As is well known, the the LQG/LTR method owes to two theoretical results. One is the attractive robustness properties of LQ regulators and Kalman filters[4, 7], and the other is the maximally achievable accuracy properties of LQ regulators and Kalman filters[8]. Based on these two results, the LQG/LTR method utilizes the robustness or sensitivity recovery procedure in LQG regulator designs.

There have been made some efforts to analyze the robustness properties of LQ regulators and Kalman filters for delayed systems, W.H. Kwon [9] has first derived the circle condition of LQ regulators. Later, K. Uchida and E. Shimemura [10] and W.H. Lee and B. Levy[11] derived the same circle condition independently. Using the robust stability condition for ordinary systems[5], W.H. Lee and B. Levy[11] showed that LQ regulators for state-delayed systems guarantee the same stability margins as those for ordinary systems. W.H. Kwon and S.J. Lee[12] derived the robust stability condition for delayed systems, and

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showed that LQ regulators and Kalman filters for state-delayed systems guarantee the same robustness margins as those for ordinary systems. In addition, W.H. Kwon and S.J. Lee[12] proved the cheap control property of LQ regulators for state-delayed systems and extended the LQG/LTR method to state-delayed systems.

For input-delayed systems, S.J. Lee et al.[13] derived guaranteed stability margins of LQ regulators in terms of system parameters, and extended the LQG/LTR method to systems with delayed-input only. For systems with delays in both the state and the input, W.H. Kwon and S.J. Lee[14] analyzed guaranteed stability margins of LQ regulators and generalized the LQG/LTR method.

To the authors' knowledge, however, there have been reported no literatures concerning the robustness property of Kalman filters or the LQG/LTR method for system with delay in the output. In this paper, the robustness property of Kalman filters for systems with delays in both the state and the output is investigated. More precisely, the circle condition concerning the return difference matrix of Kalman filters will be derived. Based on the circle condition together with the robust stability condition[12], it will be shown that Kalman filters for systems with delays in both the state and the output guarantee the same nondivergence margins as those for ordinary systems. The results in this paper might be expected to make theoretical background on extending the LQG/LTR method to systems with delays in both the state and the output.

2. Kalman filters with delays in state and output

We consider the following system :

$$\frac{d}{dt}x(t)=A_0x(t)+A_0x(t-h)+w(t) \tag{1}$$

$$y(t)=C_0x(t)+C_1x(t-h)+v(t) \tag{2}$$

where $x(t)$ is an n-dimensional state vector, $y(t)$ is an m-dimensional output vector ; A_0, A_1, C_0, and C_1 are constant matrices with appropriate dimensions; the processes $w(t)$ and $v(t)$ are zero mean white Gaussian noise processes with covariance intensity Q and R, respectively. Furthermore, the noise processes $w(t)$, $v(t)$, and the initial condition are assumed to be independent of each other. The Kalman filters for the systems(1)~(2) is well known and given by[15]

$$\begin{aligned}\frac{d}{dt}\hat{x}(t|t)=&A_0\hat{x}(t|t)+A_1\hat{x}(t-h|t)\\&+[P_0C_0^T+P_1^T(-h)C_1^T]R^{-1}\\&[y(t)-C_0\hat{x}(t|t)-C_1\hat{x}(t-h|t)]\end{aligned} \tag{3}$$

$$\begin{aligned}\hat{x}(t-h|t)=&\hat{x}(t-h|t-h)+\int_{-h}^{0}[P_1(-\sigma-h)C_0^T\\&+P_2(-\sigma-h,\ -h)C_1^T]R^{-1}\\&\cdot[y(t+\sigma)-C_0\hat{x}(t+\sigma|t+\sigma)\\&-C_1\hat{x}(t+\sigma-h|t+\sigma)]d\sigma\end{aligned} \tag{4}$$

with $\hat{x}(\theta|0)=0$ for $-h\le\theta\le0$, where P_0 is an $n\times n$ constant matrix, $P_1(\theta)$ and $P_2(\theta,\ \eta)$ are piecewise absolutely continuous $n\times n$ matrices such that

$$\begin{aligned}0=&A_0P_0+P_0A_0^T+A_1P_1(-h)+P_1^T(-h)A_1^T\\&+Q-[P_0C_0^T+P_1^T(-h)C_1^T]R^{-1}\\&[C_0P_0+C_1P_1(-h)]\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{d}{d\theta}P_1(\theta)=&-P_1(\theta)A_0^T-P_2(\theta,\ -h)A_1^T\\&+[P_1(\theta)C_0^T+P_2(\theta,\ -h)C_1^T]R^{-1}\\&[C_0P_0+C_1P_1(-h)]\end{aligned} \tag{6}$$

$$\begin{aligned}\left(\frac{\partial}{\partial\theta}+\frac{\partial}{\partial\eta}\right)P_2(\theta,\ \eta)=&[P_1(\theta)C_0^T\\&+P_2(\theta,\ -h)C_1^T]\cdot R^{-1}[C_0P_1^T(\eta)\\&+C_1P_2^T(\eta,\ -h)]\end{aligned} \tag{7}$$

with the boundary conditions

$$P_1(0)=P_0,\ P_2(\theta,\ 0)=P_1(\theta) \tag{8}$$

and the symmetry conditions

$$P_0^T=P_0,\ P_2^T(\theta,\ \eta)=P_2(\eta,\ \theta) \tag{9}$$

for $-h\le\theta\le0$ and $-h\le\eta\le0$

3. Robustness Properties of Kalman Filters

The block diagram of the Kalman filter for systems with delays in both the state and the output is shown in Fig. 1, where $\nu(t)$ denotes the innovation vector

$$\nu(t)=y(t)-C_0\hat{x}(t|t)-C_1\hat{x}(t-h|t) \tag{10}$$

The output estimate $\hat{y}(t)$ and the innovation $\nu(t)$ have the following relation in the frequency

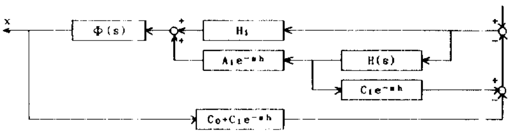

Fig. 1 Block diagram of Kalman filters.

domain.

$$\hat{y}(s)=[(C_0+C_1e^{-sh})\Phi(s)(A_1e^{-sh}H_2(s)+H_1)+C_1e^{-sh}H_2(s)]\nu(s) \quad (11)$$

where

$$H_1=(P_0C_0^T+P_1^T(-h)C_1^T)R^{-1} \quad (12)$$

$$H_2(s)=(\bar{P}(-s)C_0^T+\bar{P}_2(-s)C_1^T)R^{-1} \quad (13)$$

$$\bar{P}_1(s)=\int_{-h}^{0}e^{s\theta}P_1(\theta)d\theta \quad (14)$$

$$\bar{P}_2(s)=\int_{-h}^{0}e^{s\theta}P_2(\theta,\ -h)d\theta \quad (15)$$

$$\Phi(s)=(sI-A_0-A_1e^{-sh})^{-1} \quad (16)$$

The return difference matrix $T(s)$ at the filter input is defined by

$$T(s)=I+(C_0+C_1e^{-sh})\Phi(s)(A_1e^{-sh}H_2(s)+H_1)+C_1e^{-sh}H_2(s) \quad (17)$$

To derive the circle condition concerning the return difference matrix, we will transform the operator-type Riccati equations (5)～(7) to algebraic ones in the frequency domain.

LEMMA 1 : The operator-type Riccati equations (5)～(7) with the boundary condition (8) can be represented in the frequency domain as follows:

$$0=-\Delta(s)P_0-P_0\Delta^T(-s)-A_1P_0e^{-sh}-P_0A_1^Te^{sh}+A_1P_1(-h)+P_1^T(-h)A_1^T+Q-[P_0C_0^T+P_1^T(-h)C_1^T]R^{-1}[C_0P_0+C_1P_1(-h)] \quad (18)$$

$$0=\bar{P}_1(s)\Delta^T(s)+(\bar{P}_1(s)-\bar{P}_2(s)e^{sh})A_1^Te^{-sh}+P_1(-h)e^{-sh}-P_0+[\bar{P}_1(s)C_0^T+\bar{P}_2(s)C_1^T]R^{-1}[C_0P_0+C_1P_1(-h)] \quad (19)$$

$$0=\bar{P}_1^T(s)-\bar{P}_2^T(s)e^{sh}+\bar{P}_1(-s)-\bar{P}_2(-s)e^{-sh}-[\bar{P}_1(-s)C_0^T+\bar{P}_2(-s)C_1^T]R^{-1}[C_0\bar{P}_1^T(s)+C_1\bar{P}_2^T(s)] \quad (20)$$

where

$$\Delta(s)=sI-A_0-A_1e^{-sh} \quad (21)$$

Proof : See Appendix

Now we can state the following theorem concerning the return difference matrix.

THEOREM 1 : The return difference matrix $T(s)$ satisfies the following relation in the frequency domain:

$$T(jw)RT^*(jw)=R+(C_0+C_1e^{-jwh})\Phi(jw)Q\Phi^*(jw)(C_0+C_1e^{-jwh})^* \quad (22)$$

where* denotes the complex conjugate transpose.

Proof : See Appendix.

For investigating the robustness property of Kalman filters, it is convenient to represent the uncertainties such a multiplicative perturbation factor as shown in Fig. 2, where $[I+L(s)]^{-1}$ is the uncertainty of the system reflected to the loop breaking point X. Since the left-hand side of the equation (22) is hermitian and $(C_0+C_1e^{-jwh})\Phi(jw)Q\Phi^*(jw)(C_0+C_1e^{-jwh})^*\geq 0$, we can obtain the circle condition.

$$T(jw)RT^*(jw)\geq R \quad (23)$$

Note that for single-output systems the relation (23) becomes $|T(jw)|^2\geq 1$, from which it can be seen that Kalman filters guarantee such nondivergence margins as (1/2, ∞) gain margin and ±60° phase margin. For multivariable systems, the relation (23) is inadequate for rebustness analysis, since a bound on $\sigma_{min}[T(jw)]$ is difficult to obtain from the relation (23) where $\sigma_{min}(\cdot)$ denotes the minimum singular value. Hence, we define a modified return difference matrix

$$\hat{T}(jw)=R^{-1/2}T(jw)R^{1/2} \quad (24)$$

which satisifes

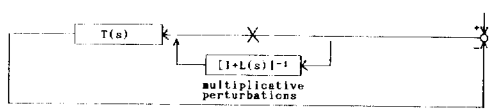

Fig. 2 Kalman filter configuration for robustness analysis.

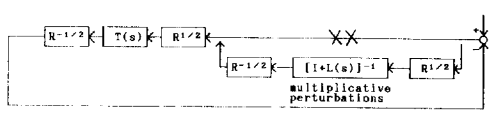

Fig. 3 Modified Kalman filter configuration for robustness analysis.

$$\hat{T}(jw)\hat{T}^*(jw) \geq I \quad (25)$$

for $0 \leq w < \infty$. The configuration with the modified return difference matrix *in* shown in Fig. 3. The nondivergence margins at the loop breaking point X in Fig. 2 turn out to be the same as those at the loop breaking point XX in Fig. 3 if $[I+L(s)]^{-1}$ and R are diagonal, since Fig. 2 with the perturbation $[I+L(s)]^{-1}$ inserted at the loop breaking point X is equivalent, in view of filter divergence, to Fig. 3 with the perturbation $R^{-1/2}[I+L(s)]^{-1}R^{1/2}$ is inserted at the loop breaking point XX under the condition that $[I+L(s)]^{-1}$ and R are diagonal[4].

With the configuration in Fig. 3, we will present the guaranteed nondivergence margins of Kalman filters. Henceforth, for simplicity we assume the system(1) ~ (2) has no poles on the imaginary axis.

THEOREM 2 : The Kalman filters(3) ~ (4) for the system (1) ~ (2) guarantee the same nondivergence margins as those for ordinary systems, that is, $(1/2, \infty)$ gain margin and $\pm 60°$ phase margin in the presence of noninteracting perturbations at each input loop when R is diagonal.

Proof : See Appendix.

Note that the result in Theorem 2 is not dual to that in [14] concerning the guaranteed stability margins of LQ regulators for systems with delays in both the state and the input. If Kalman filters are derived using such an innovation vector as

$$\nu(t) = y(t) - C_0\hat{x}(t|t) - C_1\hat{x}(t-h|t-h) \quad (26)$$

instead of the innovation vector(10), the result might be expected dual to that in [14]. Finally, it should be mentioned that for systems with delay in the output only, that is, in case of $A_1 = 0$, the same result as in theorem 2 is obtained.

4. Conclusions

In this paper, it is shown that Kalman filters for systems with delays in both the state and the output guarantee the same nondivergence margins as those for ordinary systems. For this analysis, the circle condition concerning the return difference matrix for Kalman filters is derived. The results in this paper generalize the well-known ones for ordinary systems and can be extended to more general delayed systems.

Finally, it should be mentioned that the LQG/LTR method for systems with delays in the output, regardless of the existence of delay in the state, remains for further investigations.

Appendix

Proof of Lemma 1 :

The equation (18) can be directly derived from the equation(5). Multiplying both sides of the equation(6) by $e^{s\theta}$ and intergrating from $-h$ to 0 with respect to θ yeild the euqation(19). Multiplying both sides of the euqation(7) by $e^{s(\eta-\theta)}$ and integrating from $-h$ to 0 with respect to η and θ yield the euqation(20). This completes the proof.

Proof of Theorem 1 :

From the return difference matrix equation(17) and (11) ~ (14), it follows that

$$\begin{aligned} T(s)RT^T(-s) &= [I+(C_0+C_1e^{-sh})\Phi(s)(A_1e^{-sh}H_2(s)+H_1) \\ &\quad + C_1e^{-sh}H_2(s)] \cdot R[I+(H_2^T(-s)A_1^Te^{sh} \\ &\quad + H_1^T)\Phi^T(-s)(C_0^T+C_1^Te^{sh}) \\ &\quad + H_2^T(-s)C_1^Te^{sh}] \\ &= R+(C_0+C_1e^{-sh})\Phi(s)(A_1e^{-sh}H_2(s) \\ &\quad + H_1)R + C_1e^{-sh}H_2(s)R + R(H_2^T(-s) \\ &\quad A_1^Te^{sh}+H_1^T)\Phi^T(-s)(C_0^T+C_1^Te^{sh}) \\ &\quad + RH_2^T(-s)C_1^Te^{sh} + (C_0+C_1e^{-sh}) \\ &\quad \Phi(s)(A_1e^{-sh}H_2(s)+H_1)R(H_2^T(-s) \\ &\quad A_1^Te^{sh}+H_1^T)\Phi^T(-s) \cdot (C_0^T+C_1^Te^{sh}) \\ &\quad + (C_0+C_1e^{-sh})\Phi(s)(A_1e^{-sh}H_2(s) \\ &\quad + H_1)R(H_2^T(-s)C_1^Te^{-sh}) + C_1e^{-sh} \\ &\quad H_2(s)R(H_2^T(-s)A_1^Te^{sh} \\ &\quad + H_1^T)\Phi^T(-s)(C_0^T+C_1^Te^{sh}) + C_1H_2(s) \\ &\quad RH_2^T(-s)C_1 \end{aligned} \quad (27)$$

Using Lemma 1, (27) becomes

$$\begin{aligned} T(s)RT^T(-s) &= R+(C_0+C_1e^{-sh})\Phi(s)Q\Phi^T(-s)(C_0^T \\ &\quad + C_1^Te^{sh}) \end{aligned} \quad (28)$$

Therefore, the euqation(22) is obtained from the euqation(28) by replacing s by jw. This complete the proof.

Proof of Theorem 2 :

Let $G(s)$ be a loop transfer function matrix which is stable given by

$$G(s)=(C_0+C_1e^{-sh})\Phi(S)(A_1e^{-sh}H_2(s)+H_1)+C_1e^{-sh}H_2(s) \qquad (29)$$

and $\tilde{G}(s)$ be perturbed system such that

$$\tilde{G}(s)=G(s)(I+L(s))^{-1} \qquad (30)$$

where $L(s)=\text{diag}[l_1(t)l_2(s)\cdots l_m(s)]$ and R is also diagonal. From the circle condition(25), it follows that

$$\sigma_{\min}(\hat{T}(jw))\geq 1 \qquad (31)$$

for $0\leq w<\infty$. Using the condition(31) and the robust stability condition for the delayed system [12], it can be seen that the perturbed system is stable if the condition

$$|l_i(jw)|<1 \ \ \forall 0\leq w<\infty,\ i=1,2,\cdots,m \qquad (32)$$

is satisfied.

In order to obtain gain margins, let

$$l_i(s)=l_i,\ l_i : \text{real} \qquad (33)$$

then Equation(32) becomes

$$\frac{1}{2}<\frac{1}{l_i+1} \qquad (34)$$

Thus, the guaranteed gain margin is $(1/2, \infty)$.

Similary, to obtain phase margins, let

$$\frac{1}{1+l_i(s)}=e^{j\phi_i(s)},\ \phi_i(s) : \text{real} \qquad (35)$$

then Equation(32) becomes

$$\cos\phi_i(s)>1/2 \qquad (36)$$

From Equation(36), it can be seen that the guaranteed phase margin is $\pm 60^\circ$

This completes the proof.

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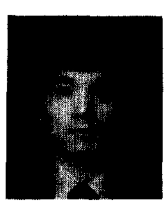

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