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Dynamics of Interacting Multiple Autonomous Mobile Robots

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Abstract- This paper deals with the global dynamic behavior of multiple autonomous mobile robots with suggested navigation strategies within unbounded and bounded spatial domain. We derive some navigation strategies of robots with complete detectability with finite range to reach their goal states without collision which is motivated by Coulomb's law regarding repulsive and attractive forces between electrical charges. An analysis of the dynamic behavior of the interacting robots with the suggested navigation strategies under the assumption that communication is not permissible between robots is made and some examples are illustrated by computer simulation. The convergence of robot motions to their goal states under certain conditions is established by considering their global dynamic behavior even when some objects are close to their goal points.

1. Introduction

Navigation for a mobile robot means finding a path between its starting position and goal position without collision with other objects such as other robots and obstacles. In real-time operation the robot may encounter unanticipated objects when moving to its goal point. In this paper, we develop collision-free strategies of robots motivated by Couloumb's law within both bounded and unbounded spatial domain and analyze their dynamic behaviors.

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Many papers have been published on movement of manipulators without collision. At the present time, relatively little work has been done on the navigation of mobile robots, in particular, autonomous mobile robots(AMRs) although they have some degree of adaptability to cope with unexpected events. Wang[1, 2] described a strategy for AMRs employing a vision system for navigation around other robots within a specified domain. Okotomi and Mori[3] introduced a method that determined robot movement by means of potential fields, and proposed two new concepts : a state space description of the movement of a robot, and an oval potential characterization of the potential field. Lumelsky[6] provided an algorithm that guarantees reaching the goal by modelling a robot with tactile sensors as a point among arbitrarily shaped obstacles. Freud and Hoyer[10] presented a method for real time path planning in multi-robot systems by using a hierarchical coordinator.

In this paper, we consider the navigation of robots based on the dynamic interaction between a robot and objects, and a robot and its goal position. The following basic assumptions are introduced:

1) All the objects are spherically shaped. 2) The mass distribution of the robot is spherically symmetric. 3) Every robot is equipped with sensors which are sufficient to gather the necessary information regarding the environment. 4) Each robot knows its goal position. 5) Rotation of a robot with respect to its center of mass is not considered.

Our proposed approach is motivated by the interactive force between electric charges. The interactive force is repulsive if two charges are of the same sign, and the interactive force is attractive if two charges are of the opposite sign. For a robot, all other objects are considered to be point charges of the same sign. A goal point is considered as a point charge whose sign is opposite to that of a robot.

To model the robot dynamics, we consider each robot to be a point mass at the center of a spherical domain. The equation of motion of robot i has the following form where the dot denotes differentiation with respect to t.

$$m_i \ddot{r}_i(t) + b_i \dot{r}_i(t) = f_i(t) \qquad (1)$$

where m_i denotes the mass, $b_i(>0)$, the friction coefficient, and f_i, the control force. Here, we assume that the admissible control force f_i is a piecewise continuous function of t.

Our main concerns are:

1) the derivation of navigation strategies of robots to avoid collision when multiple robots are interacting with each other.

2) the analysis of the global dynamic behavior of robots for given navigation strategies.

2. Navigation Stratgies of AMR

This chapter is devoted to the derivation of navigation strategies of a robot in the presence of detectable robots.

2.1 Homing Navigation Strategy

This section focuses on the simplest case in which neither objects nor a domain boundary are detectable from a robot. Let $e(t)=r_g-r(t)$ denote the position error of the robot at time t, where r_g is the goal position of the robot. The motion for a robot is assumed to be describable by

$$m\ddot{r}(t)+b\dot{r}(t)=f(t) \qquad (2)$$

where the homing navigation strategy is given by

$$f(t) = \begin{cases} f_c\left(\dfrac{e(t)}{\|e(t)\|} - \alpha g(\dot{r}(t))\right) & \text{if } e(t) \neq 0 \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

where f_c is a positive constant, and α is a given positive rate coefficient. The function $g=g(\dot{r})$on $R^3 \rightarrow R^3$ satisfies$<\dot{r}, \ g(\dot{r})>>0$ for all $\dot{r} \neq 0$, and $g(0)=0$ where $<, \ >$ denotes inner product. From the practical standpoint, it is highly desirable for a robot to have a nonoscillatory approach to its goal state. This requirement can be fullfilled by introducing velocity feedback in the navigation strategy.

Let $x_1=r-r_g$ and $x_2=\dot{x}_1=\dot{r}$. Then the state space equations are given by

$$\dot{x}_1=x_2 \qquad (4)$$

$$\dot{x}_2=-\frac{b}{m}x_2-\frac{f_c}{m}\frac{x_1}{\|x_1\|}-\frac{f_c\alpha}{m}g(x_2) \qquad (5)$$

2.1.1 Convergence to Goal State:

In this section, we shall establish the convergence of the robot motion to the goal state. Consider the real valued positive definite C^1 function $V=V(x)$ defined on R^6:

$$V(x)=\frac{1}{2}m\|x_2\|^2+f_c\|x_1\| \qquad (6)$$

Its derivative with respect to t along a trajectory of (4) and (5) is given by

$$\begin{aligned}\dot{V}(x) &= <x_2, \ \dot{x}_2> + f_c\frac{\langle x_1, \ x_2\rangle}{\|x_1\|} \\ &= -b\|x_2\|^2 - f_c\alpha\langle x_2, g(x_2)\rangle \leq 0\end{aligned} \qquad (7)$$

Thus V is a Lyapunov function of $x=f(x)$ on R^6. By inspection, the origin of R^6 is the unique equilibrium state of (4) and (5). Consider the set E given by

$$E=\{x\in R^6 : \dot{V}(x)=0\}$$
$$=\{x\in R^6 : x_2=0\} \tag{8}$$

Then, M, the largest invariant set in E, is the set containing only the origin. By LaSalle's Inveriance Principle[8, 9], the region of attraction of the origin is the whole space R^6. Thus, a robot starting with any initial state x_0, tends its goal state as $t\rightarrow\infty$.

2.2 Navigation Strategies in the Presence of Multiple Objects

This section is devoted to the derivation of navigation strategies of a robot in the presence of multiple objects. We shall derive navigation strategies of robots when there is no communication between robots. The control force for robot i is given by

$$f_i(t)=f_{gi}\frac{r_{gi}-r_i}{\|r_{gi}-r_i\|}-f_{gi}\alpha_i g(\dot{r}_i)$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{1} f_{ij}\left(\frac{r_i-r_j}{\|r_i-r_j\|^3}-\frac{r_{gi}-r_j}{\|r_{gi}-r_j\|^3}\right) \tag{9}$$

where 1 $(\neq 0)$ denotes the number of objects within the detectable range of robot i and α_i is a given positive coefficient. $f_i(t)$consists of attractive force between robot i and its goal position, damping force and repulsive force between robot i and the other objects. Since we assume that the robots navigate without communication, each robot does not know the goal positions of the others. It is desirable to introduce a heavier weighting coefficient f_{ij} depending on the nature of the objects, on an obstacle than a robot since an obstacle does not have a navigation strategy.

We set $x_1^{(i)}=r_i-r_{gi}$ and $x_2^{(i)}=\dot{r}_i$ for $i=1,\cdots,n$, where n is the number of robots under consideration.

Consider the following equations for robot i.

$$\dot{x}_1^{(i)}=x_2^{(i)}, \tag{10}$$
$$\dot{x}_2^{(i)}=-\frac{b_i}{m_i}x_2^{(i)}+\frac{1}{m_i}\Bigg\{-f_{gi}\left(\frac{x_1^{(i)}}{\|x_1^{(1)}\|}+\alpha_i g(x_2^{(i)})\right)$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{1} f_{ij}\left(\frac{x_1^{(i)}+r_{gi}-x_1^{(j)}-r_{gj}}{\|x_1^{(i)}+r_{gi}-x_1^{(j)}-r_{gj}\|^3}-\frac{r_{gi}-x_i^{(j)}-r_{gj}}{\|r_{gi}-x_1^{(j)}-r_{gj}\|^3}\right)\Bigg\} \tag{11}$$

It is notable that the term including $\sum$ becomes zero when robot i can not detect robot j.

2.2.1 Convergence to Goal State :

We shall determine the domains of attraction of the origin for two simple cases :

1) Robot 1 detects robot 2, but robot 1 is not detectable from robot 2.

2) Two robots are mutually detectable. (This will be extended to n robot case)

Case1 : Robot 1 detects robot 2, but robot 1 is not detectable from robot 2. In this case, robot 2 applies the homing navigation strategy (2) to reach its goal state. The main difficulty in applying the Lyapunov Direct Method is the lack of systematic procedures for constructing the function V. Here, we consider the real-valued C_1 function $V=V(x)$ given by

$$V(x)=\sum_{i=1}^{2}\left(\frac{1}{2}m_i\|x_2^{(i)}\|^2+f_{gi}\|x_1^{(i)}\|\right)$$
$$+m_1\langle x_1^{(1)},\ x_2^{(1)}\rangle \tag{12}$$

Let Ω_1 denote the subset of R^{12} defined by

$$\Omega_1=\{x_1\in R^{12} : \|x_1^{(1)}\|<\frac{2f_{g1}}{m_1}\} \tag{13}$$

Then, $V(x)$ is positive definite on Ω_1.
$\dot{V}$ in view of (10)-(11), is given by

$$\dot{V}(x)=\sum_{i=1}^{2}(m_i\langle x_2^{(i)},\ x_2^{(i)}\rangle+f_{gi}\frac{\langle x_1^{(i)},\ x_2^{(i)}\rangle}{\|x_1^{(i)}\|})+m_1\langle x_2^{(i)},\ x_2^{(i)}\rangle$$
$$+m_1\langle x_1^{(1)},\ x_2^{(1)}\rangle\leq -q_1\|x_2^{(1)}\|^2-b_2\|x_2^{(2)}\|^2$$
$$-\sum_{i=1}^{2}f_{gi}\alpha_1\langle x_2^{(i)},\ g(x_2^{(i)})\rangle$$
$$+s_1(\|x_1^{(1)}\|-\frac{f_{g1}}{2s_1})^2-\frac{f_{g1}^2}{4s_1} \tag{14}$$

where $q_1=b_1/2-m_1-(f_{g1}\alpha_1 k)/2-f_{12}/2$ and $s_1=b_1/2+(f_{g1}\alpha_1)/2+3f_{12}/2$. Let the subset Ω_2 be a subset of R^{12} defined by

$$\Omega_2=\{x\in R^{12} : \|x_1^{(1)}\|\langle f_{g1}/s_1\} \tag{15}$$

Let $E=\{x\in\overline{\Omega}_2 : \dot{V}(x)=0\}$ where $\overline{\Omega}_2$ denotes the closure of Ω_2.

We wish to construct Ω, a domain of attraction of the origin. The largest invariant set M in E is $\{x_{eq}\}$, where x_{eq} is the equilibrium state of (10) and (11) closest to the origin. Since robot 2 applies its homing navigation strategy, and the origin is the unique equilibrium state, hence M consists of only the origin. Thus, for $q_1>0$, a domain of attraction of the origin is $\Omega=\Omega_1\cap\Omega_2$. The motions of both robots initiated inside Ω converge to the origin as $t\to\infty$.

Case 2 : Two robots are mutually detectable.

In this case, each robot is assumed to be in the detectable range of the other. A convergence condition to the origin of the system consisting of two robots is established first before we extend it to the n-robot case.

Here, we choose a Lyapunov function candidate on R^{12} given by

$$V(x)=\sum_{i=1}^{2}\left(\frac{1}{2}m_i x_2^{(i)2}+m_i<x_1^{(i)}, x_2^{(i)}>\right) \tag{16}$$

Let $\Omega_1=\{x\in R^{12}:\|x_1^{(i)}\|<2f_{gi}/m_i \text{for } i=1,\ 2\}$. We observe that V is positive definite on Ω_1. Differentiating V with respect to t results in

$$\begin{aligned}\dot V(x)=&\sum_{i=1}^{2}\Big(m_i\langle x_2^{(i)},\ x_2^{(i)}\rangle\\&+f_{gi}\frac{\langle x_1^{(i)},\ x_2^{(i)}\rangle}{\|x_1^{(i)}\|}\Big)\\&+\sum_{i=1}^{2}(m_i\langle x_2^{(i)},\ x_2^{(i)}\rangle\\&+m_i\langle x_1^{(i)},\ \dot x_2^{(i)}\rangle)\\&\le-\sum_{i=1}^{2}(f_{gi}\alpha_1\langle x_2^{(i)},\ g(x_2^{(i)})\rangle\\&+q_i\|x_1^{(i)}\|^2)\\&+\sum_{i=1}^{2}\left\{s_i\left(\|x_1^{(i)}\|-\frac{f_{gi}}{2s_i}\right)^2-\frac{f_{gi}}{4s_i}\right\}\end{aligned} \tag{17}$$

where $q_1=b_1/2-m_1-(f_{g1}\alpha_1k_1)/2-f_{23}/2$, $q_2=b_2/2-m_2-(f_{g2}\alpha_2k_2)/2-f_{21}/2$, $s_1=b_1/2+(f_{gi}\alpha_1k_1)/2+3f_{12}/2$, and $s_2=b_2/2+(f_{g2}\alpha_2k_2)/2+3f_{21}/2$. k_i is a positive real number that satisfies Lipschitz condition. Define $\Omega_2=\{x\in R^{12}:\|x_1^{(i)}\|<f_{gi}/s_i$ for $i=1,\ 2\}$. V is a Lyapunov function on $\Omega_1\cap\Omega_2$. Here, a region of attraction of the origin is $\Omega=\Omega_1\cap\Omega_2$. Therefore, the motions of robots starting with initial states $x_0^{(i)}$ in Ω tends to their goal states as $t\to\infty$.

Now, we wish to extend the foregoing result to the case of n robots in the three-dimensional position space. Consider the real-valued positive-definite C^1 function

$$\begin{aligned}V(x)=&\sum_{i=1}^{n}\Big(\frac{1}{2}m_i\|x_2^{(i)}\|^2+f_{gi}\|x_1^{(i)}\|\\&+m_i<x_1^{(i)},\ x_2^{(i)}>\end{aligned} \tag{18}$$

Define

$$\begin{aligned}\Omega_1=&\{x\in R^{6n}:\ \|x_1^{(i)}\|<2f_{gi/mi}\\&\text{for } i=,\cdots,n\}\end{aligned} \tag{19}$$

Clearly V is positive definite on Ω_1. Taking the derivative of V with respect to t yields

$$\begin{aligned}\dot V(x)=&\sum_{i=1}^{n}\Big(m_i\langle x_2^{(i)},\ \dot x^{(i)}\rangle\\&+f_{gi}\frac{\langle x_1^{(i)},\ x_2^{(i)}\rangle}{\|x_1^{(i)}\|}\Big)\\&+\sum_{i=1}^{n}(m_i\langle x_2^{(i)},\ x_2^{(i)}\rangle\\&+m_i\langle x_1^{(i)},\ x_2^{(i)}\rangle)\\&\le-\sum_{i=1}^{n}(f_{gi}\alpha_i\langle x_2^{(i)},\ g(x_2^{(i)})\rangle\\&+q_i\ \|x_1^{(i)}\|^2+\sum_{i=1}^{n}\{s_i\left(\|x_1^{(i)}\|-\frac{f_{gi}}{2s_i}\right)^2-\frac{f_{gi}^2}{4s_i}\}\end{aligned} \tag{20}$$

Consider the subset Ω_2 defined by

$$\begin{aligned}\Omega_2=&\{x\in R^{6n}:\ \|x_1^{(i)}\|\\&\langle f_{gi}/s_i \text{for } i=1,\cdots,n\}\end{aligned} \tag{21}$$

Then, V, is negative definite on Ω_2. A region of attraction of the origin is $\Omega=\Omega_1\cap\Omega_2$. Thus, the motions of all the robots initiated inside Ω converge to their goal states.

2.3 Navigation Strategy among Stationary Obstacles within Boundary Domain

Navigation strategy of a robot among stationary obstacles can be considered a special case of case 1 in section 2.2. We only need to set $x_1^{(j)}=0$ in equation (11) since r_{gj} is fixed. Its convergence is simpler than that in case of section 2-2. The proposed navigation strategy may be used among moving obstacles but the convergence of a robot can be considered only for special cases. Collision avoidance among obstacles is another

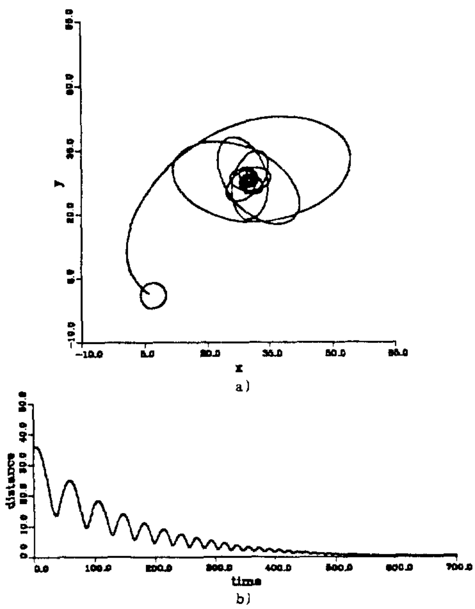

Fig. 1 Trajectories of the robot with homing navigation strategy
a) Trajectory of the robot when $b=0.1$ and $a=0$
b) Distance between the robot and its goal position $(x(0),\ y(0),\ \dot{x}(0),\ \dot{y}(0))=(7, 1, -10, 5), (x_g,\ y_g)=(30, 28)$ robot parameters : $m=1kg, f_c=10N$, $r=1m$

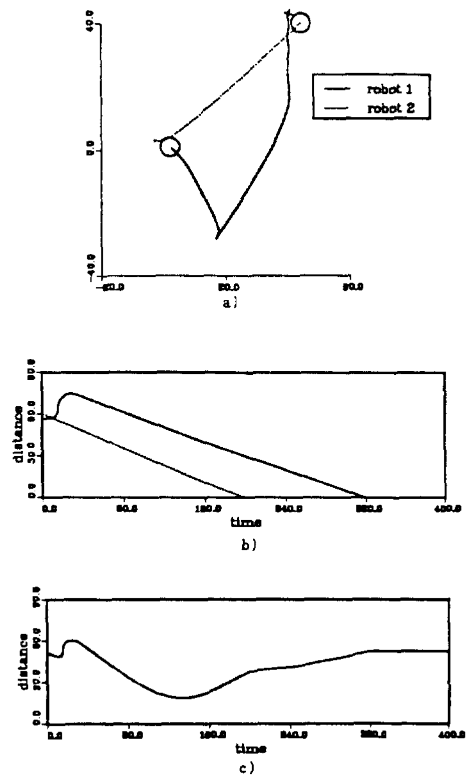

Fig. 2 Trajectries of two robots (case 1)
a) Robot 2 is detectable from robot 1 but robot 1 is out of detectable range of robot 2.
b) Distance between robot 1 and its goal point.
c) Distance between the two robots $(x^{(1)}(0),\ y^{(1)}(0),\ \dot{x}^{(1)}(0),\ \dot{y}^{(1)}(0))=(2, 1, -3, 2)$, $(x_{g1},\ y_{g1})=(40, 43)$ $(x^{(2)}(0),\ y^{(2)}(0),\ \dot{x}^{(2)}(0),\ \dot{y}^{(a)}(0))=(44, 40, -2, 2)$, $(x_{g1},\ y_{g1})=(-3, 3)$

topic in robotics. In robot navigation, boundary can be regarded as a stationary obstacle. The convergence of a robot within boundary domain is omitted since it can be easily deduced from the previous case.

3. Simulation Results

Since it is impossible to determine analytically the dynamic behavior of robots with the proposed navigation strategies, we resort to computer simulation studies. For simplicity, we consider only the navigation of robots whose motions are restricted to a plane.

First, we study the performance of a single robot with velocity feedback $a=0$. It can be shown that the robot motion is oscillatory in nature. Next we consider a pair of identical robots with the same navigation strategy given by (10) and (11). Figure 2 and 3 show the trajectories of two robots without communication for case 1 and case 2 respectively. Since they can not communicate with each other, each robot can only use the information about the position of the other robot acquired by sensors when the other robot is within the detectable range. Robot 2 adopts its

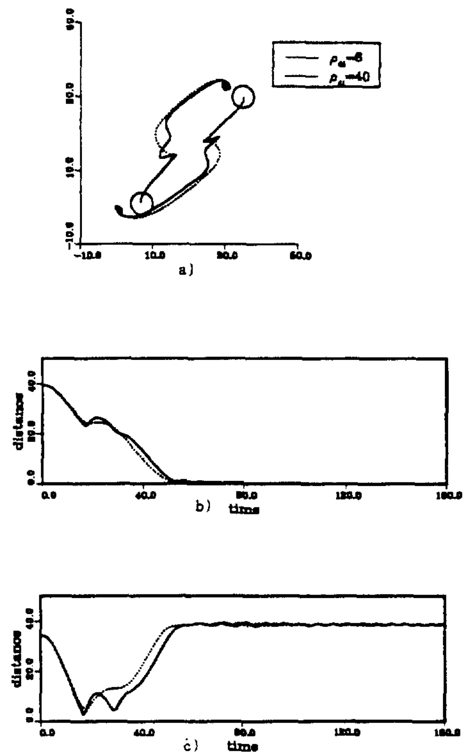

Fig. 3 Trajectries of two robots with various detectable ranges

a) Trajectories of two robots

b) Distance between robot 1 and its goal position.

c) Distance between the two robots

$(x^{(1)}(0),\ y^{(1)}(0),\ \dot{x}^{(1)}(0),\ \dot{y}^{(1)}(0))=(7,\ 1,-3,\ 2)$, $(x_{g1},\ y_{g1})=(30,\ 33)$ $(x^{(2)}(0),y^{(2)}(0),\dot{x}^{(2)}(0),\ \dot{y}^{(2)}(0))=(35,\ 30,\ 3,\ -2)$, $(x_{g1},\ y_{g1})=(58,\ 34)$

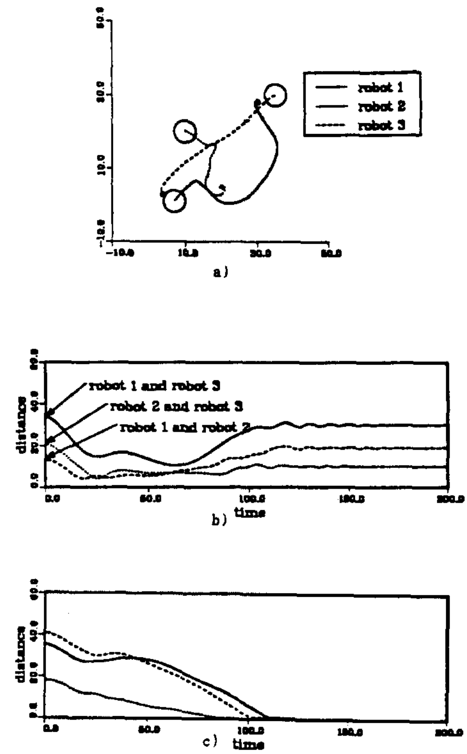

Fig. 4 Trajectries of three robots($\alpha=0.1$)

a) Trajectories of three robots

b) Distance between robots.

c) Distance between the robots and respective goal positions

$(x^{(1)}(0),\ y^{(1)}(0),\ \dot{x}^{(1)}(0),\ \dot{y}^{(1)}(0))=(7,\ 1,-0,\ 1)$, $(x_{g1},\ y_{g1})=(30,\ 28)$ $(x^{(2)}(0),\ y^{(2)}(0),\ \dot{x}^{(2)}(0)\ \dot{y}^{(2)}(0))=(10,\ 20,-1,\ 3)$, $(x_{g2},\ y_{g2})=(20,\ 5)$ $(x^{(3)}(0),\ y^{(3)}(0),\ \dot{x}^{(3)}(0),\ \dot{y}^{(3)}(0))=(35,\ 30,\ -2,\ 0)$, $(x_{g1},\ y_{g1})=(4,\ 3)$

homing navigation strategy. But the movement of robot 1 depends on the position of robot 1 depends on the position of robot 2. We observe from figure 3 that both robots with various detectable ranges interact throughout their motion. Since each robot does not know the goal position of the other robot, each robot only makes use of the current position of the other robot. It is evident that a robot having higher dectectable range reaches its goal state faster than one with a low detectable range, since it is capable of detecting obstacles in advance. Figures 4 and 5 show the trajectories of three identical robots with various α. Figure 6 shows that the proposed navigation strategies can be available among stationary obstacles in a bounded domain by considering them as stationary robots. Generally, we can consider 2^n-1 possible interacting cases for $n(\ >1)$robots in free space. When the velocity feedback coefficients α's are small, the dynamic behaviors of the interacting robots could become

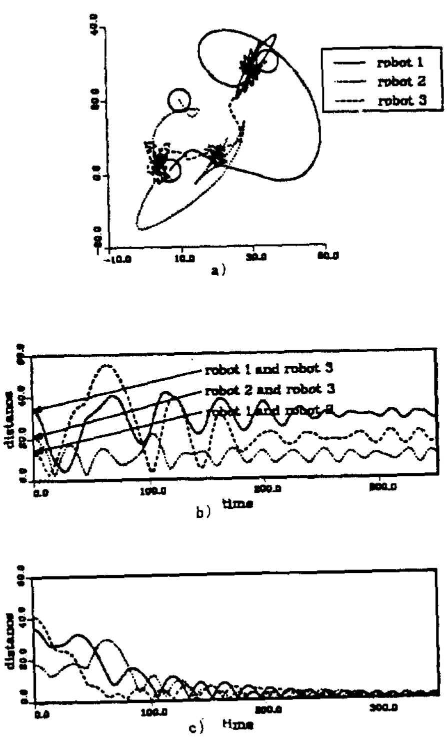

Fig. 5 Trajectries of three robots($\alpha=0.01$)
a) Trajectories of three robots
b) Distance between the robot.
c) Distance between the robots and respective goal positions

very complex as shown in figure 5.

4. Conclusions

In general, it is impossible to derive navigation strategies which are effective for all situations. Here, the strategies for collision avoidance are derived by introducing appropriate repulsive forces between the objects and the robots when they are close to each other, while the homing navigation strategy is derived by introduting appropriate attractive forces between the goal point and the robot.

In an unbounded spatial domain, the proposed navigation strategies give satisfactory dynamic performance under most situations. The oscillatory approach to the goal point of the robot can be avoided by introducing appropriate velocity feed-

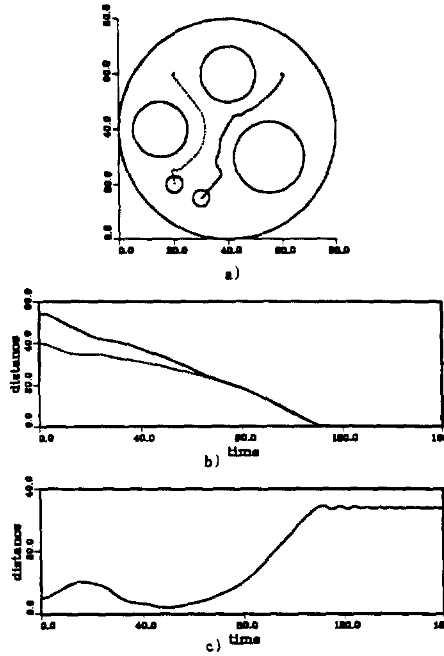

Fig. 6 Trajectries of two robots among obstacles within a specified domain
a) Trajectories of the robots
b) Distance between robots and respective goal positions.
c) Distance between the robots

back.

The effectiveness of the proposed navigation strategies depends on the detectable range of the robot. A robot having high detectable range reaches its goal state faster than one with low detectable range, since it is capable of detecting objects in advance.

The proposed approaches for the navigation of AMRs may be applied to many physical situations such as space station assembly and automatic collision avoidance of vehicles.

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