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Adaptive Parametric Estimation and Classification of Remotely Sensed Imagery Using a Pyramid Structure

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Abstract

An unsupervised region based image segmentation algorithm implemented with a pyramid structure has been developed. Rather than depending on traditional local splitting and merging of regions with a similarity test of region statistics, the algorithm identifies the homogenous and boundary regions at each level of pyramid, then the global parameters of each class are estimated and updated with values of the homogenous regions represented at that level of the pyramid using the mixture distribution estimation. The image is then classified through the pyramid structure. Classification results obtained for both simulated and SPOT imagery are presented.

I. Introduction

The earth resources observation satellites, such as Thematic Mapper and SPOT acquire imagery at high spatial and temporal resolution, thereby providing a huge number of data points to be analyzed. With the current interest in problems associated with global change, it is now necessary to analyze data from large regions over a long time history. Therefore, a reduction in the number of operations performed on each high resolution image is required. In many applications it is desired to segment and classify the regions of an image either as a means of data compression or as an end product for analysis.

If the objects or regions of interest are relatively large compared to the pixel size, then the prob-ability of the groups of contiguous pixels being in the same class is much greater than the probability of being in different classes(spatial redundancy). For large homogeneous interior regions, it may not be necessary to process all the pixels to identify the class label of that region.

More pixels are needed to identify the exact boundary only for areas near the boundaries between two or more classes. By identifying homogeneous interior regions without analyzing the responses of all the pixels and rather concentrating on boundaries, the number of operations and computational cost can be reduced. One means of efficiently implementing this idea involves a *multilevel approach* which can be applied naturally through a pyramid structure [Figure 1].

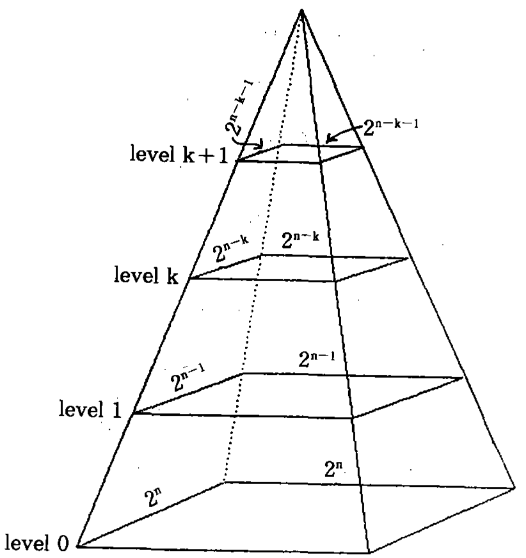

Figure 1. The Pyramid Data Structure.

The concept of unsupervised region based image segmentation has been employed in a new algorithm. It does not, however, depend on the traditional local merging and splitting of regions based on similarity tests of region statistics. Instead, global parameters of each class are estimated and updated with representative values of the classes using the mixture distribution technique. The image is then classified through a pyramid structure. Unlike traditional region based approaches, this new approach is based on global estimators and does not have ordering problems with merging of regions.

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nique. The image is then classified through a pyramid structure. Unlike traditional region based approaches, this new approach is based on global estimators and does not have ordering problems with merging of regions.

The algorithm has been applied to both simulated data and multispectral SPOT imagery. Classification results are reported for a variety of patterns and noise characteristics.

Ⅱ. Background

The algorithm developed in this paper utilizes results from statistical estimation theory and from multiresolution data structures.

A. Image Segmentation with Multiresolution Structures

Traditionally, image segmentation techniques with multiresolution structure are developed with either the quadtree or the pyramid structure.

Image segmentation with the quadtree structure has been implemented through three different approaches[10]: region merging or growing, region splitting, and region split-merge. These approaches incorporate spatial information by merging only spatially adjacent regions and using merging criteria based on region wide spatial features.

Pyramids provide successively condensed representations of the information in the whole image. The most obvious advantage of the pyramid representation is that it provides the opportunity for reducing the computational cost of various image operations using 'divide and conquer' principles.[6] Image segmentation with the pyramid structure has been developed through the pyramid node linking method.[4]

All these region based image segmentation approaches incorporate spatial information by merging only spatially adjacent regions and using merging criteria based on region-wide spatial features. Unfortunately, there are some common problems with these methods.[10] Because the region merging process is sequential in nature, the ultimate aggregated regions depend on the order in which regions are merged. Also, typical merging approaches often fail to find the best merge match. The characteristics of regions can change during the region growth process. Subregions that actually belong in the larger regions might be rejected in the early stages of the merging or vice versa. Generally, since these approaches use only regional(local) information, they all have the problem of lack of global information.

B. Mixture Distribution Estimation

A parametric family of finite mixture distributions is of the from

$$p(\mathbf{x} \mid \Phi)=\sum_{k=1}^{K} \alpha_k p_k(\mathbf{x} \mid \phi_k), \ \mathbf{x}=(\mathbf{x}_1, \cdots, \mathbf{x}_n)\in \mathrm{R}^n, \cdots\cdots(1)$$

where each α_k is nonnegative and $\sum_{k=1}^{K} \alpha_k=1$, where each p_k is a density function of $\mathbf{x}$ parametrized by $\phi_k \in \Omega_k$.

B.1. Clustering Based on Mixture Distribution Estimation

The mixture distribution estimation model has been considered as a clustering[8] or unsupervised learning problem[2]. Since the clustering with mixture distributions is based on the likelihood function of unordered data, it has the advantage of extracting infomation from all the pixels regardless of their locations and avoiding the computation of all pairwise distances or similarities.

The EM algorithm was adapted[5, 8] in the estimation of mixture distributions. The general properties of the EM algorithm ensure that the sequence of estimates of parameters converge to the maximum likelihood estimates.

The mixture distribution estimation problem involves samples which are expressible as one or a stochastically independent union of samples of three distinct types.[8] In modeling a mixture distribution, a sample observation of the mixture is labeled if its component density of origin is known with certainty, otherwise it is unlabeled. The three types of samples are:

Type 1. Suppose that $\{x_i\}_{i=1, \cdots, N}$ is an independent sample of N unlabeled observations on the mixture, i. e., a set of N observations on independent, identically distributed random variables with density $p(x \mid \Phi^*)$, where Φ^* is a set of parameters characterizing the mixture distribution. Then $S_1=\{x_i\}_{i=1, \cdots, N}$ is a sample of *Type* 1.

Type 2. Suppose that J_k, $k=1, \cdots, k$ are nonnegative integers and that, $\{y_{ki}\}_{i=1, \cdots, J_k}$ is an independent sample of observations on the *kth* component population, i.e., a set of J_k observations on independent, identically distributed random variables with density $p_k(x \mid \Phi^*)$. Then $S_2=\bigcup_{k=1}^{K} \{y_{ki}\}_{i=1, \cdots, J_k}$ is a sample of *Type* 2.

Type 3. Suppose that an independent sample of L unlabeled observations is drawn on the mixture, that these observations are subsequently labeled, and that, for $k=1, \cdots$, K, a set $\{z_{ki}\}_{i=1, \cdots, L_k}$ of them is associated with the kth component population with $L=\sum_{l=1}^{K} L_k$. Then $S_3=\bigcup_{k=1}^{K} \{z_{ki}\}_{i=1, \cdots, L_k}$ is a sample of *Type* 3.

A totally unlabeled sample of S_1 of *Type* 1 is the sort of sample considered in general mixture distribution problems. The major difference between two types of completely labeled samples S_2 and S_3 of *Type* 2 and 3 is that the L_k of *Type* 3 contains information about the mixing proportions, while J_k of *Type* 2 does not.

B.2. Maximum Likelihood Estimates of the General Mixture Distribution

Suppose that a set $\mathbf{x}=\{x_1, x_2, \cdots x_n\}$ of n independent samples is drawn independently from the mixture distribution and the class or origin of each sample is not known,

$$p(x_i \mid \Phi)=\sum_{k=1}^{K} \alpha_k p_k(x_i \mid \phi_k)$$

where each $\alpha_k \geq 0$, $\sum_{k=1}^{K} \alpha_k=1$ and $\Phi=(\alpha_1, \cdots \alpha_K, \phi_1, \cdots \phi_K)$. The likelihood function is

$$p(\mathbf{x} \mid \Phi)=\prod_{i=1}^{n} p(x_i \mid \Phi)$$

where the parameter vector Φ is fixed but unknown. The log-likelihood of the observed sample is

$$L=\sum_{i=1}^{n} \log p(x_i \mid \Phi)$$

The constraint $\sum_{k=1}^{K} \alpha_k=1$ is introduced via the Lagrange multiplier λ,

$$L^*=L-\lambda(\sum_{k=1}^{K} \alpha_k-1)$$

Assuming that $p(\mathbf{x} \mid \Phi)$ is differentiable wrt Φ,

$$\nabla_{\alpha k} L^* = \frac{\sum_{i=1}^{n} p_k(x_i \mid \phi_k)}{p(x_i \mid \Phi)} - \lambda = 0 \quad \cdots\cdots (1)$$

$$\nabla_{\phi k} L^* = \sum_{i=1}^{n} \alpha_k \frac{\nabla \phi_k p_k(x_i \mid \phi_k)}{p(x_i \mid \Phi)} = 0 \quad \cdots\cdots (2)$$

where ∇ is a partial derivative. Multiplying (1) by α_k and summing over k gives

$$n - \lambda = 0$$

The posterior probability of the occurrence of the kth class is

$$p(k \mid x_i) = \frac{\alpha_k p_k(x_i \mid \phi_k)}{p(x_i \mid \Phi)} = \frac{\alpha_k p_k(x_i \mid \phi_k)}{\sum_{k=1}^{K} \alpha_k p_k(x_i \mid \phi_k)} \quad \cdots\cdots (3)$$

Multipling (1) by α_k, the maximum likelihood estimate $\hat{\alpha}_k$ of α is

$$\hat{\alpha}_k = (1/n) \sum_{i=1}^{n} p(k \mid x_i) \quad \cdots\cdots (4)$$

i.e. the maximum likelihood estimate of the mixing proportion for class k is given by the sample mean of the conditional probabilities that x_i comes from class k. (2) can be expressed in terms of the posterior probability

$$\sum_{i=1}^{n} p(k \mid x_i) \nabla \phi_k \log p_k(x_i \mid \phi_k) = 0 \quad \cdots\cdots (5)$$

The maximum likelihood equations for estimating the parameters ϕ_k are then a weighted average of the maximum likelihood equations from each component considered separately. The weight for the ith sample is an estimate of how likely it is that x_i belongs to the kth component.

Analytic solutions can not be obtained in general for (4) and (5), so they must be solved approximately via some type of iterative computational procedure. If the EM algorithm[7)] is used, and $\Phi^r = (\alpha_1^r, \cdots, \alpha_K^r, \phi_1^r, \cdots, \phi_K^r)$ is the maximizer of the log likelihood function of rth itera tion, then the next approximate maximizer $\Phi^{r+1}=(\alpha_1^{r+1}, \cdots, \alpha_K^{r+1}, \phi_1^{r+1}, \cdots, \phi_K^{r+1})$ satisfies

$$\alpha_k^{r+1}=\frac{1}{n}\sum_{i=1}^{n}\frac{\alpha_k^r\, p_k(x_i \mid \phi_k^r)}{p(x_i \mid \Phi^r)}$$

$$\phi_k^{r+1}\in max_{\phi k}\sum_{i=1}^{n}\frac{\alpha_k^r\, p_k(x_i \mid \phi_k^r)}{p(x_i \mid \Phi^r)}\log p_k(x_i \mid \phi_k)$$

for $k=1, \cdots, K$. Note that each weight $\alpha_k^r\, p_k(x_i \mid \phi_k^r)/p(x_i \mid \Phi^r)$ is the posterior probability that x_i originated in the *kth* component, given the current approximate maximumlikelihood estimate Φ^r. This general model can be applied to *d*-dimensional multivariate normal populations.

B.3. S_1(Unlabeled) Samples only

Consider the multivariate normal mixture distribution,

$$p_k(\mathrm{x} \mid \phi_k)=N_d(\mu_k, \Sigma_k) \quad k=1, 2, \cdots K$$

$$p_k(\mathrm{x} \mid \phi_k)=\frac{1}{(2\pi)^{d/2}\mid \Sigma_k \mid^{1/2}}\exp\{-1/2(\mathrm{x}-\mu_k)'\Sigma_k^{-1}(\mathrm{x}-\mu_k)\}$$

where $\phi_k=(\mu_k, \Sigma_k)$, $k=1, \cdots, K$. From (4) and (5) of Section 2. 4. 3

$$\hat{\alpha}_k=\frac{1}{n}\sum_{i=1}^{n}\hat{p}(k \mid \mathrm{x}_i)$$

$$\nabla_{\mu_k}\log p_k(x_i \mid \phi_k)=\Sigma_k^{-1}(x_i-\mu_k)$$

so

$$\hat{\mu}_k=\frac{\Sigma_{i=1}^{n}\hat{p}(k \mid \mathrm{x}_i)\, x_i}{\Sigma_{i=1}^{n}\hat{p}(k \mid \mathrm{x}_i)}$$

$$\hat{\Sigma}_k=\frac{\Sigma_{i=1}^{n}\hat{p}(k \mid \mathrm{x}_i)(\mathrm{x}_i-\hat{\mu}_k)(\mathrm{x}_i-\hat{\mu}_k)'}{\Sigma_{i=1}^{n}\hat{p}(k \mid \mathrm{x}_i)}$$

The maximum likelihood estimates can be found directly from the EM algorithm.

B.4. Union of S_1 and S_3(Labeled and Unlabeled) Samples

This case is the union of labeled and unlabeled samples. The labeled samples contribute to estimation of mixing proportions because of the proportion information contained in the labeled observations. Suppose that

$$L=\{z_{ki}: k=1, \cdots, K,\ i=1, \cdots, L_K\}$$

is available, where $z_{k1}, \cdots, z_{kL_k}$ are from observations in component k.

$$Lik_{com}=Lik_{labelled}\ Lik_{unlabeled}$$
$$=\prod_{k=1}^{K}\prod_{i=1}^{L_k} \alpha_k p_k(z_{ki} \mid \phi_k) \prod_{i=1}^{n} \sum_{k=1}^{K} \alpha_k p_k(x_i \mid \phi_k)$$

$$\nabla_{\phi k} L_{com}=\sum_{i=1}^{L_k} \nabla_{\phi k} \log p_k(z_{ki} \mid \phi_k)+\sum_{i=1}^{n} p(k \mid x_i)\nabla_{\phi k} \log p_k(x_i \mid \phi_k)$$

where $L_{com}=\log Lik_{com}$.

$$\hat{\alpha}_k=\frac{1}{n+L}\left\{L_k+\sum_{i=1}^{n} p(k \mid x_i)\right\} \text{ where } \sum_{k=1}^{K} L_k=L$$

$$\hat{\mu}_k=\frac{\sum_{i=1}^{L_k} z_{ki}+\sum_{i=1}^{n} \hat{p}(k \mid x_i)x_i}{L_k+\sum_{i=1}^{n} \hat{p}(k \mid x_i)}$$

$$\hat{\Sigma}_k=\frac{\sum_{i=1}^{L_k} (z_{ki}-\mu_k)(z_{ki}-\mu_k)^t+\sum_{i=1}^{n} \hat{p}(k \mid x_i)(x_i-\hat{\mu}_k)(x_i-\hat{\mu}_k)^t}{L_k+\sum_{i=1}^{n} \hat{p}(k \mid x_i)}$$

Ⅲ. Structure of Algorithm

A. Definition of Types of Regions and Decision Categories

Three types of regions, $R_{i,j,l}$ and associated decision categories are required at any particular level of the process, where i, j denotes the region location at level l. Regions are designated as

a) *Unspecified homogeneous*: A region $R_{i,j,l}$ is said to be unspecified homogeneous if it is composed of a class unknown to the procedure at that level, or something other than one of the K classes which are recognized at a particular level.

b) *Specified homogeneous*: A region $R_{i,j,l}$ is said to be specified homogeneous if it is composed entirely of only one of the K classes which are recognized at a particular level.

c) *Boundary*: A region $R_{i,j,l}$ is said to be boundary if it is composed of more than one class at a particular level.

The segmentation procedure therefore requires the following decisions for a given region $R_{i,j,l}$:

a) At a particular level, it must be determined whether $R_{i,j,l}$ is a homogeneous or boundary region.

b) If $R_{i,j,l}$ is a homogeneous region, then $R_{i,j,l}$ is labeled with one of the K known classes or as an unknown class.

c) If $R_{i,j,l}$ is a boundary region, then next finer resolution level is considered.

B. Construction of the Multiresolution Pyramid Structure

The full resolution image, 2^m by 2^m, is placed on the bottom level of the pyramid(level 0). To define the reduced-resolution versions of the image, an exponentially tapering 'pyramid' of arrays of sizes 2^{m-1} by 2^{m-1}, 2^{m-2} by 2^{m-2}, $\cdots$, 4 by 4 and 2 by 2 is used, so that the lth level has size 2^{m-l} by 2^{m-l}. Each node in the pyramid is indexed by a triple(i, j, l), where l is its array level and i and j are its row and column numbers. Two kinds of pyramids, non-overlapping average pyramid and SSCP pyramid, are constructed. The SSCP pyramid is constructed to carry the variability of regions.

C. The Homogeneity Test of Unlabeled Regions, $R^u_{i,j,l}$

Detection of homogeneous regions is a critical part of the algorithm. At each level, the homogeneity tests are performed to determine whether a region falls in the interior of a class or lies on the boundary between classes. These homogeneous regions are then treated as a combined sample for the estimation of parameters.

The homogeneity test in this algorithm is actually implemented using following series of hypotheses:

H_0: $R_{i,j,l}$ is composed of one class.

H_1: $R_{i,j,l}$ entirely belongs to one of the known classes.

H_2: $R_{i,j,l}$ is composed of more than one class.

It is important to note that under hypothesis H_0, the class characteristics are not known, while under the hypothesis H_1, the class characteristics are known. The homogeneity of a region is determined from the following tests.

Test 1: H_0 versus H_2, to differentiate the homogeneous regions from the boundary regions.

Test 2: H_1 versus H_2, to differentiate the homogeneous regions belonging to the previously known classes from the boundary regions.

Test 1 involves a homogeneity test of a region, which has four subregions at a particular level. It simply tests whether a region comes from the interior region of one class. Test 2 is a membership homogeneity test, which tests whether the unlabeled subregion $R^u_{i,j,l-1}$ belongs entirely to one of the previously known classes.

Statistical tests are used for these homogeneity tests, where it is assumed that $p_i(x \mid i)$ is multivariate normal, $N(\mu_i, \Sigma_i)$.

C.1. Test 2: Four-Subregion Homogeneity Test

The purpose of the four-subregion homogeneity test is to identify the very 'homogeneous' regions not lying on the boundary regions at the top level, L. At each level, each region consists of four equal sized subregions. To decide whether a region, $R_{i,j,l}$ is homogeneous, both equality of covariance matrices and equality of mean vectors of the four subregions are tested. Only regions which pass both tests are considered as homogeneous at this level. The parameters are then estimated with using data from these homogeneous regions. The test statistics can be easily and efficiently calculated with the non-overlapping average and SSCP pyramid.

For the test of equality of covariance matrices of four subregions, the χ^2 test is used. If some of the classes have the same covariance matrices, the equality test of covariance matrices is not adequate to detect the homogeneity of a region. Therefore, the equality test of mean vectors is applied to the regions which pass the equality of covariance test. The χ^2 test is used for the multivariate test of equality of mean vectors.

C.2. Test 2: Membership-Homogeneity Test

After the parameters are reestimated with projected labeled regions at level l(detailed in F) these parameters are used to test the membership of unlabeled regions at the same level, l. An unlabeled region $R^u_{i,j,l}$ at level l can be tested to determine whether it belongs to one of the previously recognized classes or lies on the boundary of the classes.

First, a test of a membership of class to an unlabeled region $R^u_{i,j,l}$ is performed using the parameter values at level, l. Because of the averaging effect of the pyramid, the variability of a region has not been considered. Therefore, the test of SSCP of a region is performed to check the variability of a region against the variability of each class.

The χ^2 test is used in order to test whether an unclassified region $R^u_{i,j,l}$ belongs to one of the previously recognized classes. For a region which passes the membership test for one of the known classes, k, the SSCP of an unlabeled region is compared the $\Sigma'_{k,l}$ (see G) of the known class, k to determine whether it is homogenous or falls in the boundary of more than one class. The χ^2 test is used.

The regions which pass both tests for one of the known classes are considered homogeneous and included in the estimation of parameters.

D. Estimation of Parameters at Level, l

Parameters are estimated for the homogenous regions using the EM iteration method.

At the top level, L, the homogeneous regions are identified with four-subregions homogeneity test. *Case 1* estimation is used for the totally unlabeled homogenous regions. Since the variability is averaged out though the pyramid structure and only homogeneous regions are included to the estimation, the classes are more separated. Therefore, convergence is rapid and accurate estimates can be obtained.[5)]

From level $L-1$ to level 1, where the labeled regions, $R^c_{i,j,l}$ are available from the previous level, these regions can be utilized in the estimation of parameters. These labeled regions are combined with the newly identified homogeneous regions at this level for the estimation. *Case 3* estimation is used for updating the parameters. The labeled regions not only provide good initial values for the estimation, but also result in an increased convergence rate and improved estimates.

E. Labeling at Level l

After the parameters of each class have been estimated and updated, the homogeneous regions are labeled with the class which gives the maximum posterior probability. A label of a class map, $C_{i,j,l}$ is obtained by observing

$$\frac{\alpha_k p_k(x_{i,j,l} \mid \phi_k)}{\sum_i^K \alpha_i p_i(x_{i,j,l} \mid \phi_i)}$$

which is the posterior probability that $x_{i,j,l}$ comes from class k.

F. Projection to Level, $l-1$

After the homogeneous regions are labeled on the basis of the estimated parameters at level l, the label map and homogeneity map are projected to the next lower level, l-1. The mean vectors, covariance matrices and class proportions of each class are reestimated with all the classified regions at level l-1. The loss of variability of regions due to the pyramid structure can be recovered with this projection and reestimation at lower levels. Let $x_{k,l}^c$ be the mean value of a region classfied into class k and $SS_{k,l-1}^{t,c}$ be the SSCP of a region classified into class k at level l-1.

$$\alpha_{k,l-1} = \frac{n_k}{\sum_{k=1}^K n_k}$$

$$\mu_{k,l-1} = \frac{1}{n_k}\sum_{i=1}^{n_k} x_{k,l-1}^c$$

$$\Sigma_{k,l-1} = \frac{1}{n_k}\left[\sum_{i=1}^{n_k}(x_{i,l-1}^c - \mu_{k,l-1})(x_{i,l-1}^c - \mu_{k,l-1})'\right]$$

$$\Sigma'_{k,l-1} = \frac{1}{n_k \cdot n}\left[\sum_{i=1}^{n_k} SS_{k,l-1}^{t,c} + n\sum_{i=1}^{n_k}(x_{i,l-1}^c - \mu_{k,l-1})(x_{i,l-1}^c - \mu_{k,l-1})'\right]$$

where n_k is the number of regions classified into class k and n is the number of pixels in a region at level l-1.

G. Optional Extensions

Once a region is identified as homogeneous and labeled at a higher level, the homogeneity and class label of a region are carried to the bottom level of the pyramid. The algorithm does not have a systematic way to correct the *Type*-Ⅰ error of homogeneity test or the mislabeling of a regions at the higher level of the pyramid. Optional tests have been implemented to handle this problem.

G.1. Back-homogeneity Test at level, l

A homogeneity test is applied to the previously labeled regions to correct the *Type*-Ⅰ errors. At level l, after the class parameters are updated with the previously labeled regions and newly included homogeneous regions, the updated parameters are used for the membership-homogeneity test of the previously labeled regions.

For regions of class label, k, the membership of the region is tested with the updated parameters of class, k. If it fails the test, a label of that region is deleted from the label and homogeneity maps, and the class parameters are adjusted accordingly. The test provides the algorithm the chance to define the boundaries more accurately. However, it may also give a more noisy classified image.

G.2. Relabeling at level, l

Since the class parameters are updated through the pyramid structure, there should be the capability to change the class label of a region which is labeled at a higher level. If relabeling is allowed at each level with updated parameters, it gives a more accurate label of a region corresponding to the updated class parameters. Unfortunately, the advantage of region labeling is essentially completely lost. However, it still has the advantage of reduction of data operations for the purpose of estimation.

Ⅳ. Application and Evaluation of the Algorithm

A. Application of Algorithm to Simulated Data

The new adaptive parametric estimation and classification algorithm with pyramid structure was implemented and tested on simulated images with a variety of characteristics. Two and three class 64×64 images with two noise covariance structures were simulated. In Case A, the classes were fairly well separated, each class had same variance structure, and channels were not correlated. The images in Case B had classes which were poorly separated, each class had a different variance structure, and the channels were correlated(Table 1). The performance of the algorithm was evaluated using the total misclassification error rate.

Table 1. Parameters of 2 Simulated Cases of 2 Class and 3 Class Images with 2 Channels

Class	Case	μ (d, k)	Σ(d, d)
2 Class	Case A	$\begin{pmatrix}5\\8\end{pmatrix}$ $\begin{pmatrix}7\\10\end{pmatrix}$	$\begin{pmatrix}1.0 & 0.0\\0.0 & 1.0\end{pmatrix}$ $\begin{pmatrix}1.0 & 0.0\\0.0 & 1.0\end{pmatrix}$
	Case B	$\begin{pmatrix}5\\9\end{pmatrix}$ $\begin{pmatrix}7\\12\end{pmatrix}$	$\begin{pmatrix}2.0 & 0.7\\0.7 & 2.5\end{pmatrix}$ $\begin{pmatrix}2.3 & 1.0\\1.0 & 3.5\end{pmatrix}$
3 Class	Case A	$\begin{pmatrix}5\\8\end{pmatrix}$ $\begin{pmatrix}7\\10\end{pmatrix}$ $\begin{pmatrix}9\\12\end{pmatrix}$	$\begin{pmatrix}1.0 & 0.0\\0.0 & 1.0\end{pmatrix}$ $\begin{pmatrix}1.0 & 0.0\\0.0 & 1.0\end{pmatrix}$ $\begin{pmatrix}1.0 & 0.0\\0.0 & 1.0\end{pmatrix}$
	Case B	$\begin{pmatrix}5\\9\end{pmatrix}$ $\begin{pmatrix}7\\12\end{pmatrix}$ $\begin{pmatrix}9\\15\end{pmatrix}$	$\begin{pmatrix}2.5 & 1.1\\1.1 & 3.0\end{pmatrix}$ $\begin{pmatrix}2.0 & 0.7\\0.7 & 2.5\end{pmatrix}$ $\begin{pmatrix}2.5 & 1.0\\1.0 & 3.5\end{pmatrix}$

The algorithm was run on two patterns of two and three class images. Bayes' classification was performed for comparison of the misclassification errors. As a supervised method, the true parameter values were used instead of selecting training sets from the image. Therefore, the results from the Bayes' classification were the best that could be possibly be obtained.

The multivariate statistical tests were performed at $\alpha=0.1$ for the homogeneity test of each level. The misclassification errors were considerably smaller than for Bayes' classification(Table 2) The average of reduction in misclassification errors compared to the corresponding Bayes' classification errors of Cases A and B were 0.24 percent and 0.23 percent respectively. The algorithm performed particularly well in the homogenous regions. In Case B, the high variability of each class, the differing variance structure and correlation between channels resulted in more misclassification errors than Case A.

Table 2. Comparison of Misclassification rate of Bayes classifier and Algorithm(at $\alpha=0.1$)

(%)

Class	Classifier	Case A	Case B
2 Class	Bayes' Classifier	7.91	16.47
	Algorithm	2.07	4.33
3 Class	Bayes' Classifier	10.99	22.15
	Algorithm	2.49	4.68

Table 3 shows the reduction in data operations from using the pyramid structure. It shows the reduction of data operations for Case A of the three class pattern. The dimension of the original image, at level 0, was 64×64. It was reduced to 8×8 at level 3. The algorithm started at level 3, where the total number of regions was 64. 34 of 64(53 percent)were recognized as homogeneous regions at level 3, and these regions were included for estimation and classified. At level 2, those 34 regions were projected and 67 regions were newly included as homogeneous regions. 79 percent of the image was classifed at this level. At level 1, 91 percent of the image and at level 0, the whole image was classified.

Table 3. The Number of Data Operations at Each Level

3 Classes, Case A				
	#of Nodes	Projected	Newly Included	Classified # (percent)
level 3	64	0	34	34(53)
level 2	256	136	67	203(79)
level 1	1024	812	120	932(91)
level 0	4096	3728	368	4096(100)

B. Implementation of Algorithm on SPOT Date

The new algorithm was implemented and tested on a study area (256×256) in a SPOT image located between Weslaco and Edcouch/Elsa, Texas. The enhanced true color images (using red, green, blue color components) of June 4, 1989, are displayed in Figure 2.

The new algorithm was run on the original three band image. The classification results were evaluated using qualitative analysis. The algorithm was run on the image with 8 classes. The homogeneity tests failed for a lot of regions because of the spatial dependency due to the high spatial resolution.

To recover from errors in the homogeneity tests at higher levels, the algorithm with "back-homogeneity" test was run on the images. The "back-homogeneity" test helped to define more detailed boundaries of the agriculture fields. The algorithm with "relabeling" was also run on the images to adjust the class labels corresponding updated the class parameters [Figure 3]. The classified images with "relabeling" are displayed with RGB color components. The mean value of each class was substituted to each band. The classified images with "relabeling" showed strong resemblence to the original images.

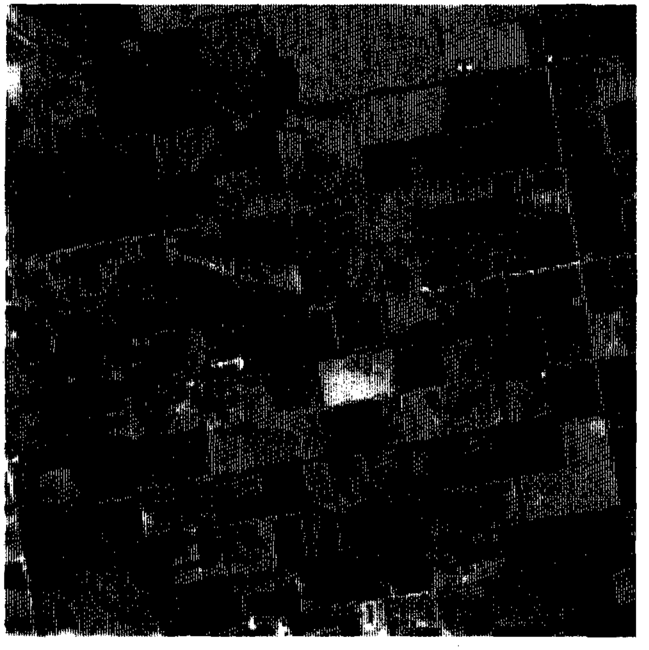

Figure 2. The SPOT enhanced image of June 4, 1989 in Weslaco, Texas.

V. Conclusions and Recommendations

This study concentrated on development of a computationally efficient classification algorithm implemented with the pyramid structure. Unlike traditional region based image segmentation approaches with multiresolution structures, which depend on local merging and splitting of regions based on the similarity of neighboring regions, this algorithm identifies the homogenous and boundary regions at each level of the pyramid using region statistics. The global parameters of each class are estimated based on values of the homogeneous regions represented at that level of the pyramid using mixture distribution estimation. These estimated parameters are implicitly used as a merging criterion of homogeneous regions. Therefore, in addition to achieving computational efficiency with a multilevel approach, this algorithm overcomes the problems of ordering of merging regions and the lack of global information of region based image segmentation approaches.

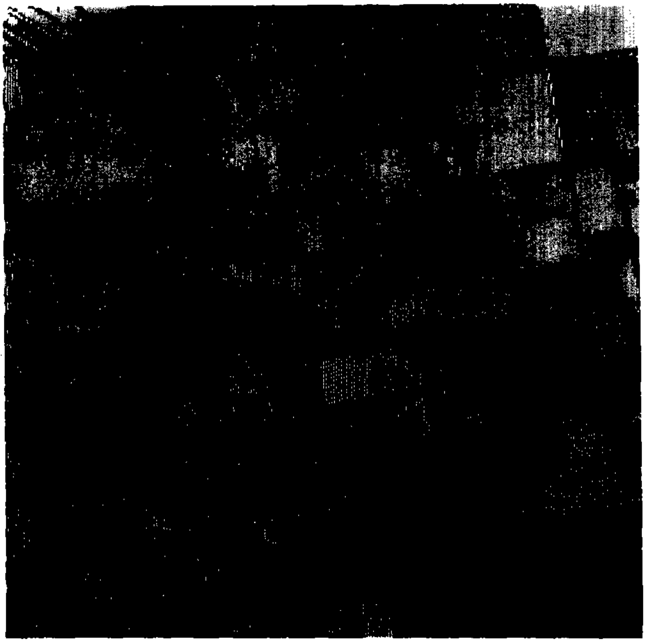

Figure 3. Classified Image of June 4, 1989 in Weslaco, Texas.

The statistical tests are developed under the assumption that the observations are samples of random variables which are normally and independently distributed. The statistical tests are vulnerable to any violation of these assumptions. Nonparametric test can be considered if the normality assumption is to be relaxed.[1] However, both parametric and nonparametric tests require the assumption of independence of samples. The randomization test[3] could be considered for the case where assumption of independence is violated.

The estimation procedure used by the algorithm assumes the normality of the data. Usually for large amounts of data, the normality can be assumed safely. However, the estimation procedure could be modified to handle estimation of nonparametric probability density, such as Parzen density estimation.[7]

References

1) W. W. Daniel, *Applied Nonparametric Statistics*, PWS-KENT Publishing Company, Boston, 1978.

2) R. O. Duda and P. Hart, *Pattern Classification and Scene Analysis*, John Wiley and Sons, New York, 1973.

3) E. S. Edgington, *Radomization Tests*, Marcel Dekker, New York, 1980.

4) T. Hong, K. A. Narayanan, S. Pelig, A. Rosenfeld and T. Silberberg, "Image smoothing and segmentation by multiresolution pixel linking: further experiments and extensions," *IEEE Transaction on Systems, Man, and Cybernetics*, Vol. SMC-12, No. 5, pp. 611-622, 1982.

5) R. A. Render and H. F. Walker, "Mixture densities, maximum likelihood and the EM algorithm," *SIAM Riew*, Vol. 26, No. 2, pp. 195-239, 1984.

6) A. Rosenfeld, "Some useful properties of pyramids," *Multiresolution Image Processing and Analysis*, A. Rosenfeld, Eds., New York, pp. 2-5, 1984.

7) B. W. Silverman, *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, 1986.

8) D. Titterington, A. Smith and U. Makov, *Statistical Analysis of Finite Mixture Distributions*, John Wiley & Sons, New York, 1985.

9) J. Wolfe, "Pattern clustering by multivariate mixture analysis," *Multivariate Behavioral Research*, Vol. 5, pp. 329-350, 1970.

10) S. W. Zucker, "Region Growing: Childhood and adolescence," *Computer Graphics and Image Processing*, Vol. 4, pp. 382-399, 1976.