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PRIMORDIAL BLACK HOLES IN THE VERY EARLY UNIVERSE

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ABSTRACT

Energy density evolution of primordial black holes(PBHs) due to quantum gravitational tunneling effect in the very early Universe is calculated for the four cases of GUTs(grand unified theories) (SM, SUSY SM, SUSY SU(5), SU(5)). For the three of them(SM, SUSY SM, SUSY SU(5)), it is confirmed that there are a considerable amount of PBHs and so it may give a firm support to Lindley's paper(1981) in which he tried to solve the baryon asymmetry problem.

It is shown that the formation of PBHs increases the cosmic scale factor R and decreases the total energy density ρ_t faster than in the usual radiation dominated era.

I. INTRODUCTION

Since Zel'dovich(1967) and Hawking(1971) referred to primordial black holes(PBHs) due to statistical density fluctuation, much progress has been made in relation to baryogenesis(Krass 1982), missing mass problem(MacGibbon 1987), galactic nucleation(Carr 1975) etc.

Also, recently Gross *et al.* (1982) showed that there is another mechanism, that is, quantum gravitational tunneling effect to form very tiny PBHs in the very hot and flat space. Later, using the classical approach similar to vapor-to-water phase transition, Kapsta derived the black hole(BH) generation rate which had been derived by Gross *et al.*(1982) so that he gave them a firm support.

With this PBH generation rate, and assuming that PBHs do not affect the evolution of the Universe, Hayward and Pavon(1989) calculated the epoch when the PBH energy density is equal to the radiation energy density. Also, they calculated the duration of this period on the assumption that the ratio of PBH to radiation energy density is a constant.

In the present paper, we investigated the evolution of PBH density with regard to its effect upon the Universe.

In section II, we introduce the fundamental equations and assumptions. In section III, we briefly investigate the effect of PBH density on the evolution of the Universe, in section IV we show the method of calculation and in section V and VI the results and discussion.

II. FUNDAMENTAL EQUATIONS

1. Radiation Dominated Era

The equations of standard model in the radiation dominated era are, in the unit of $\hbar=c=k_b$ (Boltzmann constant) $=G=1$,

$$\left[\frac{dR}{dt}\right]^2=\frac{8\pi}{3}\rho R^2 \tag{1}$$

$$\frac{d^2R}{dt^2}=-\frac{4\pi}{3}(\rho+3p)R, \tag{2}$$

$$p=\frac{\rho}{3}, \tag{3}$$

$$\rho=\frac{\pi^2}{30}NT^4 \tag{4}$$

As usual in the early Universe we approximated $k=0$.

From equations (1)—(4), we get equations (5)—(7), where N is the total number of effective massless degrees of freedom,

$$\rho=\frac{3}{30\pi t^2}, \tag{5}$$

$$R\propto t^{1/2} \tag{6}$$

$$T=\kappa t^{-1/2} \tag{7}$$

where

$$\kappa\equiv\left[\left(\frac{45}{N\pi}\right)^{1/2}\frac{1}{4\pi}\right]^{1/2} \tag{8}$$

2. Initial Mass of PBH and its Time Evolution

As PBH forms due to quantum instability, we may safely assume that PBH forms with Hawking temperature T at $t=\tau$,

$$M_{bh}(\tau)=\frac{1}{8\pi T(\tau)} \tag{9}$$

For a freely evaporating BH, we have by Stephan-Boltzmann's law

$$\frac{dM_{bh}}{dt}=\frac{1}{8\pi}\frac{d}{dt}\left[\frac{1}{T}\right]=-\frac{N}{(8\pi)^3M_{bh}^2} \tag{10}$$

Integration yields the mass of PBH at t,

$$M_{bh}(t,\tau)=\frac{1}{8\pi}\left[\left(\frac{1}{T(\tau)}\right)^3+3N(\tau-t)\right]^{1/3}. \tag{11}$$

3. BH Nucleation Rate

BH nucleation rate per unit volume and unit time is (Gross *et al.* 1982)

$$\Gamma(T)=1.75\,T\left[\frac{\mu}{T}\right]^{\theta}\frac{1}{64\pi^3}\exp[-1/(16\pi T^2)] \qquad (12)^{1)}$$

where μ is a constant close to the Planck mass, T is the radiation temperature and θ is a number which depends on N_s, the number of spin fields accessible to the system, defined as follows

$$\theta=\frac{1}{45}\left[212\,N_2-\frac{233}{4}N_{3/2}-13N_1+\frac{7}{4}N_{1/2}+N_0\right]. \tag{13}$$

N_s, N, and θ for the four GUT's are shown in Table 1.

Table 1. N_s is the number of spin fields with spin s, N is the sum of N_s and θ is defined by the equation (13).

	N_2	$N_{3/2}$	N_1	$N_{1/2}$	N_0	N	θ
SM	1	0	12	45	4	62	3.08
SUSY SM	1	1	12	65	53	132	3.65
SUSY SU(5)	1	1	24	103	113	242	3.00
SU (5)	1	0	24	45	34	104	0.28

4. Density Equation of PBH

Now, we can obtain the density equation of PBH,

$$\rho_{bh}(t)=\frac{1}{R(t)^3}\int_{t*}^{t}R(\tau)^3M(t,\tau)\Gamma(T(\tau))\,d\tau \tag{14}$$

where t_* is the formation time of a BH which would have evaporated at the time t.

III. PBH's EFFECT ON THE EVOLUTION OF THE UNIVERSE

As the formation of PBH affects the equation of state, the evolution of the Universe becomes a little different from that of the usual radiation dominated era.

Let the total density be ρ_t

$$\rho_t=\rho_{bh}+\rho_{\rm rad}=(1+\beta)\rho_{\rm rad} \tag{15}$$

where $\beta=\rho_{bh}/\rho_{\rm rad}$. Now, we write the equations (1)—(3)

1) In equation (9), 1.75 is corrected by Kapsta(1984) from 0.87 in Hayward and Pavon (1989).

$$\frac{dR}{dt}=\left[\frac{8\pi}{3}\rho_t\right]^{1/2}R,$$

$$d\rho=-3(\rho_t+p)\frac{dR}{R}, \tag{16}$$

$$p=\frac{\rho_{\rm rad}}{3}=\alpha\,\rho_t.$$

In relation to equation (15),

$$\alpha=\frac{1}{3(1+\beta)}. \tag{17}$$

If we suppose β as a time-independent constant to estimate the effect of PBH on the evolution of the Universe and solve equation (16), we get

$$\rho_t=\frac{1}{6\pi(1+\alpha)^2t^2}, \tag{18}$$

$$R\propto t^{2/3(1+\alpha)}. \tag{19}$$

And using equation (4) and the third equation in equation (16) we get

$$T=\left[\frac{15\alpha}{N\pi^3(1+\alpha)^2}\right]^{1/4}t^{-1/2}\equiv\kappa' t^{-1/2} \tag{20}$$

Comparing equations (18)—(20) with equations (5)—(7), it is noted that the evolution of the Universe becomes different due to the PBH formation.

As the PBH density becomes high, β becomes high. So, α in equation (17) will approach 0, and R evolves as $\sim t^{2/3}$ as in the matter dominated era. Also the total density as shown in equations (18) and (19) evolves not as $\sim R^{-4}$ but $\sim R^{-3}$.

IV. NUMERICAL CALCULATION

1. Assumptions

We made three assumptions following Hayward and Pavon(1989).

First, we suppose that the mass of PBH formed with temperature T is given as equation (9), which is highly plausible because BH temperature is defined as $T=1/(8\pi M)$ and the formation mechanism is quantum instability of hot flat space.

Secondly, to consider interaction between PBH and surrounding radiation we introduce the relative velocity of neighboring PBHs due to Hubble expansion

$$v_{\rm exp}=\frac{d}{dt}n^{-1/3}=n^{-1/3}\frac{(dR/dt)}{R}$$

$$=n^{-1/3}\left[\frac{8\pi}{3}(p_{\rm rad}+nm)\right]^{1/2} \tag{21}$$

where n is the number density of PBHs, m is the average mass of PBH, and the velocity required for a PBH to escape from the gravitational pull of its nearest neighbor is

$$v_e = (4mn^{1/3})^{1/2}. \tag{22}$$

Comparing these two equations, we are to find $v_{\exp} \geq v_e$ for all possible $\rho_{\rm rad}$ and ρ_{bh}.

Finally, in order to examine how any interaction between PBHs and surrounding radiation affects the evolution of PBH mass, the average time of interaction may be expressed as

$$\tau_{\rm in} = \frac{1}{n\sigma} = \frac{1}{36\pi\, m\, \rho_{bh}} = \frac{8\pi}{3\beta}\ \kappa'(1+\alpha)^2(1+\beta)t^{3/2} \tag{23}$$

and the dynamic time due to Hubble expansion

$$t_{\exp} = \frac{R}{(dR/dt)} = \frac{3}{2}\ (1+\alpha)t \tag{24}$$

If $\tau_{\rm in} > t_{\exp}$, that is, if

$$t > \left[\frac{9}{16}\ (1-3\alpha)\right]^2 \left[\frac{15\pi\,\alpha}{N}\ (1+\alpha)^2\right]^{-1/2}, \tag{25}$$

PBH will begin to evaporate without interaction immediately after formation. Even though equation (25) is not satisfied, the immediate evaporation may be assumed, since accretion is negligible (Hawking 1971; Carr and Hawking 1974).

If we think as usual that the Universe is created with the size of Planck scale to avoid the singularity, initial values are $t_0 = 1$, $R_0 = 1$ and $T_0 = \kappa$. We assume there is no PBH ($\beta = 0$) at t_0.

2. $\beta(n)$

We solve equations (14)—(16) by iteration method because of mutual dependence of β and R or T.

In the n-th step on integration of equation (14), we put temporarily $\beta(n-1)$ as $\beta(n)$ only to get $R(n)$ and $T(n)$. Through $R(n)$ and $T(n)$ we get new $\beta(n)$ and then we put $\{\beta(n)+\beta(n-1)\}/2$ as $\beta'(n)$, we make the table of $\beta(n-3)$, $\beta(n-2)$, $\beta(n-1)$, and $\beta'(n)$. By the Lagrangian interpolation β between $\beta(n-1)$ and $\beta'(n)$ can be obtained. We evolve R and T by Runge-Kutta method and get $\beta''(n)$. We repeat this procedure until we get self-consistent β, that is, $\beta^{(n-1)}(n) = \beta^{n}(n)$ to the desirable accuracy. When $n = 0, 1, 2, 3$, we simply use linear interpolation instead of Lagrangian interpolation.

3. Ω_{pbh}

It has been argued that evaporating BHs may actually leave stable Planck mass residues(Bunch 1981). Indeed, MacGibbon(1987) suggested that such relics of PBHs may account for the "missing mass". In this view, in the absense of inflation we can calculate Ω_{pbh} from quantum gravitational tunneling effect.

The total number density of formed PBH is represented as

$$N_t = \frac{1}{R^3(t_f)} \int_1^{t_f} R^3(\tau)\Gamma(T)\,d\tau. \tag{26}$$

We can determine t_f at which the value in the integral is sufficiently small.

If all evaporating PBHs leave stable Planck mass residues, the time-independent ratio of the number density of PBH residues to photon number density is

$$\frac{N_t(t_0)}{\rho_{\rm rad}(t_0)\,/\,T(t_0)} = \frac{N_t(t_f)}{\rho_{\rm rad}(t_f)\,/\,T(t_f)}. \tag{27}$$

Then Ω_{pbh} is

$$\Omega_{pbh} = \rho_{pbh}/\rho_c = \frac{N_t(t_f)\,M_p}{\rho_{\rm rad}(t_f)\,/\,T(t_f)}\,(\rho_{\rm rad}(t_0)\,/\,T(t_0))\,/\rho_c. \tag{28}$$

In this equation $M_p = (\hbar c/G)^{1/2}$ is the Planck mass and in the unit of this paper is 1.

4. Relation to inflations

The exponential expansion of inflation dilutes all prior products. So we need to know when inflation occurred so as to know the exact effect upon the Universe. Unfortunately we are not sure when and how many times inflation occurred. Only it is certain that at least above 10^{14} GeV inflation could occur.

In order for the PBH residues to play the role of dark matter, the last inflation should occur above the certain temperature of the Universe because inflation dilutes all prior products and the decrease of temperature reduces PBH generation rate, so we are able to calculate how early the last inflation should occur to make $\Omega_{pbh} = 1$ as shown in the next section.

V. RESULTS

1. PBH density evolution (β)

In Figures 1a—1d, for all cases, PBH density increases rapidly at first because of high PBH generation rate and then, after attaining the peak, slowly decreases due to decrease of the generation rate and to Hawking radiation. As the temperature of the Universe goes down, the mass of forming BH increases, hence the curves show the longer tail than the initial increase. Also it is very sensitive to N and θ, with the maximum of β depending more upon θ than N.

For the SU(5) case, ρ_{bh} never attains $\rho_{\rm rad}$, and the model remains radiation dominated from the Planck era onward. In comparison with Hayward and Pavon's result, there is a slight difference because of their neglect of PBHs effect upon the evolution of the

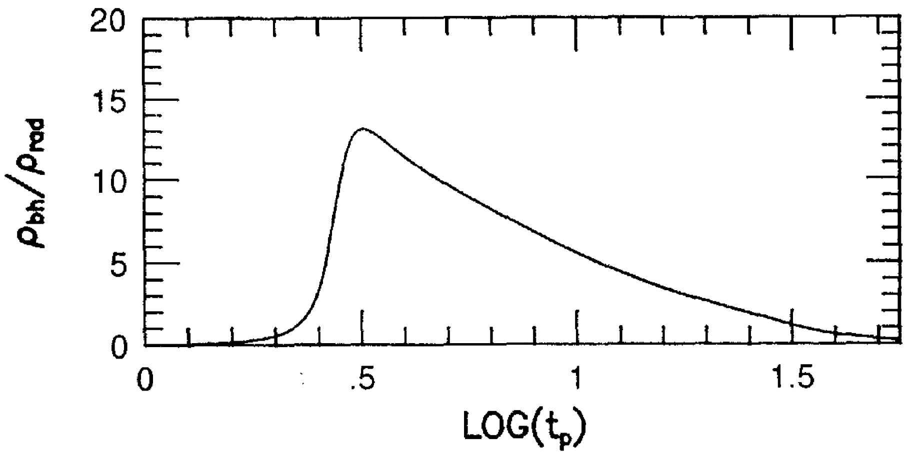

Fig. 1a. β evolution for the case of SM($N = 62$, $\theta = 3.083$). The maximum of β is about 13 and PBH density dominates the radiation density from $t_p = 2.2$ to 33.2.

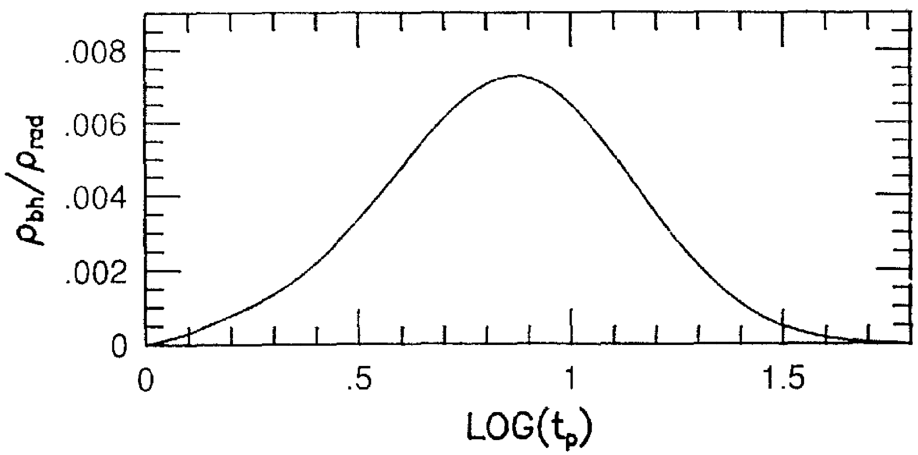

Fig. 1c. β evolution for the case of SUSY SU(5) ($N = 242$, $\theta = 3,000$). The maximum of β is about 1.7 and PBH density dominates the radiation density from $t_p = 2.7$ to 7.4.

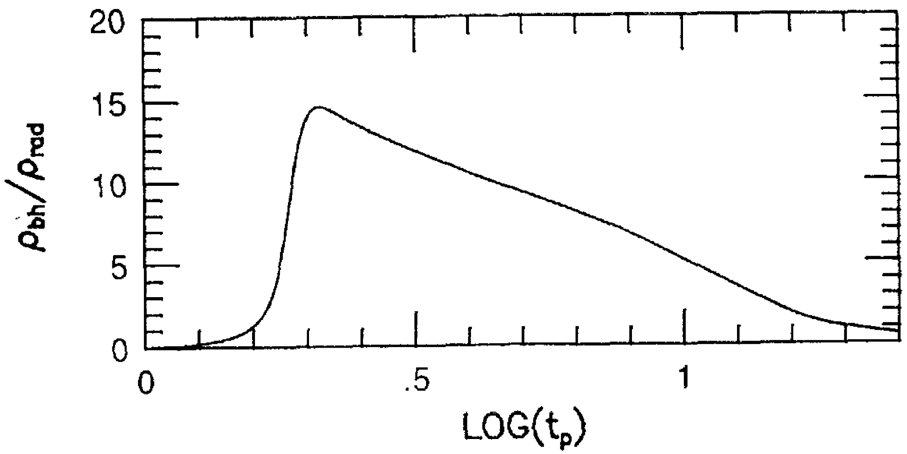

Fig. 1b. β evolution for the case of SUSY SM($N = 132$, $\theta = 3.65$). The maximum of β is about 15 and PBH density dominates the radiation density from $t_p = 1.5$ to 2.1.

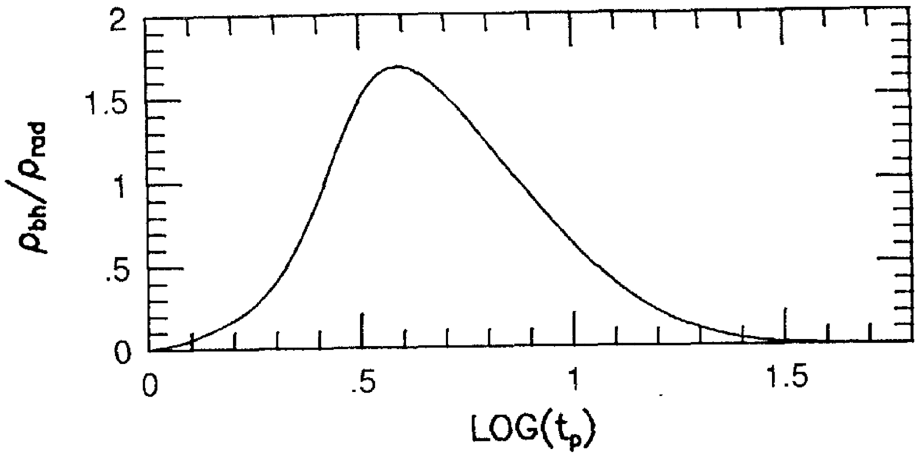

Fig. 1d. β evolution for the case of SU(5) ($N = 104$, $\theta = 0.283$). The maximum of β is about 0.007 and this model remains the radiation dominated from the Planck era.

evolution of the Universe and of a small error in their calculation.[2)]

2. Evolution of R and ρ_t

In Figures 2a-2d, and Figures 3a-3d, with the change of equation of state from equation (3) to the third equation of (16), the Universe evolves a little different from the ususal radiation dominated era. The sacle factor R increases more rapidly than that of radiation dominated era because equation of state approaches that of the matter dominated era ($p=0$, $k=0$, $R \propto t^{2/3}$). At the same time, even though the total density evolves nearly proportional to R^{-3}, R evolves more closely proportional to $t^{2/3}$ so that the total density decreases more rapidly than in the radiation dominated case (dashed line).

For the cases of large β, (SM, SUSY SM, in Figure 1) the feature of R, ρ_t evolution in the PBH dominated era is more distinct. For the case of SU (5), we can hardly find any difference from the radiation dominated era because of the smallness of PBH formation.

3. Ω_{pbh}

Table 2 shows that Ω_{pbh} calculated by equation (28) on assumption of Planck mass residues, is immensely large. With theses large Ω_{pbh}'s, it may be either that BHs leave very few Planck mass residues or that inflation must start early ($T > T(t_{fi})$) before the Universe collapses in $\sim \pi H^{-1} \Omega_{pbh}^{-1/2}$.

Table 2. Ω_{pbh} for the four cases of GUT's

	$T(t_{fi})$	$\rho_{\mathrm{rad}}(t_{fi})$	N_t	Ω_{pbh}
SM	2.56×10^{-2}	8.76×10^{-6}	7.77×10^{-5}	$2.84\times10^{26}h^{-2}$
SUSY SM	2.13×10^{-2}	8.94×10^{-6}	8.73×10^{-5}	$2.60\times10^{26}h^{-2}$
SUSY SU(5)	1.80×10^{-2}	8.36×10^{-6}	8.24×10^{-5}	$2.22\times10^{26}h^{-2}$
SU(5)	2.19×10^{-2}	7.87×10^{-6}	1.13×10^{-6}	$3.93\times10^{24}h^{-2}$

4. Relation to inflation

We define the critical temperature at which the last inflation occurred so that Planck mass residues of PBH could make $\Omega_{pbh}=1$. They are shown in Table 3 for the four cases of GUT's in the first column in the unit of $h/(2\pi)=c=k_b=G=1$ and in the second in the unit of GeV.

For all the four cases, inflation should occur in the rather early Universe when we recall that the minimum temperature at which inflation can occur is about 10^{14} GeV.

2) The is a slight difference from Hayward and Pavon's owing to the different constant in $\Gamma(T)$ (Refer to Note 1.) by error.

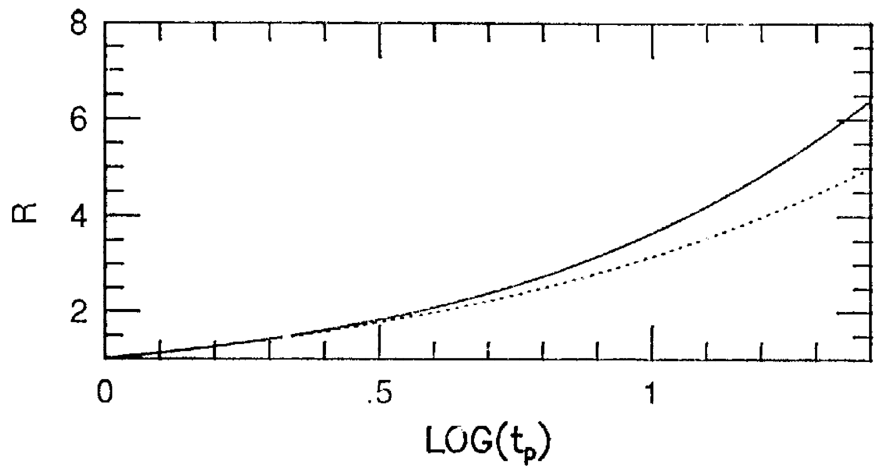

Fig. 2a. Evolution of R for the case of SM ($N=62$, $\theta=3.083$) The dashed line is for the usual radiation dominated era.

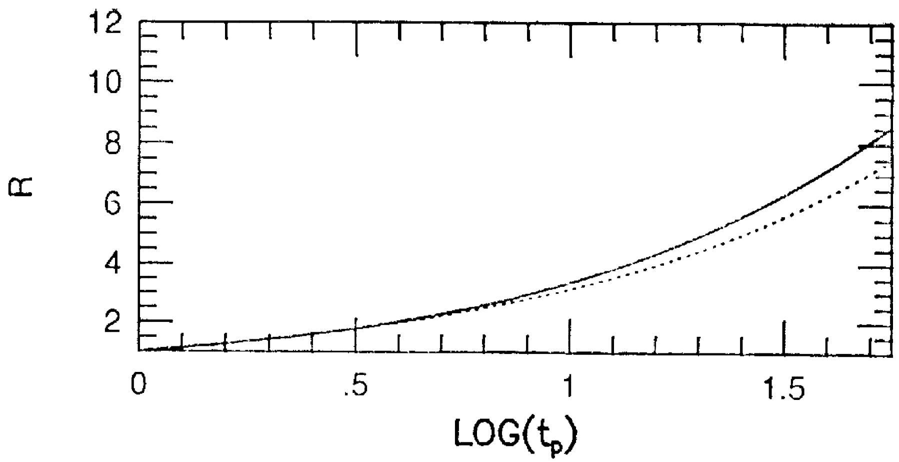

Fig. 2c. Evolution of R for the case of SUSY SU (5) ($N=242$, $\theta=3.000$).

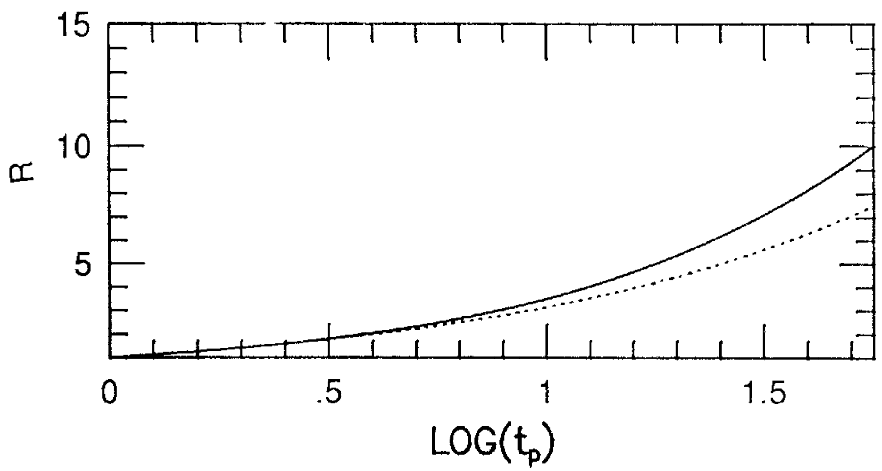

Fig. 2b. Evolution of R for the case of SUSY SM ($N=132$, $\theta=3.656$).

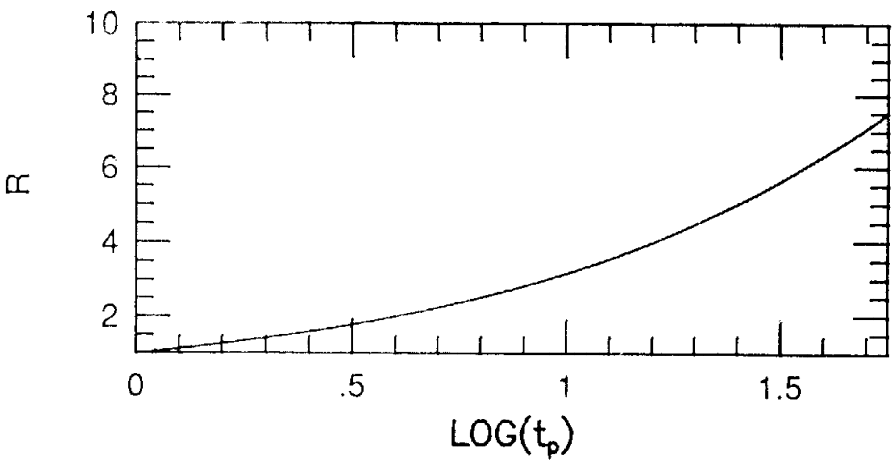

Fig. 2d. Evolution of R for the case of SU (5) ($N=104$, $\theta=0.283$). We can hardly find any difference from the radiation dominated era.

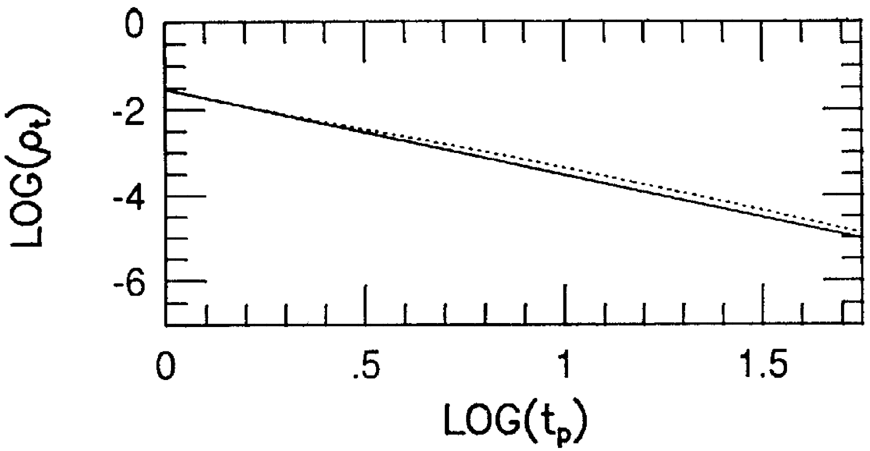

Fig. 3a. Evolution of ρ_t for the case of SM ($N=62$, $\theta=3.083$). The dashed line is for the usual radiation dominated era.

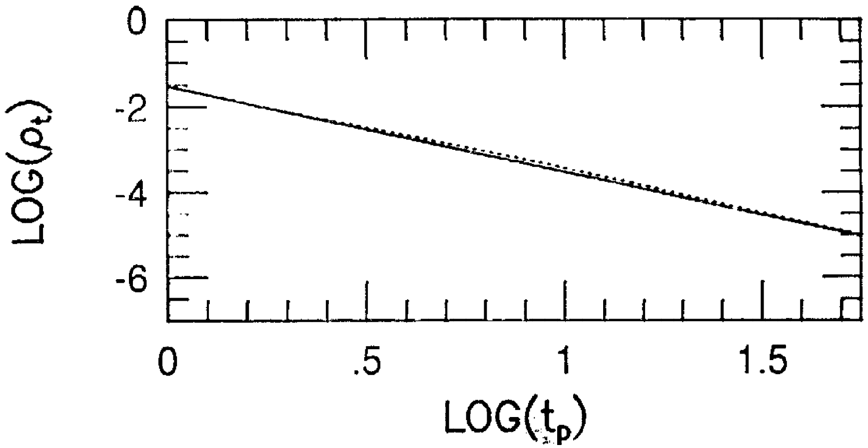

Fig. 3c. Evolution of ρ_t for the case of SUSY SU (5) ($N=242$, $\theta=3.000$).

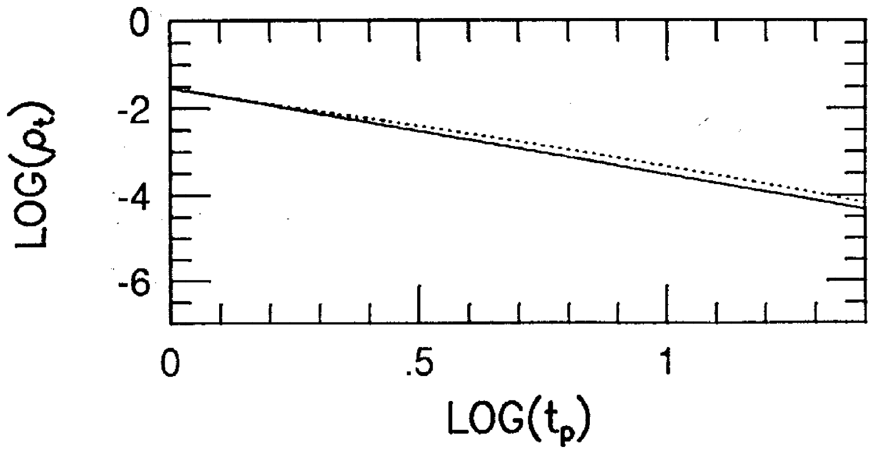

Fig. 3b. Evolution of ρ_t for the case of SUSY SM ($N=132$, $\theta=3.656$).

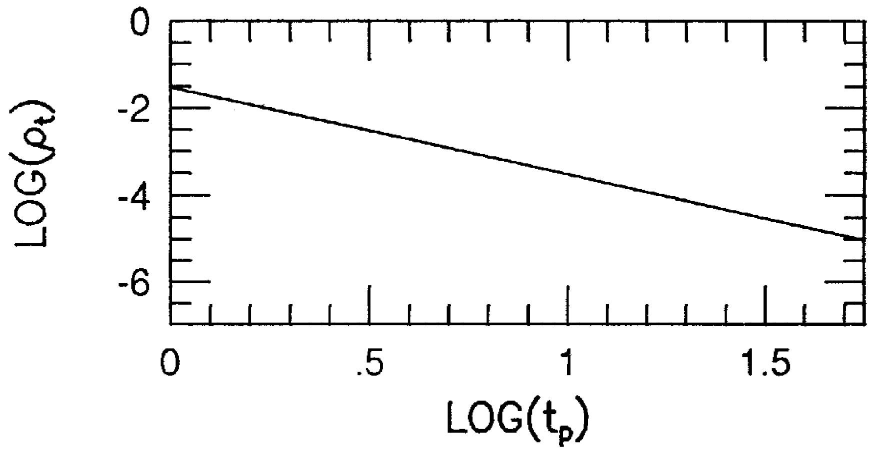

Fig. 3d. Evolution of ρ_t for the case of SU (5) ($N=104$, $\theta=0.283$). We can hardly find any difference from in the radiation dominated era in figure.

Table 3. Critical temperatures for the four cases of GUT's. They are presented in the first column in the unit $h/(2\pi)=c=k_b=G=1$ and in the second in the unit of GeV.

	Critical Temperature	Critical Temperature(GeV)
SM	1.65×10^{-2}	2.01×10^{17}
SUSY SM	1.63×10^{-2}	1.99×10^{17}
SUSY SU (5)	1.68×10^{-2}	2.05×10^{17}
SU (5)	1.81×10^{-2}	2.21×10^{17}

VI. CONCLUDING REMARKS

In this study, we investigated the density evolution of PBH due to quantum gravitational tunneling as treated semi-classically and its influence in terms of PBHs effect upon the evolution of the Universe. It varies with different GUT, with the maximum ratio of PBH density to radiation energy density ranging from 0.007 for SU (5) to about 15 for SUSY SM. It may be thought very important cosmologically that except SU (5) there existed a brief period similar to the matter dominated Universe.

In the absence of inflation we came to the conclusion that calculated large Ω_{pbh} reveals that either inflation starts very early or there is no Planck mass residue. Also if inflation occur above 2×10^{17} GeV, PBH Planck mass residues can make $\Omega_{pbh}\geq1$.

Perhaps the most interesting application of our results is to the baryon asymmetry problem. According to the GUT, X-boson of about 10^{15} GeV is expected to be responsible for baryon non-conservation. Here we have a basis to explain baryon number density to photon number density N_b/N_r. But only thermal X-bosons may not explain N_b/N_r (Lindley 1981) and so PBH is very recommendable on the ground that PBH can produce non-thermal X-boson through Hawking radiation when thermal X-boson cannot be produced due to the temperature drop of the Universe. Lindley(1981) has observed that N_b/N_r can be obtained if the Universe is assumed to be dominated by BHs at 1—100 t_p. During this period when it has been found there were many PBHs due to quantum instability. Hayward and Pavon's paper and our investigation seem to suggest the plausibility of Lindely's *ad hoc* proposal.

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