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An Approximation Method in Bayesian Prediction of Nuclear Power Plant Accidents

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Abstract

A nuclear power plant can be viewed as a large complex man-machine system where high system reliability is obtained by ensuring that sub-systems are designed to operate at a very high level of performance. The chance of severe accident involving at least partial core-melt is very low but once it happens the consequence is very catastrophic. The prediction of risk in low probability, high-risk incidents must be examined in the context of general engineering knowledge and operational experience. Engineering knowledge forms part of the prior information that must be quantified and then updated by statistical evidence gathered from operational experience. Recently, Bayesian procedures have been used to estimate rate of accident and to predict future risks. The Bayesian procedure has advantages in that it efficiently incorporates experts opnions and, if properly applied, it adaptively updates the model parameters such as the rate or probability of accidents. But at the same time it has the disadvantages of computational complexity. The predictive distribution for the time to next incident can not always be expected to end up with a nice closed form even with conjugate priors. Thus we often encounter a numerical integration problem with high dimensions to obtain a predictive distribution, which is practically unsolvable for a model that involves many parameters. In order to circumvent this difficulty, we propose a method of approximation that essentially breaks down a problem involving many

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integrations into several repetitive steps so that each step involves only a small number of integrations.

1. Introduction and Background

Predictions of accidents in nuclear power plants cannot be made without understanding the safety characteristics of complex nuclear power plants. Predictors should carefully examine the most critical factors threatening the safe operation of nuclear power plants, particularly how these factors could interact and lead to a nuclear accident. Engineering knowledge forms part of the prior information that must be quantified and then updated by statistical evidence gathered from operational experience.

Although the nuclear accidents occur in innumerable ways, they fall into only a few categories from the stand point of public safety. Evans and Hope[1984], the group of experts of the Nuclear Energy Agency in Paris(NEA[1986]), and Tat Chi Chow, and R.M. Oliver[1988] proposed their classification schemes. The classification schemes are very subjective and rather arbitrary, but they are all similar in that they all use the likelihood of accidents to release radioactivity as the criterion. In this paper, we calssify the historical incidents to take care of two important points in modeling. One is the importance of human reliability when a nuclear power plant is considered as a large man-machine system ; the other is the interaction between low and high severity indicdents. To utilize the fact that one severity level incident may contain information helpful in predicting the other, we classify the incidents into different groups depending on their severity. The number of different severity groups to be classified is determined based on the purpose of the analysis and the availability of information contained in data. We start with three different severity groups : minor, significant, and severe incidents. These groups are defined as follows :

· serve incidents : incidents that include core melt or partial core melt by which the reactor suffers fairly extensive damage and some quantity of radioactivity is released into the atmosphere.

· significant incidents : incidents that include major contamination of an area or workers or threat to a final defensive system in the nuclear reactor.

· minor incidents : all precursors selected by Accident Sequence Precursor Program in U.S.A. except those selected as severe or significant incidents.

We introduce another classification of incidents : level 0, level 1, and level 2. Level 2 incidents consist of serve incidents, level 1 incidents of level 2 incidents and significant incidents, and level 0 incidents consist of level 1 incidents and minor incidents. Thus level 0 incidents include all precursors, level 1 incidents are a subset of level 0 incidents, and level 2 incidents are a subset of level 1 incidents. Based on such classificatioins, the U.S. Nuclear power industry has witnessed 235 level 0 incidents with a total operating experience of 504.682 Effective Reactor Years(ERYs), where ERYs is defined as the Reactor Years(RYs) multiplied by the portion of time during which a reactor is on line. Of the 235 level 0 incidents, 7 were classi fied as level 1 incidents. Of the 7 level 1 incidents, 1 was classified as a level 2 incident.

For each incident, we also examine causes that can be atributed to the incident initiation and causes that escalate the incident from minor to significant or severe incident status. The causes of incident initiation include many factors such as machine failures, operator errors in manual control, communication or decision, and fire, earthquakes, etc. To pay special attention to the human reliability, we treat operator errors as one type of causes of incident initiation and all the other causes as another type. Once an incident is initiated it can escalate to a more severe incident through another series of machine failures or operator errors or combination of failures and errors, while causes rather than operator errors or machine failures are negligible. Thus operator errors and machine failures are two major causes considered in incident escalation.

The above classification enables us to extract three pieces of information from a single incident: count of incident, severity level, and causes of incident initiation and escalation. Now we define notations and assumptions that will be used throughout this paper.

Notation

subsystem j: group of machines and operators whose failure or error is responsible for the incident escalation from level $j-1$ to level j, $j=1, 2$

T: effective reactor years

$n_0^{hb}(T)$: number of level 0 incidents over a time period (0, T) initiated involving operator errors

$n_0^c(T)$: number of lvel 0 incidents over a time period (0, T) initiated without involving operator errors

$n_0(T)$: number of level 0 incidents over a time period (0, T), $n_0^{hb}(T)+n_0^c(T)$

$n_j^h(T)$: number of level j incidents over a time period (0, T) that involve operator errors only when escalating from level $j-1$

$n_j^m(T)$: number of level j incidents over a time period (0, T) that involve machine failures only when escalating from level $j-1$

$n_j^b(T)$: number of level j incidents over a time period (0, T) that involve both machine failures and operator errors when escalating from level $j-1$

$n_j^{mb}(T)$: number of level j incidents over a time period (0, T) that involve machine failures when escalating from level $j-1$, $n_j^m(T)+n_j^b(T)$

$n_j^{hb}(T)$: number of level j incidents over a time period (0, T) that involve operators errors when escalating from level $j-1$, $n_j^h(T)+n_j^b(T)$

$n_j(T)$: number of level j incidents over a time period (0, T), $n_j^m(T)+n_j^h(T)+n_j^b(T)$

x_j^h: time to next level j incident that involves operator errors only when escalating from level $j-1$

x_j^m: time to next level j incident that involves machine failures only when escalating from level $j-1$

x_j^b: time to next level j incident that involves both machine failures and operator errors when escalating from level $j-1$

x_j: time until next level j incident, $\min[x_j^m, x_j^h, x_j^b]$

ϕ_j: probability of machine failures in sub-system j

θ_j: probability of operator errors in sub-system j

λ_0^{hb}: arrival rate of level 0 incidents initiated involving operator errors

λ_0^c : arrival rate of level 0 incidents initiated without involving operator errors

λ_0 : arrival rate of level 0 incidents, $\lambda_0^{hb}+\lambda_0^c$

λ_j : arrival rate of level j incidents

$LN(\mu, \sigma^2)$: Lognormal distribution with parameters μ, σ^2

$P(\lambda)$: Poisson distribution with parameter λ

$\Gamma(\alpha, \beta)$: Gamma distribution with parameters α, β

$Be(a, b)$: Beta distribution with parameters a, b

Assumptions

1. Since the accident in nuclear power plants is rare, we assume that the counting process of $n_0^{hb}(T)$, $n_0^c(T)$ follows Poisson distributions given parameters λ_0^{hb}, λ_0^c, respectively.

2. Model parameters are assumed to be independent of one another.

Based on the classification of three different severity levels and causes of incident initiation and escalation selected as above, the number of incidents over time is summarized in Table 1.

When we perform a Bayesian prediction, we assume prior distributions on model parameters and update them as we acquire more data, and finally get predictive distribution by integrating out unobservable model parameters :

$$p(x \mid D)=\int\cdots\int p(\Phi \mid D)\ p(x \mid \Phi)d\Phi \quad \cdots\cdots\cdots (1)$$

where Φ denote the vector parameters in a prediction model, and D denotes data, and x denotes

Table 1. Number of Incidents over Time

End of Year	Cum. ERYs	Level 0			Level 1				Level 2			
		n_0^c	n_0^{hb}	n_0	n_1^m	n_1^h	n_1^b	n_1	n_2^m	n_2^h	n_2^b	n_2
1969	12.344	0	1	1	0	0	0	0	0	0	0	0
1970	16.387	2	2	4	1	0	0	1	0	0	0	0
1971	24.379	8	4	12	1	0	0	1	0	0	0	0
1972	35.181	11	7	18	1	0	0	1	0	0	0	0
1973	50.094	14	10	24	1	0	0	1	0	0	0	0
1974	68.719	17	11	28	1	0	0	1	0	0	0	0
1975	98.884	24	15	39	2	0	0	2	0	0	0	0
1976	133.547	27	18	45	2	0	0	2	0	0	0	0
1977	173.575	39	22	61	2	0	0	2	0	0	0	0
1978	219.279	49	29	78	3	0	0	3	0	0	0	0
1979	261.275	65	38	103	3	0	1	4	1	0	0	1
1980	303.624	85	44	129	4	0	1	5	1	0	0	1
1981	348.836	97	50	147	4	1	1	6	1	0	0	1
1984	395.765	122	56	178	4	1	1	6	1	0	0	1
1985	451.206	154	67	221	4	1	2	7	1	0	0	1
1986	504.682	167	68	235	4	1	2	7	1	0	0	1

the time to next incident. Level 0 incidents are influenced by two parameters λ_0^{hb}, λ_0^c. Level j incidents are influenced by 2(j+1) parameters, λ_0^{hb}, λ_0^c, ϕ_i, and θ_i, i=1,⋯, j. In order to obtain the predictive distribution of level j accident, we have to solve 2(j+1) dimensional numerical integration problem which is impractical when j becomes large. Therefore we are in need of developing a methodology that can cope with such a difficulty.

2. Approximation Method

2-1. Lognormal Priors

To assess prior distributions for model parameters we use expert opinion as well as available data. Among the available data arranged in Table 1, we will use the portion up to the end of 1980 for the prior information and the remaining recent data will be used to obtain and analyze the behavior of posterior distributions. Expert opinions are available from previous works in nuclear power plant industry or some related areas such as WASH-1400 Nuclear Safety Study[1975], Predicting Nuclear Incidents by Tat-Chi Chow, and R.M. Oliver[1988], and Handbook of Human Reliability Analysis with Emphasis on Nuclear Power Plant Applications[1980], etc.

A normal distribution is not a good candidate for a distribution of failure rates. There have been several studies including Wechsler[1952], and Green and Bourne[1972] that have shown that most human traits and abilities do not conform to a normal distribution. Works such as WASH-1400 Nuclear Safety Study[1975], Tat-Chi Cfhow et al. [1988] have suggested that arrival rates of various severity level incidents are lognormally distributed. The rationale of using lognormal distributions is that a safety system provided with many reducndancies tends to bunch up towards the low probability of failure. The plot of the arrival rates of λ_0^{hb} and λ_0^c from the data also shows the tendency of positive skewness. Thus we start with lognormal distribution for arrival rates of level 0 incidents. Using the weighted mean and variance from the data in Table 1, we firstly obtain

$$\lambda_0^{hb} \sim Ln(-2,\ 0.2) \qquad \lambda_0^c \sim LN(-1.4,\ 0.3)$$

A lognormal distribution can be approximated by a Gamma distribution when the ratio of mean to standard deviation of a lognormal distribution is greater than 1. In this case a bell-shaped Gamma distribution is guaranteed. Vesely[1977] has performed a sensitivity analysis using a Monte Carlo procedure to see if the assumption of different kinds of distributions for the human failure estimates would materially affect the failure of various subsystems in safety-related systems. It was found that the predicted failure rate did not differ materially no matter what distribution was assumed. Mills and Hatfield[1974] have also shown that the forecasts of a failure rate is insensitive to the distributional assumptions. We adopt the convention that if a random variable λ follows a Gamma distribution with parameters α and β, the probability density function is expressed as

$$p(\lambda) = \Gamma(\alpha,\ \beta) = \frac{\beta(\beta\lambda)^{\alpha-1}e^{-\beta\lambda}}{\Gamma(\alpha)},\quad E[\lambda] = \frac{\alpha}{\beta}$$

The above lognormal priors are approximated by

$$\lambda_0^{hb} \sim \Gamma(2.39,\ 16.3) \quad \lambda_0^c \sim \Gamma(2.2,\ 9.6)$$

Fig. 1 plots the lognormal and Gamma distributions

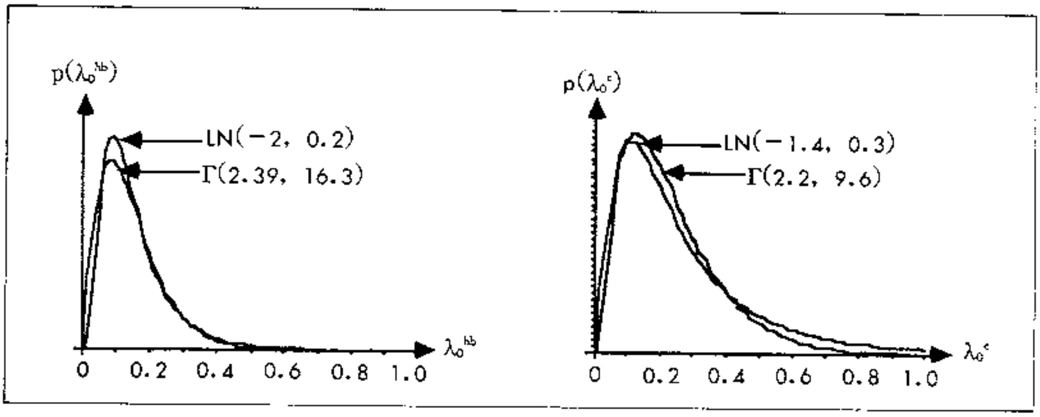

Fig. 1. Arrival Rates Approximated by Gamma Distributions.

for arrival rates and it can be seen that the approximation by the Gamma distributions is quite close.

The motivation to introduce a Gamma approximation is that the use of the Gamma prior brings us several simplifications in calculation. If the distribution of λ is $\Gamma(\alpha, \beta)$ and we have observed n events in T units of time where the number of events given parameter λ follows a Poisson distribution, then the distribution of λ is closed under observations. In other words, the posterior distribution is also $\Gamma(\alpha', \beta')$, where (′) denotes posterior parameters :

$$\alpha' = \alpha + n,$$

$$\beta' = \beta + T$$

In addition to this advantage in parameter updating, the predictive distribution of the time to next incident, x, can be obtained in a closed form, which is a shifted pareto distribution :

$$p(x \mid D) = \int p(x \mid \lambda)\ p(\lambda \mid D) d\lambda$$

$$= \left(\frac{\beta'}{\beta'+x}\right)^{\alpha'} \left(\frac{\alpha'}{\beta'+x}\right)$$

$$= \left(\frac{\beta+T}{\beta+T+x}\right)^{\alpha} \frac{\alpha+n}{\beta+T+x} \cdots\cdots\cdots\cdots (3)$$

2-2. Arrival Rate of Level 0 Incidents

The arrival rate of level 0 incidents, λ_0, is the sum of two arrival rates λ_0^{hb} and λ_0^{c}, which have independent Gamma distributions. The distribution of λ_0 is obtained by a convolution of the distributions of λ_0^{hb} and λ_0^{c}. Let two independent random variables X and Y, representing arrival rates, follow a Gamma distribution with parameters α_1, β_1 and α_2, β_2, respectively. And let Z denote the sum of X and Y. Then the distribution of Z is obtained by

$$f_{X+Y}(z) = \int_0^z \frac{\beta_1(\beta_1 x)^{\alpha_1-1} e^{-\beta_1 x}}{\Gamma(\alpha_1)} \frac{\beta_2(\beta_2(z-x))^{\alpha_2-1} e^{-\beta_2(z-x)}}{\Gamma(\alpha_2)} dx$$

$$= \frac{\beta_1^{\alpha_1}\beta_2^{\alpha_2} e^{-\beta_2 z}}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_0^z x^{\alpha_1-1}(z-x)^{\alpha_2-1} e^{-(\beta_1-\beta_2)x} dx$$

When arrival rates are small and/or β_1 is close to β_2,

$$e^{-(\beta_1-\beta_2)x} \approx e^0 = 1$$

Thus

$$f_{X+Y}(z) \approx \frac{\beta_1^{\alpha1}\beta_2^{\alpha2}e^{-\beta2z}}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\int_0^z x^{\alpha1-1}(z-x)^{\alpha2-1}\,dx$$

We substitute $u=x/z$ with $dx=z\,du$, then

$$f_{X+Y}(z) = \frac{\int_0^1 u^{\alpha1-1}(1-u)^{\alpha2-1}du}{\Gamma(\alpha_1)\Gamma(\alpha_1)}\beta_1^{\alpha1}\ \beta_2^{\alpha2}\ z^{\alpha1+\alpha2-1}\ e^{-\beta2z}$$

$$= C\ \beta_1^{\alpha1}\ \beta_2^{\alpha2}\ z^{\alpha1+\alpha2-1}\ e^{-\beta2z}$$

The constant C is obtained from the relation $\int f(z)dz=1$.

$$C = \frac{1}{\beta_1^{\alpha1}\ \beta_2^{\alpha2}\int_0^\infty z^{\alpha1+\alpha2-1}\ e^{-\beta2z}\ dz}$$

$$= \frac{1}{(\frac{\beta_1}{\beta_2})^{\alpha1}\ \Gamma(\alpha_1+\alpha_2)}$$

So we get

$$f_{X+Y}(z) = \frac{\beta_2^{\alpha1+\alpha2}\ z^{\alpha1+\alpha2-1}\ e^{-\beta2z}}{\Gamma(\alpha_1+\alpha_2)} \sim \Gamma(\alpha_1+\alpha_2,\ \beta_2)$$

or

$$f_{X+Y}(z) = \frac{\beta_1^{\alpha1+\alpha2}\ z^{\alpha1+\alpha2-1}\ e^{-\beta1z}}{\Gamma(\alpha_1+\alpha_1)} \sim \Gamma(\alpha_1+\alpha_2,\ \beta_1)$$

Therefore we approximate the arrival rate of level 0 incidents with a Gamma distribution where the value of parameter β_0' is obtained by the mean of β_0^{hb}, and β_0^c:

$$\lambda_0 \mid D \sim \Gamma(\alpha_0',\ \beta_0^\circ) = \Gamma(108.59,\ 214.01)$$

Fig. 2 shows the distributions of the arrival rate of level 0 incidents obtained by convolution and by Gamma approximation.

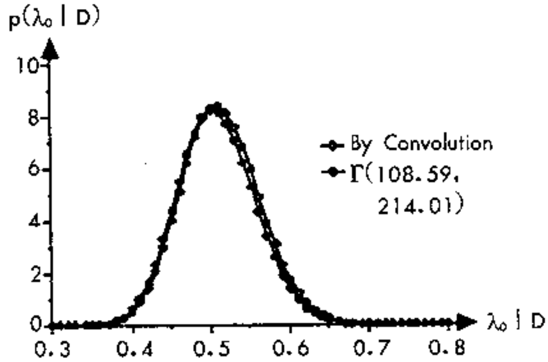

Fig. 2. Distribution of λ_0 Approximated by a Gamma Distribution.

3. Arrival Rate of Level j Incidents

The predictive distribution for a time to next level 1 incident is obtained as follows:

$$p(x_1 \mid D) = \iiiint p(x_1 \mid \lambda_0^{hb},\ \lambda_0^c,\ \phi_1,\ \theta_1)$$
$$p(\lambda_0^{hb},\ \lambda_0^c,\ \phi_1,\ \phi_1 \mid D)\ d\lambda_0^{hb}\ d\lambda_0^c d\phi_1 d\theta_1$$

Consider the event tree in Fig. 3 that shows the escalation process up to level 1 only. The counts at the end of each sequence can be considered as randomly partitioned numbers from the total number of level 0 incidents with probabilities of $(1-\phi_1)(1-\theta_1)$, $(1-\theta_1)\theta_1$, $\phi_1(1-\theta_1)$, and $\phi_1\theta_1$ from the top to the bottom sequence, respectively. Then the number of incidents that pass through each sequence follows a Poisson distribution with a rate of λ_0 times the associated probabilities.

Therefore the time to next incident is exponentially distributed with the appropriate rates:

$$x_1^h \mid \lambda_0,\ \phi_1,\ \theta_1 \sim Exp(\lambda_0(1-\phi_1)\theta_1)$$

$$x_1^m \mid \lambda_0,\ \phi_1,\ \theta_1 \sim Exp(\lambda_0\phi_1(1-\theta_1))$$

$$x_1^b \mid \lambda_0,\ \phi_1,\ \theta_1 \sim Exp(\lambda_0\phi_1\theta_1)$$

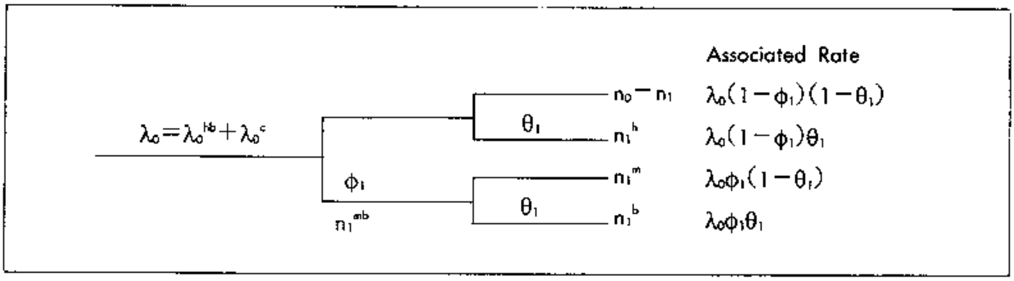

Fig. 3. Event Tree Showing the Escalation Process to Level 1.

where

$\lambda_0 = \lambda_0^{hb} + \lambda_0^c$

Since the next incident may follow any of those sequences, time to next level 1 incident x_1 is minimum of x_1^h, x_1^m, and x_1^b. It can be shown that the minimum of exponentially distributed independent random variables is also exponential with arrival rate the sum of all individual arrival rates. Thus we obtain

$$(x_1 \mid \lambda_0, \phi_1, \theta_1) = \min[x_1^h, x_1^m, x_1^b \mid \lambda_0, \phi_1, \theta_1]$$
$$\sim \text{Exp}(\lambda_0(\phi_1 + \theta_1 - \phi_1\theta_1))$$

Now generally consider level j incidents. The predictive distribution for the time to next level j incident is obtained by

$$p(x_j \mid D) = \int \cdots \int p(x_j \mid \Phi_j) p(\Phi_j \mid D) d\Phi_j$$

where $\Phi_j = (\lambda_0^{hb}, \lambda_0^c, \phi_1, \cdots, \phi_j, \theta_1, \cdots, \theta_j)$

Because of the repetitive property of sub-models, we can only consider the sub-event tree in Fig. 4 that shows the escalation process from level $j-1$ to level j incidents.

For the same reasons as discussed above, time to next level j incident is exponentially distributed as follows :

$$x_j^h \mid \lambda_{j-1}, \phi_j, \theta_j \sim \text{Exp}(\lambda_{j-1}(1-\phi_j)\theta_j)$$
$$x_j^m \mid \lambda_{j-1}, \phi_j, \phi_j \sim \text{Exp}(\lambda_{j-1}\phi_j(1-\theta_j))$$
$$x_j^b \mid \lambda_{j-1}, \phi_j, \theta_j \sim \text{Exp}(\lambda_{j-1}\phi_j\theta_j)$$

where

$\lambda_0 = \lambda_0^{hb} + \lambda_0^c$

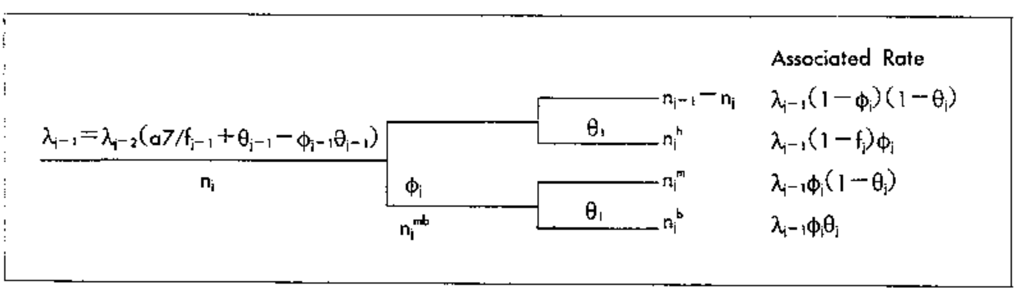

Fig. 4. Sub-Tree Showing Escalation Process From Level $j-1$ to Level j.

$\lambda_{j-1}=\lambda_{j-2}(\phi_{j-1}+\theta_{j-1}-\phi_{j-1}\theta_{j-1})$, $j=2,3,\cdots$

Thus we obtain

$(x_j \mid \Phi_j)=\min[x_j^h,\ x_j^m,\ x_j^b \mid \Phi_j]$

$\sim \mathrm{Exp}(\lambda_{j-1}(\phi_j+\theta_j-\phi_j\theta_j))$

The arrival rate of level $j-1$ incidents, λ_{j-1}, can be recursively obtained from λ_0. For a multi-level model, level j incidents are influenced by $2(j+1)$ unobservable parameters. The predictive distribution for the time to the next level j incident in equation (1) can not always be expected to end up with a nice closed form even with conjugate priors. Thus we encounter a numerical integration problem with $2(j+1)$ dimensions to obtain a predictive distribution for level j incidents, which is practially unsolvable for large j. In order to circumvent this difficulty, we propose a method of approximation that essentially breaks down a problem involving many integrations into several repetitive steps so that each step involves only a small number of integrations.

To find the distribution of λ_1, let λ_1, λ_0, ϕ_1, θ_1 follow distributions of F_1, F_0, G_1, and H_1, respectively. Then the cumulative distribution of λ_1 is

$$F_1(z)=\mathrm{Prob.}\ \{\lambda_1=\lambda_0(\phi_1+\theta_1-\phi_1\theta_1)\le z\}$$
$$=\iiint_{\lambda_0(\phi_1+\theta_1-\phi_1\theta_1)\le z} f_0(\lambda_0)g_1(\phi_1)h_1(\theta_1)d\lambda_0 d\phi_1 d\theta_1$$
$$=\int_0^1\int_z^{\frac{z}{\theta_1}}\int_0^{\frac{z-\lambda_0\theta_1}{\lambda_0(1-\theta_1)}} f_0(\lambda_0)g_1(\phi_1)h_1(\theta_1)d\phi_1 d\lambda_0 d\theta_1$$
$$=\int_0^1\int_z^{\frac{z}{\theta_1}} f_0(\lambda_0)\ G_1\left(\frac{z-\lambda_0\theta_1}{\lambda_0(1-\theta_1)}\right)\ h_1(\theta_1)d\lambda_0 d\theta_1 \quad \cdots\cdots (4)$$

By differencing equation (4), we obtain

$$f_1(z)=\frac{d}{dz}F_1(z)=\int_0^1\int_z^{\frac{z}{\theta_1}} f_0(\lambda_0)g_1\left(\frac{z-\lambda_0\theta_1}{\lambda_0(1-\theta)}\right)h_1(\theta_1)\frac{1}{\lambda_0(1-\theta_1)}d\lambda_0 d\theta_1 \quad \cdots\cdots (5)$$

Now we want to find a closed form distribution that approximates the distribution obtained by equation (5) so that prediction is simple by using equation (3). Let $\lambda_0\sim\Gamma(\alpha_0,\beta_0)$ as we have already approximated, and $E[\phi_1+\theta_1-\phi_1\theta_1]=\mu_1$, and $\mathrm{Var}[\phi_1+\theta_1-\phi_1\theta_1]=\sigma_1^2$. Since ϕ_1 and θ_1 are independent of each other, μ_1 and σ_1^2 are obtained by

$$\mu_1=E[\phi_1]+E[\theta_1]-E[\phi_1]E[\phi_1]$$
$$\sigma_1^2=\mathrm{Var}[\phi_1]+\mathrm{Var}[\theta_1]+\mathrm{Var}[\phi_1\theta_1]-2\ \mathrm{Cov}[\phi_1,\ \phi_1\theta_1]-2\ \mathrm{Cov}[\theta_1,\ \phi_1\theta_1]$$
$$=\mathrm{Var}[\phi_1]+\mathrm{Var}[\theta_1]+\{E[\phi_1]\}^2\ \mathrm{Var}[\theta_1]+\{E[\theta_1]\}^2\ \mathrm{Var}[\phi_1]+\mathrm{Var}[\phi_1]\ \mathrm{Var}[\phi_1]-2\ E[\phi_1]\ \mathrm{Var}[\phi_1]-2E[\phi_1]\ \mathrm{Var}[\phi_1] \quad \cdots\cdots (6)$$

The mean and variance of the arrival rate of level 1 incidents are calculated from

$$E[\lambda_1]=\frac{\alpha_0}{\beta_0}\mu_1$$

$$\mathrm{Var}[\lambda_1]=\frac{\alpha_0}{\beta_0^2}\mu_1^2+\frac{\alpha_0^2+\alpha_0}{\beta_0^2}\sigma_1^2$$

For small σ_1/μ_1, $(\phi_1+\theta_1-\phi_1\theta_1)$ approximately behaves like a constant and it can be shown that a constant times a Gamma random variable is also Gamma, λ_1 can be approximated by $\Gamma(\alpha_1,\beta_1)$. We obtain parameters α_1, β_1 from equation(7) :

$$\alpha_1=\frac{\alpha_0\mu_1^2}{(\alpha_0+1)\sigma_1^2+\mu_1^2},\quad \beta_1=\frac{\beta_0\mu_1}{(\alpha_0+1)\sigma_1^2+\mu_1^2}$$

For similar reasons we can recursively obtain that the arrival rate of higher level incidents follows a Gamma distributions ;

$\lambda_j=\lambda_{j-1}(\phi_j+\theta_j-\phi_j\theta_j)\sim\Gamma(\alpha_j,\ \beta_j)$, $ˊ=1,2,3,\cdots$

where

$$\alpha_j=\frac{\alpha_{j-1}\mu_j^2}{(\alpha_{j-1}+1)\sigma_j^2+\mu_j^2},\quad \beta_j=\frac{\beta_{j-1}\mu_j}{(\alpha_{j-1}+1)\sigma_j^2+\mu_j^2} \qquad (8)$$

and μ_j, σ_j^2 are obtained from equation (6) by replacing subscript 1 with j.

The branch probabilities θ_j, ϕ_j are assumed to follow a Beta distribution since it is a quite flexible distribution covering almost all forms of distributions between 0 and 1. We adopt the prior parameters in Heejoon Yang[1989]:

$\theta_j \sim Be(6, 600) \quad \phi_j \sim Be(4, 67) \quad j=1, 2$

When the prior distribution of branch parameters are assumed to be a Beta distribution and the likelihood of the counts passing through the down branch in the event tree is Binomial conditional on a corresponding branch parameter and the total counts passing through the upstream fork in the event tree, the posterior distribution of the branch parameters is also a Beta distribution with new parameters reflecting the sum of all accident sequences which pass through "up" and "down" branches sharing a common parametergsee Fig. 4. :

$\theta_j \sim Be(a_j, b_j) \quad \phi_j \sim Be(c_j, d_j)$

$\theta_j \mid D \sim Be(a_j+n_j^h+n_j^b,\ b_j+n_{j-1}-n_{j-1}+n_j^m)$

$\phi \mid D \sim Be(c_j+n_j^{mb},\ d_j+n_{j-1}+n_j^{mb})$

Using the above mentioned updating scheme and equation(8), we obtain

$\lambda_1 \mid D \sim \Gamma(68.47,\ 426.17)$

$\lambda_2 \mid D \sim \Gamma(3.24,\ 2542.35)$

Fig. 5 shows the posterior distributions of arrival rates for level 1 and 2 incidents approximated by Gamma distributions.

The above approximations are based on the independence assumption among model parameters, similar value of β_o^{hb} and β_o^c, and small σ_j/μ_j. Even though we may start with fairly different β_o^{hb} and β_o^c, these parameters are updated by adding the total operating experience T. Thus as T becomes large, the initial difference between β_o^{hb} and β_o^c becomes relatively unimportant. Furthermore, as we acquire more and more data, the distributions of ϕ and θ become sharper and sharper, resulting in smaller values of σ_j/μ_j. Therefore we can expect more accurate approximations as we observe more

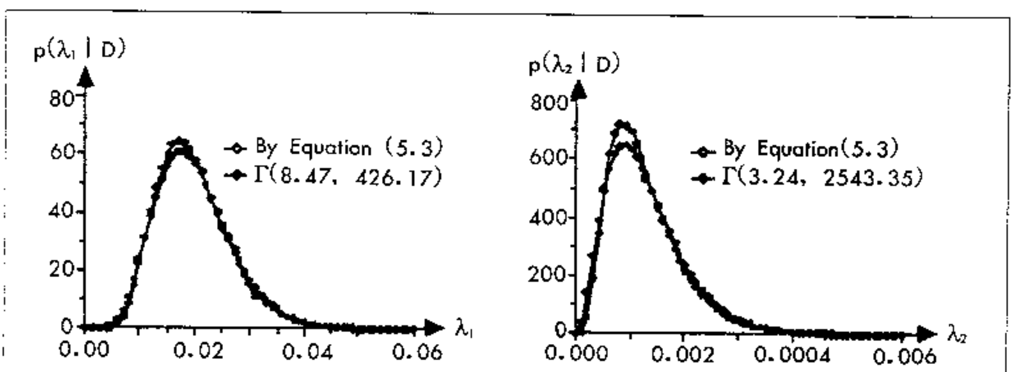

Fig. 5. Distribution of λ_1 and λ_2 Approximated by Gamma.

data.

Due to the approximation method and the properties of the Gamma prior and the Poisson likelihood, the predictive distribution of time to next level j incident that needs a 2(j+1) dimensional numerical integration in equation (1) reduces to a nice closed from as in equation (3). The expected value, which is the conventionally used point predictor, does not well represent the predictive distribution since the distribution has a long tail. Thus we can use a median and quantiles instead.

Let

$$p = \text{Prob.}\{\text{time to next incident} \leq z_p \mid D\}$$

Then z_p denotes quantiles where $z_{0.5}$ denotes a median. The Gamma prior and Poisson likelihood again simplify the equation for quantiles in a closed form as in equation (9), and Table 2 summarizes the results of prediction.

$$z_p = \beta_2\{(\frac{1}{1-p})^{1/\alpha_2} - 1\} \qquad (9)$$

4. Summary

We develop an approximation method to solve high dimension numerical integrations. The approximation method essentially breaks down a problem involving many integrations into several repetitive steps so that each step involves only a small number of integrations. It enables us to easily obtain posterior distributions in closed forms. We show that distributions obtained by approximation methods are quite close to the real ones obtained by numerical integrations.

Due to the approximation method, the forecast can be easily done ; starting with assessed priors and assumed likelyhoods we update model parameters as we acquire more data, and then we follow the approximation procedure to obtain the parameter values of the Gamma distribution for the arrival rate of level j incidents, and finally substitute the obtained posterior parameters of the Gamma distribution in the derived equation that finds the quantiles of time to next incident. Also we show that we can expect more accurate approximations as we observe more data.

Table 2. Qunatiles of Time to Next Sever Incident

Units	5%	25%	50%	75%	95%
ERY	41	236	606	1358	3867
RY	61	349	896	2007	5715

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