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A Study on Operations in Single-Card KANBAN System with a General-Type-Structure Production Process

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일반 형태의 생산구조 단일카드 KANBAN 시스템의 운영 최적화

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Abstract

In this paper we study a mathematical programming model for the single-card KANBAN system in a multi-stage capacitated general-type-structure production. Until now this production type setting has not been studied. The modeling of this problem results in a complex integer programming which can be modified to the more simple integer programming model. We present a heuristic method and some numerical examples. Though the presented method doesn't always find an optimal solution, this method guarantees to find a feasible solution. We expect this work to be practised in the real fields.

1. Introduction

Recently a great deal of attention has been arised to the Japanese production and inventory management techniques. In particular, the just-in-time (JIT) system with KANBAN has received most of this interest. The KANBAN system is a multi-stage production and inventory control system. The basic idea of KANBAN system is JIT. The JIT denies any inventory(safety stock, cycle stock etc.) and it is similar to zero inventory. Now, the KANBAN system and JIT system become fundamental notions in field production and inventory system. Many researchers study on this systems.

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In the KANBAN systems, the production is ordered and delivered by 'KANBAN' card('KANBAN' means a card or a tag in Japanese). At each stage inventory level is determined by the number of 'KANBAN' cards (we write 'KANBAN card' by KANBAN). The optimal operating of KANBAN system minimizes the number of KANBAN in a feasible production planning schedule. As the number of KANBAN circulating at a stage is lessened, the operating efficiency of that stage is increased.

The operating of KANBAN system is divided by the single-card and the dual-card system. Dual-card system : At each stage, there are 2 kinds of KANBAN. Production card and delivery card are divided and operated mutual dependently. The principle of this system is the same as that of single-card system. Single-card system(Fig. 1) : At each stage the card is one kind. 1) KANBAN is attached to empty container in P^n and container is filled with item n in P^n. When container is filled fully, it is sent to I^n. 2) When the first piece of a full container in I^n is used in S(n)(the production process of the immediate successor), the KANBAN is detached and kept aside. 3) At the end of each time period, all the KANBAN that are detached during the time period are collected and sent back to P^n. 4) Then these KANBAN serve as the new production order in P^n.(*Unless S(n) use any piece of container in I^n, then P^n cannot produce) Accordingly, if any stage is halted, then all stages are halted.

*Four important natures of KANBAN system. 1) The total number of KANBAN circulating between P^n and I^n is unchanged over time(the change of the number of KANBAN is only possible by outsider to input or to drain into KANBAN system). 2) The maximum inventory level in I^n is limited by the number of KANBAN circulating between P^n and I^n. 3) The movement of KANBAN between P^n and I^n is triggered by the inventory withdrawal from I^n in S(n)(i.e the immediate successor). 4) By circulating KANBAN within every stage, all the stages in a production setting are chained together. The upstream stages can actually be self-operated. As described above, the optimal operating of KANBAN system depends on the number of KANBAN determined at the starting time.

The KANBAN system was originally designed

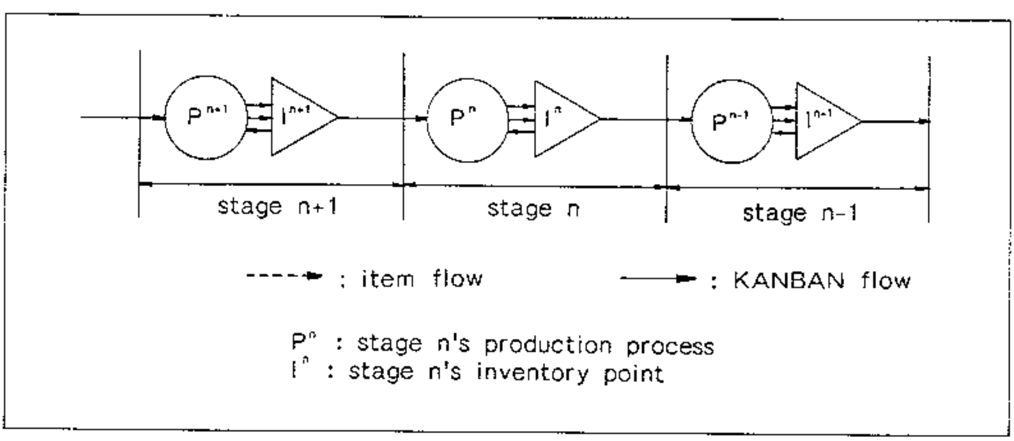

Fig. 1 single-card KANBAN system operating by Toyoda to realize JIT production. After success of KANBAN system, there have been several articles of KANBAN system compared with MRP system[3], simulation analysis of KANBAN system and MRP system[2, 7], and an endeavor to integrate KANBAN system and MRP system[6, 7], etc. Many qualitative analysis papers have been continuously presented. But there were not many mathematical approaches to KANBAN system. Kimure and Terada formulated the pull system and gave a simulation model of fluctuation in production and inventory[2]. Bitran and Chang presented a mathematical programming model for the KABAN system in a determinated multi-stage capacitated assembly-tree-structure production setting and discussed solution procedure to the problem and addressed three special cases of practical interest[1]. Among the above articles, Bitran and Chang's

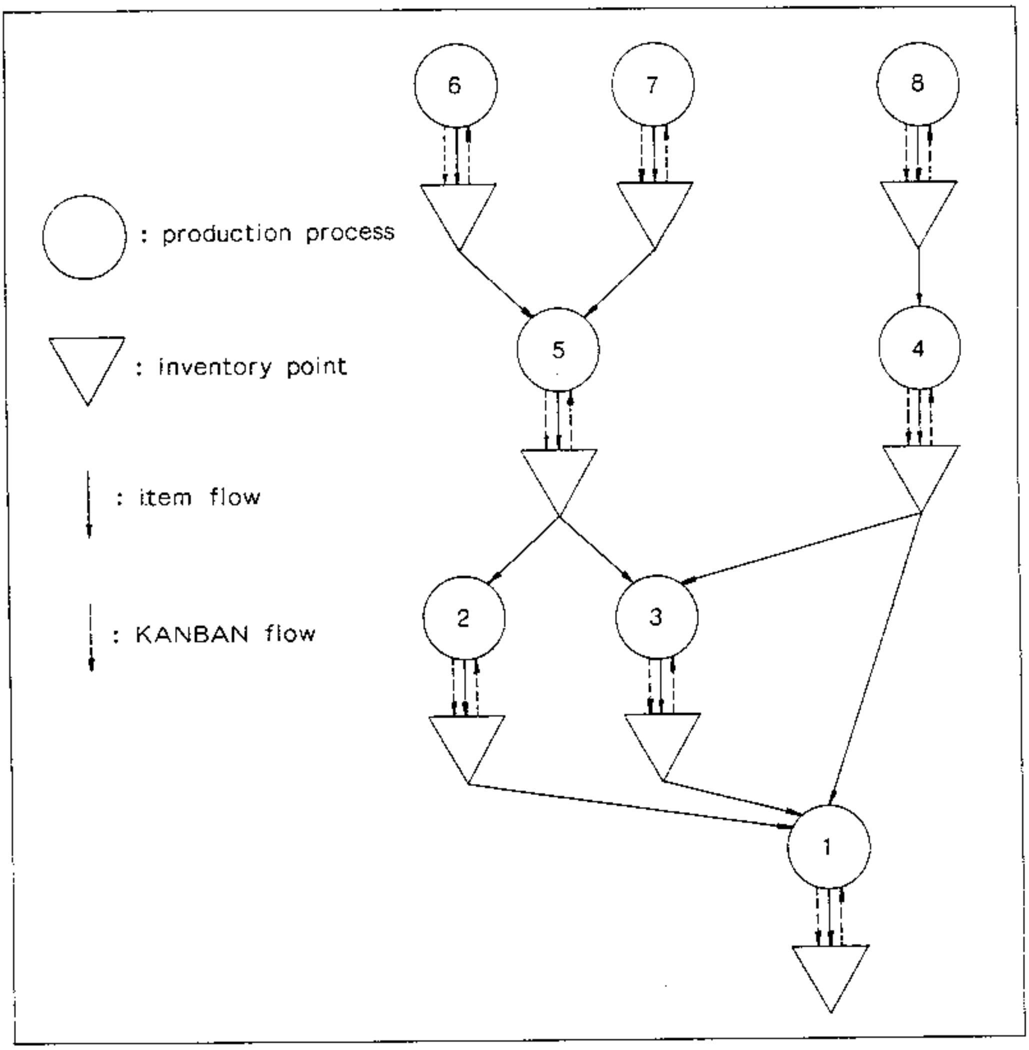

Fig. 2 general-structure production process study was epochal in the field of mathematical analysis of KANBAN system. The optimal operation KANBAN system was studied in serial-production setting and in assemble-tree-structure production setting. But until now general-type-structure production setting has not been studied. It is more applicable in real manufacturing situation. This paper extends Bitran and Chang's assemble-tree-structure to the general-type-structure.

2. Model Description

In the general-type-structure production setting system, any stage can send items to several successors of stages and any stage can receive items by several predecessors of stages. The model in this paper is presented in Fig. 2.

Assumptions

1) Transfer time of KANBAN is zero. 2) KANBAN detached in P^n in period t are available as production orders in period $t+1$. 3) In each stage P^n, the planned amount in period t is always produced. 4) There is no stop by unexpected trouble. 5) There is no distinction among containers in the same stage. But there are distinctions among containers in other stages. 6) The stage of complete production is one.

Parameters

α^n : container capacity, the number of units of item n in a full container. α^n $\{1, 2, \cdots\}$

β_t^n : production capacity, the number of full containers of item n, at P^n in period t. β_t^n $\{0, 1, \cdots\}$ $(n=1, \cdots, N;\ t=1, \cdots, T)$

$S(n)$: set of immediate successive stages of stage $n(n=1, \cdots, N)$, ex) $S(5)=\{2, 3\}$

$P(n)$: set of immediate preceding stages of stage $n(n=1, \cdots, N)$, ex) $P(5)=\{6, 7\}$

$N\{S(n)\}$: the cardinality of $S(n)$, ex) $S(5)=\{2, 3\}$ $N\{S(5)\}=2$

$N\{P(n)\}$: the cardinality of $P(n)$, ex) $P(5)=\{6, 7\}$ $N\{P(5)\}=2$

$e^{n,s}$: the number of units of item n which are required to make one unit of item in $S(n)$; $e^{n,s}$ $\{1, 2, \cdots\}(n=1, \cdots, N)$

$\sum_s e^{n,s}$: the number of units of item n which are required to make one unit of each item of the stages in $S(n)$

V_0^n : the number of full containers having a KANBAN attatched to it$(n=1, \cdots, N)$ i.e. initial inventory

$W_0^{n,s}$: the number of units of item n which are used in $S(n)$. The units are remained in a container which are filled partially, whose KANBAN has been detached, at the end of period 0 ; $W_0^{n,s}$ $\{0, 1, \cdots \sim \alpha^n-1\}(n=1, \cdots, N)$ i.e. initial inventory

- $W_0^{n,s} < \alpha^n - 1$
- $\sum_s W_0^{n,s} < N\{S(n)\}(\alpha^n-1)$

X_t^1 : production requirement, in terms of the number of full containers of item 1, at stage 1 in period t.

Q^n : the number of full container production quotas whole planning horizon is defined as $Q^n = \max\{0, \Sigma[(e^{n,s} \cdot \alpha^s/\alpha^n) \cdot Q^s - (W_0^{n,s}/\alpha^n)] - V_0^n\}$

Variables

X_t^n : the number of detached KANBAN of item n which respectively trigger the production of a full container in P^n in period $t(n=1, \cdots, N;\ t=1, \cdots, T)$.

Y_t^n : the number of KANBAN of item n which are detached from their associated containers in I^n in period t(n=1,⋯, N ; t=1,⋯, T).

U_t^n : the number of detached KANBAN of item n which are available at the end of period t and have not triggered any production yet(n=1,⋯, N ; t=1,⋯, T).

V_t^n : the number of full containers of item n which are available in I^n at the end of period t. (n=1,⋯N ; t=1,⋯, T).

$W_t^{n,s}$: the number of units of item n which are remained in a partially filled container, whose KANBAN has been detached, in I^n at the end of period t. (n=1,⋯, N ; t=1,⋯, T).

U_0^n : the number of detached KANBAN of item n which are injected into P^n by management at the beginning of the planning horizon(n=1,⋯N N). (*decision variable)

We will use 「Z」 to denote the smallest integer which is larger than or equal to Z and 「Z」 to denote the largest integer which is smaller than or equal to Z. ex) 「1.2」=2 「1.2」=1

The general-type structure KANBAN system model has several characters.

Initial inventory level of stage n is $V_t^0 \cdot \alpha^n+\sum_s^S W_0^{n,s}$

X_t^0, β_t^n are variables in each period, at set-up time let $\beta_t^n=0$ i.e. in that period production doesn't occur.

$Q^n=\sum^T x_t^n$ can be calculated retroactively by Q^1. If initially inventories are sufficient, then $Q^n=0$.

Total number of KANBAN which are used in circulating P^n and I^n is fixed.

$$U_{t-1}^n+Y_{t-1}^n-X_t^n-U_t^n=0,$$
$$n=1,\cdots, ; P\ t=1,\cdots,T \quad \cdots\cdots (1)$$

$$V_{t-1}^n+X_t^n-Y_t^n-V_t^n=0,$$
$$n=1,\cdots, N ;\ t=1,\cdots, T \quad \cdots\cdots (2)$$

At the state n, production amounts are restricted to available number of KANBAN, capacities and avariable amounts of items of immediate predecessors of stage n(=p(N)).

$$X_t^n \leq U_{t-1}^n+Y_{t-1}^n$$
$$\leq b_t^n$$
$$\leq Q^n-\sum_{t=1}^{t-1} X_t^n$$
$$\leq \left(\frac{\alpha^n \cdot V_{t-1}^k+W_{t-1}^{k,n}+\alpha^k \cdot X_t^k}{e^{k,n} \cdot \alpha^n}\right) \text{ any } k\in P(n),$$
$$n=1,\cdots, N ;\ t=1,\cdots, T \quad \cdots\cdots (3)$$

At complete stage(stage 1), production schedule must be accomplished.

$$\alpha^n \cdot V_{t-1}^k+W_{t-1}^{k,0}+\alpha^s \cdot X_t^s \geq e^{s,0} \cdot \alpha^0 \cdot X_t^0$$
$$\text{any } k\in P(0),\ t=1,\cdots, T \quad \cdots\cdots (4)$$

The number of detached KANBAN during period t equals the number of used KANBAN which are immediate successors of stage n(=S(n)).

$$Y_0^n=0\ n=1,\cdots, N ;\ t=1,\cdots, T \quad \cdots\cdots (5)$$
$$Y_t^n=\left[\sum_s^{S(n)}(e^{n,s} \cdot \alpha^s \cdot X_t^s-W_{t-1}^{n,s})/\alpha^n\right]$$

The inventory level of I^n is calculated at each period.

$$\sum_s^{S(n)}[W_{t-1}^{n,s}-W_t^{n,s}-e^{n,s} \cdot \alpha^s \cdot X_t^s]+\alpha^n \cdot Y_t^n=0 \quad \cdots (6)$$

U_0^n : nonnegative integer n=1,⋯, N ⋯ (7)

Theorem 1. If $\{U_t^n, V_t^n, W_t^{n,s}, Y_t^n\}$ satisfies above restrictive formula (1)−(7), then the following is satisfied.

a) U_t^n, V_t^n, $W_t^{n,s}$, X_t^n, Y_t^n are nonnegative integers.

b) $Y_t^n \leq X_t^n+Y_{t-1}^n$, $\sum_s^{S(n)} W_t^{n,s}+(X_t^n+V_{t-1}^n) \cdot \alpha^n \geq \sum_s^{S(n)}(W_{t-1}^{n,s}+e^{n,s} \cdot \alpha^s \cdot X_n^t)$

c) $U_0^n+V_0^n=U_t^n+V_t^n+Y_t^n$

d) $\sum_{s}^{S(n)} W_t^{n,s} < \alpha^n$

Theorem 1 has the following properties : a) all variables are integers, b) no shortage occurs at any stace c) the number of KANBAN always equals to initial number without withdrawal or input by outsider, d) at any period, the sum of partial inventory is less than capacity one container in that stage.

Objective function is defined as

$$\text{Min} \sum_{n=1}^{N} C^n[U_0^n + V_0^n + D^n(1 - 1/\alpha^n)] \quad \cdots\cdots\cdots\cdots (8)$$

C^n : the accumulated value of one full container of item n.

It includes labors, materials, production and inventory cost etc. $D^n = N(S(n))$

Above formula is upper bound of the total value in the system at any time t, and it means that inventory level is maximum. Since $\langle U_0^n \rangle$ is a unique variable among of the objective function variables, we can change $\sum^{N} C^n[U_0^n + V_0^n + D^n(1 - 1/\alpha^n)]$ to $\sum^{N} C^n U_0^n$. If the number of KANBAN is small, the KANBAN circulate more faster and the efficiency will increase. It is consist of the basic objective of KANBAN. We define Model M.

$$\text{Model M Min} \sum_{n=1}^{N} C^n U_0^n$$
$$\text{s.t. } (1) - (7)$$

Above problem is a complex integer programming. We try to make more simple integer programming than above problem. If initial number of KANBAN $\langle U_0^n \rangle$ is determinated, then KANBAN system operates automatically. KANBAN system satisfies the following properties. If $\langle$U V W X Y$\rangle$ and $\langle \bar{U}\ \bar{V}\ \bar{W}\ \bar{X}\ \bar{Y} \rangle$ are the feasible solution of Model M and $\langle U_0 \rangle = \langle \bar{U}_0 \rangle$, then $\langle$U V W X Y$\rangle = \langle \bar{U}\ \bar{V}\ \bar{W}\ \bar{X}\ \bar{Y} \rangle$. If $\langle U_0^n \rangle$ is determined as particular value, other variables are automatically determined uniquely. $\langle U_0 \rangle$ is a decision variable. Same as above, $\langle U_0 \rangle$ is the partial feasible solution of feasible solution $\langle$U V W X Y$\rangle$. When Model M is changed to model M1, the following properties are satisfied. Model M has a feasible solution ⟷ Model M1 has a feasible solution. If Model M1's partial feasible solution $\langle U_0^n \rangle$ is a partial feasible solution of Model M, then two model have the same optimal value $\langle U_0 \rangle$. Model M1 can be rearranged as

$$\text{Model M1 Min} \sum_{n=1}^{N} C^n U_0^n$$

$$\text{s.t. } \sum_{s}^{S(n)} W_0^{n,s} + \alpha^n \cdot V_0^n + \alpha^n \sum_{i=1}^{t} X_i^n$$
$$- \sum_{s}^{S(n)} \sum_{i=1}^{t} e^{n,s} \cdot \alpha^s \cdot X_i^s \geq 0 \quad \cdots\cdots\cdots\cdots (M1-1)$$

$$U_0^n - \sum_{i=1}^{t} X_i^n + \sum_{s}^{S(n)} \left(\sum_{i=1}^{t} e^{n,s} \cdot \alpha^s \cdot X_i^s - W_0^{n,s} \right) \left(\frac{1}{\alpha^n} \right)$$
$$+ 1 - \varepsilon \geq 0 \quad \cdots\cdots\cdots\cdots (M1-2)$$

$$\left(\frac{\alpha^k}{\alpha^n} \right) (V_0^k + \sum_{i=1}^{t} X_i^k) - \left(\frac{\alpha^k}{\alpha^n} \right) \sum_{s}^{S(n)} \sum_{i=1}^{t} (e^{n,s} \cdot \alpha^s \cdot X_i^s - W_0^{n,s})$$
$$- e^{k,n} \cdot X_i^n + e^{k,n} \cdot \alpha^n - \alpha^n + \left(\frac{1}{\alpha^n} \right)$$
$$+ \alpha^k - 1 \geq 0 \quad \text{any } k \in P(n) \quad \cdots\cdots\cdots\cdots (M1-3)$$

$$X_i^n = \{0, 1, \cdots, \beta_i^n\} \quad \cdots\cdots\cdots\cdots (M1-4)$$

$$U_0^n = \{0, 1, 2, \cdots, \} \quad \cdots\cdots\cdots\cdots (M1-5)$$

$$\varepsilon = \min\left(\frac{1}{\alpha^n} \right), \ t = 1, \cdots, T; \ n = 1, \cdots, N$$

Theorem 2. 1. Model M has a feasible solution ⟨-⟩ Model M1 has a feasible solution. 2. Two Models have the same optimal value.

3. Solution Method

In practical KANBAN system, the system is provided with enough KANBAN to run the system at starting time. When the system goes stable, the operator cut down the number of KANBAN until optimal number is reached. Model M1 is a complex integer programming and it is hard to find its optimal solution. So we search a heuristic method. Model M1 is an equation to find the initial value of KANBAN system. The following method concentrates on finding a feasible solution. Model M1 is divided by thress steps.

STEP 1. find a feasible solution $\langle X_t^n \rangle$ in equation $\langle M1-1 \rangle$ of Model M1.
Supplementary Solution Method 1 (SSM1)

STEP 2. check if $\langle X_t^n \rangle$ satisfied equation $\langle M1-3 \rangle$ of Model M1.
Supplementary Solution Method 2 (SSM2)

STEP 3. find the minimum of $\langle U_0^n \rangle$ in equation $\langle M1-2 \rangle$ of Model M1.
Supplementary Solution Method 3 (SSM3)

SSM1 (searching a fasible solution)

STEP 0. $X_t^0 \rightarrow X_t^0$ for $t=1, \cdots, T$ $n \leftarrow 1$ $m \leftarrow n$
$\langle X_t^t \rangle$ is given as a production plan.

STEP 1. $Q_1^n \leftarrow MAX\ \{0, \frac{1}{\alpha^n}[\sum_{s}^{S(n)} e^{n,s} \cdot \alpha^s \cdot X_1^s - \alpha^n \cdot V_0^n - \sum_{s}^{S(n)} W_0^{n,s}]\}$

$Q_t^n \leftarrow MAX\{0, (\frac{1}{\alpha^n})[\sum_{s}^{S(n)} \sum_{i=1}^{T} e^{n,s} \cdot \alpha^s \cdot X_i^s - \alpha^n \cdot V_0^n - \sum_{s}^{S(n)} W_0^{n,s}] - \sum_{i=1}^{t-1} Q_i^n)\}$

$t=2, \cdots, T$

$t \leftarrow T$

$\langle Q_t^n \rangle$ can be found by recursive calculation if $\langle X_t^t \rangle$ is given.

STEP 2. $X_t^n \leftarrow MIN\{Q_t^n, \beta_t^n\}$ if $t=1$, go to STEP 3, else $t \leftarrow t-1$, go to STEP 2.

STEP 3. if $\beta_t^n < Q_t^n$, then infeasible. If $\beta_t^n > Q_t^n$ and $n=N$, check if all solutions of P(n) is made. If all of them are found, then apply Supplementary Solution Method 2, and set $m \leftarrow m+1$ and go to STEP 1. If all the solution of P(n) are not found, then set $n \leftarrow n+1$, go to STEP 1.

SSM2. (check if there are shortages and supplement)

STEP 0. get $\langle X_t^n \rangle$ found in SSM 1

STEP 1. select a k in P(n) P(n) ← P(n) − k

STEP 2. check if the following equation is satisfied

$$\{\frac{\alpha^k}{\alpha^n}(V_0^k + \sum_{i=1}^{t} \overline{X}_i^k) - \frac{\alpha^k}{\alpha^n} \sum_{s}^{S(n)} \sum_{i=1}^{t} (e^{n,s} \cdot \alpha^s \cdot X_i^s - W_0^{n,s}) - e^{k,n} \cdot \overline{X}_t^n + e^{k,n} \cdot \alpha^n - \alpha^n \geq 0\}$$

ignore $\frac{1}{\alpha^n} - \alpha^k - 1$, there is no trouble.

If satisfied, go to STEP 3,

else $X_n^t = X_n^t - \{\alpha^k(V_0^k + \sum_{i=1}^{t} \overline{X}_i^k) - (\frac{\alpha^k}{\alpha^n}) \sum_{s}^{S(n)} \sum_{i=1}^{t} (e^{n,s} \cdot \alpha^s \cdot X_i^s - W_0^{n,s}) - e^{k,n} \cdot \overline{X}_t^n + e^{k,n} \cdot \alpha^n - \alpha^n\}$

go to STEP 3.

STEP 3. if P(n)=0, go to SSM1, else go to STEP 1.

SSM 3. (find the minimum of $\langle U_0^n \rangle$)

STEP 0. get $\langle X_t^n \rangle$ found in SSM 1 $n \leftarrow 1$

STEP 1. $t \leftarrow 1$ $U_0^n \leftarrow 1$

STEP 2. if $U_0^n < 1$, then $U_0^n \leftarrow 1$

if $U_0^n > 1$, then $U_0^n = \text{MAX}\{U_0^n, U_0^n\}$

$t \leftarrow t+1$,

$U_0^n = \sum_{i=1}^{t} X_i^n - \sum_{s}^{S(n)} (\sum_{i=1}^{t} e^{n,s} \cdot \alpha^s \cdot X_i^s - W_0^{n,s})$

$(\frac{1}{\alpha^n}) - 1 +$ if $t < T$, go to STEP 2.

if $t > T$, $n \leftarrow n+1$ if $n < N$, go to STEP 1, else terminate.

The above method finds a feasible solution but not optimal. Since there is no method to find a feasible solution in general-type structure KANBAN system. This work is practical in the real field.

Example

We apply this method to a general-type-structure production process which is shown at 〈Fig. 2〉. The basic data is shown at 〈Table 1〉. The basic assumptions are same to those of Model M. By SSM1 and SSM2, the feasible solution is obtained (〈Table 2〉).

4. Conclusion

The operation of general-type KANBAN system is studied in this paper. The general-type KANBAN system is modeled in integer programming whose optimal solution is very hard to find. A heuristic solution method for general-type KANBAN system that can be used in the real fields is proposed. But this model ignores the processing time and the fixed period time which must be considered in the real fields. Considering those factors in modeling and developing a method for an optimal solution remains as further study areas. Also we expect this model to be improved by using Network modeling method.

Table 1. Example Data

stage	1	2	3	4	5	6	7	8
α^n	10	15	10	20	10	10	15	20
$E^{n,s(n)}$		$e^{2,1}=1$	$e^{3,1}=2$	$e^{4,1}=1$	$e^{5,2}=2$	$e^{6,5}=3$	$e^{7,5}=2$	$e^{8,4}=2$
					$e^{5,3}=1$		$e^{7,4}=1$	
$W^{n,s(n)}$	$w^{2,1}=5$		$w^{3,1}=0$	$w^{4,1}=5$	$w^{5,2}=0$	$w^{6,5}=5$	$w^{7,5}=5$	$w^{8,4}=5$
					$w^{5,3}=5$		$w^{7,4}=0$	
S(n)		1	1	1	2,3	5	4,5	4
N(S(n))		1	1	1	2	1	2	1
P(n)	2,3,4	5	5	7,8	6,7			
N(P(n))	3	1	1	2	2			
V^n	2	3	5	3	6	2	8	5
Q^n	240	240	480	240	960	2880	2160	480
$\lceil Q^n / \alpha^n \rfloor$	24	16	48	12	96	228	144	24

Table 2. Feasible Solution

stage	no	1	2	3	4	5	6	7	8
period	1	10	10	15	10	30	18	50	20
	2	10	10	15	10	30	18	50	20
β_i^n	3	10	10	15	10	30	18	50	20
	4	10	10	15	10	30	18	50	20
	5	10	10	15	10	30	18	50	20
period	1	3	0	1	0	0	0	0	0
	2	4	2	8	1	9	24	5	0
Q_i^n	3	5	3	10	2	20	60	30	1
	4	6	4	13	3	24	72	36	6
	5	6	4	12	4	24	72	36	6
period	1	3	0	1	0	1	0	0	0
	2	4	2	8	1	16	24	5	0
X_i^n	3	5	3	11	2	20	60	30	1
	4	6	4	13	3	24	72	36	6
	5	6	4	12	4	24	72	36	6
U_0^n			1	1	1	3	1	1	1

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