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A Multi-Criteria Decision Making Method Based on Fuzzy Outranking Relation

Kwun-Ik Cho* and Soung-Hie Kim*

부정확한 대안평가하에서 모호선호관계를 이용한 다기준 의사결정 기법

조권익* · 김성희*

Abstract

In selecting the best project in multi-criteria decision making problems, the imperfect information of consequence and the vague preference of the deicision maker(DM) would make the decision analysis more complex together with the conflict of several criteria. In this context, a method is proposed to deal the fuzzy information mentioned before instead of reducing it to a single representative value. And, based on the given imprecise information, projects are ranked completely or partially according to DM's vague preference.

The procedure consists, for each pair of projects, of calculating the degree of supporting over all criteria and the degree of opposing with respect to each criterion for their outranking relation. Together with weights for each criterion, these indices produce fuzzy outranking relations for each pair of projects. And a complete or partial ranking of projects is obtained according to outranking degrees considering the interdependence among projects.

1. Introduction

In multi-criteria decision problems, the existence of several criteria makes the search for the best decision considerably difficult because of their conflicts. Furthermore, the analysis is getting more difficult in the case of known only imprecise information ; the imprecise evaluations of projects for each criterion. It should be noted that the decision maker(DM) may think two distinct values in-

* Department of Industrial Engineering, Korea Advanced Institute of Science and Technology

different. This means that certain difference between two evaluations can be treated insignificant. And difference between two evaluations on one criterion can result his preference ignorance between two projects, although one project has higher evaluations on the others criteria. These situations should be reflected on analysis and determination of the best project.

Many multi-criteria methods have been proposed to analyze the problems where the evaluations of actions for each criterion are imprecise. There are many methods which adopt the fuzzy approach on the contrary to the scarcity of the probabilistic approach. In d'Avignon and Vincke[1], the imprecision of data is treated as the probabilistic. Other methods can be found, which deal the imprecise information using fuzzy membership function [2, 5, 6, 7, 11, 16]. These did not reflect on the eventual incomparability between projects which is real phenomenon. That is, the vague nature of DM's preference could not have no regard. As another side, approaches such as in [4, 11, 14] take into account of the fuzzy nature of preference by using the fuzzy outranking concept. But these are lacking in full reflection of fuzzy evaluation : rather using the representative of fuzzy evaluation.

A method extended in this paper is presented which has the following features : first, the imprecise data, i.e. evaluations of projects for each criterion, are handled in the form of their membership functions in order to use as much as possible information. Second, the DM's preference with respect to each criterion will be expressed by thresholds which allow the difference of evaluations to be significant or insignificant. Finally the interdependence among projects is considered in ordering the projects. This inter-dependence should be considered as for the reason that the outranking degree for each project is calculated based on the outranking relation between the considered projects.

2. Description of the Concerned Problem

Suppose that the DM has a multi-criteria decision problem as follows : DM has to decide the best project in a finite set of m projects, $A=\{A_1, A_2, \cdots, A_m\}$ considering n criteria, $C=\{C_1, C_2, \cdots, C_n\}$. For each criterion C_k, every project is assessed over a scale E_k. The interdependence among criteria is assumed to be negligible.

In situations where only the imprecise information is known, the assessment of every project with respect to each criterion results in fuzzy evaluation. Lex X_{ik} be the fuzzy number which represent the evaluation of project A_i according to criterion C_k. In this work, the membership function of X_{ik} is assumed to have the form of $(\alpha/\beta, \gamma/\delta)$ where $\alpha \leq \beta \leq \gamma \leq \delta$, as shown in Fig. 1. Representing X_{ik} in this way can facilitate expressing and dealing

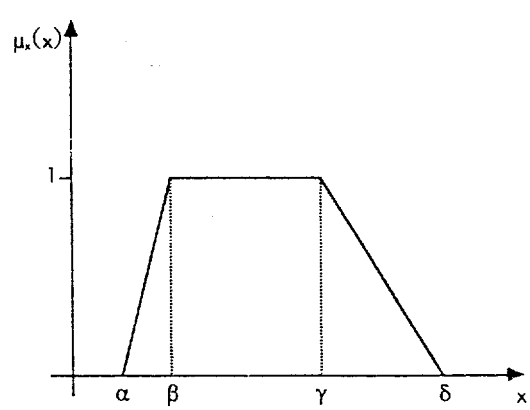

Fig. 1. The Membership Function of Evaluations.

the imprecise information ; "approximately equal to 7" may be represented as (6/7, 7/8) and "nearly between 63 and 68" as (62/64, 67/70).

3. The Fuzzy Outranking Relation

As outranking preference structure is useful to represent the real preference structure, it will be used to express DM's preference for values and deal the imprecise information[13]. In Roy[14], the fuzzy outranking was used to reflect the vagueness of DM's preference, which is characterized by a degree of outranking associating to a pair of projects in closed interval [0, 1].

The criterion to appraise the outranking of A_j by A_i is a degree of credibility which is computed from two conflicting indices : the supporting index and the opposing index. The supporting and opposing index translate respectively the same concepts as do the 'concordance' and 'discordance' index of Roy[14]. The difference is the fact that while his indices deal with punctual evaluations, this work is concerned with the fuzzy evaluations.

3-1. The Supporting Index

Supporting index measures the belief toward the proposition that one project outranks the other project for a pair of projects. In order to do this, firstly, for each criterion, a partial supporting index for this proposition should be derived based on the fuzzy evaluations for these projects. By the way, DM may think it insignificant or significant for a certain difference of evaluations with respect to a criterion. The maximum allowable difference which is thought insignificant is called an indifference threshold, and the minimum one which is considered significant is a preference threshold[15]. Since these thresholds can be determined due to DM's subjectivity concerning relevance for preference of evaluations, these may have the fuzzy nature. In this work, these thresholds are assumed to be the fuzzy number of which membership functions are the form $(\alpha/\beta, \gamma/\delta)$ as described before, and not depend on values[15].

For the propositioin "A_i outranks A_j" with respect to a criterion C_k, when the possibility of the difference between evaluations for A_i and A_j according to C_k is greater than the preference threshold for criterion C_k, s_k, one could assign a value of 1 to this partial supporting index. On the other hand, while this difference is smaller than the indifference threshold for criterion C_k, q_k, the partial supporting index should have a value of 0. Therefore the partial supporting index can be defined, for criterion C_k, as follows :

$$\mu s_k(A_i, A_j) = \{P(X_{ik} - X_{jk} \geq s_k) + P(X_{ik} - X_{jk} \leq q_k)\}/2 \quad \cdots\cdots\cdots (1)$$

where "P" denotes the possibility measure[9]. As the fuzziness of X_{jk} increases, the degree of partial supporing for the proposition "A_i outranks A_j" would be non-decreasing to reflect the imperfection of information about X_{jk}.

The normalized weight for each criterion, which stresses the relative importance of the corresponding criterion can play the role of contribution of each criterion to the belief for the given proposition. These weights can be obtained either directly from the DM, or indirectly with the existing procedures.

Combining all these partial supporting indices with these weights, a supporting index for "A_i outranks A_j", denoted by $\mu_S(A_i, A_j)$, can be defined as follows :

$$\mu_S(A_i, A_j) = \Sigma_{k=1}^{m} \mu_{S_k}(A_i, A_j) \cdot w_k \quad \cdots\cdots\cdots\cdots (2)$$

The basic notion in (2) represents that the more weight sum of the criteria in proportion to the degree of partial supporting is, the greater the belief of outranking of A_j by A_i is.

3-2. The Opposing Index

The distrust still may exist toward "A_i outranks A_j" even though A_i has a relatively higher degree of supporting than that of A_j. When the difference between evaluations for these projects is too unfavorable at least one criterion, these two projects may be regarded as incomparable even though the other criteria confirm the superiority of A_i over A_j. Therefore the difference between evaluations which is too significant must be incorporated into deriving the dergree of outranking between projects together with the supporting index in this study. The significance of difference is taken into account through the notion of veto threshold, $\hat{V}_k$, suggested by Roy[13]. And since this threshold also has the fuzzy nature, it is assumed to be fuzzy number of the form$(\alpha/\beta, \gamma/\delta)$.

If, for one or more criteria, opposing index toward the proposition "A_i outranks A_j" is high enough, the belief towards this proposition is reduced. The opposing index which may cause to reduce the belief is expressed as follows :

$$\mu_{D_k}(A_i, A_j) = \begin{cases} 0 & \text{if } \hat{X}_{jk} - \hat{X}_{ik} \leq \hat{s}_k \\ \{(\hat{X}_{jk} - \hat{X}_{ik} - \hat{v}_k)^2 + (\hat{v}_k - \hat{s}_k)^2\} / (\hat{v}_k - \hat{s}_k)^2 & \text{if } s_k \leq \hat{X}_{jk} - \hat{X}_{ik} \leq \hat{v}_k \\ 1 & \text{if } \hat{X}_{jk} - \hat{X}_{ik} \geq \hat{v}_k \end{cases} \quad \cdots\cdots\cdots (3)$$

where "∧" denotes the representative for a fuzzy number[8], which is obtained as :

$$\hat{X} = \{\int x\mu_X(x)dx\} / \{\int \mu_X(x)dx\}$$

By combining the supporting and opposing index, we can obtain the degree of credibility which reveals the degree of outranking between the pair of each projects according to considered criteria. We porpose the following formula for the degree of credibility which is same concept as in ELECTRE III method[14].

$$\mu_C(A_i, A_j) = \begin{cases} 0 & \text{if } \lambda = 1 \\ (1-\lambda)\mu_S(A_i, A_j) / \{1 - \mu_S(A_i, A_j)\} & \text{if } \lambda > \mu_S(A_i, A_j) \\ \mu_S(A_i, A_j) & \text{if } \lambda \leq \mu_S(A_i, A_j) \end{cases} \quad \cdots\cdots\cdots (4)$$

where $\lambda = \max_k \mu_{D_k}(A_i, A_j)$. This results that when the degree of opposing is negligible comparing with that of supporting, the degree of credibility to the given proposition is not reduced. And for the high degree of opposing, the degree of belief is reduced in proportion to the degree of opposing. Furthermore, when the degree of opposing with respect to a certain criterion is the value of 1, this criterion plays the roles of a perfect opponent to the given proposition.

4. Ordering the Projects

In the case of $\mu_C(A_i, A_j) < \mu_C(A_j, A_i)$, A_j can be considered to be at least as good as A_i to the degree of $\mu_C(A_j, A_i) - \mu_C(A_i, A_j)$ and incomparable to A_i with the degree of $\mu_C(A_i, A_j)$. Thus the strength of A_i in outranking terms against all the other projects, i.e. the degree of outranking, is described as :

$$\mu_S(A_i)=1-\max_{A_j\neq A_i}\{0,\ \mu_C(A_j,\ A_i)-\mu_C(A_i,\ A_j)\} \quad \cdots\cdots (5)$$

By the way, we should take into account the fact that slight difference of the degree of outranking has not to be treated significant, since some inaccuracy may be incorporated into analyzing the fuzzy information. Therefore, a classifiaction threshold is incorpoarated into concluding to outranking.

Given a classification threshold λ, A_i outranks A_j only when $\mu_S(A_i)-\mu_S(A_j)>\lambda$; they are treated as incomparable when $|\mu_S(A_i)-\mu_S(A_j)|\leq\lambda$. Let R_k be the k^{th} ranked set which contains one or more incomparable projects defined as follows :

$$R_k=\{A_i \mid \mu_S^*(\bar{R}^k)-\mu_S(A_i)<\lambda,\ \text{for all}\ A_i\in\bar{R}^k\} \quad \cdots\cdots (6)$$

where $\mu_S^*(\bar{R}^k)=\max_{A_j\in\bar{R}^k}\mu_S(A_j)$, and $R^k=\cup_{\ell=1}^{k-1}R_\ell$, and $\bar{R}^k$ is complement of R^k.

We should notice the strong interdependence among projects while ranking projects as shown in Eq. (5). This is due to the fact that the degree of outranking for each project is computed based on the degree of credibility comparing with the others. Therefore the interdependence between the one project and the others which are already ranked must be ignored. Consider the following : we are about to determine the k^{th} ranked projects, where

$$\mu_S(A_j)=\max_{A_\ell\in\bar{R}^k}\{1-\max_{A_j\neq A_i}\{0,\ \mu_C(A_k,\ A_\ell)-\mu_C(A_\ell,\ A_k)\}\},$$

$$\mu_S(A_i)=\max_{A_\ell\in\bar{R}^k}\{1-\max_{A_k\neq A_\ell,\ A_k\in\bar{R}^k}\{0,\ \mu_C(A_k,\ A_\ell)-\mu_C(A_\ell,\ A_k)\}\}.$$

A_j has the higher degree of outranking than that of A_i in the case of considering the interdependence among the already ranked projects, but A_i does when considering only interdependence among the not-yet ranked projects. Suppose that at the begining we pay no regard the projects which already have been ranked, then it is obvious that A_i outranks A_j. Therefore, A_i must be ranked as the k^{th} project from viewpoint of interdependence. In the process of ranking projects, we will take account of the interdependence among not-yet ranked projects.

The iterative procedure of ranking is summarized as follows.

Step 0 Let $R_0=\phi$ and set $k=1$.

Step 1 Construct R^k such that $R^k=\cup_{\ell=0}^{k-1}R_\ell$

Step 2 If $\bar{R}^k=\phi$, then stop with a partial or complete ordered set of projects.

Step 3 Calculate the strength of each project in outranking terms against all projects in $\bar{R}^k$ using Eq. (5).

Step 4 Find the k^{th} ranked project(s), R_k, using Eq. (6). Set $k=k+1$ and go to step 1.

5. Illustrative Example

Consider the following multi-criteria project selection problem : DM wants to select the best one among six projects according to three criteria. We have the DM's preference information to criteria as shown in Table 1 ; indifference, preference, veto threshold, and weights for each criterion. He/She thinks that the more is the better for evaluation with respect to each criterion. Evaluation of each project according to criterion would be given by DM as shown in Table 2.

By applying the proposed method, degreee of credibility for each pair of projects is derived by synthesizing the supporting and opposing index as in Table 3. Let the classification threshold $\lambda=0.1$

Table 1. Information to Each Criterion

Criterion	C_1	C_2	C_3
weight	0.4	0.5	0.1
q_j	(4/5, 5/5)	(0/1, 1/1)	(0.4/0.5, 0.5/0.5)
s_j	(4/5, 5/5)	(5/6, 6.5/7)	(1/1, 1/2)
v_j	(25/30, 30/35)	(50/55, 55/60)	(4/5, 5/5)

Table 2. Evaluation of Each Project

	C_1	C_2	C_3
A_1	(75/80, 80/85)	(88/90, 90/95)	(4/5, 5/6)
A_2	(65/65, 65/70)	(55/58, 58/60)	(0/1, 1/2)
A_3	(80/83, 83/86)	(58/60, 60/60)	(6/7, 8/9)
A_4	(38/40, 40/40)	(75/78, 80/83)	(9/10, 10/10)
A_5	(50/52/ 54/58)	(70/72, 72/75)	(7/8, 8/9)
A_6	(90/94, 94/100)	(92/95, 95/100)	(5/6, 6/6)

Table 3. The Degree of Crdibility for Each Alternative

	A_1	A_2	A_3	A_4	A_5	A_6
$\mu_C(A_1, \cdot)$	-	1.0000	0.9458	0.0000	0.9167	0.3381
$\mu_C(A_2, \cdot)$	0.0000	-	0.0000	0.0000	0.0000	0.0000
$\mu_C(A_3, \cdot)$	0.2494	1.0000	-	0.3576	0.5000	0.0298
$\mu_C(A_4, \cdot)$	0.0000	0.0404	0.0000	-	0.6000	0.0000
$\mu_C(A_5, \cdot)$	0.0019	0.6000	0.0001	0.7375	-	0.0000
$\mu_C(A_6, \cdot)$	1.0000	1.0000	0.9750	0.4900	0.9250	-

which should be predetermined in ranking process. In order to determine which project(s) is(are) ranked first, the strength of each alternative in outranking is calculated based on the degree of credibility against all the others :

$\mu_S(A_1)=0.3381$,
$\mu_S(A_2)=0$,
$\mu_S(A_3)=0.0548$,
$\mu_S(A_4)=0.5100$,
$\mu_S(A_5)=0.0750$,
$\mu_S(A_6)=1$.

Therefore the first ranked project is only A_6 since the degree of outranking for A_6 is the value of 1 and the others are less than 0.9 $(=\mu_S(A_6)-\lambda)$. Next, it should be excluded the interdependence between A_6 and the others. Then, the degree of outranking for each project, i.e. $\bar{R}^2=\{A_1, A_2, A_3, A_4, A_5\}$, is newly calculated from the degreen of credibility :

$\mu_S(A_1)=1$,

$\mu_S(A_2)=0$,

$\mu_S(A_3)=0.3035$,

$\mu_S(A_4)=0.6424$,

$\mu_S(A_5)=0.0852$,

It is worth noting that the degree of outranking for A_1 is the greatest one excluding the interdependence between the already ordered project, i.e. A_6, while considering this interrelation A_4 is. This is due to the fact that A_1 is highly dependent on A_6 than A_4. Since A_6 is already ranked project, the dependence against A_6 should be ignored for determining the next ranked project. Therefore, the second ranked project is A_1. After repeating, this procedure can result the projects ordering as follows :

$A_6 \gtrsim A_1 \gtrsim A_3 \gtrsim A_5 \gtrsim \{A_2, A_4\}$,

where '$\gtrsim$' denotes the outranking relation.

6. Conclusion

Inaccuracy in anticipated consequence of each project would be caused by the weakness of information acquisition. And since the DM's appreciations for the difference between evaluations are somewhat vague, we should incorporate this fact into analyzing process. Therefore, under multiple criteria and fuzzy information, a desirable method must not make a fetish of rigorous precision and mathematical formalism, but rather allow for imprecision and partial truth.

In this study, multi-criteria analysis is carried out with imprecise data and DM's vague preference. The fuzzy set theory makes it possible to handle the fuzzy distributive evaluations as a whole, instead of some representative for the distribution ; in which case a loss of information would occur. Moreover, since the numerical appreciation is somewhat imprecise, the use of thresholds seems appropriate to avoid concluding too hastily that an outranking relation exists. Ordering the projects the outranking interdependence among projects can be considered successfully.

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