<事例硏究>

An Aggregate Production Planning for An Optical Instrument Industry

Hark-Hwang*, Myung-Joo Oh** and Seong-Beak Lee*

Abstract

This paper presents a linear programming formulation of the aggregate production planning problem for an optical instrument manufacturing company. Taking into account of the various requirements of the company, an L.P model is developed whose objective is to minimize the total cost of production during the planning horizon. The application results and its implications are well understood by the management and expected to be used as a guide line for future production planning.

1. Introduction

The production planning problem is concerned with specifying the optimal production quantity, inventory level, the use of outside capacity and work force to meet the known demand schedule of a prescribed planning horizon. Many models have been proposed with Saad[8] and Silver[10] giving good reviews of these models. Among those, linear cost formulations and quadratic cost formulations are well known for aggregate planning models.

For linear cost formulation, the transportation method was proposed by Bow-

* Department of Industiral Engineering, Korea Advanced Institute of Science & Technology
** Executive Managing Director, Samsung Aerospace Industries, LTD.

man[2] and the linear programming formulation by Hansman and Jess[5]. Basic extensions of these models are the goal programming formulation proposed by Goodman[4] where multiple objective function is used. The linear programming model was applied by Eisseman and Young [3] in a textile mill, by Wai-Hing Chin[7] in a packing industry and by Tang et al.[9] in a heavy manufacturing industry.

For quadratic cost formulations, Holt and Muth et al.[6] developed a model in which they minimize a quadratic cost function, and come up with a linear decision rule that solves for optimal production and inventory decisions. Many extensions of this model have been developed since then, e.g., Van de panne[12] has considered the case when some coefficient estimates are not available. Bergestrom and Smith[1] have extended the formulation to incorporate the case of multi-item production planning.

The objective of this paper is to formulate an aggregate production planning model in the context of the optical division of Samsung Aerospace Industries Ltd. The model is based on linear programming and considers various requirements of the company. The division manufactures a variety of high-tech optical items including laser pickups, VTR camera lenses, binoculars, lenses and cameras. Cameras, in particular, have been quite successful on the domestic market and recently the sales volume has increased dramatically in the overseas market.

Due to the following production and sales related environments, the manager of the optical division faces the problem of establishing plan in a more systematic way:

i) A wide range of camera models are produced on three different assembly lines; high quality, medium quality and subcompact camera lines. New models are going to be introduced and production of several models will be discontinued according to the current management plan.

ii) Monthly demand forecasts for the upcoming year show drastic seasonal variations which make the production scheduling problem a more difficult one.

iii) Due to the improvements of standard of living, employees especially in major industries are reluctant to work overtime.

iv) Many subcompact models are being assembled on contract basis by a number of small sized subcontractors. Regardless of the company's work load, a certain amount of job orders has to be given to the subcontractors to maintain stable relations with them. It is expected that the proportion of the products assembled outside will increase substantially in the near future.

v) A number of camera parts are imported from a Japanese firm with whom the company has a technical tie-up. The purchase lead time is estimated to be three months.

vi) As its management policy, the company begins to adopt the life time employment. In case of decrease of demand, some workers would be transferred to other divisions. This practice eliminates the option of dismissing or laying off any workers to absorb demand fluctuations.

vii) Shortages are not allowed due to fierce competitions among camera manufacturers in the domestic market.

2. Production Costs

Since the objective of the aggregate production planning is to minimize the total production cost within a planning horizon, we shall have to investigate as to which costs effect the total cost of production. Fixed production costs such as the cost of raw materials are not relevant since they are independent of the production scheduling during the planning horizon. Thus the following variable costs are considered:

i) Direct payroll costs
ii) Overtime costs
iii) Hiring costs
iv) Inventory costs
v) Subcontract costs

Direct payroll costs are calculated by taking the average wage of a worker and multiplying it with the number of workers employed during the period. Salaried staff and management costs are excluded, since they are considered to be relatively fixed during the planning horizon. Overtime costs are calculated by multiplying the total man-months of overtime by the regular pay and the overtime payment factor.

Hiring costs include the cost of interview, medical examination, training, aptitude test and paper work. Inventory costs are the sum of handlin cost, insurance cost, interest on tied capital. Subcontract costs are calculated by multiplying the average hourly wage of the worker of subcontractors with the estimated working hours associated with the work load.

3. Model Development

A linear programming model to minimize the production costs over 12 months period is developed from which optimal solutions for work force, production and inventory level for each month are found. By consolidating similar products into product families and different labor skills to labor center, a wide range of camera models are aggregated into 12 groups. An this stage, the production capacities of each assemble line and subcontractors, the amount of safety stock required at the end of each month, and other constraints resulted from the existing managerial plan are integrated in the model development.

Notation

The following notation will be used:

W_t : workers employed at the begining of period t (number of workers)

$P_{t,g}$: production rate of group g during period t (unit)

$I_{t,g}$: the amount of inventory of group g at the end of period t (unit)

$X_{t,g}$: regular production hours for group g during period t (hour)

$Y_{t,g}$: overtime hours for group g during period t (hour)

$Z_{t,g}$: subcontractor production time for group g during period t (hour)

R_t : regular hours available per worker during period t (man-hour)

O_t : overtime hours available per worker during period t (hour/worker)

K_g : average man-hour required to assemble a unit of group g items (man-hour/unit)

$D_{t,g}$: forcasted sales of group g during period t (unit)

$S_{t,g}$: safety stock of group g required at the end of period t (unit)

$U_{t,g}$: upper limit of subcontractor production time for group g during period t (hour)

$a_{t,g}$: portion of group g products which is asembled by subcontractors during period t (%)

CREG: *monthly salary per worker*

COVER: *average hourly overtime cost*

CHIRE: *hiring cost per worker*

CINV: *monthly inventory cost in percent*

CFCg: *average unit factory price of group g items*

CSUB: *average hourly subcontract cost*

Among these, W_t, $P_{t,g}$, $I_{t,g}$, $X_{t,g}$, $Y_{t,g}$, and $Z_{t,g}$ are decision variables. Note that these variables must satisfy nonnegativity constraint and are not independent each other.

The problem can then be formulatred as:

Minimize

$$\sum_{t=1}^{12} CHIRE \times (W_t - W_{t-1}) \quad \text{hiring cost} \quad \cdots\cdots (1)$$

$$+ \sum_{t=1}^{12} CREG \times (W_t) \quad \text{payroll cost} \quad \cdots\cdots (2)$$

$$+ \sum_{t=1}^{12} \sum_{g=1}^{12} COVER \times Y_{t,g} \quad \text{overtime cost} \quad \cdots\cdots (3)$$

$$+ \sum_{t=1}^{12} \sum_{g=1}^{12} CINV \times CFC_g \times I_{t,g} \quad \text{inventory cost} \quad \cdots\cdots (4)$$

$$+ \sum_{t=1}^{12} \sum_{g=1}^{12} CSUB \times Z_{t,g} \quad \text{subcontract cost} \quad \cdots\cdots (5)$$

Subject to

i) relationship of monthly production rate, inventory and sales;

$$I_{t,g} = I_{t-1,g} + P_{t,g} - D_{t,g} \quad t=1,2,\cdots\cdots,12 \quad g=1,2,\cdots\cdots,12 \quad \cdots\cdots (6)$$

ii) safety stock requirements;

$$I_{t,g} \geqq S_{t,g} \quad \cdots\cdots (7)$$

iii) prohibition of laying off any workers;

$$W_t \geqq W_{t-1} \quad t=1,2,\cdots,12 \quad \cdots\cdots (8)$$

iv) upper limit of regular production time for each period;

$$\sum_{g=1}^{12} X_{t,g} \leqq R_t \times W_t \quad t=1,2,\cdots,12 \quad \cdots\cdots (9)$$

v) upper limit of overtime for each period;

$$\sum_{g=1}^{12} Y_{t,g} \leqq O_t \times W_t \quad t=1,2,\cdots,12 \quad \cdots\cdots (10)$$

vi) subcontractors' capacity;

$Z_{t,g} \leq U_{t,g}$ .... (11)

vii) constraints due to other critical resources;

$P_{t,2} \leq m_1$ .... (12)

$P_{t,11} + P_{t,12} \leq m_2$ m_1, m_2 constant ..... (13)

viii) relationship of regular time, overtime and subcontract time with production rate;

$m(X_{t,g} + Y_{t,g}) + n \times Z_{t,g} = K_g \times P_{t,g}$ .... (14)

$m \times n\left(\frac{X_{t,g} + Y_{t,g}}{100 - a_{t,g}} - \frac{Z_{t,g}}{a_{t,g}}\right) = 0$ .... (15)

Products groups can be classified into three categories depending on whether products of each group are assembled i) in the company's own line, ii) outside by subcontractors or iii) both by the company and subcontractors. For each group belonging to the last category, the ratio of the amount assembled outside to the production quantity of the group is estimated for each month and represented by $a_{t,g}$. Also, equation(15) is needed to keep balance in work loads between two production sites. Therefore, for equations(14) and(15), m=1 and n=0 are assigned for the groups of the first categories, m=0 and n=1 for the second categories, and m=1 and n=1 for the third categories.

Equation(12) and (13) imply that production rate of groups 2, 11 and 12 are limited by some manufacturing facilities whose machine hours become scarce resources.

Additionally, intial values of number of workers and beginning inventories should be provided for each group.

Table 1. The average production time and unit inventory cost per month

group number	average production time (minute)	unit inventory cost (won/month)
g=1	55.2	280.6
2	80.4	561.6
3	73.2	372.7
4	111.6	991.7
5	127.2	1078.2
6	79.2	393.8
7	85.2	633.0
8	204.0	572.2
9	152.0	792.8
10	249.6	1242.0
11	18.0	2592.0
12	55.2	3076.4

4. Application

The model developed was applied for a planning horizon of one year with 12 planning periods of one month each. All cost parameters and input variables are estimated by the production and accounting departments. Due to the requests of the company, we are allowed to release only part of the data(after some modification) and Appendix A shows the calculations

Table 2. Demand data(× × units)

period (month)	group number					
	1	2	3	4	5	6
1	3.2	6.5	-	1.0	3.2	-
2	5.3	11.0	-	2.0	3.2	-
3	5.4	11.2	-	2.0	3.7	-
4	8.3	16.8	-	3.0	4.2	-
5	9.3	17.5	10.0	3.0	4.7	-
6	8.6	14.4	27.6	3.0	2.0	9.0
7	5.7	13.7	36.9	2.5	3.0	32.0
8	4.3	10.1	45.0	2.5	2.0	38.0
9	9.9	12.5	43.5	2.5	-	38.0
20	6.2	8.9	36.5	1.0	0	37.0
11	4.7	7.3	28.0	0.5	-	36.0
12	4.1	6.4	22.5	-	-	20.0

period (month)	group number					
	1	2	3	4	5	6
1	22.8	-	-	-	1.5	3.0
2	23.8	-	-	-	2.0	3.5
3	24.0	-	-	-	2.0	4.0
4	29.3	-	-	-	2.5	4.2
5	29.3	-	-	11.2	2.5	4.5
6	29.3	6.3	5.0	32.3	2.0	3.5
7	22.8	16.6	23.0	34.0	1.5	3.7
8	17.5	27.0	23.0	3.40	1.0	3.3
9	7.3	37.2	30.0	29.2	-	3.8
10	8.4	32.8	25.0	33.1	-	3.8
11	3.3	27.6	13.0	21.1	-	4.4
12	3.2	22.5	7.7	13.1	-	4.3

involved for the estimation of cost parameters.

The average production time and unit holding cost per month are listed in Table 1 Table 2 gives the projected demand quantities for the planning horizon. The model had 732 variables and 712 constraints and was solved using APEX IV installed on IBM.

The aggregate production plan obtained from the model is shown in Figure 1. Figures 2 and 3 show production rates, inventory levels and forecasted demands for group 2 and 12, respectively.

From the results of the application, the following observations are made:

i) In the optimal production plan, worker is neither fired in accordance with the policy of the management nor hired due to the high man-power cost.

ii) the optimal production follows the demand for high quality models such as

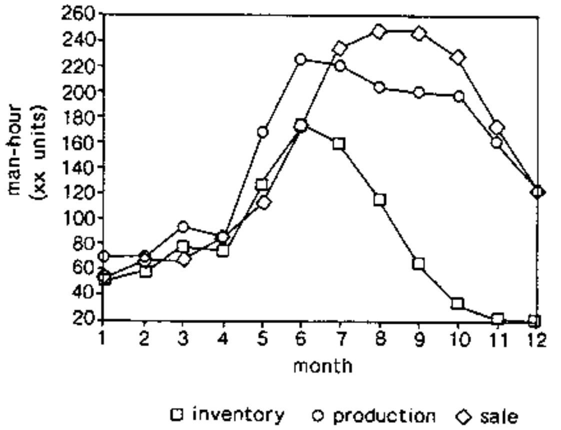

Fig. 1. Aggregate production level, demand and inventories.

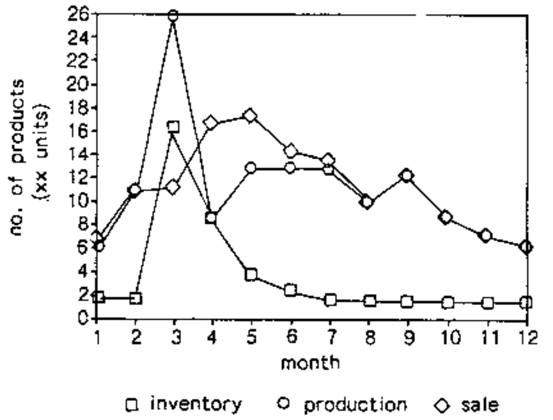

Fig. 2. Production plan for group 2.

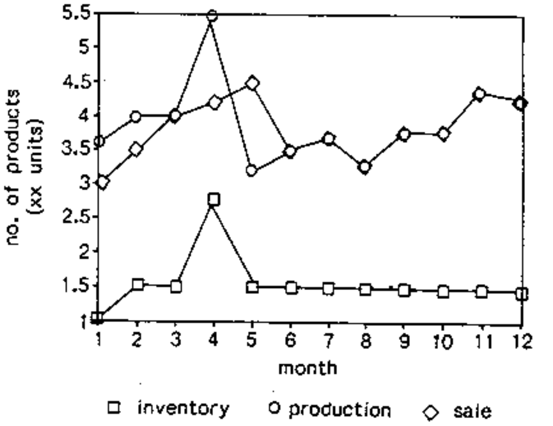

Fig. 3. Production plan for group 12.

group 12 which has expensive unit-inventory cost.

iii) Inventories are utilized for low priced models, especially in subcompact models, to absorb fluctuation in demand so that resources can be reserved for high quality models.

iv) The occurrence of under times are observed at the early and late periods of the planning horizon.

v) Overtimes are fully untilized during periods of June through October which show high demand rate.

vi) The amount of subcontract increase gradually to reach full capacity in June, and then decrease.

5. Conclusions

This study demonstrate the applicability of the linear programming technique to the aggregate production planning problem in an optical instruments industry.

One of the basic weakness of this approach is the assumption of deterministic demand: there is considerable uncertainty in the forecasts of demand, especially when a major portion of demand comes from overseas market. An another shortcoming of L.P. models is the requirement of linear cost function. However, the possibility of piecewise linearity can improve the validity at a cost of additional computational effort.

We acknowledge that significant gaps exist between this model and practice and hope to devise a way to bridge these gaps in the future study.

Appendix A.
Estimation of Cost Parameters

CREG : monthly salary per worker

wage and salary	256,600won
bonus	112,800won
retirement allowance	72,000won
fringe benefit	133,200won
total	573,600won

COVER : hourly overtime cost 3,504won

CHIRE : hiring cost per worker

interviews and selection cost	6,840won
test	120won
medical examinations	10,800won
training	24,960won
paper work	2,640won
other expense	3,600won
total	48,960won

CINV : monthly inventory carrying cost

interest on capital blocked	1.25 %
insurance	0.014%
warehouse operation and material handling cost	0.035%
total	1.299%

References

[1] Bergestorm, G.L.and Smith, B.E., "Multi-item Production Planning, an Extension of HMMS Rules," *Manag. Sci.* (1970), pp.614-619.

[2] Bowman, E.H., "Production Scheduling by Transportation Method of Linear Programming," *Operation Res.*, Vol. 4(1956), pp.100-103.

[3] Eise-Man and Young, "Linear Programming Model for Production Planning of a textile Mill," *Manag. Sci.*, Vol. 15(1969).

[4] Goodman, D.A., "A Goal Programming Approach to Aggregate Production Planning of Production and Workforce,"

Manag. Sci., Vol. 20(1975), pp.1569-1575.

[5] Hansman, F. and Jess, S., "A Linear Programming Approach to Production and Employment Scheduling," *Mgmt. Tech.*, Vol. 1(1960).

[6] Holt, C.C., Modigliani, F., Muth, J.F. and Simon, H.A. *Planning Production Inventories and Work Force*, Prentice Hall, Inc. 1960.

[7] Wai-Hing Chin, "A Planning Model for a Petrochemical Complex in Thailand," Msater Thesis, No. 599(1972), *AIT*, Bangkok.

[8] Sadd, G.H., "An Overviw of Production Planning Models: Structural Classification and Empirical Assessment," *Int. J. Prod. Res.*, Vol. 20(1982), pp.105-114.

[9] Tang, C.S., Adulban, P., and Zubair, T., "An Aggregate Production Planning for A Heavy Manufacturing Industry," E.J. O.R., Vol. 7(1981), pp.22-29.

[10] Silver, E.A., "A Tutorial on Production Smoothing and Work-force balancing," *Operations Res.*, Vol. 15(1967).

[11] Taubert, W.H., "Search Decision Rules for Aggregate Scheduling Problem," *Manag. Sci.*, Vol. 14(1967), pp.343-359.

12. Van De Panne, C., and Posje, P., "Sensitivity analysis of lost coefficient estimates: the case of linear decision rules for employment and production," *Manag. Sci.*, Vol. 9(1962).