論　文

Three Dimensional Analysis of the Solidification Order of Some Common Casting Sections

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주물교차부 응고순서의 3차원해석

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개　요

주물의 응고순서는 주물의 품질을 결정하는데 가장 중요한 요소이다. 이런 응고를 결정하는데 가장 큰 역할을 하는것은 시보리노프의 식인데 이 식을 이용하여 주형에서의 용탕의 통로나 압탕의 크기 및 위치를 결정하는데 효과적으로 사용하고 있다.

그러나 실제의 복잡한 주물에서는 각 부분에서의 모듀러스를 결정하는데 어려움이 있고 그 결과 여러가지 모듀러스를 결정하는 방법들을 제시하기에 이르렀다. 가장 보편적인 방법으로서는 주물의 단면을 무한대한 판이나 봉 또는 T 또는 L형이라고 생각하고 모듀러스를 결정하는 것이다.

본 연구의 목적은 상업적으로 쓰이고 있는 주물을 몇가지 단순한 모양으로 분할하고 이때 모듀러스에 미치는 R의 크기, 주형재료, 단면의 두께 등이 응고순서에 미치는 영향을 콤퓨터에 의해 계산하였다.

이 결과는 주물의 압탕을 설계하거나 위치를 결정하는데 뿐만 아니라 주물의 설계를 콤퓨터화 하는데도 중요한 역할을 할것이다.

1. Indroduction

Perhaps the most reliable method used to evaluate the order of solidification among sections of a casting, and to be used as the basis of a riser calculation method, is that originally proposed by Chvorinov. On the basis of experiments with "skin forming" alloys it was determined that the time for solidification of a casting was proportional to the square root of the modulus of the casting (where the modulus was the ratio of the casting volume to the casting surface area). Others have elaborated upon Chvorinov's rule, and it is now recognized that it is best applied to sections of the casting in order to identify casting feeding paths, and to estimate riser dimensions and riser placement.

There are difficulties, however, in determining the modulus of the individual

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segments of complex castings and, as a result, procedures have been proposed to estimate the modulus values. Most common among these is to evaluate the modulus of casting sections by considering them to be infinite plates, bars, T and/or L sections, etc. The aim of this investigation was to evaluate the effect of the radius of curvature on the volume to surface area ratio for a number of simple geometric shapes commonly encountered in commercial castings. Further more, an analysis was made of the necessity of considering the radius of curvature in these estimates, and the advantages of using a more accurate technique to obtain reliable modulus values.

This information is of particular value, not only in determining the direction of solidification within a casting for riser design and placement, but in the computerized analysis of casting design (the so called CAD/CAM, or CAE, procedures) where the need to consider the radius of curvature of the casting makes the analysis more rigorous.

A series of frequently encountered casting intersections will be considered. These include two types of T sections, an L section, two types of X sections, the case of a boss, or top riser, on a plate section, and a side riser attached to plate section. These geometric shapes represent the majority of casting sections wherein shrinkage is observed in castings - particularly where isolated, unfed porosity forms.

While Chvorinov's analysis was originally developed for skin forming alloys, it is generally accepted that the concept of the solidification time - modulus relationship applies to all solidification processes, although it may be complicated by variations in initial temperature, the presence of adjacent sections, etc. This analysis is, then, applicable for the range of casting alloys produced. Procedure for Computation of Solidification Order.

Computer techniques were used to construct a series of design graphs based upon the three dimensional analysis of the solidification process, i.e., the section moduli. The procedure for the computation and plotting of design graphs is as follows:

1. Define the variables, or parameters, involved in a section being considered.
2. Define the limits, or boundaries, of each region involved in that section.
3. Calculate the volume to surface area ratio of each region.
4. Equilabrate the modulus equations of the two adjacent being compared to each other.
5. Where necessary, assume a fixed relationship between variables or parameters so that only two variables, such as T_1 and T_2, remain.
6. Plot the equilibrium conditions existing between adjacent sections as a function of the remaining variables, i.e., T_1 and T_2.

2. T Sections

T Section A

The first of the two types of T sections to be considered here has been designated T section A and is illustrated in Figure 1. This section is the result of rotating a T section around an axis perpendicular to the supporting leg. In this case, T_1 is the section thickness of the supporting leg, T_2 and T_3 are the section thicknesses of the cross arms, R_1 and R_2 are the fillet radii between legs T_1 and T_3 and legs T_1 and T_2, respectively, and R is the inside radius of the casting. An analysis was made of the four segments of this casting: the supporting leg (1), the cross arms (2 and 3), and the T section (4).

Since the supporting leg, T_1, and the cross arms, T_2 and T_3, are considered to be semi-infinite in length the section moduli are $T_1/2$ $T_2/2$ and $T_3/2$, respectively. To determine the section modulus of the T section (4), one section thickness length beyond the fillet radius was included in the computation of the ratio of volume to surface area.

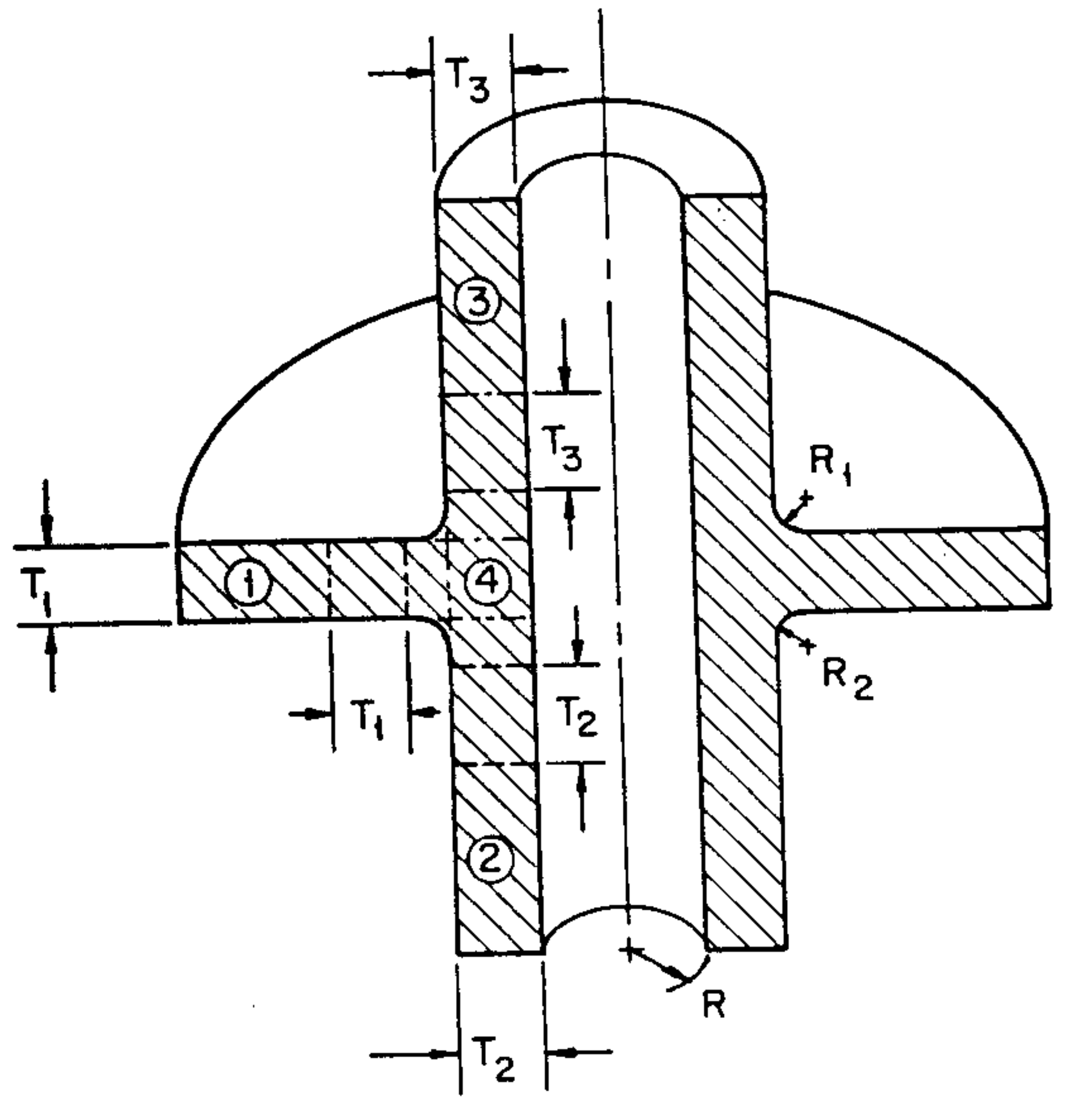


Fig. 1. Schematic representation of T section A. For purposes of this analysis the flange on this hub is considered to be of "infinite" outside diameter, and the hub sections 2 and 3 are similarly of "infinite" length so that end effects are not involved. Comparisons are made, then, of the modulus values of the T section (4) with those of the flange (1) and the hub (2 and 3), as a function of T_1, T_2, T_3, R, R_1 and R_2.

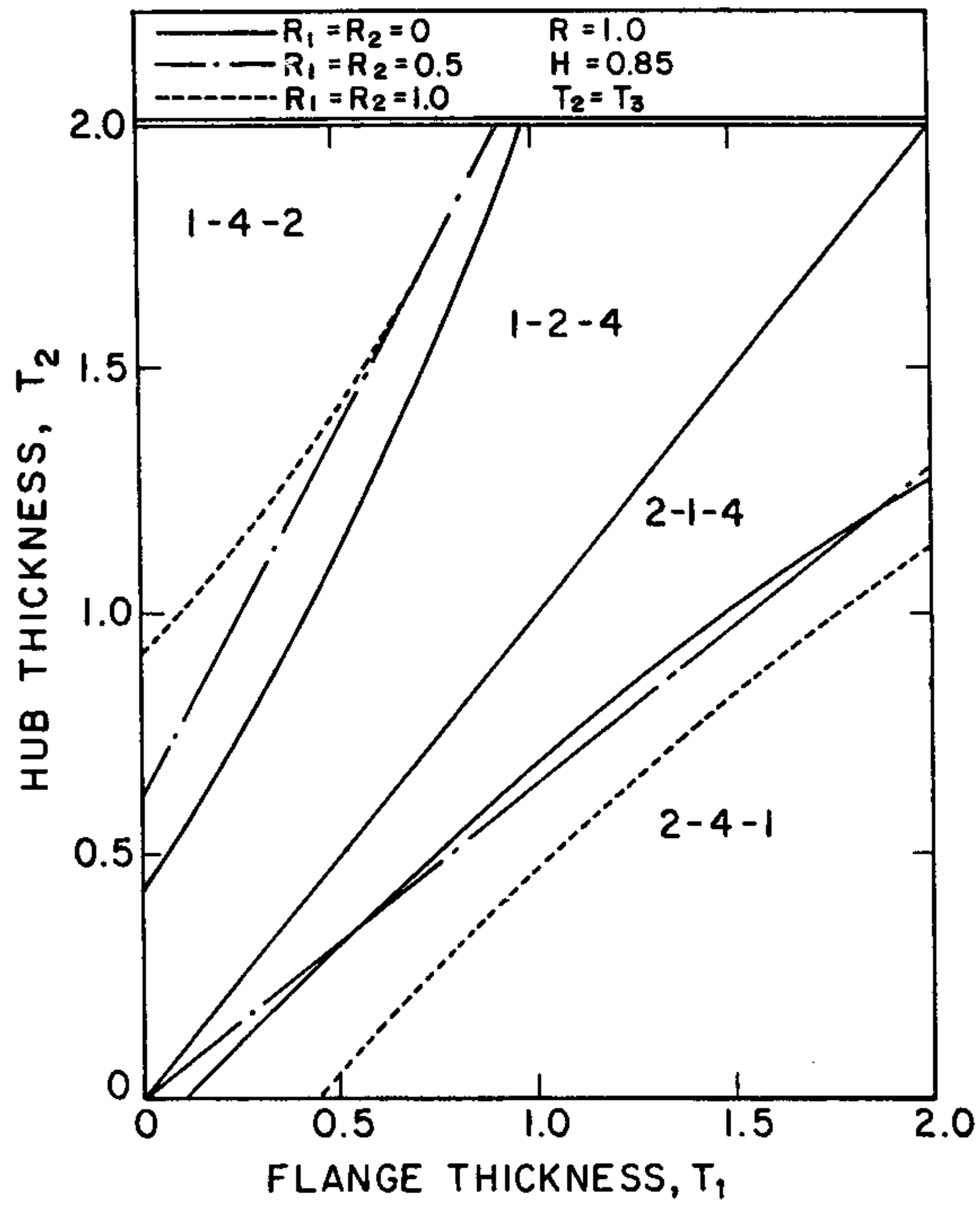


Fig. 2. Graph showing the effect of changes in the fillet radius ($R_1=R_2$) on the order of solidification in T section A, Figure 1. The inside radius of T section A has been held constant, R=1.0, and it is assumed that a core (H=0.85) has been used. A constant hub thickness has been assumed ($T_2=T_3$) so that the moduli of sections 2 and 3 are the same. (The notation 1-2-4 indicates that section 1 will solidify first and section 4 will solidify last).

The effect of the lower heat extraction capability of a core used to form the I.D. of this hub shaped casting otherwise produced in a sand mold can be realized by multiplying the surface area of the casting which is in contact with the core by a constant, say 0.85. This effectively reduces the heat transfer across that surface—an effect analagous to that of the core.

Figure 2 illustrates the effect of variation in the fillet radius on the order of solidification in a casting of geometry T section A where the core diameter is 1.0 and hub uniform thickness ($T_2=T_3$). Note that the boundry lines change as the fillet radius is increased, e., for a given hub thickness shrinkage can be expected at the T section over a broader range of flange thicknesses.

When the effect of varying the inside diameter of the hub is considered, the results are shown in Figure 3.

The influence of a relative difference in the molding media used for the core and for the mold is illustrated in Figure 4. The values for H=1.0 would apply where the mold and core were made for the same material (green sand, air set, shell, etc.). For a casting having a hub thickness of 1.0, T section shrinkage would be anticipated for flange thicknesses between about 0.6 and 1.35. Flange thicknesses less than 0.6 would result in the hub solidifying last, while a flange in excess of

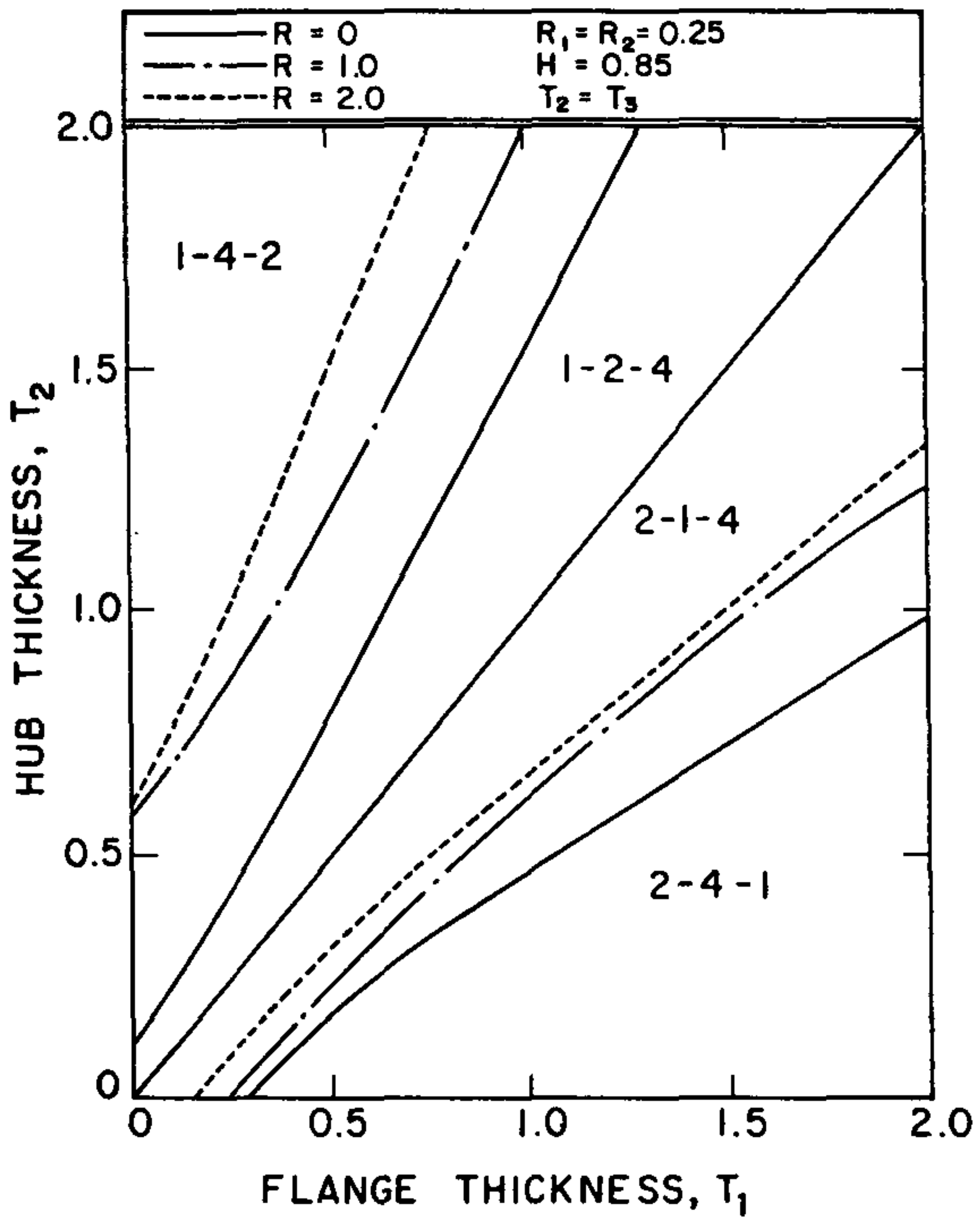


Fig. 3. The influence of variation in the hub inside diameter on the order of solidification in T section A, Figure 1, is shown here. This graph has been established for a constant fillet radius, a uniform hub thickness, and the use of a core (H=0.85).

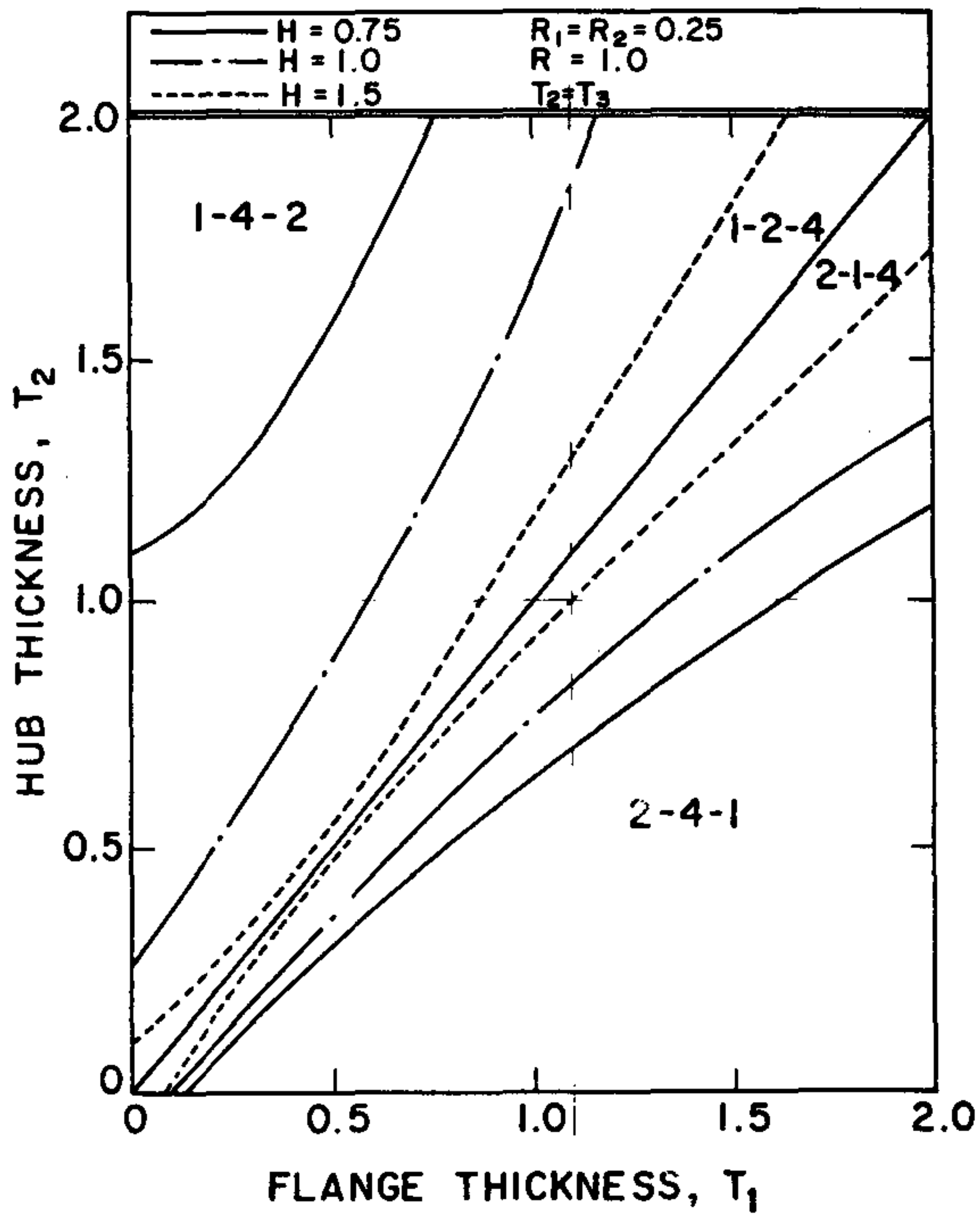


Fig. 4. Vatation in the relative abihy pf core and molding media to cool the casting during solidification are illustrated by this graph for T section A.

1.35 would be required for the flange to solidify last. Using a more insulating core (H=0.75) causes T section shrinkage to flange thicknesses in excess of 1.5. In other words, the use of a core increases the chances for T section shrinkage. As might be anticipated, if the molding media is more insulating than the core, H=1.5 the tendency for T section shrinkage is markedly reduced (occurring only with flange thicknesses of 0.85 to 1.10 for the case cited).

Figure 5 illustrates the effect of a non uniform hub thickness on the solidification order in T section A. This is accomplished with a constant hub inside diameter. As the thickness of one portion of the hub, T_3, is reduced relative to the other, T_2, the range of flange

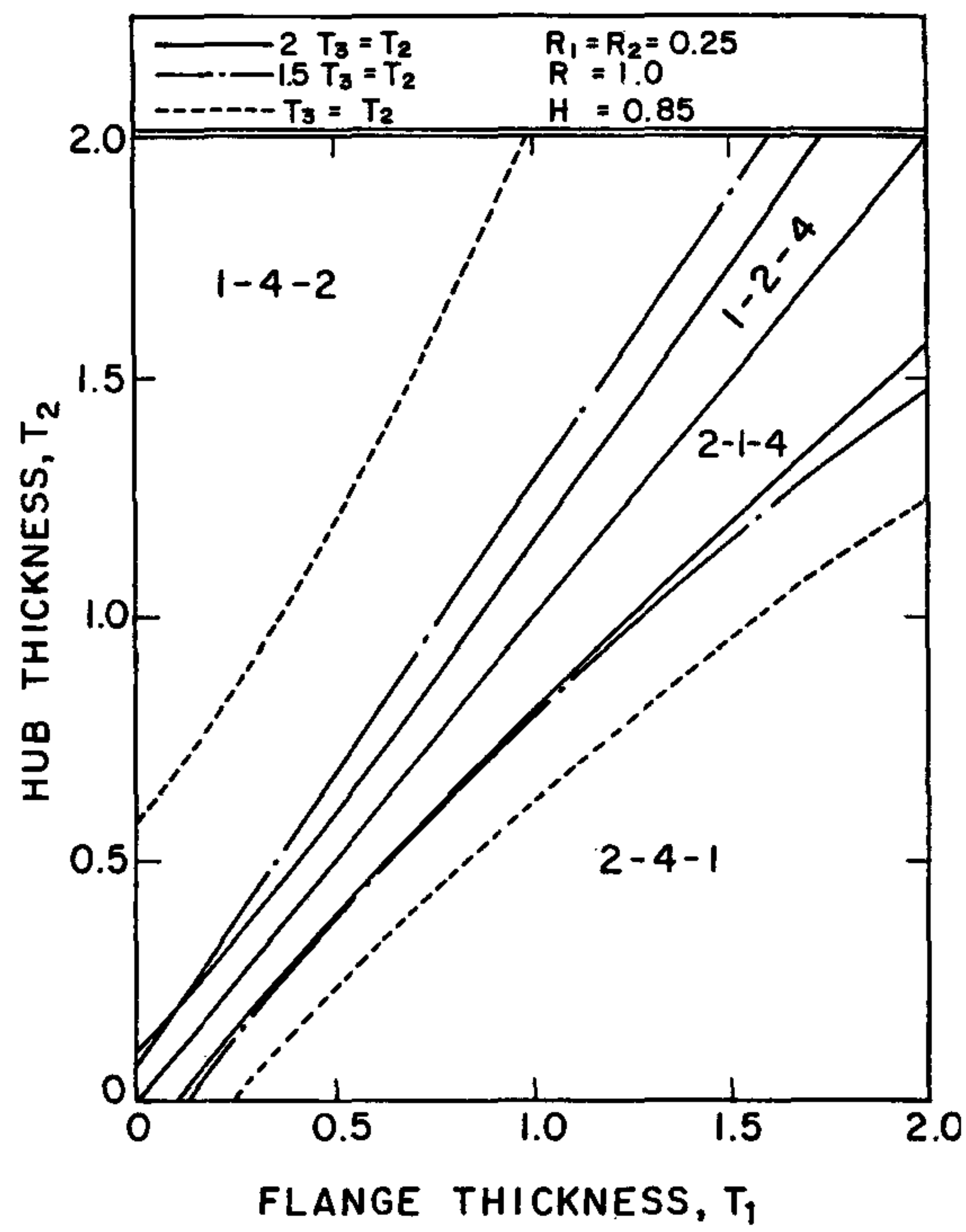


Fig. 5. The effect of a non-uniform hub thickness on the solidification order in T section A castings is presented in this graph.

thicknesses resulting in T section shrinkage is narrowed significantly.

The need to consider the radius of curvature, T of the hub casting should also be evaluated. At an infinite value of R the analysis is the same as the conventional two dimensional analysis of the solidification order. If the two dimensional analysis result and that of three dimensional analysis were within 10% of each other, the more elementary two dimensional analysis of the solidification order should suffice. A comparison of these two approaches is summarized in Figure 6 where

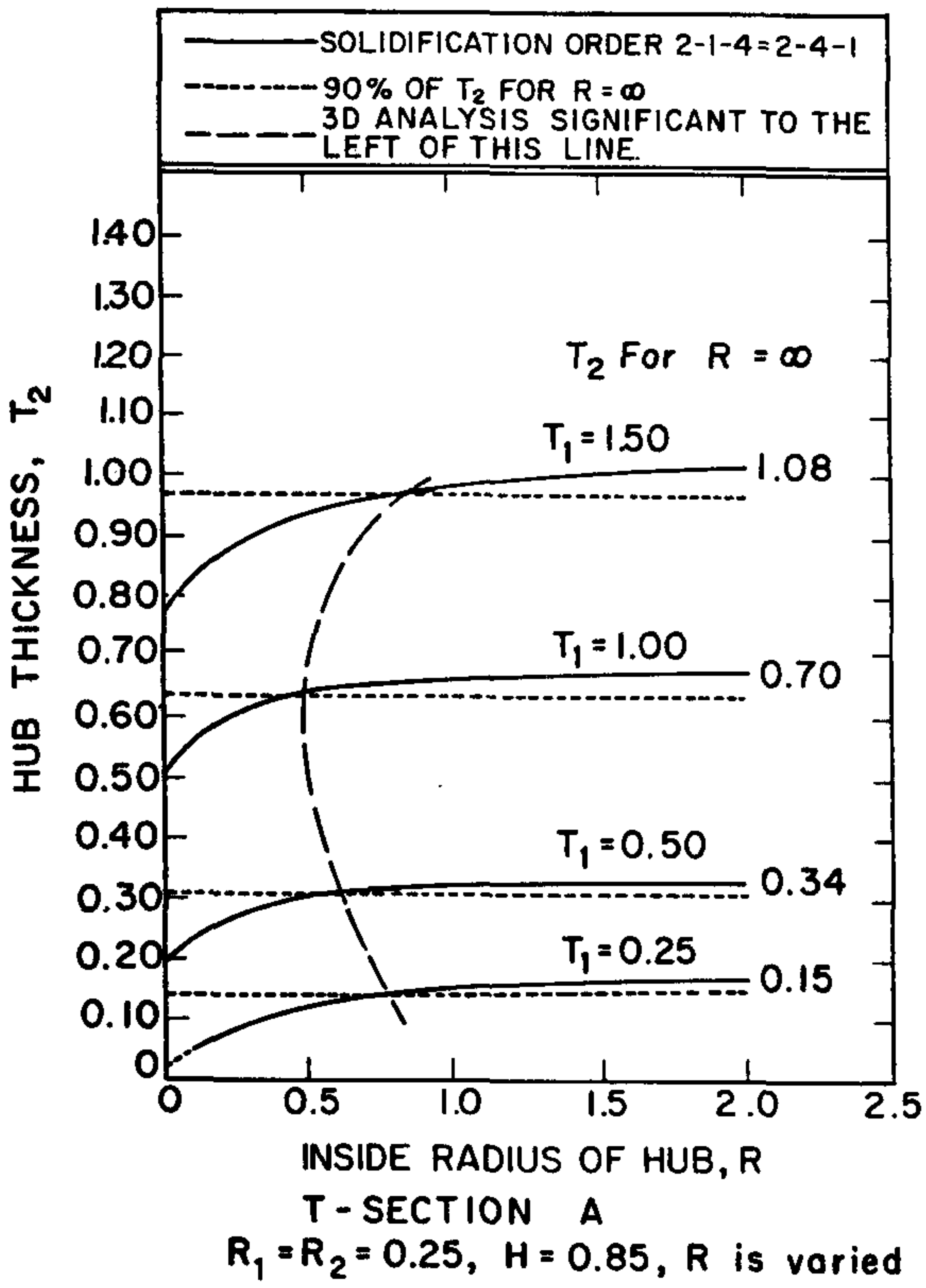


Fig. 6. Comparison of the analysis of T section A geometry by two and three dimensional analysis. Two dimensional analysis results for 2−1−4 and 2−4−1 equilibrium are indicated by R= ∞. Agreement within 10% is considered satisfactory. This limit is illustrated by the dashed line.

the effect of the radius of curvature. R. on the hub thickness, T_2, and flange thickness T_1, where equilibrium is achieved between a 2−4−1 and a 2−1−4 solidification order. In general, where the inside diameter of the T section A hub exceeds 1.0 to 1.5 the two dimensional analysis should be satisfactory. However, where the core diameter is small it is apparent that substantial error can accompany the two dimensional approach and the more involved three dimensional analysis summarized in Figures 1 through 5 should be utilized.

T Section B

The second type of T section considered has been rotated about an axis parallel to the supporting leg, Figure 7. The method used to evaluate the order of solidification in this section was similar to that described for T section A, except that an end segment, L, has been considered as a constant. In the example analysis to be discussed, the section L has been assigned a value of 1.5.

Variation in the radius of the fillet, R_1 and R_2, on the solidification order is illustrated by Figure 8. Increasing the fillet radius has a strong effect on the solidification order, and greatly increases the tendency to obtain shrinkage porosity in the T section. For

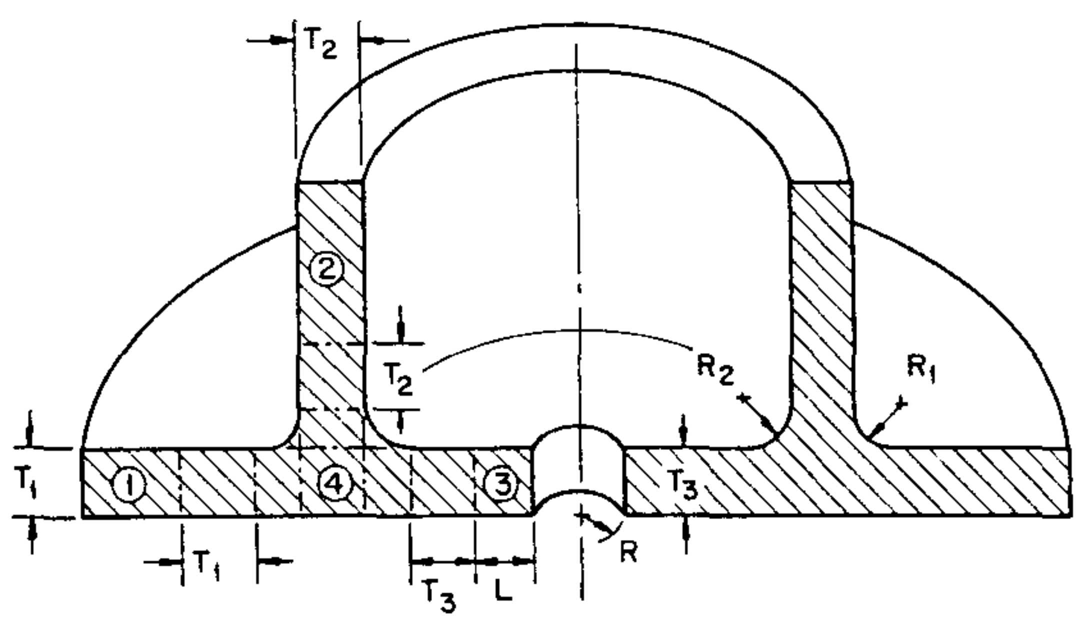


Fig. 7. Schematic representation of T section B. Analysis of the solidification order is similar to that of T section A, Figure 1, except for the inclusion of a segment, L.

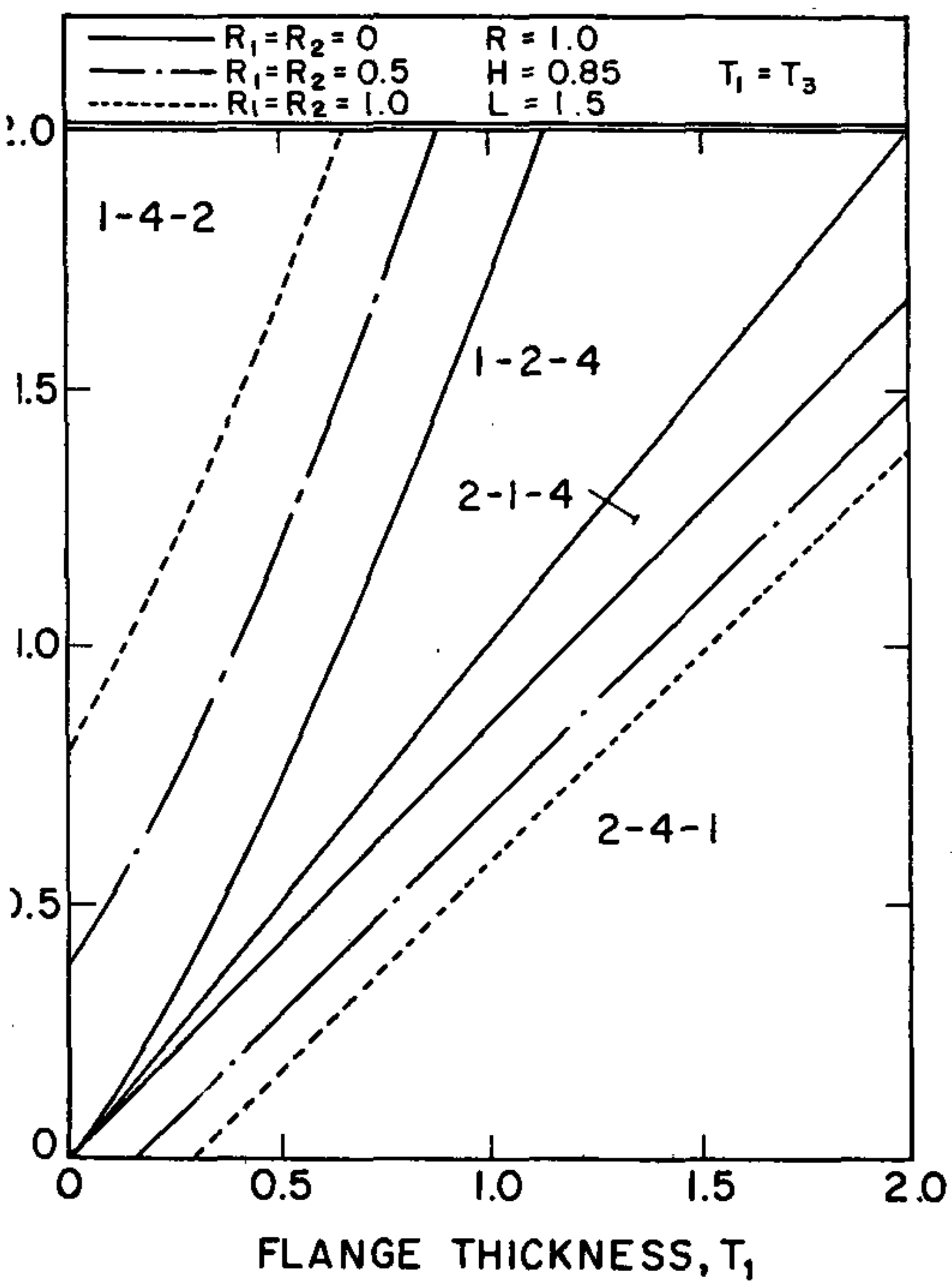


Fig. 8. Relationship of hub thickness, T_2, and flange thickness, T_1, for T section B as a function of fillet radius, R_1 and R_2. Constant values for R (1.0) and L (1.5) have been used in this presentation, as has a value of H=0.85 for the core material.

example, consider a casting with a 1.0 hub thickness. With a sharp ($R_1=R_2=0$) fillet radius, the flange thicknesses expected to exhibit shrinkage porosity range from about 65 to 1.2 while a 0.5 fillet radius expands this range to 0.4 to 1.4.

Changes in the value of the radius, R, are illustrated by Figure 9. Note that in this case the inside radius of the hub is not R, but is the sum of $R+L+T_3+R_2$. Increasing the value of R results in a significant reduction in the tendency for shrinkage to occur in the T section.

Variations in the thickness of the arms of the flange section, T_1 and T_3, are illustrated by the graph presented in Figure 10. Changes in the thickness of these flange arms was considered with a "flat" bottom on this casting.

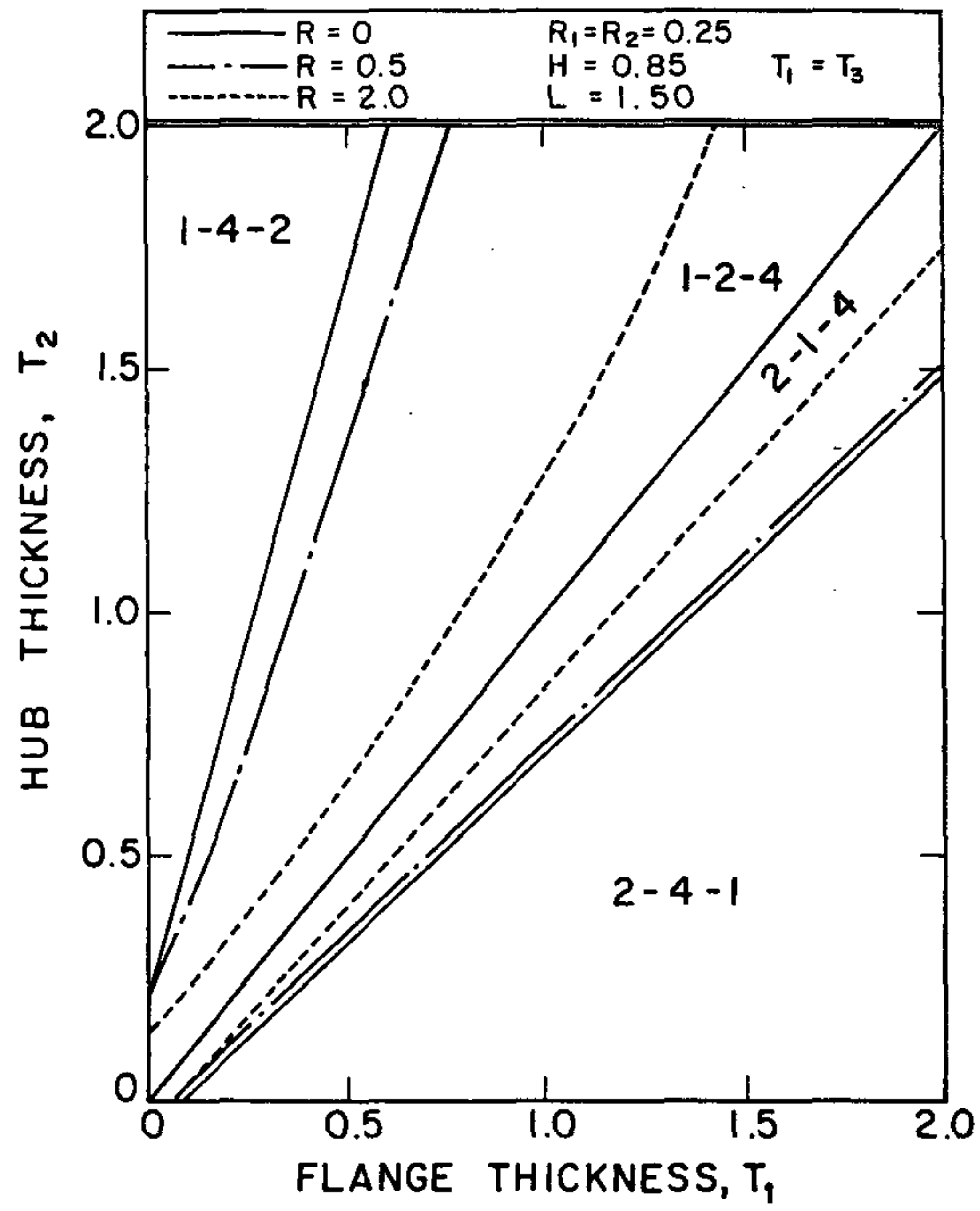


Fig. 9. Effect of radius, R_1 on the solidification order of T section B for variations in the hub (T_2) and flange (T_1) thickness.

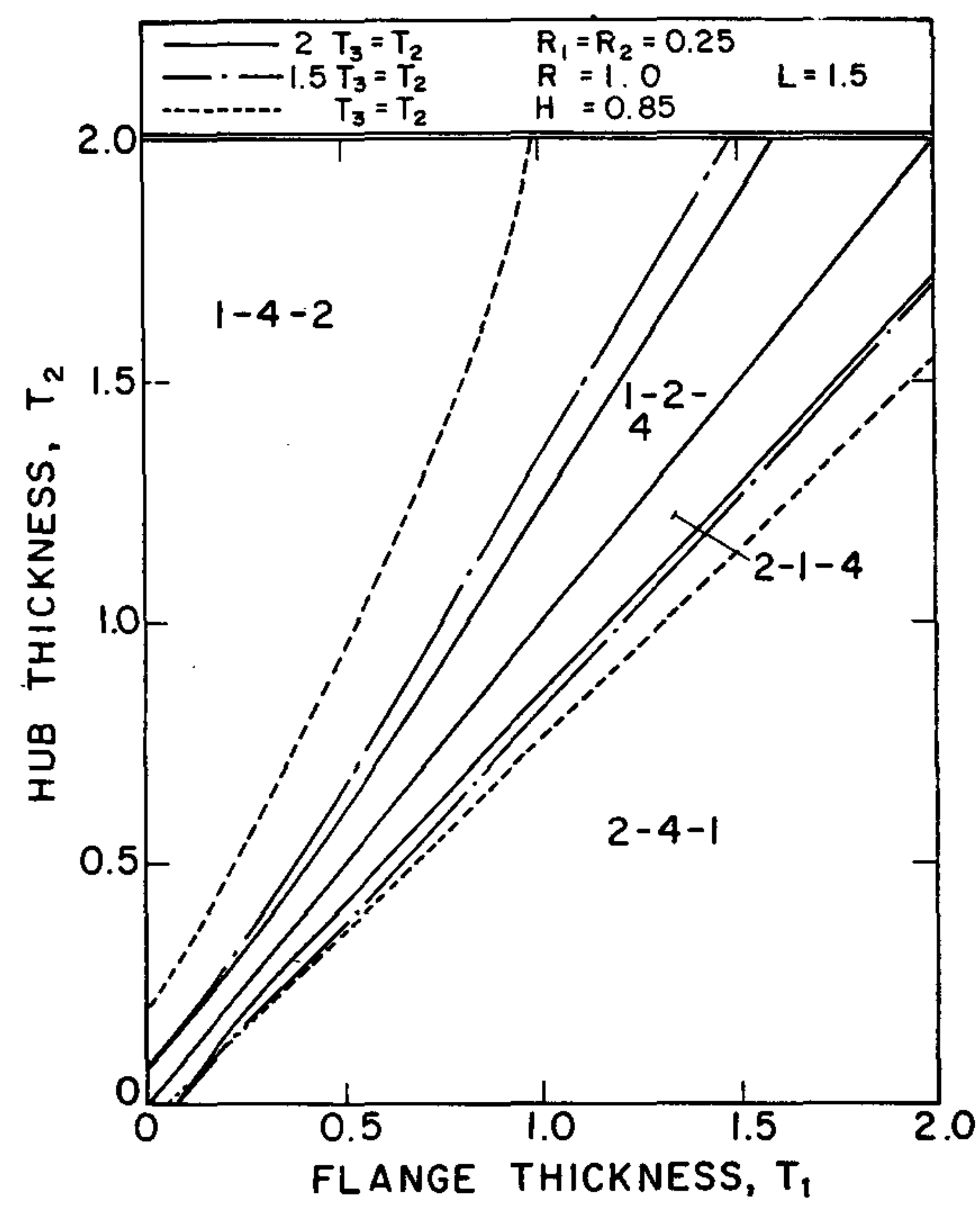


Fig. 10. Order of solidification of T section B as affected by a non-uniform flange geometry

Maintaining a uniform cross arm section minimizes the occurrence of T section shrinkage.

3. L Sections

Another frequently encountered casting geometry is the L section illustrated in Figure 11 as rotated about an axis parallel to one leg with the other leg toward the outside diameter. This casting is divided into three segments: a flange (considered semi-infinite) of thickness T_1 (1), a hub (also semi-infinite) of thickness T_2 (2), and an L-shaped junction with an inside radius R_1 and outside radius R_2. Since the hub and flange are semi-infinite the moduli of these sections are $T_2/2$ and $T_1/2$, respectively. The modulus of the L junction was determined in a manner similar to that previously described for T section geometries.

Figure 12 presents the effect of variations in the fillet radius on the solidification order of the L section. An increase in the fillet radius increases the range of geometries over which L section shrinkage might be anticipated, but the effect does not appear to be as great as for T section A, Figure 2. The effect of different values of the inside and outside radii can be determined by comparison of this data with others in the literature, 10 and by noting a few of the examples plotted in Figure 13. As expected, increasing the size of the in-

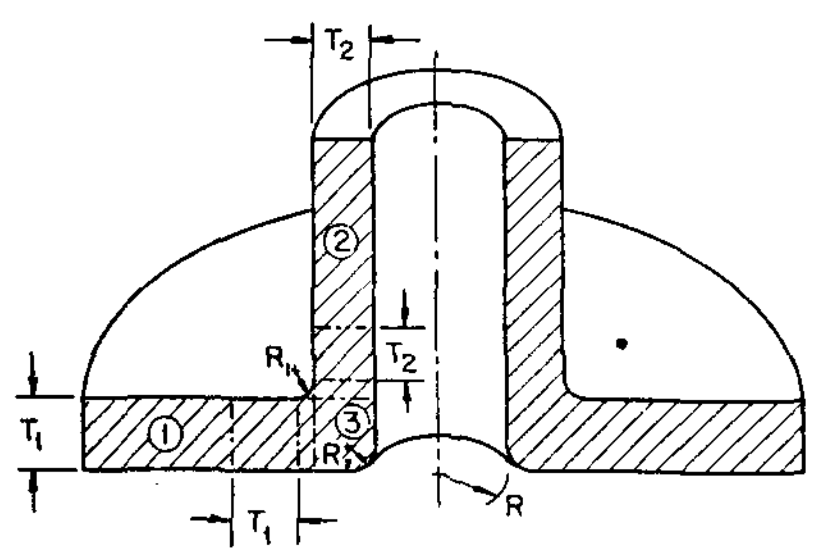


Fig. 11. Schematic representation of the L section. The analysis of the solidification order is similar to that used for T section A, Figure 1.

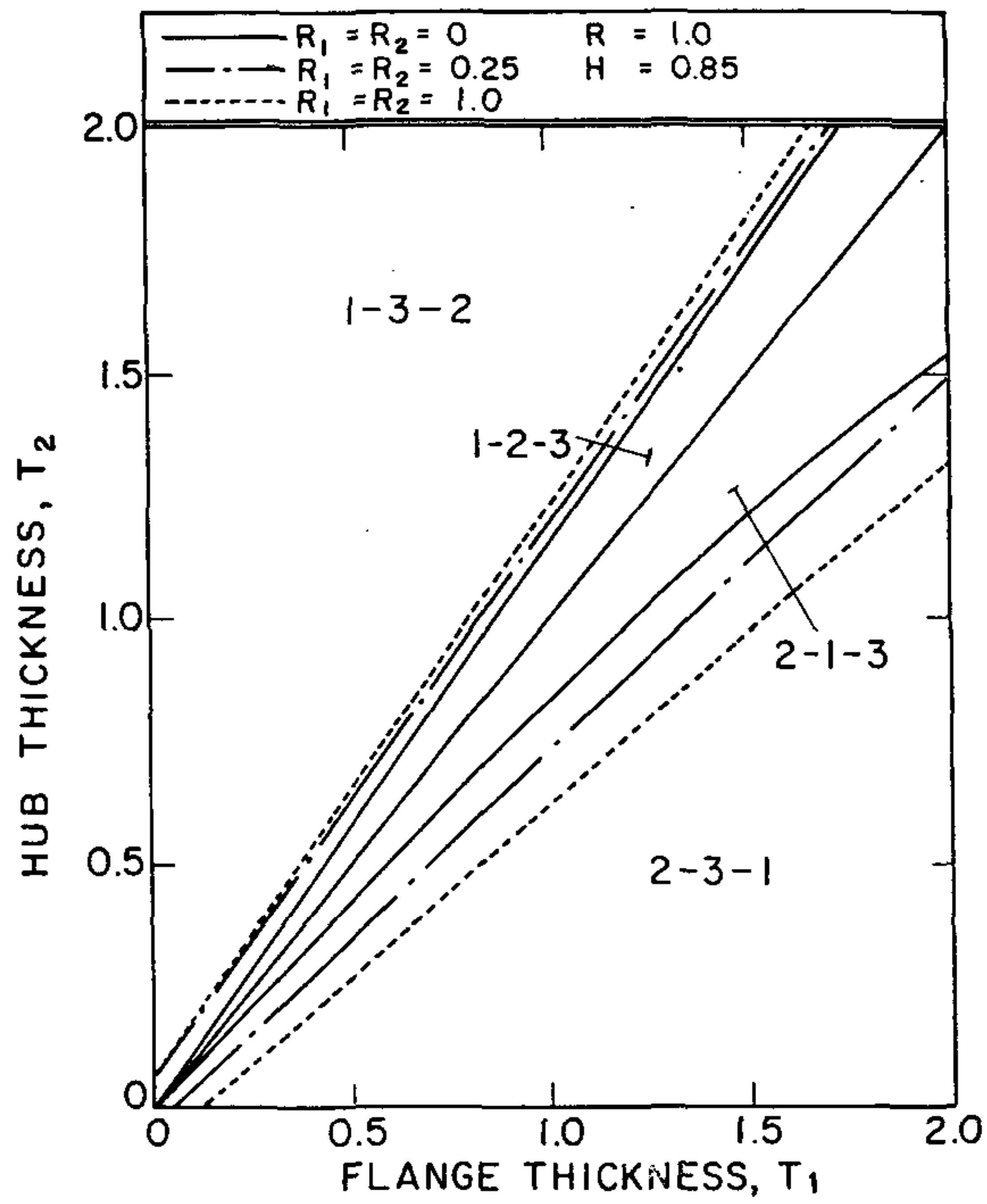


Fig. 12. Relationship of fillet radius, R_1 and R_2, to the solidification order of the L section.

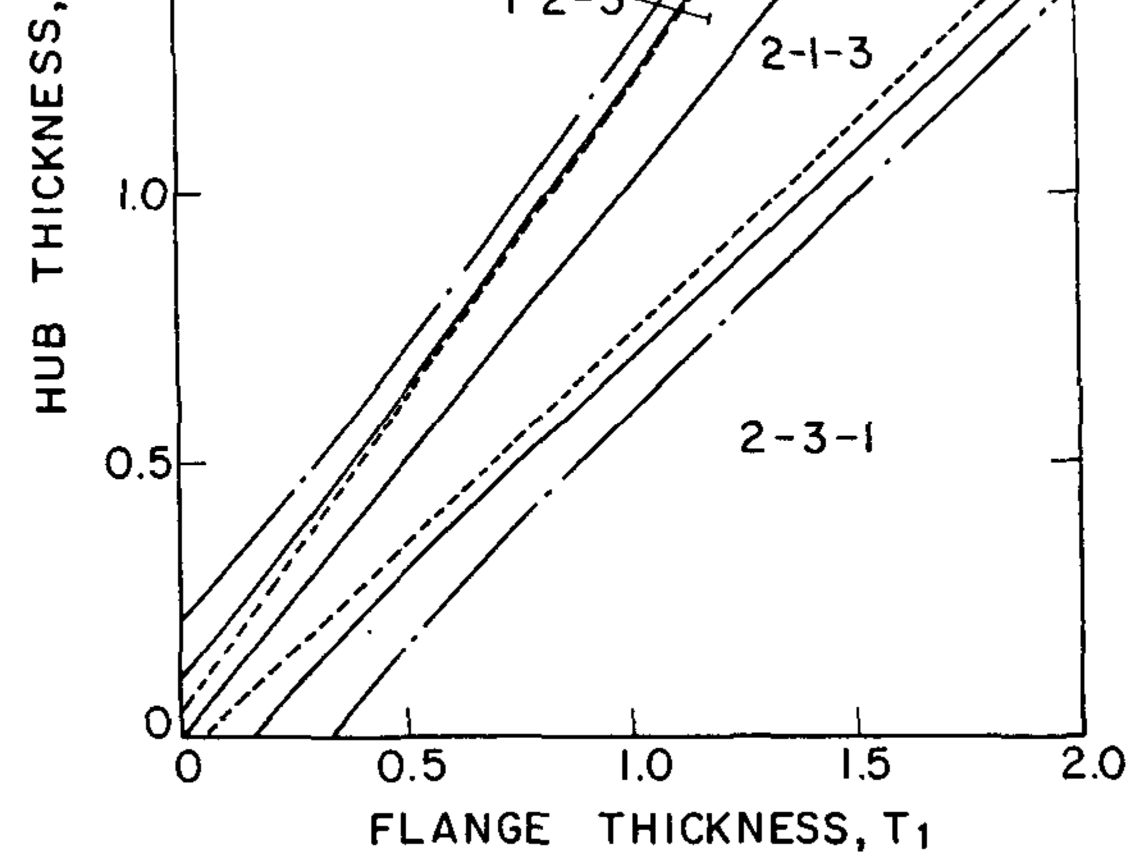


Fig. 13. Effect of varying the inside and outside fillet radii on the solidification order of the L section casting. In this case the inside fillet radius is examined when equal to, or larger than, the outside radius.

side fillet radius with respect to the outside fillet radius increases the modulus of the L section. It would also be expected that increasing the outside fillet radius compared to the inside would decrease the L section modulus, see Figure 14.

The effect of increasing the inside hub radius, R, on the solidification order of the L section is illustrated in Figure 15. As the radius of curvature of the L section hub is increased the modulus of the L section is decreased so that a more limited range of casting geometries will exhibit shrinkage porosity.

A shift in the solidification order from 2-1-3 to 2-3-1 is affected by the three dimensional analysis only for casting where R is less than 0.5. However, the shift from the 1-3-2 solidification order to a 1-2-3 solidification order is not affected by the method of analysis. In other words, for L section castings having an inside diameter greater than 1.0, a two dimensional analysis will suffice. Where feeding is through the hub. not the flange, a two dimensional analysis will produce satisfactory results at all values of R.

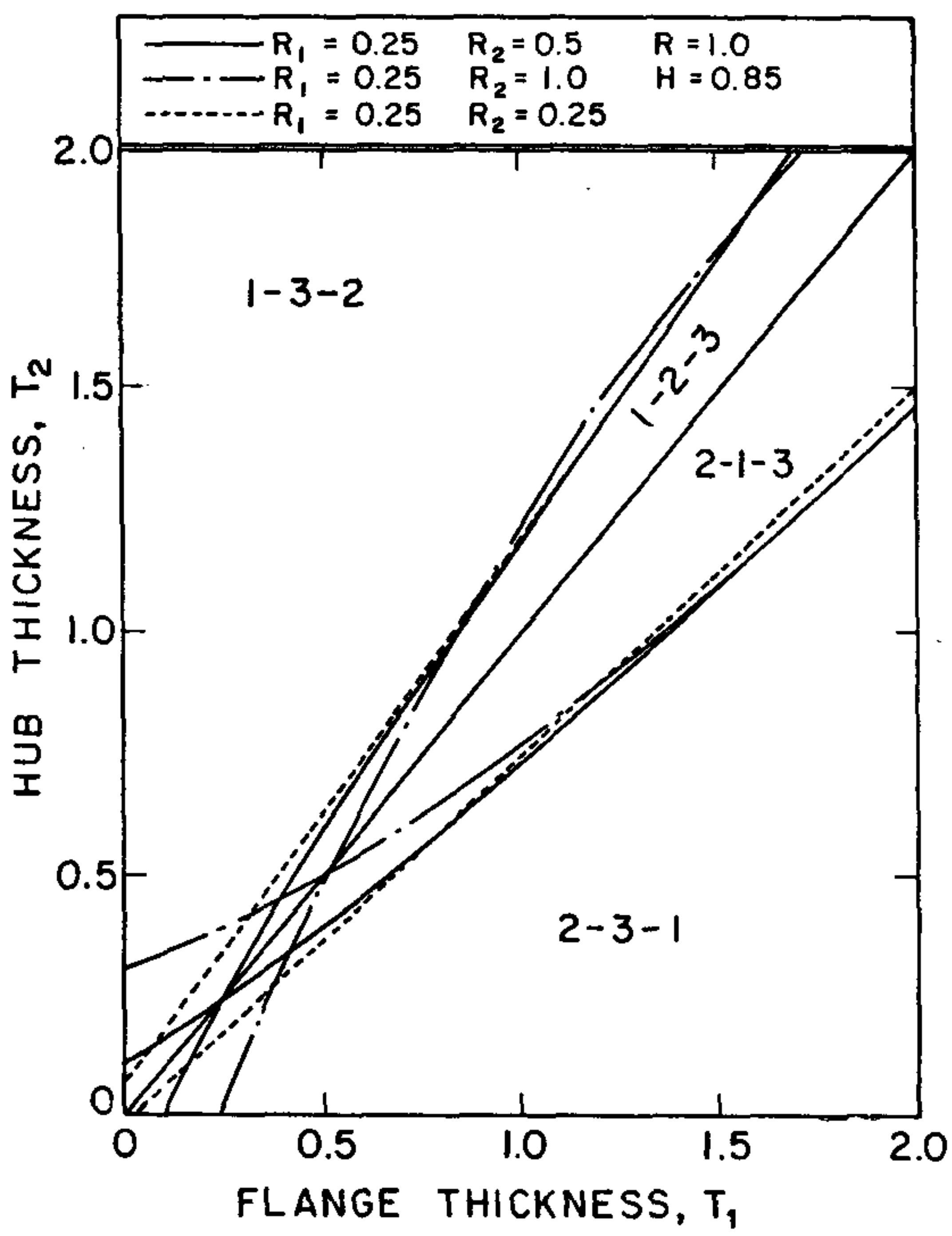


Fig. 14. Schematic representation of the effect of increasing the size of the outside fillet radius with respect to the inside fillet radius of the L section casting. Where the curves over lap at the lower left hand portion of the graph represents geometries which are not realistic.

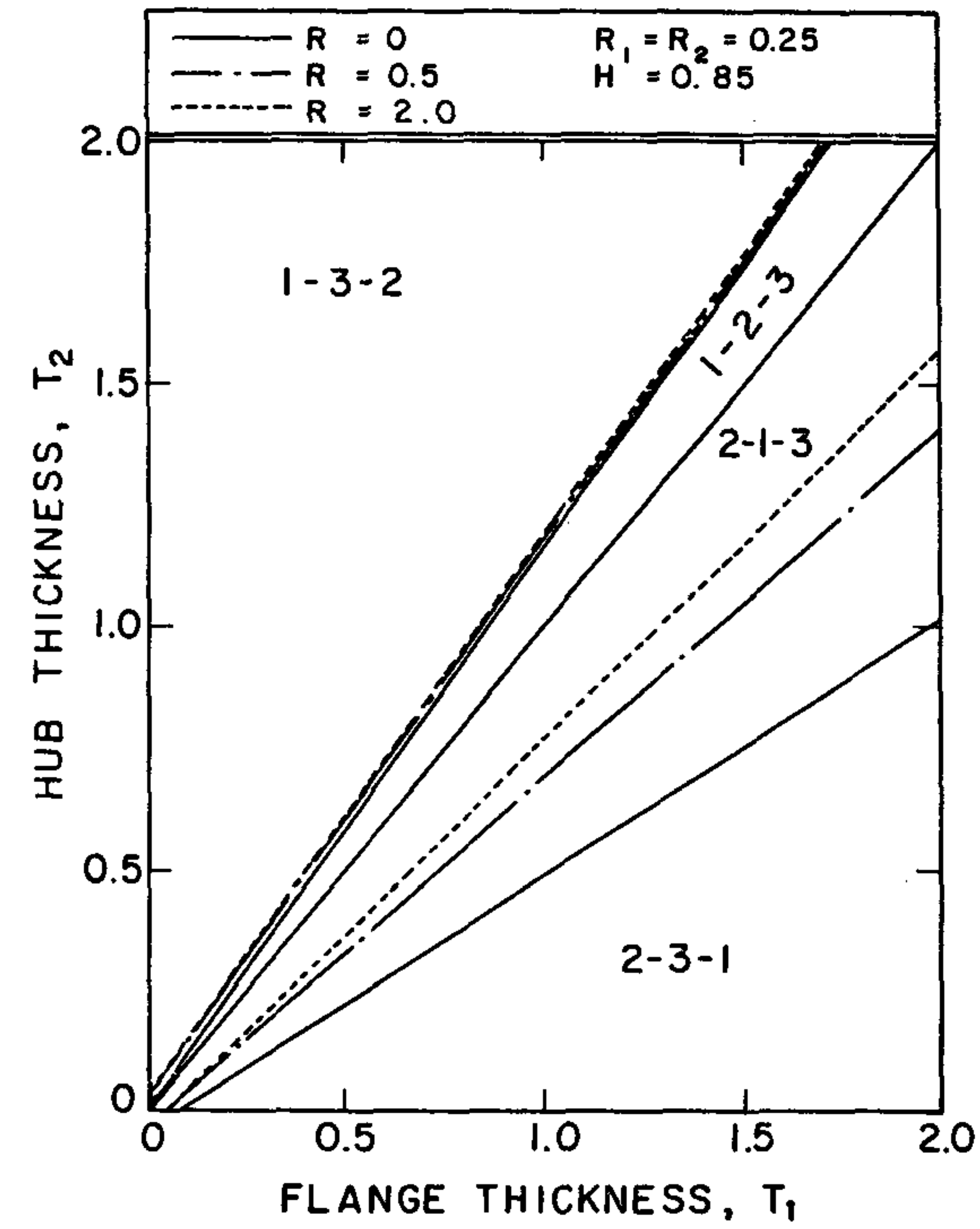


Fig. 15. Effect of variations in the inside diameter on the solidification order of the L section.

4. X Sections

Section A, Opposite Legs Equal

Figure 16 presents a sketch of X section A, a casting geometry of an X section where the legs opposite each other are of equal thickness. This geometry, and its analysis of solidification order, is related to that of T section B, presented in figure 7. The segment, L, has also been included here. In this analysis

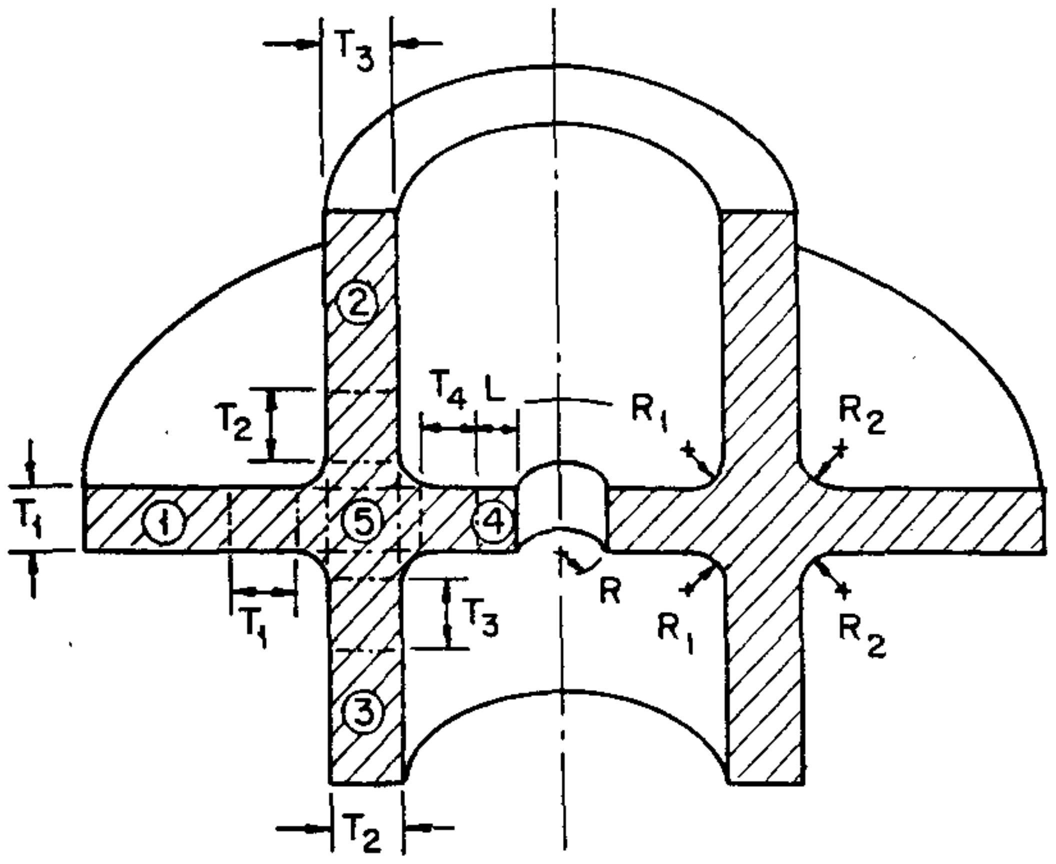


Fig. 16. Diagram showing the geometry of X section A where the two opposite legs are maintained equal to each other. Analysis of the solidification order is similar to that used for T section B, Figure 7.

the X section has been designated 5. Section 4, because of end effects, will usually solidify prior to section 1, but where the value of R is quite large (beyond that considered here) it will have a modulus equivalent to section 1.

Variation in the size of the fillet radius on the X section A solidification oredr is show in Figure 17. It may be noted that this geometry effectively promotes the possible shrinkage at the X junction and that, in comparison, the effect of the fillet radius is small. Similarly the inside radius, R, has a minor effect, Figure 18.

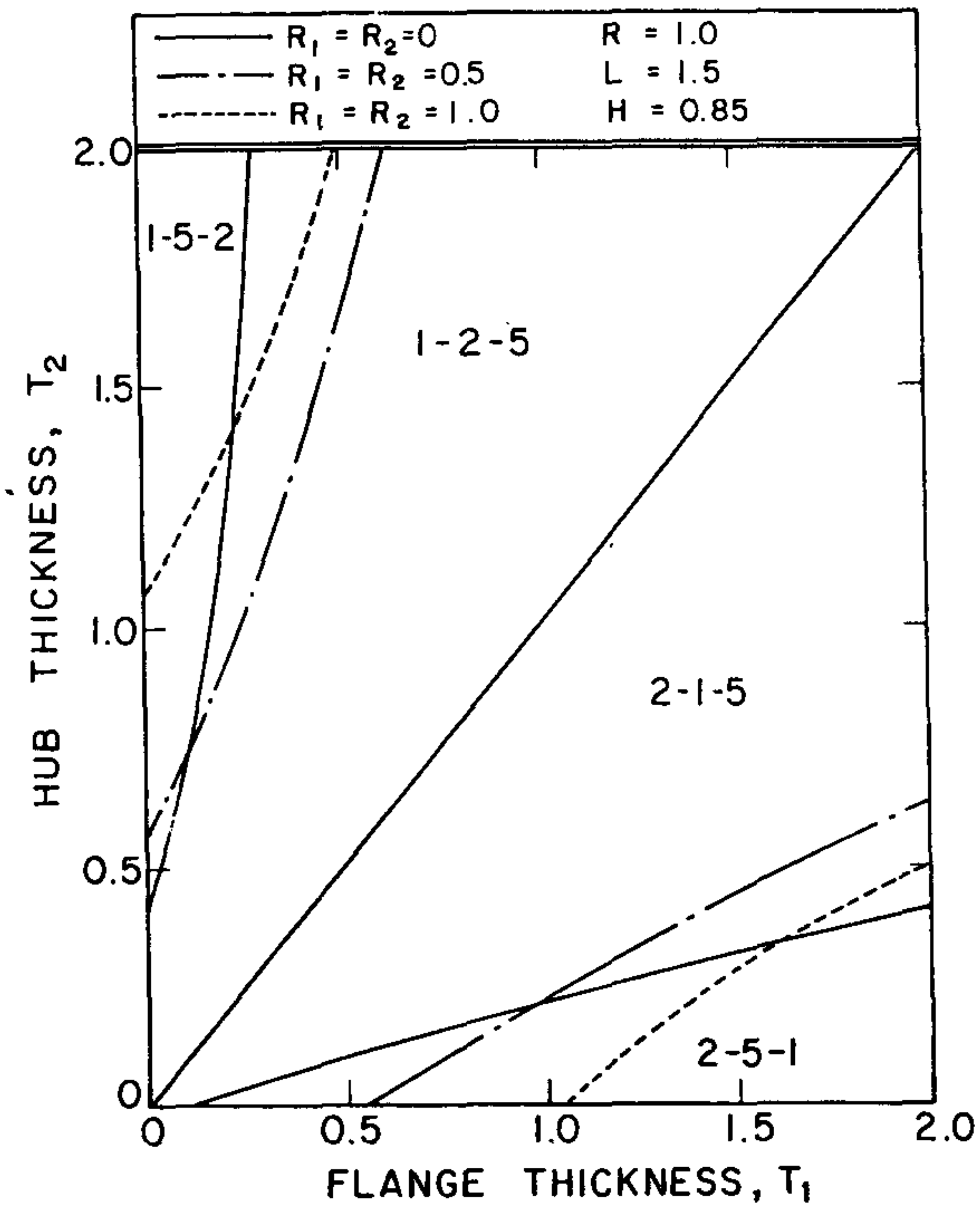


Fig. 17. Effect of variation in the fillet radius on the solidification order of X section A.

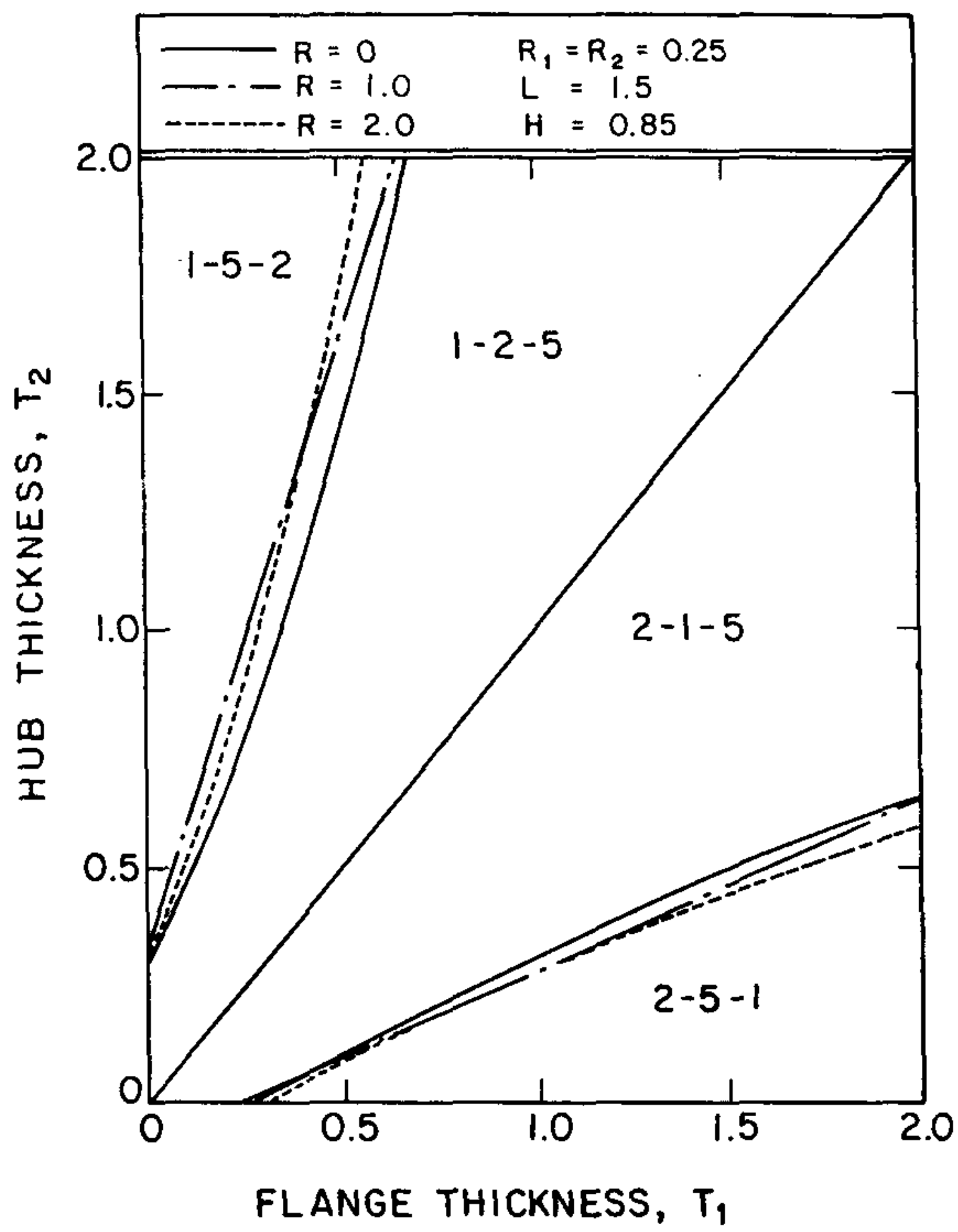


Fig. 18. Effect of variation in the hub diameter on the solidification order of X section A.

X Section B, Flange Legs and One Hub Leg Equal

A casting geometry of an X junction having three legs equal is presented in Figure 19. In this case the two flange legs and one hub leg are always of equal thickness. The analysis of the solidification order of this casting is similar to that described for T section B, Figure 7.

Variations in the size of the fillet radius, R_1 and R_2, on the solidification order of X section B is presented in Figure 20 The effect of

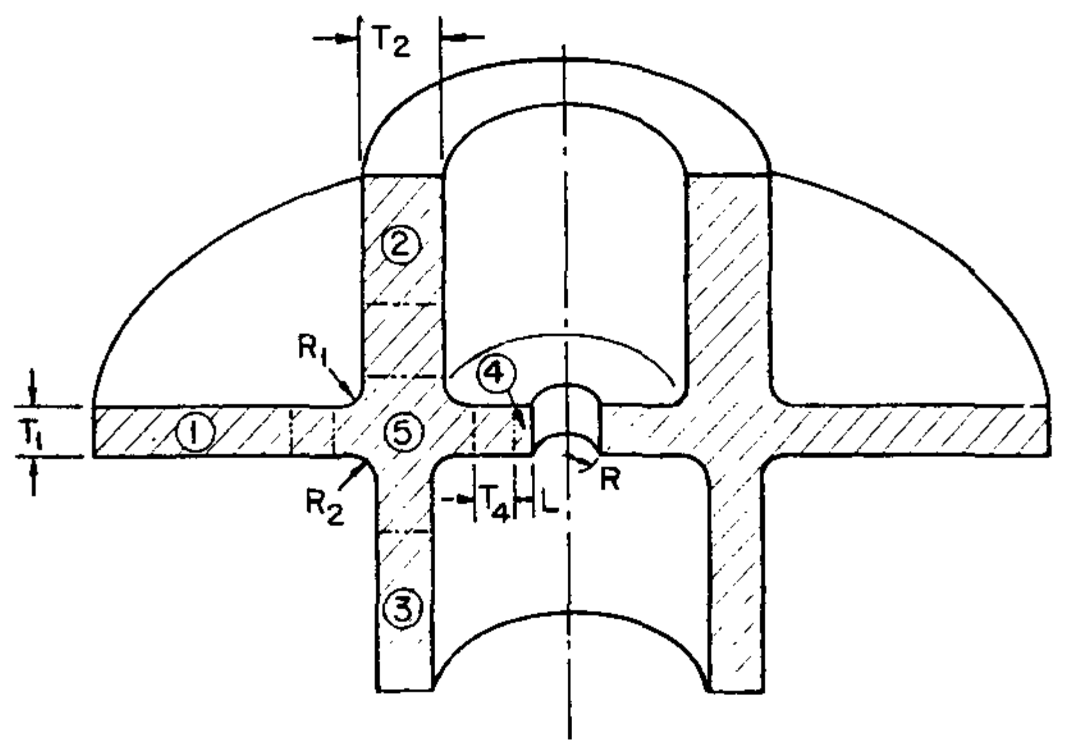


Fig. 19. Sketch of X section B where the two flange legs and one hub leg are maintained equal.

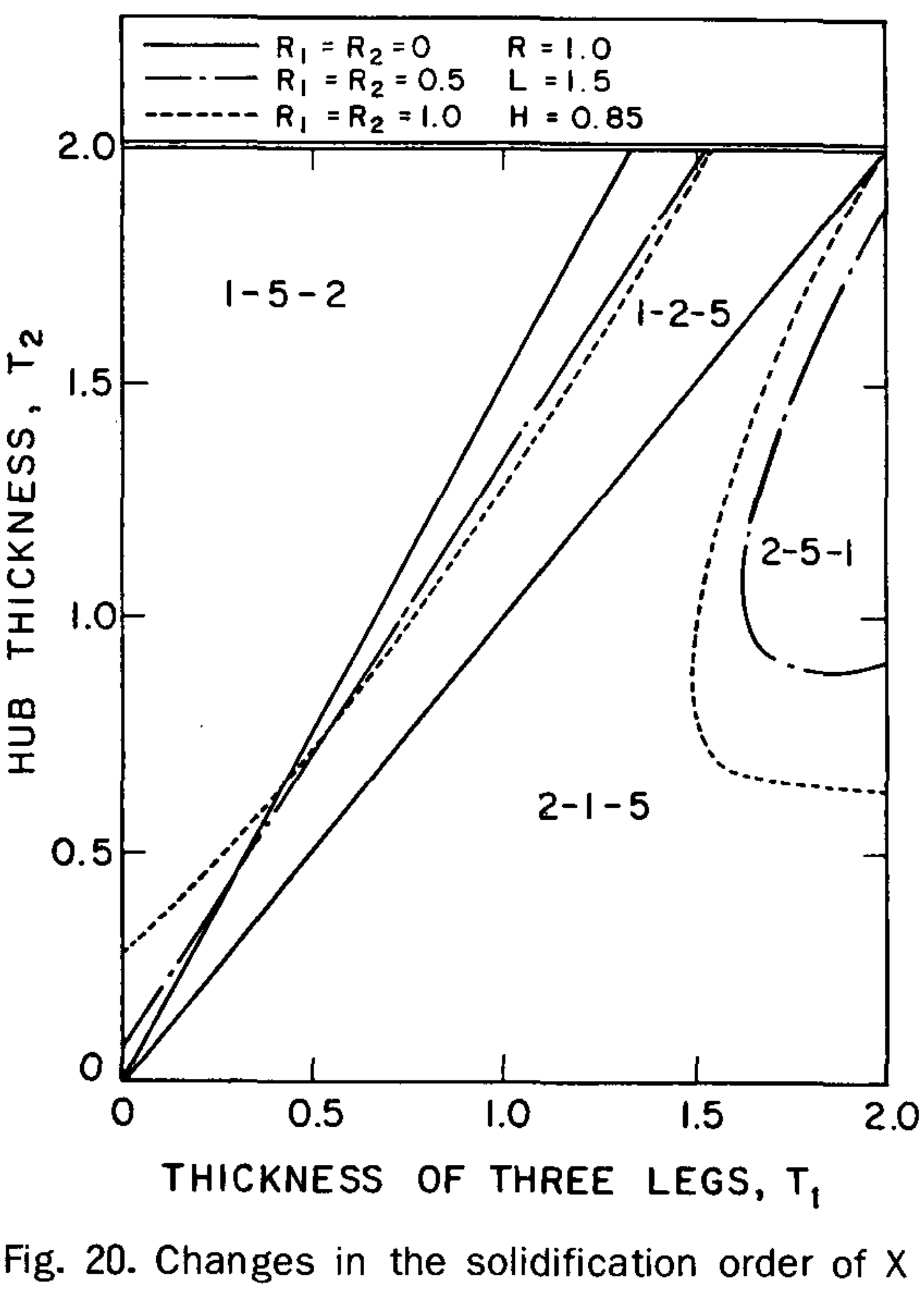


Fig. 20. Changes in the solidification order of X section B as a result of variations in the fillet radius.

changing the inside hub radius is shown in Figure 21.

For certain of these geometries, the influence of radius of curvature, or the inside diameter, is seen to be rather striking. In these cases a three dimensional analysis of the

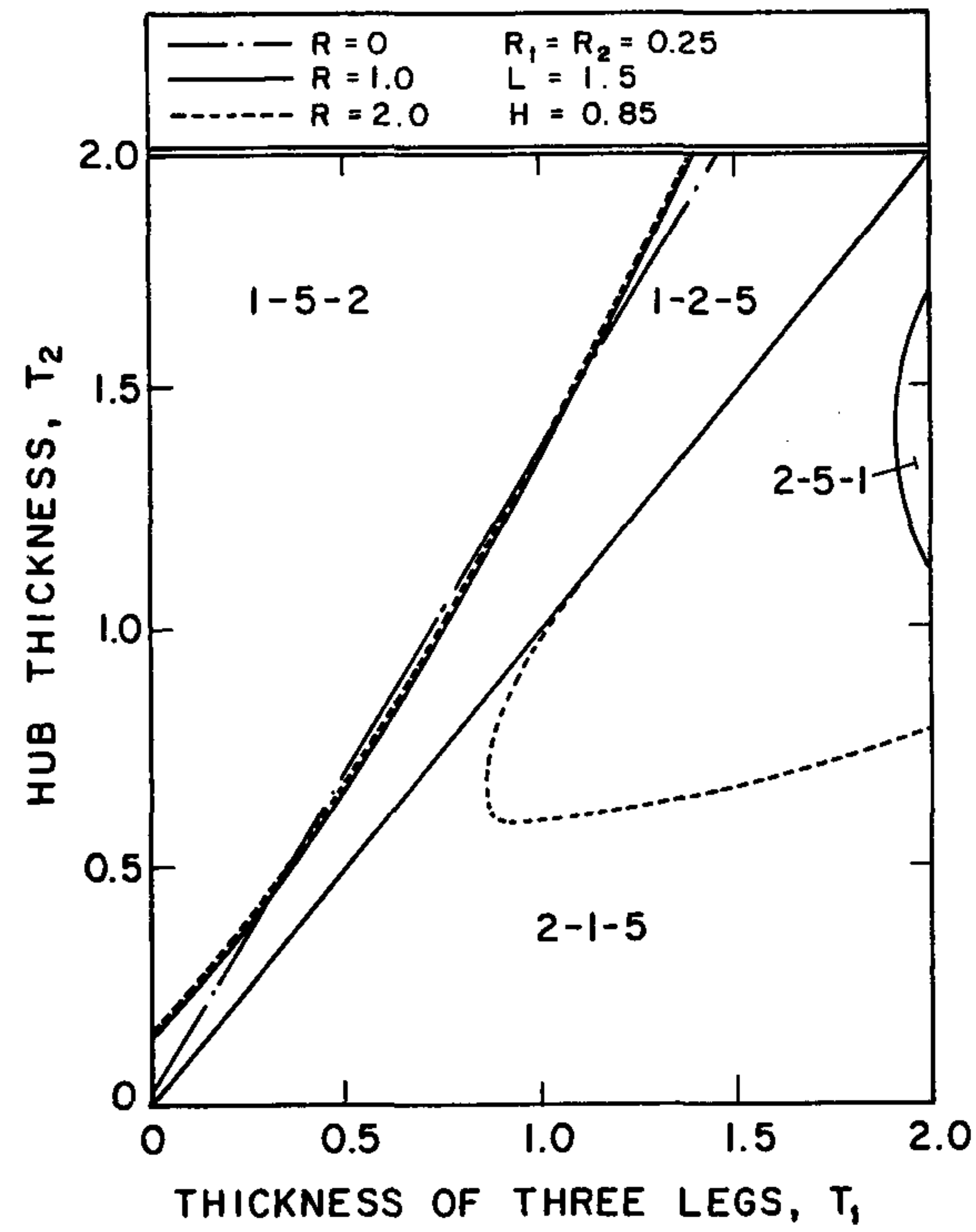


Fig. 21. Effect of variations of the inside hub diameter on the solidification order of X section B.

solidification order must be used.

5. Top riser on plate

A particularly troublesome geometry for casting is that of a boss on a plate, or of the use of a top riser on a plate shaped casting. The foundryman realizes that all too often the placement of a top riser on a plate section results in some feed metal transfer to the casting, but also causes shrinkage porosity just below the riser in the casting. An analysis of this casting geometry was made using the diagram presented in Figure 22. In this case the plate thickness is T_1, and the plate is considered to be semi－infinite, i.e., its section modulus is T/2 and edge effects can be ignored. The boss, or top riser, has a diameter, Dr, and is considered to be of limited height. The riser－plate junction, 3, is to be analyzed. Variables to be studied are the fillet

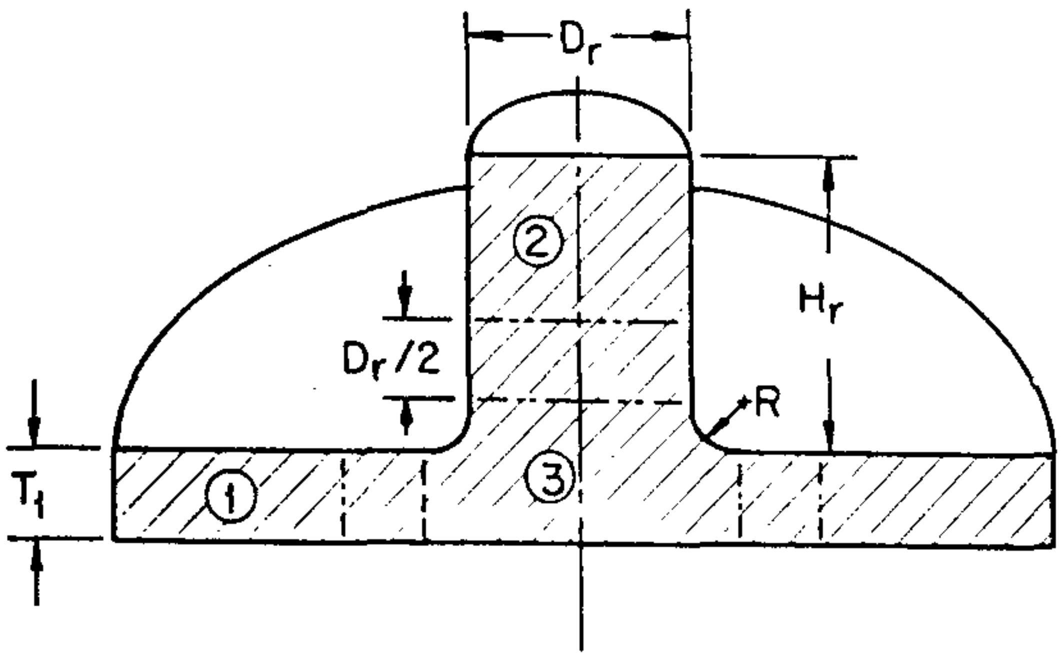


Fig. 22. Sketch of the configuration of a top riser attached to a flat plate.

radius of the connection, R, and the ratio of the riser height, HR, to the riser diameter. The height of the riser has been measured from the top of the plate casting to the top of the riser.

These geometric constraints on the analysis presented here must be understood thoroughly to appropriately read the graphical data generated. As the fillet radius is increased, the length of the riser included in the determination of the junction modulus is increased—and, the length of the remainder of the riser modulus, 2, is reduced.

The first effect to be considered is that of the effect of the fillet radius, R, Figure 23. In this case the ratio of the riser height to diameter has been maintained at 1.50. The influence of a radius at the base of the riser, such as that caused by using a slick to remove rough sand edges, can be quite strong in increasing the tendency for shrinkage to occur under the riser. (Note that Figure 23 presents data for three fillet radii, and that four solidification order regions are identified. Solidification order 1−3−2 is where the plate section solidifies first and the riser last, and is to the upper left of the three sets of curves. The solidification order 2−1−3 is located in the central part of the graph and is separated, for all three fillet radii, from the 1−3−2 region by the solidification order 1−2−3. The solidification order 2−3−1 is located to the lower right of all three sets of curves).

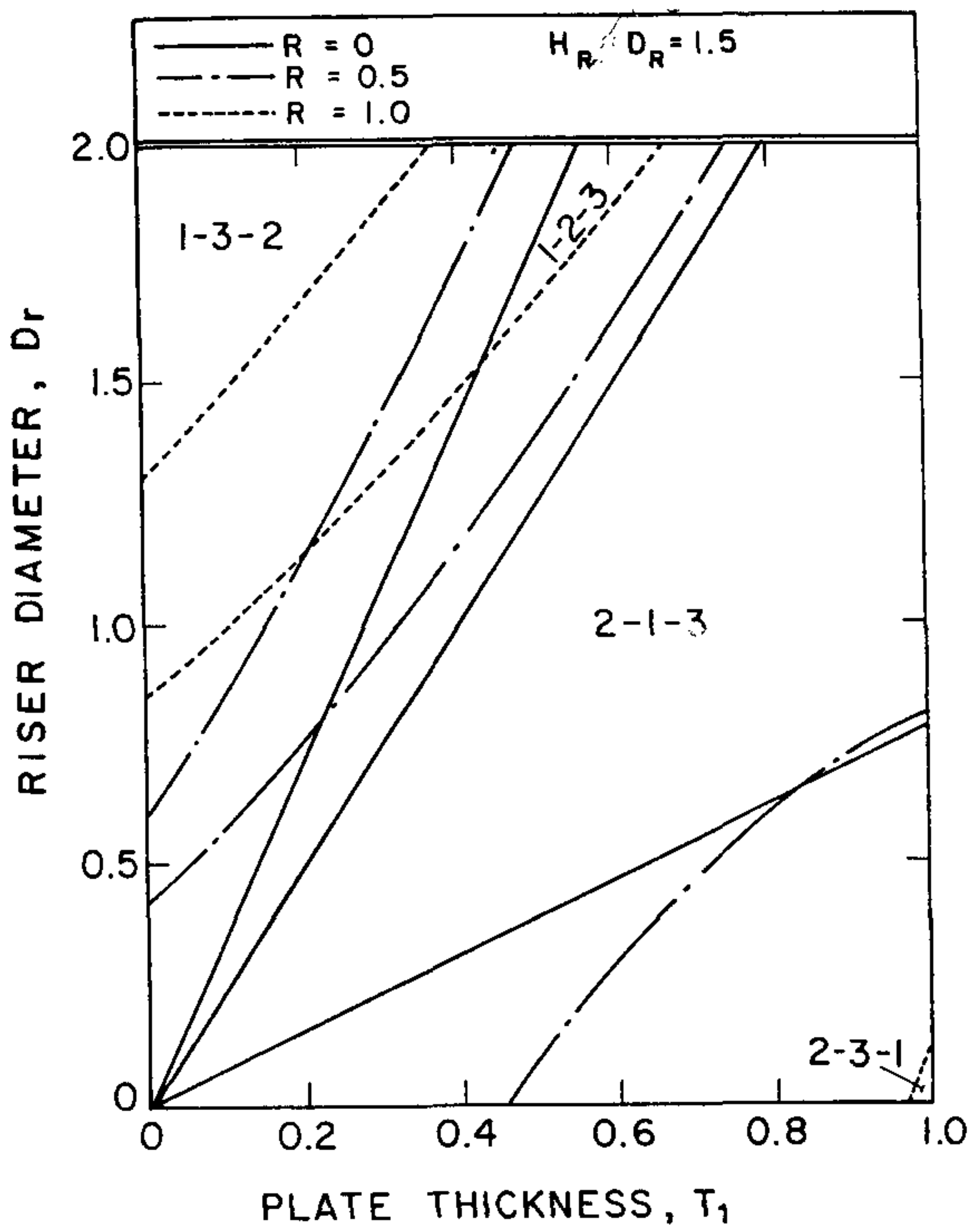


Fig. 23. Effect of plate thickness and top riser diameter on the solidification order as influenced by the fillet radius size.

Variations in the height to diameter ratio of the riser are shown in Figure 24. Increasing this ratio reduces the effect of the non−insulating riser top and broadens the region of favorable solidification ratio, 1−3−2.

It should be kept in mind that the lower right portion of Figures 23 and 24 refer to geometries where the plate section will freeze last certainly not desirable for top risers, but of great help where the geometry is that of a boss on a plate and where feeding is desired through the plate.

6. Side riser attached to a casting

Because of its significance to the casting process an analysis was also made of the geometry encountered with a side riser attached

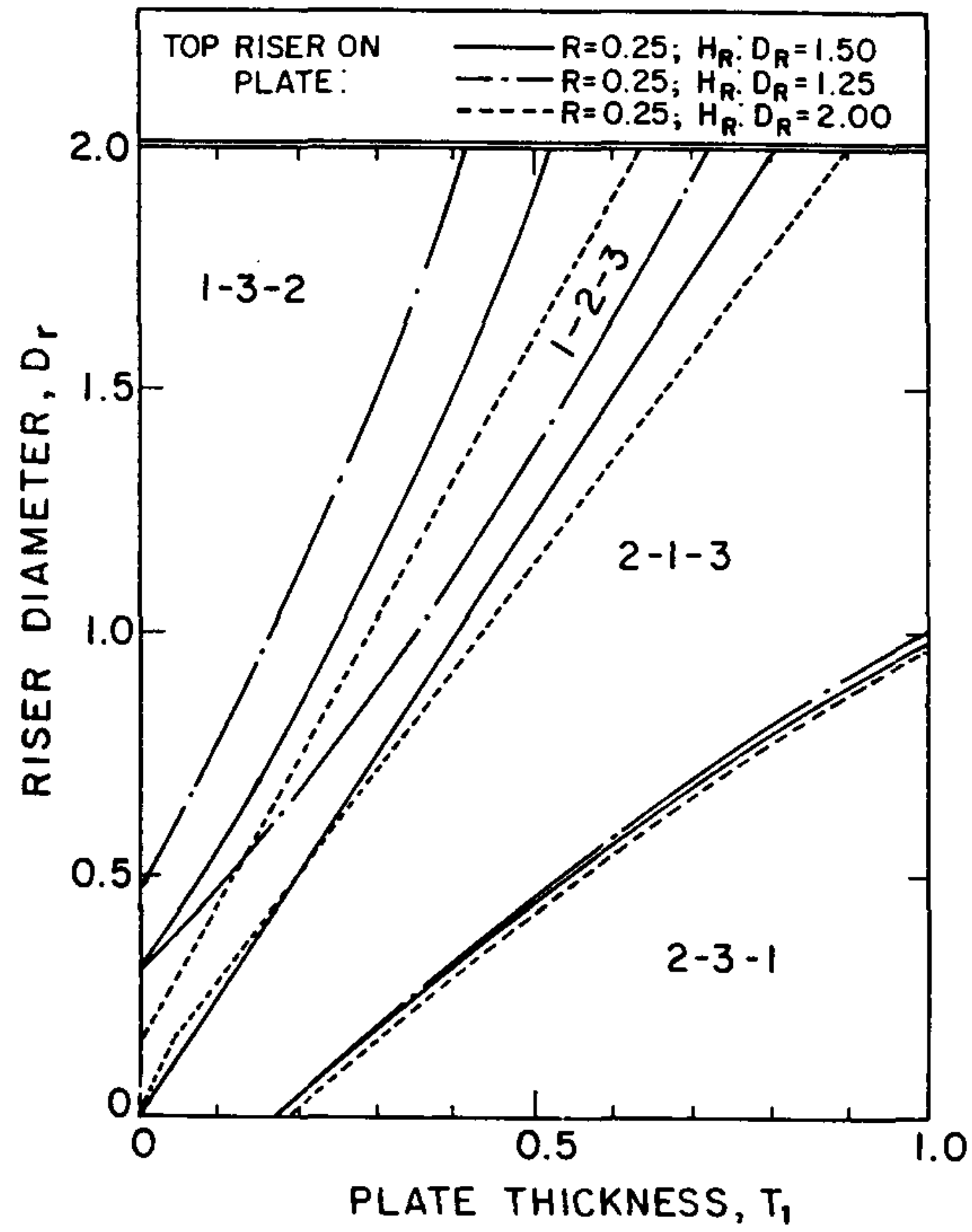


Fig. 24. Effect of the height to diameter ratio of the top riser, the riser diameter and the plate thickness on the solidification order.

to a plate section of the casting. Figure 25 presents a sketch of this junction. The plate section, 1, is considered to be semi-infinite with the contact of equal thickness to the plate and the width (or diameter) of the riser. In other words, the contact is merely an extension of the plate t the riser. The riser section, 2, is considered to be of a limited height, HR, and that it is a "blind" riser, i.e., there is molding media over the top of the riser so that heat is transferred out through the top as well as through the side walls and bottom of the riser. This is similar to the riser construction considered for the top riser discussed previously. The junction of the riser and the plate, 3, is to be analyzed with variations in the size of the fillet radius and the height to diameter ratio of the riser.

It is important to realize how the variables to be considered affect the geometry of the riser and riser connection. Note for a fillet

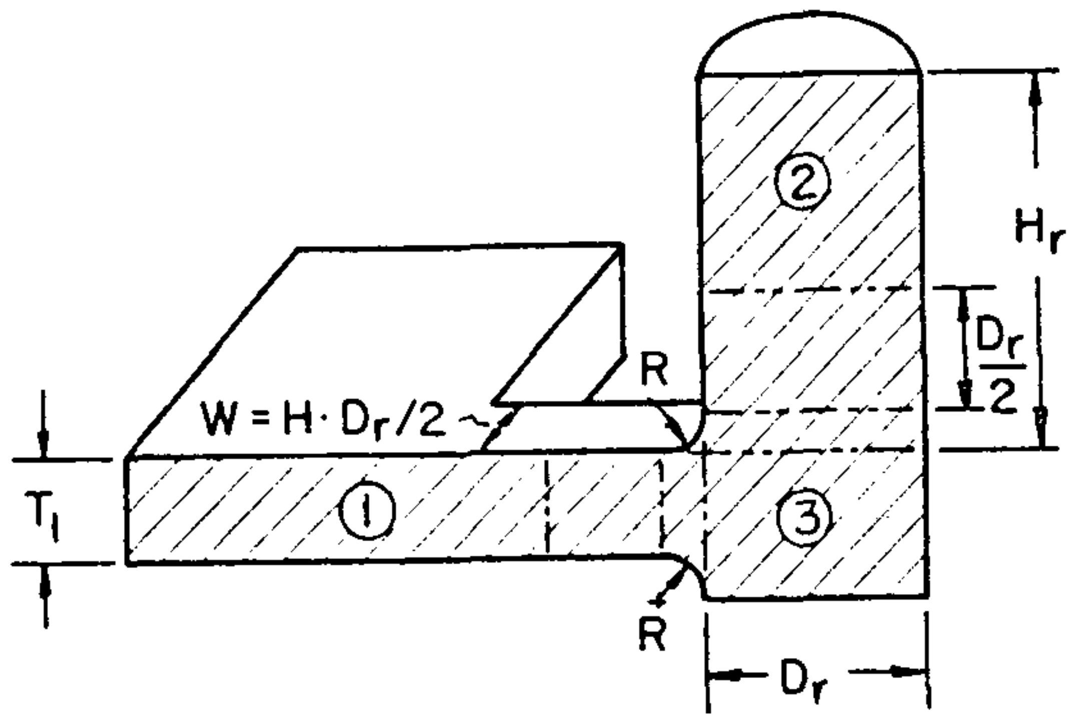
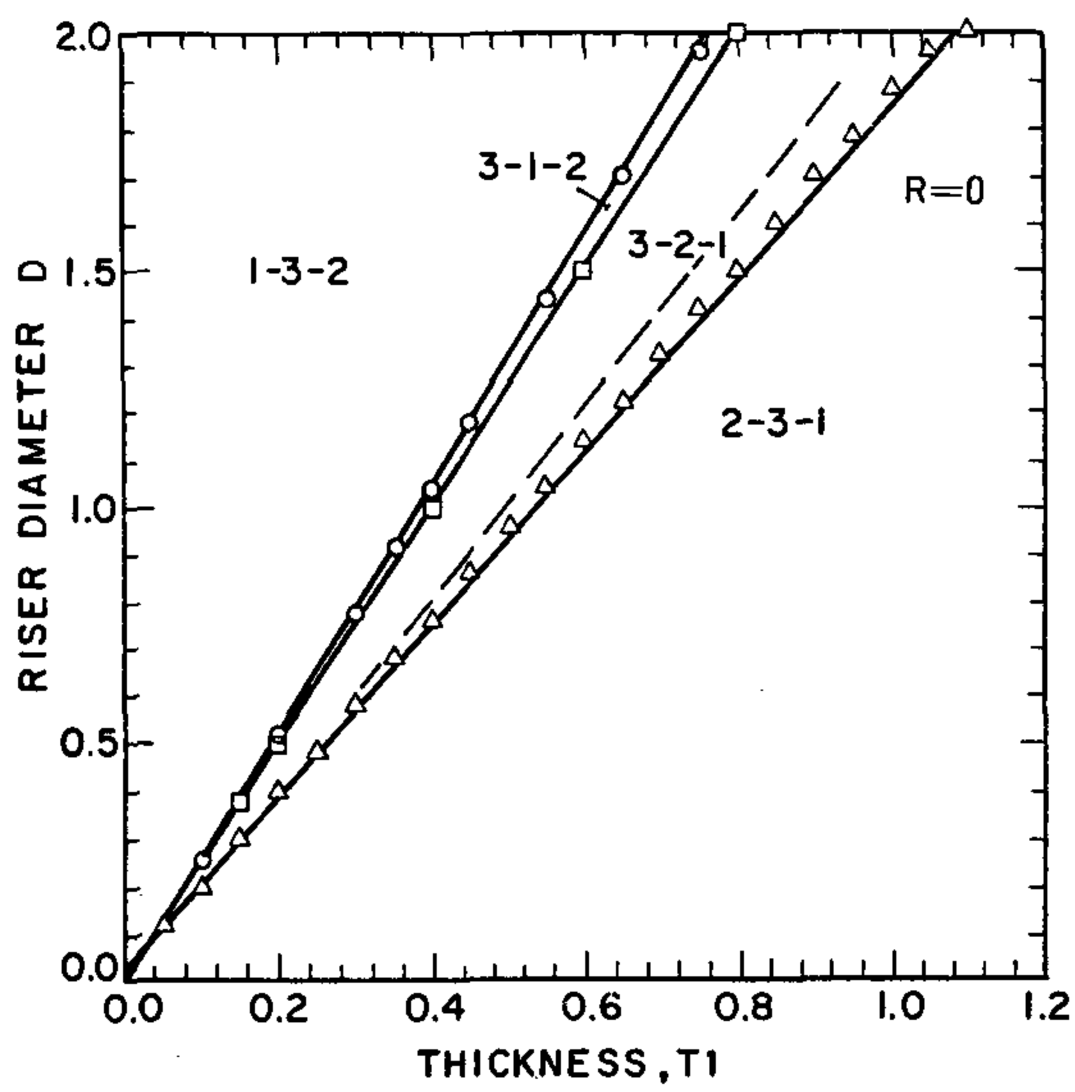


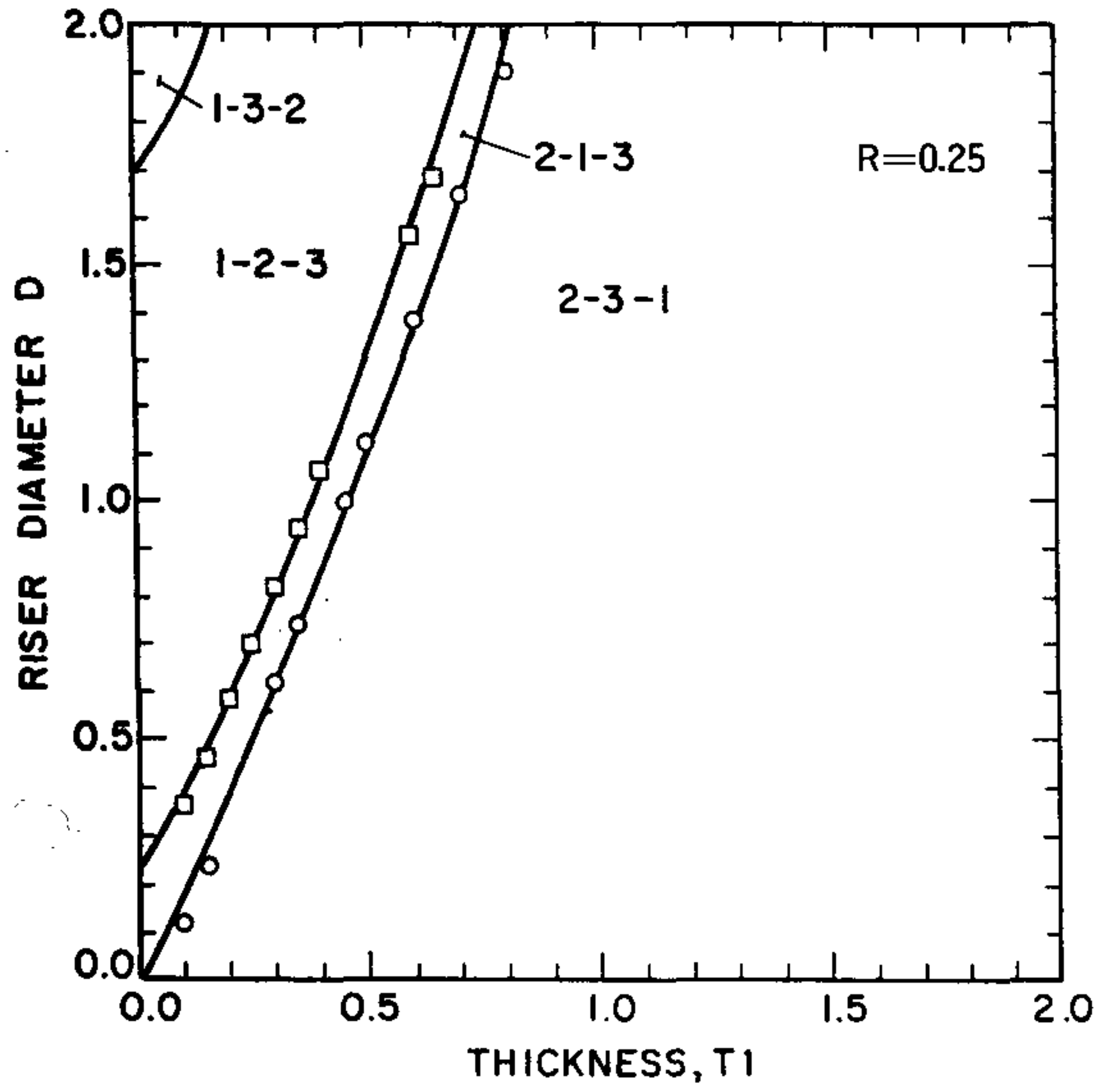
Fig. 25. Geometry of the side riser attached to a plate casting.

radius of zero, there is no riser base, i.e., the riser does not extend below the connection. Also note that as the fillet radius is increased, the depth of the base is increased. In this case, then, the riser base depth is a function of the fillet radius. Also note that the height of the riser is set from the top of the riser connection and that increasing the fillet radius does not increase the height of the riser.

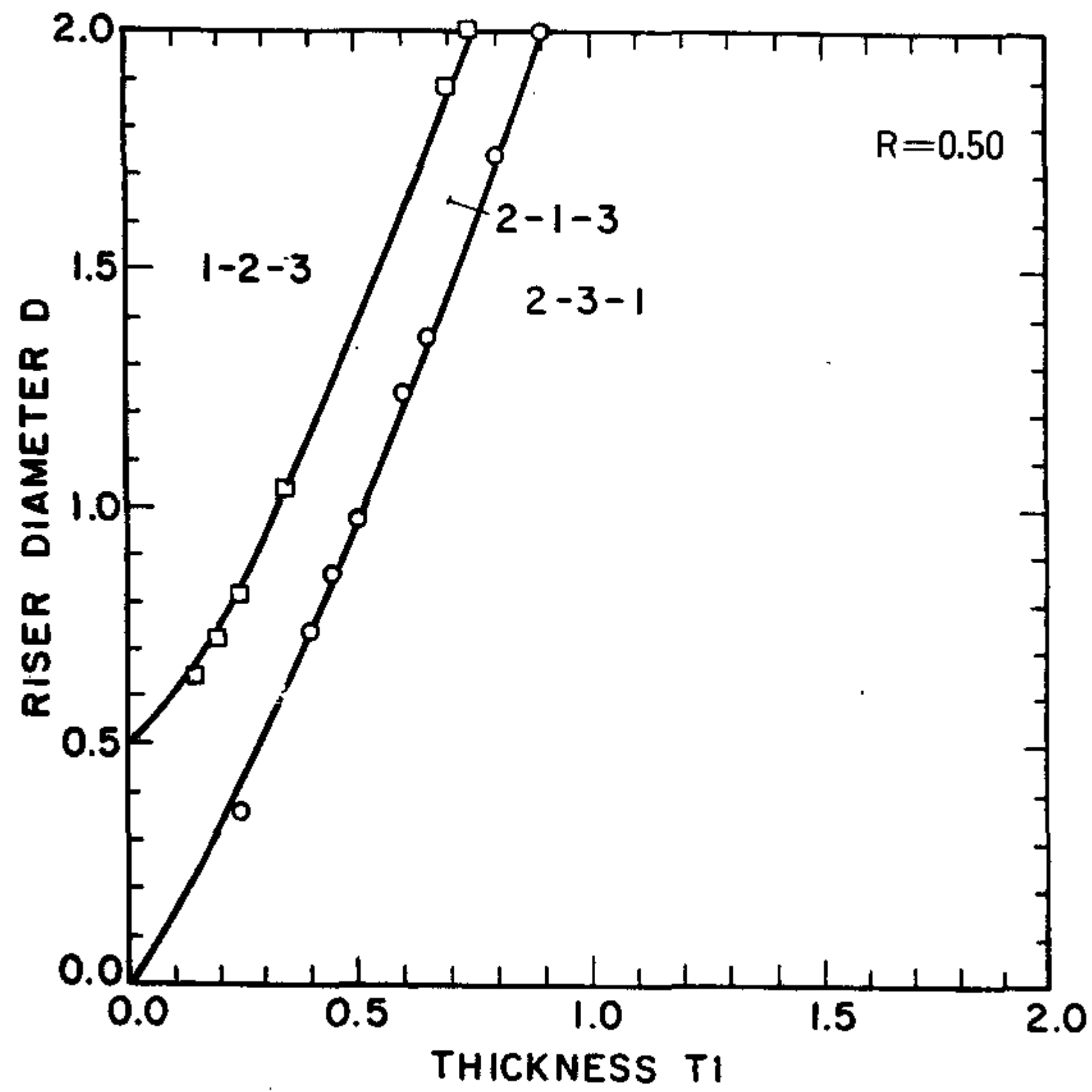
Just two cases of variations in geometry have been considered here. The effect of increasing the size of the fillet radius from R=0 to R=1.0 is demonstrated by Figures 26 a, b, c and d. Note that an increase in the size of



a)

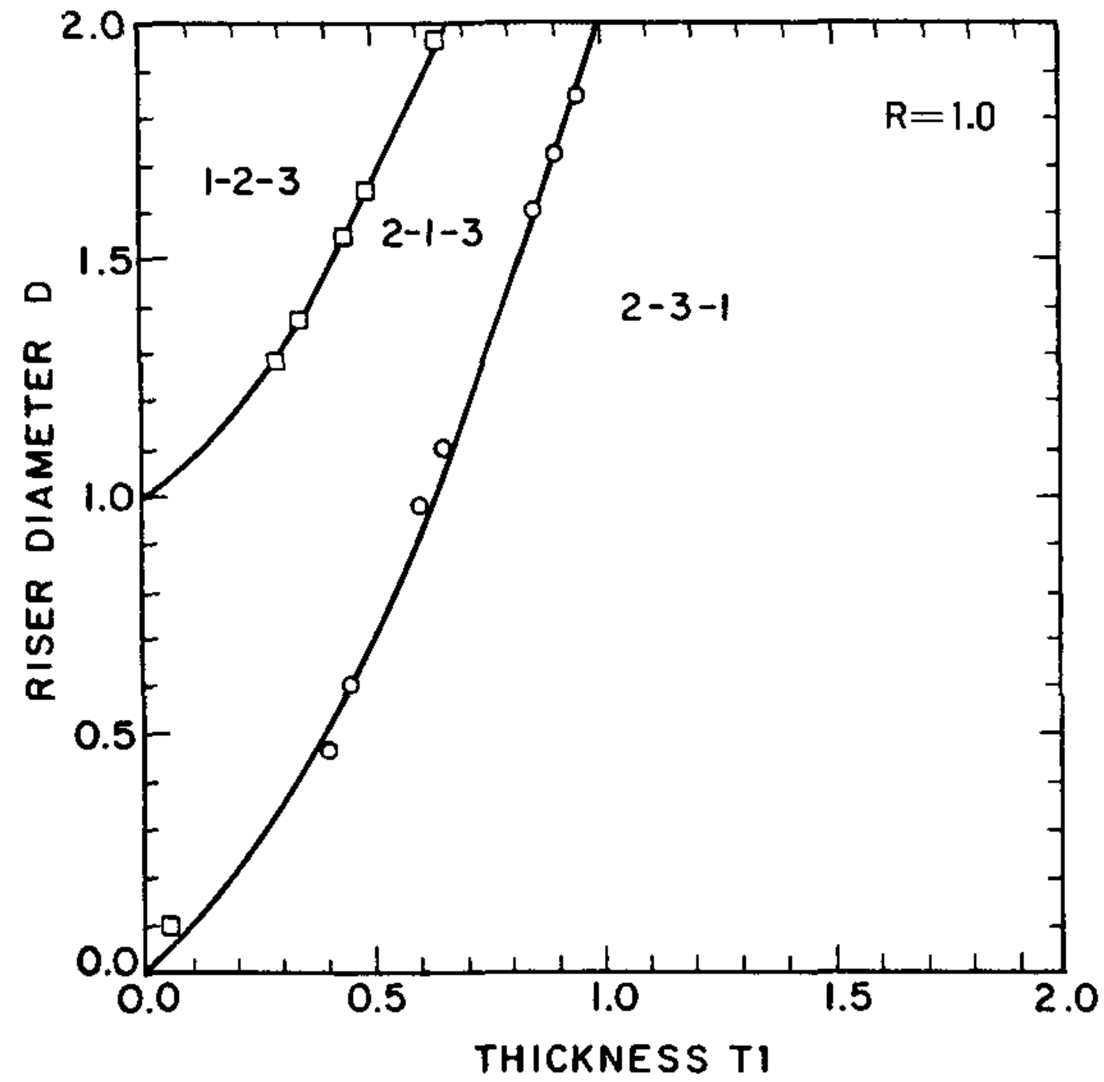


b)



c)

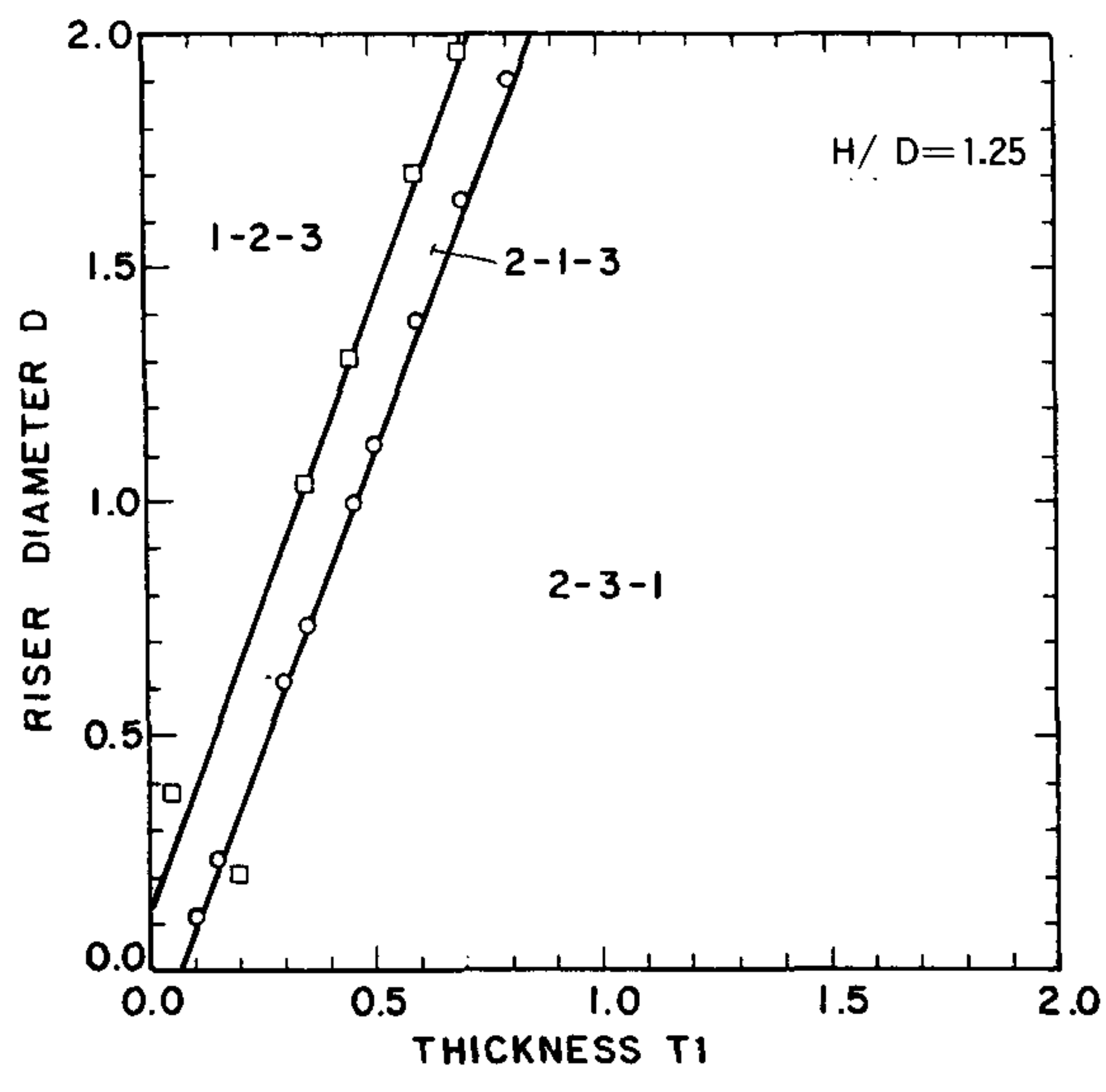
the fillet radius (along with related geometrical effects) results in a severe limitation of the 1−3−2 region of solidification order. In this region, the solidification of the casting is followed by solidification of the contact and then the riser. However, under the conditions of the geometry selected, increasing the fillet radius also increased the modulus of the connection but decreases the influence of the riser since the riser height is limited.



d)

Fig. 26. Effect of riser diameter and plate thickness on the solidification order for various fillet radii: a) R=0 in., b) R=0.25 in., C) R=0.50 in., d) R=1.0 in. (constant H/ D=1.50).

This effect can be appreciated by comparing Figures 27a, 26b, 27b and 27c where the ratio of the height of the riser to the diameter of the riser has been increased (1.25, 1.50, 2.0 and 2.5). Incerasing the value of this ratio causes the 1−3−2 solidification order to become attainable.



a)

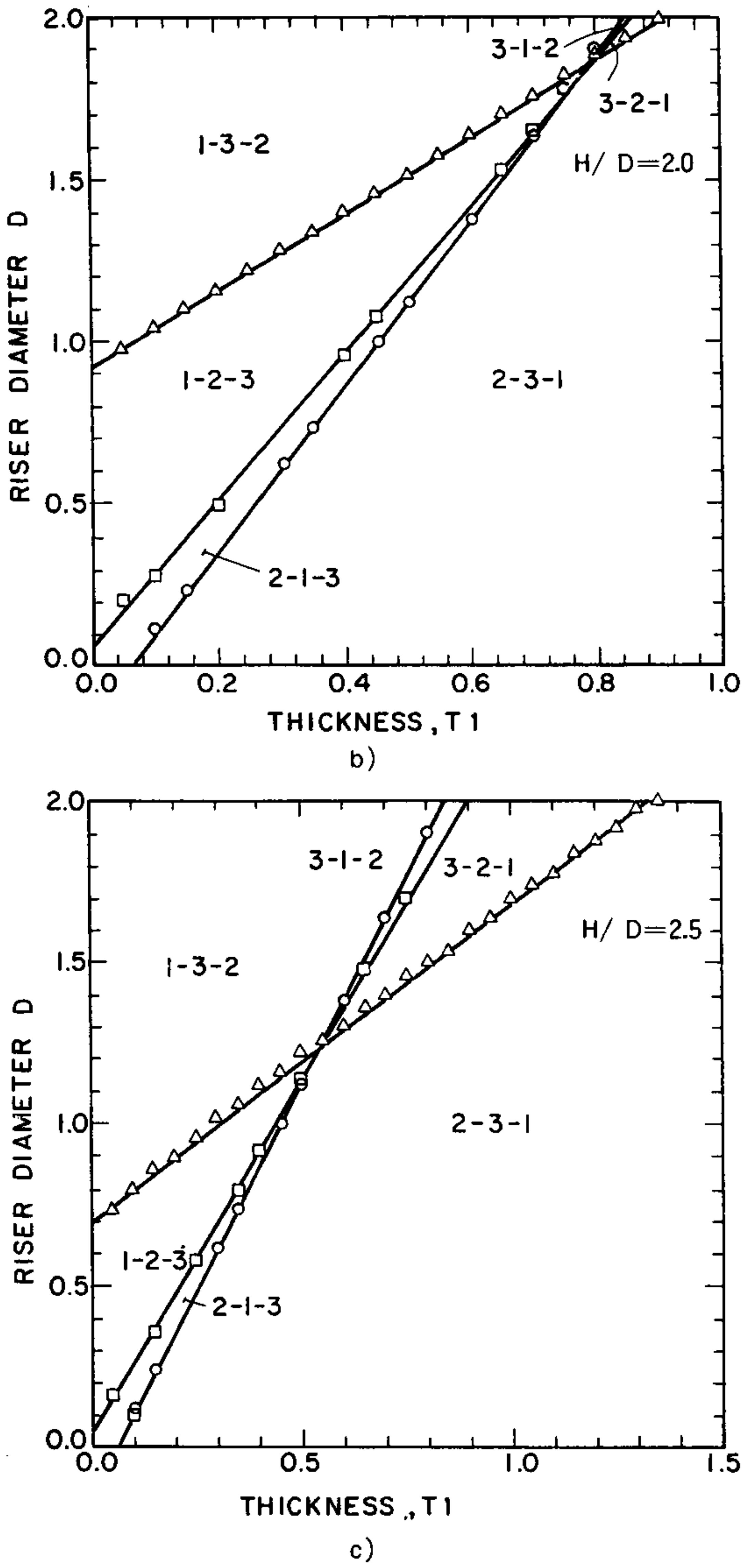


Fig. 27. Effect of riser diameter and plate thickness at a constant fillet radius, R=0.25 in on the solidification order: a) H/ D=1.25, b) H/ D=2.0, c) H/ D=2.5

7. Conclusion

While the three dimensional analysis of the solidification order of common casting shapes is quite complicated, the results of its application to a few simple geometries has demonstrated that a more straightforward two dimensional analysis will suffice in most situations all except the smaller castings, or castings where the radius of curvature is a value near that of the casting wall thickness.

Further development of the technique is required for both top and side riser geometries.

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