

# Improvement of the Accuracy of Supershort Baseline Acoustic Positioning System by Kalman Filter

Hae-Hoon PARK and Gab-Dong YOON

*Department of Fishing Technology, National Fisheries University of Pusan,  
Pusan 608-737, Korea*

Underwater acoustic navigation and position fixing systems have been extensively used not only in surface position fixing but also in underwater position fixing. In recently, application of these systems has been in the field of underwater inspection of offshore platforms, where it is vital to track the position of an unmanned submersible or diver carrying underwater cameras and nondestructive testing equipment. But these systems are included the fixing errors as results of a signal with additive noise, the attenuation of sound and the interference effects due to multipath reflection and forward scattering.

In this paper to improve the position fixing by the supershort baseline acoustic position system, a method to apply the Kalman filter to the fix of the system is proposed and the digital simulation under noise condition is conducted. The optimal positions by the Kalman filter are compared with original positions, and it is confirmed that the results of the proposed method are evidently more accurate.

## Introduction

An accurate position determination of vehicles has an importance in underwater search and navigation.

Acoustic navigation technics, utilizing bottom moored acoustic reference such as transponders or beacons can provide very precise relative positioning of both surface and subsurface objects. In recently, application of these systems has been in the field of underwater inspection of offshore platforms etc.(Milne, 1983; Nakanishi, 1988).

There are several acoustic systems for determining the position of a surface vessel relative to a reference point on the sea floor (Robert, 1985; Milne, 1983; Vestgaard et al., 1978).

The first, called the short-baseline (SBL) system, uses only a single free running sea floor beacon and the vessel position relative to the beacon is determined from a measurement of the beacon pulse arrival times at each of the three vessel-mounted hydrophones which are normally arranged

to form two orthogonal axes. To increase the accuracy of this system, three vessel-mounted hydrophones should be spaced as far apart as practical.

The second is the long-baseline (LBL) system which measures the slant range using three widely spaced transponding beacons on the sea floor. The LBL system requires that the positions of three beacons be precisely determined in terms of their ranges and true bearings with respect to sea floor reference point. Then, instantaneous ranges from the vessel to the transponders are determined by measuring the two-way propagation delays.

The final, called the supershort baseline (SSBL) system, is similar to the SBL system, but the vessel-mounted hydrophones are contained in a single housing. The SSBL system relies on a phase difference or phase comparison of the acoustic signals received by three closely spaced sensors. With SSBL system four position-computing modes are available, that is, beacon mode, transponder mode, responder mode and transponder/responder mode with depth.

The vehicle position which must be determined from the measured quantities through some nonlinear relationship, is usually inaccurate due to the received signal with additive noise, the sound speed assumed in the measurement and irregular motion of vessel by ocean waves.

In this paper to improve the position fixing by the SSBL acoustic system with beacon mode, a method to apply the Kalman filter to the fix of the system is proposed and the digital simulation under noise condition is conducted.

### SSBL system with beacon mode

The SSBL system with beacon mode is shown in Fig. 1.

Fig. 2 illustrates how the difference in the phase of an acoustic signal carrier frequency,  $f$ , received by two sensors spaced,  $b$ , apart relates to the mechanical angle of incidence,  $\theta$ , in the SSBL system. The mechanical angle is in the plane formed by the acoustic source and the two sensors. If the baseline length,  $b$ , is always much less than the distance to the signal source, the acoustic wave fronts are essentially planar and the relationship between electrical phase and mechanical angle of incidence becomes:

$$dT = k \cos \theta \tag{1}$$

where  $dT$  = electrical phase angle  
 $\theta$  = mechanical incidence angle

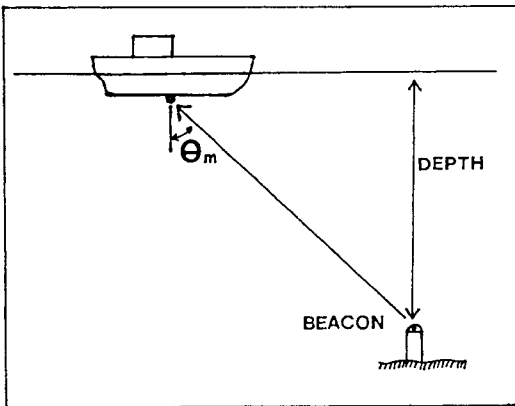


Fig. 1. The SSBL system with beacon mode.

- $k$  = stiffness factor ( $2\pi fb/c$ )
- $b$  = baseline length between sensors
- $c$  = underwater velocity of sound

Solving equation (1) for  $\theta$  gives the mechanical angle as a function of the electrical phase, therefore

$$\theta = \cos^{-1} dT/k.$$

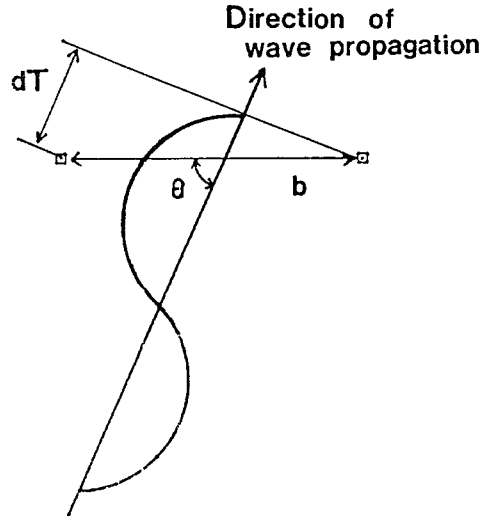


Fig. 2. Phase delay of acoustic wave as a function of mechanical angle of incidence at the hydrophone.

The  $\theta$  is the vertex angle of a conical surface with axis formed by the baseline of the two sensors. If three sensors on the hydrophone array lie on two orthogonal baselines, the direction of the acoustic signal will lie at the intersection of two conical surface with coincident vertices where the mechanical angles of incidence are  $\theta_{mx}$  and  $\theta_{my}$ . To determine the position of the beacon in 3-D as shown in Fig. 3, the vertical separation between the hydrophone and the beacon  $h$  must be known. It should be noted that the angles of incidence  $\theta_{mx}$  and  $\theta_{my}$  do not necessarily lie in the XZ and YZ plane of the array and a conversion may therefore be required before the results represent horizontal displacements away from the reference beacon. From geometrical consideration in the Fig. 3,

$$X_a = \frac{h \cos \theta_{mx}}{\sqrt{(1 - \cos^2 \theta_{mx} - \cos^2 \theta_{my})}}$$

and

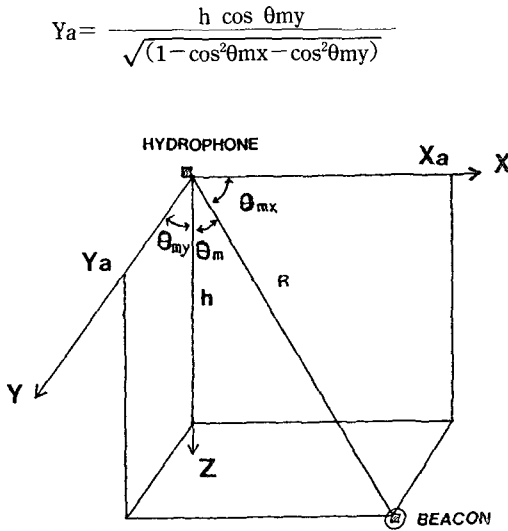


Fig. 3. Geometry of the beacon/transponder position reference.

### Optimal estimation by Kalman filter

In this section the problem of obtaining the optimal estimate of the relative position,  $X_a$ ,  $Y_a$ , based on a set of noisy measurements is considered via filtering.

We shall want to estimate the relative position, velocity, and acceleration of a ship in the  $x$ ,  $y$  rectangular coordinates respectively. These signal variables will be arranged in a column to be defined as the signal vector  $X(k)$  at time  $k$ . Measurements of only position in the  $x$ ,  $y$  coordinates are made with additive noise.

On the assumption that the ship moves on the two dimensional plane, the vehicle dynamics can be modelled by the following sixth order linear equation (Jang, 1988; Usagawa et al., 1987):

the process equation:

$$X(k) = A X(k-1) + w(k)$$

the measurement equation:

$$Y(k) = C X(k) + v(k)$$

where  $w(k)$  is a process noise sequence which is assumed to be gaussian;  $Y(k)$  is a position measurement that is corrupted with a measurement noise sequence  $v(k)$ , and

$$A = \begin{bmatrix} 1 & T & T^2/2 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & T^2/2 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$v(k) = \begin{bmatrix} v1(k) \\ v2(k) \end{bmatrix} \quad w(k) = \begin{bmatrix} 0 \\ 0 \\ w1(k) \\ 0 \\ 0 \\ w2(k) \end{bmatrix}$$

$T$  = sampling interval.

For the above model described by the state equations, the vector Kalman filter can be written as follows (Bozic, 1979):

Estimator:

$$\hat{X}(k) = A \hat{X}(k-1) + K(k)[Y(k) - C A \hat{X}(k-1)]$$

Filter gain:

$$K(k) = P1(k)C'[C P1(k)C' + R(k)]^{-1}$$

$$\text{Where } P1(k) = A P(k-1)A' + Q(k-1).$$

Error covariance matrix:

$$P(k) = [I - K(k) C(k)]P1(k) [I - K(k) C(k)]' + K(k) R(k) K(k)'$$

where,  $R(k) = E[v(k) v(k)']$

$$Q(k) = E[w(k) w(k)']$$

$$P(k) = E[\{X(k) - \hat{X}(k)\}\{X(k) - \hat{X}(k)\}']$$

' = transpose

$E[ ]$  denotes the expectation of  $[ ]$

$\hat{X}(k)$  is the estimate of  $X(k)$ .

### Results

Fig. 4 is the block diagram which shows how the ship position can be optimally estimated from a set of noisy measurements in this simulation.

The phase angle of a time signal can be determined from the Fourier transform which is a complex quantity. In digital system the fast Fourier transform (Brigham, 1974) is an algorithm that efficiently computes the discrete Fourier transform, and its basic computation is referred to as a butterfly computation when the number of sample data is an

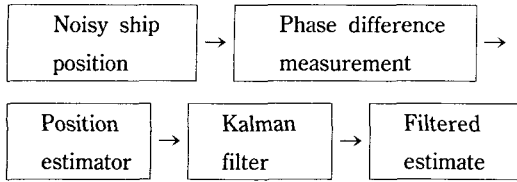


Fig. 4. Block diagram of the simulation.

teger power of 2. And because the speed of sound at sea changes with water temperature, salinity and pressure, it was calculated by means of the following equation (Clay and Medwin, 1977):

$$c = 1,449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.10T)(S - 35) + 0.016Z$$

where  $c$ =sound speed (m/s),  $T$ =temperature ( $^{\circ}$ C),  $S$ =salinity (ppt) and  $Z$ =depth (m).

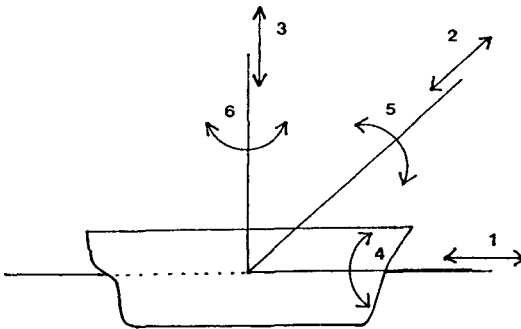


Fig. 5. Translatory and angular motions.  
 1. surge 2. sway 3. heave  
 4. roll 5. pitch 6. yaw

A ship advancing at a steady mean forward speed in a seaway will move in six degrees of freedom as shown in Fig. 5. The ship's motion can be considered to be made up of three translational components, surge, sway and heave, and three rotational components, roll, pitch and yaw (Beck, 1989; Haraguchi, 1990).

For the usual case of an unrestrained ship with port/starboard symmetry in low to moderate sea states, the longitudinal motions (surge, heave and pitch) are uncoupled from the transverse motions

(sway, roll and yaw). Among them surge and sway motions are of our interest. In this simulation it is assumed that the ratio of wave length to ship length is less than 1 and sea state is moderate. And the maneuver-noise covariance matrix is also assumed by

$$Q(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^{-4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-4} \end{bmatrix}$$

The noises  $v1(k)$ ,  $v2(k)$  which are assumed zero-mean gaussian, uncorrelated with each other, are pseudorandomly generated by independently corrupting true values with variances 14.29, 11.42 respectively.

Fig. 6 shows simulation results for the cases that ship speed is zero and 10 m/s in x direction only. The initial vehicle position in Fig. 6-(A) is at [100, 30, 170] and in Fig. 6-(B) at (300, 30, 170) in x, y, z coordinates. The assumed initial position is [250, 50, 170] in both cases.

It indicates that the filtered estimate approaches the mean position of the vehicle as the iteration number increases. The variances of the filtered position in Fig. 6 were reduced from 14.29, 11.42 to 4.74, 4.35 respectively.

### Conclusion

Acoustic navigation technics utilizing bottom moored acoustic reference can provide very precise relative positioning of both surface and subsurface objects.

A position based upon a set of noisy measurements was calculated by a supershort baseline acoustic system with beacon mode and estimated via Kalman filtering. The estimated position via the filtering was evidently more accurate. The variances of the filtered estimate were reduced from 14.29, 11.42 to 4.74, 4.35 respectively.

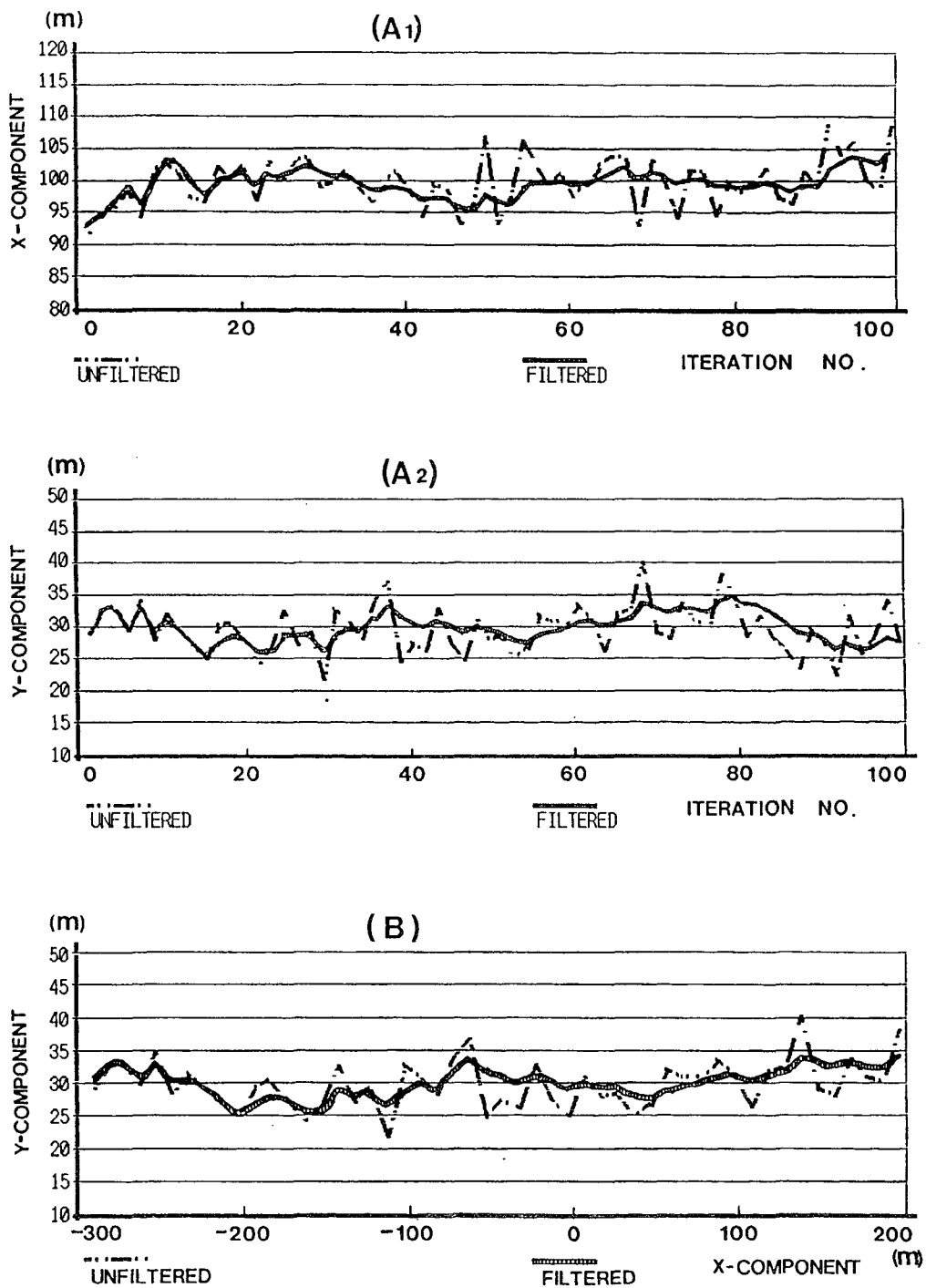


Fig. 6. Simulation results in cases that ship speed is zero in (A) and 10 m/s in (B).  
 A: X-component in (A1) and Y-component in (A2).  
 B: X-Y position of unfiltered and filtered.

References

- Beck, R. F. 1989. Ship responses to regular waves. *In* Principles of Naval Architecture. Second revision. Volume III, Motions in Waves and Controllability. V. L. Lewis, ed., The society of Naval Architects and Marine Engineers, pp. 41~45.
- Bozic, S. M. 1979. Digital and Kalman Filtering. Edward Arnold. London, pp. 115~119.
- Brigham, E. O. 1974. The Fast Fourier Transform. Prentice-Hall. Engle Cliffs, NJ, 252p.
- Clay, C. S. and H. Medwin. 1977. Acoustical Oceanography: Principles and Applications. Wiley-Interscience, New York, pp. 3~4.
- Haraguchi, T. 1990. On Measurement Results of Six Degrees Motions of Model Vessels Using two Video Trackers. J. Japan Institute of navigation, 83, 21~28 (in Japanese).
- Jang, Y. S. 1988. Orbit Estimation of a Moving Underwater Sound Source using Kalman Filter algorithm. MS thesis, Dep. of Electronic Eng., National Fisheries University of Pusan. (in Korean).
- Milne, P. H. 1983. Underwater Acoustic Positioning Systems. Gulf Pub. Co. Houston, 284p.
- Nakanishi, T. 1988. Underwater Acoustic navigation System for '6,500 m Deep Research Submersible System'. Navigation. 98, 84~95 (in Japanese).
- Robert, J. L. 1985. A Multimode Acoustic Position Indicator for Greater Accuracy and Reliability. Proc. 17th Ann. Offshore Technology Conference, Houston, 313~321.
- Usagawa, T., S. Nishimura, M. Ebata and J. Okda. 1987. Analysis of a moving sound source—Orbit estimation using microphone array—. J. Acoust. Soc. Japan, (E) 8, 23~28.
- Vestgaard, K. and K. Hansen. 1978. Super Short Baseline Hydroacoustic Navigation System. Proc. 10th Ann. Offshore Technology Conference, Houston, 449~454.

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