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SEMI-INVARIANT SUBMANIFOLDS WITH HARMONIC CURVATURE*

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0. Introduction

Semi-invariant submanifolds of a Kaehlerian manifold have been studied by Blair and Ludden [2], Ki [18], Tashiro [17], Yano [2], [18] and others. It is well known that the almost contact metric structure is induced on these submanifolds [18].

On the other hand, some studies about the non existence for the real hypersurfaces of a complex space form under the natural linear condition which can be imposed on the Ricci tensor S' or $\nabla S'$ have been made by Ki, [7], Ki, Nakagawa and Suh [8], Kim [11], Kimura [13], Kon [14] and Montiel [15], etc. In particular, it is proved in [8] that there are no real hypersurfaces with harmonic curvature of a complex space form.

By the way, the second author of the present paper [10] investigated that there are no semi-invariant submanifolds of codimension 3 with the parallel Ricci tensor of a complex hyperbolic space on which the distinguished normal is parallel in the normal bundle.

The purpose of the present paper is to extend the above result [10].

We show that there are no semi-invariant submanifolds of codimension 3 with harmonic curvature of a complex space form on which the distinguished normal is parallel in the normal bundle.

1. Preliminaries

Let $\bar{M}$ be a real $(2n+2)$-dimensional Kaehlerian manifold covered by a system of coordinate neighborhoods $\{\bar{U}\,;\,y^A\}$ with almost complex structure J and Riemannian metric G, where J is parallel and G is

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J-Hermitian.

Let M be a real $(2n-1)$-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{V ; x^h\}$ and immersed isometrically in $\overline{M}$ by the immersion $i : M \rightarrow \overline{M}$. The immersion is represented locally by $y^A = y^A(x^h)$. We now put $B_i{}^A = \partial_i y^A$, $\partial_i = \partial/\partial x^i$, then $B_i = (B_i{}^A)$ are $(2n-1)$-linearly independent vectors of $\overline{M}$ tangent to M. Three mutually orthogonal unit normals to M will be denoted by C^A, D^A *and* E^A.

Denoting by ∇_j the operator of van der Waerden-Bortolotti covariant dfferentiation with respect to the induced Riemannian metric, the equation of Gauss for M of $\overline{M}$ is obtained:

$$\nabla_j B_i{}^A = h_{ji} C^A + k_{ji} D^A + l_{ji} E^A, \tag{1.1}$$

where h_{ji}, k_{ji} and l_{ji} are the components of the second fundamental forms in the direction of normals C^A, D^A and E^A respectively.

The equations of Weingarten are also given by

$$\begin{cases} \nabla_j C^A = -h_j{}^h B_h{}^A + l_j D^A + m_j E^A, \\ \nabla_j D^A = -k_j{}^h B_h{}^A - l_j C^A + n_j E^A, \\ \nabla_j E^A = -l_j{}^h B_h{}^A - m_j C^A - n_j D^A, \end{cases} \tag{1.2}$$

where $h_j{}^h = h_{jt} g^{th}$, $k_j{}^h = k_{jt} g^{th}$, $l_j{}^h = l_{jt} g^{th}$, l_j, m_j and n_j being the components of the third fundamental tensors and $(g^{ji}) = (g_{ji})^{-1}$.

The vector field C is said to be *parallel* in the normal bundle if $\nabla_j{}^{\perp} C = 0$, that is, l_j and m_j vanish identically.

On the other hand, a submanifold M is called a *CR-submanifold* [19] of a Kaehlerian manifold $\overline{M}$ if there exists a differentiable distribution $T : x \rightarrow T_x \subset M_x$ on M satisfying the following conditions, where M_x denotes the tangent space to M at each point x in M:

(1) T is invariant, that is, $JT_x = T_x$ for each point x in M,

(2) the complementary orthogonal distribution $T^{\perp} : x \rightarrow T_x{}^{\perp} \subset M_x$ is totally real, that is, $JT_x{}^{\perp} \subset M_x{}^{\perp}$ for each x in M, where $M_x{}^{\perp}$ denotes the normal space to M at $x \in M$. In particular, M is said to be a *semi-invariant* submanifold provided that dim $T^{\perp} = 1$. Then the unit normal vector field in $JT^{\perp}$ is called the *distinguished normal* to the semi-invariant submanifold and denoted by N^A ([17]), [18]).

Let M be a semi-invariant submanifold of codimension 3 in a $(2n+2)$-dimensional Kaehlerian manifold $\overline{M}$. We take the distinguished normal N^A as C^A. Then we have ([18])

$$J_B{}^A B_i{}^B = \phi_i{}^h B_h{}^A + \xi_i C^A, \quad J_B{}^A C^B = -\xi^h B_h{}^A, \tag{1.3}$$

$$J_B{}^A D^B = -E^A, \quad J_B{}^A E^B = D^A, \tag{1.4}$$

where we have put $\phi_{ji} = G(JB_j, B_i)$, $\xi_i = G(JB_i, C)$ in M, ξ^h being associated components of ξ_h. By the properties of the almost Hermitian structure (J, G), it follows from (1.3) and (1.4) that (ϕ, g, ξ) defines an almost contact metric structure. Since J is parallel tensor, by using (1.1), (1.2) and (1.3), we find

$$\nabla_j \phi_i{}^h = -h_{ji}\xi^h + h_j{}^h \xi_i, \tag{1.5}$$

$$\nabla_j \xi_i = -h_{jr}\phi_i{}^r. \tag{1.6}$$

In the sequel we suppose that the distinguished normal C is parallel in the normal bundle. Then we can verify from (1.2) and (1.4) that

$$k_{jr}\phi_i{}^r = l_{ji}, \quad l_{jr}\phi_i{}^r = -k_{ji}. \tag{1.7}$$

Thus, it follows that

$$k_{jr}\xi^r = 0, \quad l_{jr}\xi^r = 0, \quad k = l = 0, \tag{1.8}$$

where $k = k_{ji}g^{ji}$ and $l = l_{ji}g^{ji}$.

To write our formulas in a convention form, the components $T_{ji}{}^m$ of a tensor field T^m and a function T_m on M for any integer $m(\geqq 2)$ are introduced as follows:

$$T_{ji}{}^m = T_{ji_1} T_{i_2}{}^{i_1} \dots T_i{}^{i_{m-1}}, \quad T_m = \sum_i T_{ii}{}^m.$$

In our notation, it is easily seen from (1.7) that

$$k_{jr}l_i{}^r + k_{ir}l_j{}^r = 0, \tag{1.9}$$

$$k_{ji}{}^2 = l_{ji}{}^2. \tag{1.10}$$

The ambient Kaehlerian manifold $\bar{M}$ is assumed to be of constant holomorphic sectional curvature c, which is called a *complex space form* and denoted by $M_{n+1}(c)$. Then the Gauss equation of M is obtained:

$$\begin{aligned} R_{kji}{}^h = {} & c(\delta_k{}^h g_{ji} - \delta_j{}^h g_{ki} + \phi_k{}^h \phi_{ji} - \phi_j{}^h \phi_{ki} - 2\phi_{kj}\phi_i{}^h)/4 \\ & + h_k{}^h h_{ji} - h_j{}^h h_{ki} + k_k{}^h k_{ji} - k_j{}^h k_{ki} + l_k{}^h l_{ji} - l_j{}^h l_{ki}. \end{aligned} \tag{1.11}$$

And the equations of Codazzi for M are given by

(1.12) $$\nabla_k h_{ji} - \nabla_j h_{ki} = c(\xi_k\phi_{ji} - \xi_j\phi_{ki} - 2\xi_i\phi_{kj})/4,$$

(1.13) $$D_k k_{ji} = D_j k_{ki},\quad D_k l_{ji} = D_j l_{ki},$$

where we have denoted

(1.14) $$D_k k_{ji} = \nabla_k k_{ji} - n_k l_{ji},\quad D_k l_{ji} = \nabla_k l_{ji} + n_k k_{ji}.$$

Those of Ricci are given by

(1.15) $$h_{jr}k_i^{\ r} = h_{ir}k_j^{\ r},\quad h_{jr}l_i^{\ r} = h_{ir}l_j^{\ r},$$

which together with (1.7) gives

(1.16) $$h_{ji}k^{ji} = 0,\quad h_{ji}l^{ji} = 0.$$

Let S_{ji} be the components of the Ricci tensor S of M, then the Gauss equation implies

(1.17 $$S_{ji} = c\{(2n+1)g_{ji} - 3\xi_j\xi_i\}/4 + hh_{ji} - h_{ji}^2 - 2k_{ji}^2,$$

where we have used (1.10) and the fact that $\nabla_j^{\perp}C=0$, which implies that the scalar curvature r of M is given by

(1.18) $$r = c(n^2-1) + h^2 - h_2 - 2k_2.$$

2. Submanifolds with harmonic curvature

Let M be a $(2n-1)$-dimensional semi-invariant submanifold with harmonic curvature in a complex space form $M_{n+1}(c)$, $c\neq 0$. Then the Ricci tensor S is of Codazzi type [4], [5]. Thus the Ricci formula for S gives rise to

$$\nabla_m\nabla_k S_{ji} = \nabla_j\nabla_i S_{mk} - R_{mjk}^{\ \ \ r}S_{ir} - R_{mji}^{\ \ \ r}S_{kr},$$

which together with the first Bianchi identity and the Ricci formula imply that

$$R_{mki}^{\ \ \ r}S_{jr} + R_{kji}^{\ \ \ r}S_{mr} + R_{jmi}^{\ \ \ r}S_{kr} = 0.$$

Since the distinguished normal C is parallel in the normal bundle, it is, using (1.7) and (1.17), seen that

$$\begin{aligned}&3c(\xi_j R_{mki}^{\ \ \ r}\xi_r + \xi_m R_{kji}^{\ \ \ r}\xi_r + \xi_k R_{jmi}^{\ \ \ r}\xi_r)/4\\ &= R_{mki}^{\ \ \ r}(T_{jr} - 2k_{jr}^2) + R_{kji}^{\ \ \ r}(T_{mr} - 2k_{mr}^2) + R_{jmi}^{\ \ \ r}(T_{kr} - 2k_{kr}^2),\end{aligned}$$

where we have put $T_{ji} = hh_{ji} - h_{ji}^2$. Making use of (1.7), (1.8), (1.9) and (1.11), it turns out to be

$$3\{\xi_j(h_{ki}h_{mr}-h_{mi}h_{kr})\xi^r+\xi_m(h_{ji}h_{kr}-h_{ki}h_{jr})\xi^r+\xi_k(h_{mi}h_{jr}-h_{ji}h_{mr})\xi^r\}$$
$$=(\phi_{ki}\phi_{mr}-\phi_{mi}\phi_{kr}-2\phi_{mk}\phi_{ir})T_j{}^r+(\phi_{ji}\phi_{kr}-\phi_{ki}\phi_{jr}-2\phi_{kj}\phi_{ir})T_m{}^r$$
$$+(\phi_{mi}\phi_{jr}-\phi_{ji}\phi_{mr}-2\phi_{jm}\phi_{ir})T_k{}^r+2(\phi_{ki}l_{mr}k_j{}^r+\phi_{ji}l_{kr}k_m{}^r+\phi_{mi}l_{jr}k_k{}^r$$
$$+\phi_{mk}l_{jr}k_i{}^r+\phi_{kj}l_{kr}k_i{}^r+\phi_{jm}l_{kr}k_i{}^r).$$

If we apply ξ^m to the last equation and take account of (1.8), then we have

$$3\{h_{ji}(h_{kr}\xi^r-\alpha\xi_k)-h_{ki}(h_{jr}\xi^r-\alpha\xi_j)+(h_{is}\xi^s)(\xi_k h_{jr}\xi^r-\xi_j h_{kr}\xi^r)\}$$
$$=(\phi_{ji}\phi_k{}^r-\phi_{ki}\phi_j{}^r-2\phi_{kj}\phi_i{}^r)T_{sr}\xi^s, \tag{2.1}$$

where we have defined α by $\alpha=h_{ji}\xi^j\xi^i$.

A vector field U on M is defined by $U^k=\xi^r\nabla_r\xi^k$. Then it is, making use of (1.6), clear that

$$\phi_{jr}U^r=h_{jr}\xi^r-\alpha\xi_j. \tag{2.2}$$

Transforming (2.1) by ϕ_{ji} and summing up j and i, we obtain

$$h_{kr}U^r=-(2n-1)\phi_k{}^rT_{sr}\xi^s/3 \tag{2.3}$$

because of (1.6) and the properties of the almost contact metric structure (ϕ, g, ξ). Because of the last two equations, (2.1) is reduced to

$$(2n-1)\{h_{ji}\phi_{kr}U^r-h_{ki}\phi_{jr}U^r+(h_{is}\xi^s)(\xi_k\phi_{jr}U^r-\xi_j\phi_{kr}U^r)\}$$
$$=2\phi_{kj}h_{ir}U^r+\phi_{ki}h_{jr}U^r-\phi_{ji}h_{kr}U^r. \tag{2.4}$$

Multiplying U^i to this and taking account of (2.3), we get

$$(n-1)(h_{jr}U^r\phi_{ks}U^s-h_{kr}U^r\phi_{js}U^s)=(h_{rs}U^rU^s)\phi_{kj} \tag{2.5}$$

and hence

$$(h_{rs}U^rU^s)\phi_{kt}U^t=0 \tag{2.6}$$

provided that $n>2$. From (2.4) we also have

$$(2n-3)h_{is}U^s\phi_{kr}U^r=\phi_{is}U^sh_{kr}U^r-(h_{rs}U^rU^s)\phi_{ki}. \tag{2.7}$$

We notice here that ξ is principal if and only if $\beta-\alpha^2=0$, where we have defined $\beta=h_{ji}{}^2\xi^j\xi^i$.

LEMMA 1. *Let M be a $(2n-1)$-dimensional semi-invariant submanifold with harmonic curvature and $\nabla_j{}^\perp C=0$ in $M_{n+1}(c)$, $c\neq 0$. Then ξ is principal if $n>2$.*

Proof. Let M_0 be a set of points of M at which the function $\beta-\alpha^2$ does not vanish. Suppose that M_0 is not empty. Then we have $h_{rs}U^rU^s=0$ on M_0 because of (2.2) and (2.6), and hence $(n-2)h_{is}U^s=0$ by means of (2.5) and (2.7). Thus, (2.3) tells us that $\phi_k{}^r(hh_{rs}-h_{rs}{}^2)\xi^s=0$ provided that $n>2$. By the properties of the almost contact metric structure, it is clear that

$$(h_{jr}{}^2-hh_{jr})\xi^r=(\beta-\alpha h)\xi_j$$

on M_0, which together with (1.8) and (1.17) gives

$$S_{jr}\xi^r=\delta\xi_j,\quad \delta=c(n-1)/2-\beta+\alpha h.$$

Thus, according to Proposition 2.2 of [10], M_0 is empty. This completes the proof.

LEMMA 2. *Under the same assumption as that in Lemma* 1, *we have*

$$S_{ji}=\{c(n-1)/2+h\alpha-\alpha^2\}g_{ji}. \tag{2.8}$$

Proof. Since the structure vector ξ is principal, it is, using (1.17), seen that $S_{jr}\xi^r=\{c(n-1)/2+h\alpha-\alpha^2\}\xi_j$. Thus, by transforming (2.1) by ϕ_{mk} and summing up m and k, we have (cf. [10])

$$(2n-3)S_{jr}\phi_i{}^r-S_{ir}\phi_j{}^r+\{r-c(n-1)/2-h\alpha+\alpha^2\}\phi_{ji}=0,$$

where we have used (1.11), and hence

$$2(n-1)S_{jr}\phi_i{}^r+\{r-c(n-1)/2-h\alpha+\alpha^2\}\phi_{ji}=0.$$

Therefore, by the properties of the almost contact metric structure, it follows that

$$2(n-1)S_{ji}=(r-\delta)g_{ji}+\{r-(2n-1)\delta\}\xi_j\xi_i, \tag{2.9}$$

where $\delta=c(n-1)/2+h\alpha-\alpha^2$, that is, M is of pseudo Einstein. Since S is of Codazzi type, it is clear that the scalar curvature r of M is constant. Thus, we can, making use of (2.9), easily verify that δ is constant on M (see [3], [10]).

Differentiating (2.9) covariantly along M, we obtain

$$2(n-1)\nabla_kS_{ji}=\{r-(2n-1)\delta\}(\xi_j\nabla_k\xi_i+\xi_i\nabla_k\xi_j),$$

which implies

$$\{r-(2n-1)\delta\}\{\xi_j\nabla_k\xi_i-\xi_k\nabla_j\xi_i+\xi_i(\nabla_k\xi_j-\nabla_j\xi_k)\}=0.$$

Because of Lemma 1, it is seen that $\{r-(2n-1)\delta\}\nabla_j\xi_i=0$.

By means of (1.6) and (1.12), it is not hard to see that ξ is not parallel. Accordingly we have $r=(2n-1)\delta$ and consequently $S_{ji}=\delta g_{ji}$ because of (2.9). Hence Lemma 2 is proved.

From (2.8) we have $r=\{c(n-1)/2+h\alpha-\alpha^2\}(2n-1)$, which together with (1.18) imply that

$$(2.10)\qquad 3c(n-1)/2+h^2-h_2-2k_2=(2n-1)\alpha(h-\alpha).$$

Combining (1.17) and (2.8), we obtain

$$(2.11)\qquad 2k_{ji}{}^2=(3c/4+\alpha^2-h\alpha)g_{ji}-3c\xi_j\xi_i/4+hh_{ji}-h_{ji}{}^2.$$

On the other hand, ξ being principal, we have $h_{jr}\xi^r=\alpha\xi_j$. It is proved in [9] that α is constant by using the fundamental equations on real hypersurfaces of a complex space form. Thus, making use of the Codazzi equation (1.12), we easily see that

$$(2.12)\qquad h_{jr}h_{ks}\phi^{rs}=\alpha(h_{kr}\phi_j{}^r-h_{jr}\phi_k{}^r)/2-c\phi_{kj}/4.$$

Transforming the last equation by ϕ_{jk} and summing up j and k, we find

$$(\nabla_j\xi_i)(\nabla^i\xi^j)=\alpha^2-\alpha h-c(n-1)/2$$

because of (1.6) and Lemma 1. Since we easily, using (1.6), see that $\|\nabla_j\xi_i\|^2=h_2-\alpha^2$, it follows that

$$(2.13)\qquad (1/2)\|\nabla_j\xi_i-\nabla_i\xi_j\|^2=h_2-2\alpha^2+\alpha h+(c/2)(n-1).$$

Now, if we transform (2.11) by $\phi_i{}^k$ and make use of (1.7), we can get

$$(2.14)\qquad 2k_{jr}l_i{}^r=(h\alpha-\alpha^2-3c/4)\phi_{ji}+hh_{jr}\phi_i{}^r-h_{jr}{}^2\phi_i{}^r,$$

which implies

$$(2.15)\qquad h_{jr}{}^2\phi_{ir}+h_{ir}{}^2\phi_j{}^r=h(h_{jr}\phi_i{}^r+h_{ir}\phi_j{}^r)$$

because of (1.9). Applying $h_k{}^i$ to (2.14) and taking account of (2.12), we also get

$$\begin{aligned}2k_{jr}l_s{}^rh_k{}^s=&(h\alpha-\alpha^2-3c/4)h_{kr}\phi_j{}^r+h_{jr}\phi_k{}^r\\&+(\alpha/2-h)h_{jr}h_{ks}\phi^{rs}-\alpha h_{jr}{}^2\phi_k{}^r/2,\end{aligned}$$

which joined with (1.15) yields

$$(h\alpha-\alpha^2-c)(h_{jr}\phi_i{}^r+h_{ir}\phi_j{}^r)=(\alpha/2)(h_{jr}{}^2\phi_i{}^r+h_{ir}{}^2\phi_j{}^r).$$

Because of (2.15), it follows that we have

$$(h\alpha-2\alpha^2-2c)(h_{jr}\phi_i^r+h_{ir}\phi_j^r)=0$$

and hence $(h\alpha-2\alpha^2-2c)\{h_2-\alpha h-c(n-1)/2\}=0$.

LEMMA 3. *Under the same assumptions as those in Lemma* 1, *we have* $h\alpha=2\alpha^2+2c$.

Proof. Let M_1 be a set of points at which the function $h\alpha-2\alpha^2-2c$ does not vanish. Suppose that M_1 is not empty. Then we have on M_1 $h_2=\alpha h+c(n-1)/2$, which is equivalent to $h_{jr}\phi_i^r+h_{ir}\phi_j^r=0$. Thus, (2.12) is reduced to $h_{jr}{}^2\phi_i^r=\alpha h_{jr}\phi_i^r+c\phi_{ji}/4$, which implies

$$h_{ji}{}^2=\alpha h_{ji}+(c/4)(g_{ji}-\xi_j\xi_i). \tag{2.16}$$

Thus, the shape operator A in the direction of the distinguished normal has three constant principal curvatures α, $(\alpha+\sqrt{D})/2$, $(\alpha-\sqrt{D})/2$ with multiplicities 1, n-1 and n-1 respectively, where $D=\alpha^2+c$. Consequently we have on M_1

$$h=n\alpha,\quad h_2=n\alpha^2+c(n-1)/2, \tag{2.17}$$

which together with (2.11) implies that $2k_2=(n-1)\{c-(n-1)\alpha^2\}$. Substituting this and (2.17) into (2.10), we have $\alpha=0$ on M_1. Hence, (2.11) and (2.16) mean that $h_{ji}{}^2=k_{ji}{}^2=c(g_{ji}-\xi_j\xi_i)/4$ on M_1 and consequently we see that $h_j{}^k$ and $k_j{}^h$ have the same principal curvatures $0, 1, -1$, with multiplicities 1, $n-1$ and $n-1$ respectively. It is contradictory because of (1.16). This completes the proof of the lemma.

THEOREM 4. *There are no* $(2n-1)$*-dimensional semi-invariant submanifolds with harmonic curvature in a complex space form* $M_{n+1}(c)$, $c\neq 0$ *and* $n>2$ *on which the distinguished normal is parallel in the normal bundle.*

Proof. Since we have $\alpha h=2(\alpha^2+c)$, it is seen that α is nonzero constant and hence h is constant.

Now, suppose that $\alpha^2+c=0$. Then we have $h=0$. Therefore, (2.10) and (2.13) will produce a contradiction. Accordingly, it follows that $\alpha^2+c\neq 0$ on M. If $AX=\lambda X$ for any vector field X orthogonal to ξ, then we get $(2\lambda-\alpha)A\phi X=(\alpha\lambda+c/2)\phi X$ and thus it turns out that ϕX is also a principal vector with principal curvature $\mu=(\alpha\lambda+c/2)/$

$(2\lambda-\alpha)$ because of the fact that $\alpha^2+c\neq 0$.

By the way, as in the proof of Lemma 3 we see that $\lambda\neq\mu$. Hence $\lambda+\mu=h$ by virtue of (2.15), which means $\lambda\mu=\alpha^2+5c/4$. Thus, it follows that $\lambda^2-h\lambda+\alpha^2+5c/4=0$, which joined with (1.15), (2.11) and Lemma 3 yields that the principal curvatures in the direction of the normal D vanishes identically. Hence, Lemma 3 and (2.10) tell us that

(2.18) $$h_2-h^2+(2n-1)\alpha^2+c(5n-1)/2=0.$$

By the way, α being the only principal curvature with respect to ξ, we see, using the last quadratic equation, that the shape operator A in the direction of the distinguished normal has three constant prin cipal curvatures:

$$\alpha,\quad (h+\sqrt{D})/c,\quad (h-\sqrt{D})/2,\quad D=h^2-4\alpha^2-5c.$$

And their multiplicities are respectively denoted by $1, m_1$ and m_2. Hence, the trace of the shape operator A is given by

$$h=\alpha+h(n-1)+\sqrt{D}(m_1-m_2)/2.$$

Using this fact, h_2 satisfies the following:

$$h_2-h^2=\alpha^2-\alpha h-2(n-1)(\alpha^2+5c),$$

which together with Lemma 3 gives $h_2-h^2+(2n-1)\alpha^2=-(10n-8)c$. It is contradictory because of (2.18). This completes the proof.

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