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Nonlinear Fluid Forces on Hinged Wavemakers
힌지型 造波機에 작용하는 非線形 波力

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Abstract : The nonlinear hydrodynamic pressure force and moment on hinged wavemakers of variable-draft are presented. A closed-form solution (correct to second-order) for the nonlinear wavemaker boundary value problem has been obtained by employing the Stokes perturbation expansion scheme. The physical significance of the second-order contributions to the hydrodynamic pressure moment are examined in detail. Design curves are presented which demonstrate both the magnitude of the second-order nonlinearities and the effects of the variable-draft hinge height. The second-order contributions to the total hydrodynamic force and moment consist of a time-dependent and a steady part. The sum of the first and second-order pressure force and moment show a significant increase over those predicted by linear wavemaker theory. The second-order effects are shown to vary with both relative water depth and wave amplitude. The second-order dynamic effects are relatively more important for hinged wavemakers with shallower drafts.

要 旨 : 可變吃水를 갖는 힌지型 造波機에 작용하는 非線形 波力 및 모멘트를 算定하기 위하여 非線形 境界値 문제의 2次 近似解를 Stokes의 展開技法을 利用하여 구하였다. 2次波 成分이 波力 및 모멘트에 기여하는 物理的 意味에 대해 詳述하였다. 2次波에 의한 非線形性과 造波板의 吃水에 따른 波力 모멘트 變化를 決定할 수 있도록 設計曲線을 제시하였다. 2次波 成分의 全波力 및 모멘트에 기여하는 부분은 調和成分과 常数成分으로 構成된다. 1次波와 2次波 成分을 合成한 波力 및 모멘트의 크기는 線形 造波理論의 값보다 상당한 增加를 보인다. 2次波 成分의 効果는 相對水深과 波高에 따라 달라지며 짧은 吃水를 갖는 造波機에서 相對的으로 더욱 重要하다.

1. INTRODUCTION

The basic linear wavemaker theory for forced harmonic surface gravity waves derived by Havelock (1929) and later by Biesel and Suquet (1953) has been extended to design curves by Gilbert *et al.* (1972) for three types of wavemakers which are commonly encountered in many modern wave flumes. Limited experimental verification of these design curves were discussed by Krishnamackar (1972) and by Gilbert *et al.* (1972). Hyun (1976) extended the basic theoretical solution for the periodic waves to the case of a hinged-type wavemaker with a flap of variable-draft. Experimental verifications of these wavemaker solutions for various types of wavemaker geometries have been restricted primarily to the surface wave profiles {cf., Biesel and Suquet (1953) and Ursell *et al.* (1960) for the case of monochromatic linear waves; and Madsen (1970) and Multer (1973) for the nonlinear, initial value problem formulated by Kennard (1949)}.

The solution presented by Hyun (1976) for a hinged-type wavemaker of variable-draft was extended by Hudspeth and Chen (1981) within the limits of linearized potential wave theory to wave flumes whcih consist of two constant depth regions sepa-

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rated by a gradually sloping transition region. Design curves for the wavemaker gain function, S/H, the dimensionless hydrodynamic pressure moment, M, moment arm, *l*/e, and wavemaker power, W, for this type of wave flume geometry were developed. Experimental verifications of those design curves were obtained in a large-scale wave flume at the Oregon State University Wave Research Facility (OSU-WRF) and were reported by Hudspeth *et al.* (1981).

Weakly nonlinear solutions are relatively scarce and have been largely confined to numerical solutions {cf., Multer (1970, 1973), Multer and Galvin (1967)} or approximate solutions {cf., Fontanet (1961), Madsen (1970, 1971), Daugaard (1972), Buhr Hansen and Svendsen (1974)} for piston-type wavemakers only. Waveforms of the forced waves have been mainly studied and detailed design curves for any of these solutions are nonexistent.

Flick and Guza (1980) investigated the second-order, nonlinear motion of a hinged wavemaker of variable-draft. Their approximate solution omitted both a time-independent term and the analytical, closed-form expressions for the multiplicative coefficients required for their solution. The coefficients for the propagating mode only were computed numerically. No design curves for engineering application were attempted.

The approximate solution reported by Flick and Guza (1980) will now be extended to include the following additional wave flume geometry and time-independent solution: 1) a variable-draft wavemaker hinge located either above or below the wavemaker bottom; 2) a wavemaker stroke amplitude that is measured at an arbitrary height above the hinge; and 3) a time-independent nonlinear solution which is required in order to satisfy exactly the inhomogeneous wavemaker boundary condition. The closed-form analytic solution presented will be used to develop design curves similar to those presented by Gilbert *et al.* (1971) and Hudspeth and Chen (1981).

2. VARIABLE-DRAFT HINGED WAVEMAKER THEORY

For convenience, dimensional variables will be

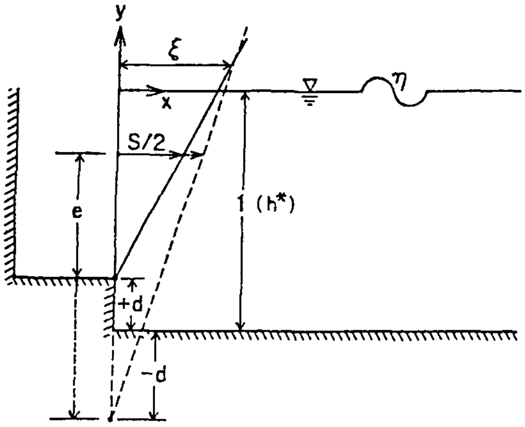

Fig. 1 Definition sketch of hinged wavemakers of variable-draft.

made dimensionaless at the outset by the following physical variables: length scale $=h^*$; time scale $=\sqrt{h^*/g^*}$; and mass scale $=\rho^*(h^*)^3$, in which $h^*=$ dimensional depth of the wave flume, $g^*=$ gravitational constant, and $\rho^*=$ fluid mass density. the relationships between dimensional variables (denoted by a superscript asterisk, * and dimensionaless variables are given by the following: $h^*\{x,y,S,d,e,H,L,w,\eta\}=\{x^*,y^*,S^*,d^*,e^*,H^*,L^*,w^*,\eta^*\}$; $\{k_m,q_m,\mu_m\}=\{k^*_m h^*;\ q^*_m h^*;\ \mu^*_m h^*\}$; $\{t,T\}=\{t^*,T^*\}\sqrt{g^*/h^*}$; $\sqrt{g^*(h^*)^3}\ \{\Phi,\Psi\}=\{\Phi^*,\Psi^*\}$; $p=p^*/\rho^*g^*h^*$; $F=F^*/\rho^*g^*(h^*)^3$; $M=M^*/\rho^*g^*(h^*)^4$; and $E=E^*/g^*h^*$.

The two-dimensional irrotational motion of an incompressible, inviscid fluid in the wave flume geometry shown in Fig. 1 may be obtained from the directional derivatives of a dimensionless scalar velocity potential, Φ (x, y, t), according to

$$u(x,y,t)=-\Phi_x,\quad v(x,y,t)=-\Phi_y \qquad \text{(1a,1b)}$$

in which the subscripts denote partial differentiation.

The dimensionless pressure, p, may be determined from the Bernoulli equation which is given by

$$p(x,y,t)=\Phi_t-\frac{1}{2}\{\Phi_x^2+\Phi_y^2\}-y+E \qquad (2)$$

in which E = the dimensionless Bernoulli constant. For the case of simple harmonic wavemaker motion with period $T=2\pi/\omega$, the resulting fluid motion will also be simple harmonic and the spatial and temporal variables may be separated. The fluid motion is also periodic in x with wave length $L=2\pi/k$.

The dimensionless velocity potential, Φ (x, y, t), must satisfy the following boundary value problem:

$$\Phi_{xx}+\Phi_{yy}=0 \quad ; x\geq 0,\ -1\leq y\leq \eta\ (x,t) \tag{3a}$$

$$\Phi_{y}=0 \quad ; x\geq 0, y=-1 \tag{3b}$$

$$\eta-\Phi_{t}+1/2[\Phi_{x}^{2}+\Phi_{y}^{2}]=\mathbf{E} \quad ; x\geq 0, y=\eta\ (x,t) \tag{3c}$$

$$\eta_{t}-\Phi_{x}\eta_{x}+\Phi_{y}=0 \quad ; x\geq 0, y=\eta\ (x,t) \tag{3d}$$

$$\frac{D\mathbf{X}}{dt}=0 \quad ; \text{on the wavemaker boundary } x=\chi(y,t) \tag{3e}$$

The material coordinate surface of the wavemaker shown in Fig. 1 may be expressed by $\mathbf{X}=x-\chi(y,t)=0$ in which $\chi(y,t)=\xi(y)\sin\omega t$ where $\xi(y)=S/2\{(y+1-d)/e\}U(y+1-d)$; in which S/2 = dimensionless amplitude of the wavemaker motion and $\mathbf{U}(\bullet)$ = Heaviside step function.

The dimensionless velocity potential, Φ, the free surface elevation, η, the hydrodynamic pressure, p, and the Bernoulli constant, $\mathbf{E}$, are expanded in the following power series: $\Phi=\sum_{n=1}^{\infty}\varepsilon^{n}{}_{n}\Phi$; $\eta=\sum_{n=1}^{\infty}\varepsilon^{n}{}_{n}\Phi$; $p=\sum_{n=1}^{\infty}\varepsilon^{n}{}_{n}p$; and $\mathbf{E}=\sum_{n=1}^{\infty}\varepsilon^{n}{}_{n}\mathbf{E}$. The small parameter $\varepsilon=Hk/2$, in which H = dimensionless wave hight. In addition, the free surface boundary conditions, Eq . (3c) and (3d) are expanded about the still water level $y=0$ in a Taylor series. The Stokes material derivative of the wavemaker displacement, $D\mathbf{X}/Dt$, may also be expanded about its mean position $x=0$ in a Taylor series;

$$\frac{D\mathbf{X}}{Dt}+(\chi)\frac{\partial}{\partial x}\left(\frac{D\mathbf{X}}{Dt}\right)+0(\varepsilon^{3})=0; \quad x=0 \tag{4}$$

By collecting the terms of the same order in Eqs.(3) and (4) {S/2 is assumed to be of the order of ε }, a set of linear boundary value problems is obtained.

3. LINEAR SOLUTION

The linear boundary value problem from the perturbation expansion is the following for order ε :

$${}_{1}\Phi_{xx}+{}_{1}\Phi_{yy}=0 \quad ; x\geq 0,\ -1\leq y\leq 0 \tag{5a}$$

$${}_{1}\Phi_{y}=0 \quad ; x\geq 0, y=-1 \tag{5b}$$

$${}_{1}\Phi_{tt}+{}_{1}\Phi_{y}+{}_{1}\mathbf{E}_{t}=0 \quad ; x>0, y=0 \tag{5c}$$

$${}_{1}\Phi_{x}=-\chi_{t} \quad ; x=0,\ -1\leq y\leq 0 \tag{5d}$$

A solution to Eq. (5) may be represented by the following eigenfunction expansion {cf., Hudspeth and Chen (1981) or Wehausen (1960)}:

$$\varepsilon_{1}\Phi(x,y,t)=\mathbf{a}_{1}\phi_{1}(ky)\sin(kx-\omega t)$$
$$+\cos\omega t\sum_{m=2}^{\infty}\mathbf{a}_{m}\phi_{m}(k_{m}y)\exp(-k_{m}x) \tag{6}$$

in which the dimensionless orthonormal eigenfunctions, $\phi_1(y)$ and $\phi_m(y)$, in the interval of orthogonality, $-1\leq y\leq 0$, are given by $\phi_1(ky)=\cosh k(y+1)/\mathbf{n}_1$; $\phi_m(k_m y)=\cos k_m(y+1)/\mathbf{n}_m$, $m\geq 2$ and the normalizing constants are $\mathbf{n}_1^2=(2k+\sinh 2k)/4k$; $\mathbf{n}_m^2=(2k_m+\sin 2k_m)/4k_m$, $m\gg 2$. The eigenvalues k and k_m are the real roots of $\omega^2=k\tanh k$, $\omega^2=-k_m\tan k_m$ for $m\geq 2$. It can be readily shown that $(2m-3)\pi/2<k_m<(m-1)\pi$ for $m\geq 2$. In Eq. (5c), ${}_1\mathbf{E}_t$ is equal to zero. The dimensionless multiplicative coefficients, $\mathbf{a}_1$ and $\mathbf{a}_m$, are determined from Eq. (5d) to be

$$\mathbf{a}_{1}=\frac{-S\omega\mathbf{D}(k)\operatorname{cosech}k}{2e\,\mathbf{n}_{1}k^{3}} \tag{7a}$$

$$\mathbf{a}_{m}=\frac{S\omega\,\mathbf{D}_{m}(k_{m})}{2e\,\mathbf{n}_{m}k_{m}^{3}}, \quad m\geq 2 \tag{7b}$$

in which $\mathbf{D}(k)=\sinh k[k(1-d)\sinh k-\cosh k+\cosh kd\cdot\mathbf{U}(d)]$ and $\mathbf{D}_m(k_m)=k_m(1-d)\cdot\sin k_m+\cos k_m-\cos\{k_m d\bullet\mathbf{U}(d)\}$.The transcendental functions that contain the Heaviside step function, $\mathbf{U}$ (d), are equal to unity when the hinge is below the wave flume bottom, i.e., $d\leq 0$.

Far away from the wavemaker ($x>3$, say), the evanescent eigenmodes are sensibly less than 1% of their value at the wavemaker, and the dimensionless linear free surface profile, $\varepsilon\,{}_1\eta$ (x, t), is given by

$\varepsilon_1\eta(x, t) = H/2\cos(kx-\omega t)$ so that dimensionless wavemaker gain function, S/H, is

$$S/H = ek^2\mathbf{n}_1^2/\mathbf{D}(k) \quad (8)$$

in agreement with Eq. (17) given by Hyun (1976). These equations may be shown to be equivalent to those given by Flick and Guza {(1980), Eq. 12} for equivalent wave flume geometries. The solution for a piston type wavemaker may be obtained by letting $d \to -\infty$

4. SECOND-ORDER SOLUTION

The boundary value problem for the dimensionless velocity potential correct to second-order, ε^2, is given by the following:

$$_2\Phi_{xx} + {_2\Phi_{yy}} = 0 \quad ; 0 \le x < +\infty,\ -1 \le y < 0 \quad (9a)$$

$$_2\Phi_y = 0 \quad ; 0 \le x < +\infty,\ y = -1 \quad (9b)$$

$$_2\Phi_{tt} + {_2\Phi_y} + {_2\mathbf{E}_t} = \frac{\partial}{\partial t}\{{_1\Phi_x^2} + {_1\Phi_y^2}\} - {_1\eta}\frac{\partial}{\partial y}[{_1\Phi_{tt}} + {_1\Phi_y}] \quad ; 0 \le x < +\infty,\ y = 0 \quad (9c)$$

$$_2\Phi_x = {_1\Phi_y}\chi_y - {_1\Phi_{xx}}\chi \quad ; x = 0,\ -1 + d \le y \le 0 \quad (9d)$$

Eq. (9d) may be shown to be equivalent to the second-order kinematic wavemaker boundary condition given by Flick and Guza {(1980), Eq. 8b} provided that the sign of the second term on the right-hand side of their Eq. (6) is changed from a plus to a minus. Flick and Guza (1980) obtained theirs by requiring continuity of the normal component of the relative velocity between the fluid on the wavemaker and the wavemaker boundary. Eq. (9d) was obtained by the Stoke's material derivative of a material coordinate using Eq. (3e) {cf., Kinsman (1965)}. Note that time-independent terms arise in addition to second-harmonic terms in the right-hand side of Eq. (9d) which require a time-independent solution in $_2\Phi$.

The analytical procedure usually employed to solve boundary value problems which include inhomogeneous boundary conditions on two nonoverlapping orthogonal boundaries {e.g., Eqs. (9c) and (9d)} involves the linear decomposition of the second-order velocity potential into two linearly independent components; viz. a Stokes free wave potential, $_2\Phi^S$, and a forced wavemaker wave potential, $_2\Phi^f$; i.e., $_2\Phi = {_2\Phi^S} + {_2\Phi^f}$.

The dimensionless Stokes free wave potential, $_2\Phi^S$, is simply the Stokes second-order wave given by

$$\varepsilon^2\,{_2\Phi^S} = -\frac{3H^2\omega}{32}\frac{\cosh\{2k(y+1)\}}{\sinh^4 k}\sin 2(kx-\omega t) \quad (10)$$

Using this linear decomposition, each dimensionless potential is now chosen to satisfy only one of the two inhomogeneous boundary conditions given by Eqs. (9c) and (9d) {cf., Wehausen (1960)}.

The inhomogeneous combined free surface boundary condition given by Eq. (9c) yields

$$_2\Phi_{tt}^f + {_2\Phi_y^f} = -\{{_2\Phi_{tt}^S} + {_2\Phi_y^S} + {_2\mathbf{E}_t}\} + \frac{\partial}{\partial t}\{{_1\Phi_x^2} + {_1\Phi_y^2}\} - {_1\eta}\frac{\partial}{\partial y}[{_1\Phi_{tt}} + {_1\Phi_y}] = 0;\ y = 0 \quad (11)$$

which gives a well-posed Sturm-Liouville problem for the dimensionless second-order forced wave potential, $_2\Phi^f$, in the y coordinate since the dimensionless second-order Stokes wave potential, $_2\Phi^S$, satisfies the inhomogeneous free surface boundary condition. The solution for the dimensionless forced wave potential, $_2\Phi^f$, is assumed to be given by

$$\varepsilon^2\,{_2\Phi^f}(x,y,t) = \Psi(x,y) + \{B_1\cos(q_1x - 2\omega t) + C_1\sin(q_1x - 2\omega t)\}\,\mathbf{Q}_1(q_1y) + \sum_{n=2}^{\infty}\{B_n\sin 2\omega t + C_n\cos 2\omega t\}\,\mathbf{Q}_n(q_ny)\cdot\exp(-q_nx) \quad (12)$$

in which the dimensionless orthonormal eigenfunctions, $\mathbf{Q}_1(q_1y)$ and $\mathbf{Q}_n(q_ny)$, in the interval of orthogonality, $-1 \le y < 0$, are $\mathbf{Q}_1(q_1y) = \cosh q_1(y+1)/\mathbf{N}_1$;

$\mathbf{Q}_n\ (q_n y) = \cos q_n\ (y+1)/\mathbf{N}_n$ for $n \geq 2$ and the normalizing constants are $\mathbf{N}_1^2 = (2q_1 + \sinh 2q_1)/4q_1$; $\mathbf{N}_n^2 = (2q_n + \sin 2q_n)/4q_n$ for $n \geq 2$. The eigenvalues q_1 and q_n are the real roots of $4\omega^2 = q_1 \tanh q_1$, $\omega^2 = -q_n \tan q_n$ for $n \geq 2$.

The dimensionless time-independent solution, $\Psi(x, y)$, must satisfy the following boundary value problem:

$$\Psi_{xx} + \Psi_{yy} = 0 \quad ; x \geq 0,\ -1 \leq y \leq 0 \qquad (13a)$$

$$\Psi_y = 0 \quad ; x \geq 0,\ y = -1 \qquad (13b)$$

$$\Psi_y = 0 \quad ; x \geq 0,\ y = 0 \qquad (13c)$$

$$\Psi_x = -(S\,\mathbf{a}_1 k/4e\,n_1)\,[\sinh k(y+1) + k(y+1-d)\cosh k(y+1)]\cdot\mathbf{U}(y+1-d) \quad ; x = 0,\ -1 \leq y \leq 0 \qquad (13d)$$

$$\Psi_x \text{ bounded} \quad ; x \gg 0 \qquad (13e)$$

A solution is given by the following eigenfunction expansion:

$$\Psi(x, y) = A_0 x + \sum_{n=1}^{\infty} A_n \psi_n(\mu_n y)\ \exp(-\mu_n x) \qquad (14)$$

in which the dimensionless orthonormal eigenfunctions, $\psi_n\ (\mu_n y)$, in the interval of orthogonality, $-1 \leq y \leq 0$, are $\psi_n(\mu_n y) = \sqrt{2}\cos \mu_n (y+1)$ provided that the dimensionless eigenvalues are $\mu_n = n\pi$; $n \geq 1$. The time-independent solution given by Eq. (14) is related with the mean fluid motion, or the Eulerian mass circulation in the wave flumes. This topic was treated in a separate paper presented by Kim *et. al.* (1986).

Flick and Guza (1980) computed the propagating eigenmode coefficients, B_1 and C_1, in Eq. (12) numerically and did not report values for the evanescent eigenmode coefficients, B_n and C_n. The dimensionless multiplicative constant coefficients in the eigenfunction expansion given by Eq. (12) are determined form Eq. (9d) and are given by the following:

$$A_0 = -S\,\mathbf{a}_1 k\{4e\,\mathbf{n}_1\}^{-1}(1-d)\,\mathbf{S}(k) \qquad (15)$$

$$A_n = \mathbf{a}_1 (k/n\pi)^2 S\{4\mu_n e \Sigma^2(k, \mu_n)\}^{-1}\{\sqrt{2}(-1)^n \phi_1(k)[2 + k_o(1-d)\Sigma(k, \mu_n)] - \psi_n(\mu_n d)\,\phi(kd)[2 + \mathbf{R}(\mu_n, k)\Delta(k, \mu_n)\,\mathbf{T}(kd)\,\mathbf{t}(\mu_n d)]\} \qquad (16)$$

$$B_1 = -S\sum_{m=2}^{\infty} \mathbf{a}_m \mathbf{R}^2(k_m, q_1)\{4q_1 e \Sigma^2(k_m, q_1)\}^{-1} \beta_{m1}(k_m, q_1) \qquad (17)$$

$$B_n = -S\sum_{m=2}^{\infty} \mathbf{a}_m \mathbf{R}^2(k_m, q_n)\{4q_n e \Delta^2(k_m, q_n)\}^{-1} \beta_{mn}(k_m, q_n) \qquad (18)$$

$$C_1 = 3H^2\omega\mathbf{R}^2(2k, q_1)\{8q_1\Delta(2k, q_1)\}^{-1}\mathbf{Q}_1(q_1)\operatorname{cosech} 2k + \mathbf{a}_1 S\,\mathbf{R}^2(k, q_1)\{4q_1 e\Delta^2(k, q_1)\}^{-1}\Gamma_1(k, q_1) \qquad (19)$$

$$C_n = 3H^2\omega\mathbf{R}^2(2k, q_n)\{8q_n\Sigma(2k, q_n)\}^{-1}\mathbf{Q}_n(q_n)\operatorname{cosech} 2k + \mathbf{a}_1 S\,\mathbf{R}^2(k, q_n)\{4q_n e\,\Sigma^2(k, q_n)\}^{-1}\Gamma_n(k, q_n) \qquad (20)$$

in which

$$\beta_{m1}(k_m, q_1) = \phi_m(k_m)\mathbf{Q}_1(q_1)[3k_o(1-d)\Sigma(k_m, q_1) - 4\Omega^2(k_m)\Delta(k_m, q_1) - 2] + \phi_m(k_m d)\mathbf{Q}_1(q_1 d)[2 - \mathbf{R}(q_1, k_m)\Delta(k_m, q_1)\,\mathbf{t}(k_m d)\,\mathbf{T}(q_1 d)] \qquad (21a)$$

$$\beta_{mn}(k_m, q_n) = \phi_m(k_m)\mathbf{Q}_n(q_n)[3k_o(1-d))\Delta(k_m, q_n) - 4\Omega^2(k_m)\Sigma(k_m, q_n) - 2]$$
$$+\phi_m(k_m d)\mathbf{Q}_n(q_n d)[2+\mathbf{R}(q_n, k_m)\Sigma(k_m, q_n)\mathbf{t}(k_m d)\mathbf{t}(q_n d)] \quad (21b)$$

$$\Gamma_1(k, q_1) = \phi_1(k)\mathbf{Q}_1(q_1)[3k_o(1-d)]\Delta(k, q_1) + 4\Omega^2(k)\Sigma(k, q_1) - 2]$$
$$+\phi_1(kd)\mathbf{Q}_1(q_1 d)[2-\mathbf{R}(q_1, k)\Sigma(k, q_1)\ \mathbf{T}(kd)\ \mathbf{T}(q_1 d)] \quad (21c)$$

$$\Gamma_n(k, q_n) = \phi_1(k)\mathbf{Q}_n(q_n)[3k_o(1-d))\Sigma(k, q_n) + 4\Omega^2(k)\Delta(k, q_n) - 2]$$
$$+\phi_1(kd)\mathbf{Q}_n(q_n d)[2+\mathbf{R}(q_n, k)\ \Delta(k, q_n)\mathbf{T}(kd)\mathbf{t}(q_n d)] \quad (21d)$$

in which $\mathbf{C}(x) = \cosh x$, $\mathbf{S}(x) = \sinh x$, $\mathbf{T}(x) = \tanh x$, $\mathbf{t}(x) = \tan x$, $k_o = \omega^2$, $\Omega(\alpha) = k_o/\alpha$, $\mathbf{R}(\alpha, \beta) = \alpha/\beta$, $\Sigma(\alpha, \beta) = 1 + (\mathbf{R})^2$, and $\Delta(\alpha, \beta) = 1 - (\mathbf{R})^2$.

The first-and the second-order solutions given by Eqs. (6) and (12) include an infinite number of evanescent eigenmodes. The truncation criteria for the linear solution was the value of L for which

$$\langle \{{}_1u_p(y) - {}_1u(o, y)\}^2 \rangle \ll \{\nu \langle |{}_1u_p(y)| \rangle\}^2 \quad (22)$$

in which $\langle \bullet \rangle$ is depth-average operator; ν = truncation criterion; ${}_1u_p(y) = \omega\ \xi\ (y)$ represents the dimensionless prescribed wavemaker motion; and ${}_1u(o, y) = {}^2a_1 k\ \phi_1(ky) + \sum_{m=2}^{L} a_m k_m \phi_m(k_m y)$ represents the dimensionless water particle velocity at the wavemaker. For the linear solution, ν is taken as 0.01.

Similarly, the second-order solution was truncated for the values of N_s (for $\sin 2\omega t$) and N_c (for $\cos 2\omega t$) according to Eq. (22) in which

$$2u_p(y) = \begin{cases} \sum_{m=2}^{L} (\mathbf{a}_m k_m S/4e\ \mathbf{n}_m)[\sin k_m(y+1) + k_m(y+1-d)\cos k_m(y+1)] \cdot \mathbf{U}(y+1-d) & \text{for } \sin 2\omega t \\ -3H^2\omega k \cosh 2k(y+1)/16\sinh^4 k - (S\mathbf{a}_1 k/4e\mathbf{n}_1)[\sinh k(y+1) + k(y+1-d) \cosh k(y+1)] \cdot \mathbf{U}(y+1-d) & \text{for } \cos 2\omega t \end{cases} \quad (23a)$$

and

$$_2u(o, y) = \begin{cases} -q_1 B_1 \mathbf{Q}_1(q_1 y) + \sum_{n=2}^{N_s} q_n B_n \mathbf{Q}_n(q_n y) & \text{for } \sin 2\omega t \\ -q_1 C_1 \mathbf{Q}_1(q_1 y) + \sum_{n=2}^{N_c} q_n C_n \mathbf{Q}_n(q_n y) & \text{for } \cos 2\omega t \end{cases} \quad (23b)$$

For the second-order solution, the truncation criterion in Eq. (22) was raised to a slightly higher percentage for shorter hinge drafts (viz., $\nu = 0.03$ for $d = 0.25$, and $\nu = 0.05$ for $d = 0.50$). The evanescent eigenmodes were found to be more important when the water depth was relatively deep (i.e., $h^*/L^*_o > 0.6$) and when the hinge draft was relatively small (i.e., increasing values of d). The values of L, N_s and N_c Varied between $5 \le L \le 28$; $14 \le N_s \le 60$; and $1 \le N_c \le 60$ over the range of $-.50 \le d \le .50$ and $.01 \le h^*/L_o^* \le 1.5$.

5. FORCE AND MOMENT ON THE WAVEMAKER

The dimensionless hydrodynamic force and moment on the wavemaker may also be expanded in a power series in ε as $\mathbf{F}=\sum_{n=1}^{\infty}\varepsilon^n{}_n\mathbf{F}$ and $\mathbf{M}=\sum_{n=1}^{\infty}\varepsilon^n{}_n\mathbf{M}$.

The dimensionless linear hydrodynamic pressure force exerted on the fluid side only of the wavemaker may be determined from

$$ {}_1\mathbf{F}(t)=-w\int_{d-1}^{o}\varepsilon\,{}_1\Phi_t\,(o,y,t)\,dy \quad (24a)$$

$$ =-{}_1F_p\cos\,\omega t-{}_1F_e\sin\,\omega t \quad (24b)$$

in which the dimensionless hydrodynamic pressure force due to the propagating and the evanescent eigenmode is given by

$$ {}_1F_p=\frac{1}{2}w\;H\;k^{-1}\,\mathrm{sech}\;k\,[\sinh\,k-\sinh\,kd] \quad (25a)$$

$$ {}_1F_e=-\frac{w\;H\;k_o k^2\mathbf{n}_1^2}{2\mathbf{D}(k)}\sum_{m=2}^{L}\frac{\mathbf{D}_m(k_m)}{k_m^4\mathbf{n}_m^2}\cdot[\sin\,k_m-\sin\,k_m d] \quad (25b)$$

in which w = dimensionless width of the wavemaker and $k_o=2\pi/L_o$, $L_o=T^2/2\pi$ being the dimensionless deep water wave length.

The dimensionless linear hydrodynamic pressure moment about the wavemaker hinge that is exerted on the fluid side only of the wavemaker may be determined from

$$ {}_1\mathbf{M}(t)=w\int_{d-1}^{o}(y+1-d)\,\varepsilon\,{}_1\Phi_t\,(o,y,t)\,dy \quad (26a)$$

$$ ={}_1M_p\cos\,\omega t-{}_1M_e\sin\,\omega t \quad (26b)$$

$$ =|{}_1M|\;\sin(\omega t+\alpha_1) \quad (26c)$$

in which the dimensionless hydrodynamic pressure moment due to the propagating and the evanescent eigenmode is given by

$$ {}_1M_p=w\;H\;k^{-2}\,\mathbf{D}(k)\;\mathrm{cosech}\;2k \quad (27a)$$

$$ {}_1M_e=w[H\;k_o k^2\mathbf{n}_1^2/2\,\mathbf{D}(k)]\cdot\sum_{m=2}^{L}\mathbf{D}_m^2(k_m)/[k_m^5\mathbf{n}_m^2] \quad (27b)$$

Eq. (25) and (27) can be readily shown to be identical with Eqs. (21) and (20) given by Hyun (1976).

The dimensionless second-order hydrodynamic pressure force exerted on the fluid side only of the wavemaker may be determined from

$$ {}_2F(t)=-w\varepsilon^2\int_{d-1}^{0}\{{}_2\Phi_t(o,y,t)-\frac{1}{2}[{}_1\Phi_x^2(o,y,t)+{}_1\Phi_y^2(o,y,t)]+{}_2\mathbf{E}\}\,dy-\frac{w\varepsilon^2}{2}{}_1\Phi_t^2\,(o,o,t) \quad (28a)$$

$$ =-{}_2F_p\cos\,2\omega t-{}_2F_e\sin\,2\omega t-F_{df} \quad (28b)$$

in which the last term in Eq. (28a) results from the pressure above the still water level and the dimensionless Bernoulli constant at second order is given by $\varepsilon^2{}_2\mathbf{E}=H^2w^2/\;(16\sinh^2k)$.

The dimensionless second-order force components are given by

$$ \begin{aligned} {}_2F_p=&-2\omega w\{-3H^2\omega[\mathbf{S}(2k)-\mathbf{S}(2kd)]/64k\mathbf{S}^4(k)+C_1[\mathbf{Q}_1'(q_1)-\mathbf{Q}_1'(q_1d)]/q_1\\ &+\mathbf{a}_1^2k[k(1-d)/\mathbf{n}_1^2-\Omega(k)\phi_1^2(k)]/\;(8\omega)-\sum_{m=2}^{N_S}B_m[\mathbf{Q}_m'(q_m)-\mathbf{Q}_m'(q_md)]/q_m\\ &+\sum_{m=2}^{L}\{\mathbf{a}_1\mathbf{a}_mk_m\phi_1(kd)\phi_m(k_md)[\mathbf{T}(kd)+\mathbf{R}(k_m,k)\mathbf{t}(k_md)]/[4\omega\Sigma(k_m,k)]\\ &+\sum_{n=2}^{L}\mathbf{a}_m\mathbf{a}_n[(2-\delta_{mn})k_o\phi_m(k_m)\phi_n(k_n)-(1-\delta_{mn})k_n\phi_m(k_md)\phi_n(k_nd)\cdot\\ &[\mathbf{t}(k_md)-\mathbf{t}(k_nd)]/[1-\mathbf{R}(k_n,k_m)]+\delta_{mn}k_nk_m(1-d)/\mathbf{n}_m\mathbf{n}_n]/(8\omega)\}\} \end{aligned} \quad (29a)$$

$$_2F_e = -2\omega w\{B_1[\mathbf{Q}_1'(q_1) - \mathbf{Q}_1'(q_1 d)]/q_1 + \sum_{m=2}^{N_c} C_m[\mathbf{Q}_m'(q_m) - \mathbf{Q}_m'(q_m d)]/q_m$$

$$-(\mathbf{a}_1/4\omega)\sum_{m=2}^{L}\mathbf{a}_m[2k_o\phi_1(k)\phi_m(k_m) - k_m\phi_1(kd)\phi_m(k_m d)\Sigma^{-1}(k_m, k)$$

$$\cdot[\mathbf{R}(k_m, k)\mathbf{T}(kd) - \mathbf{t}(k_m d)]]\} \tag{29b}$$

$$_2F_{df} = \frac{1}{4}kw\{\mathbf{a}_1^2\phi_1(kd)\phi_1'(kd) + H^2\omega^2(1-d)/[4k\mathbf{S}^2(k)]$$

$$-\sum_{m=2}^{L}\{2\mathbf{a}_1\mathbf{a}_m\mathbf{R}(k_m, k)\phi_1(kd)\phi_m(k_m d)\Sigma^{-1}(k_m, k)[\mathbf{T}(kd) + \mathbf{R}(k_m, k)\mathbf{t}(k_m d)]$$

$$-\sum_{n=2}^{L}\mathbf{a}_m\mathbf{a}_n[(1+\delta_{mn})\mathbf{R}(k_o, k)\phi_m(k_m)\phi_n(k_n) + (1-\delta_{mn})\mathbf{R}(k_n, k)\phi_m(k_m d)\phi_m(k_n d)$$

$$[\mathbf{t}(k_m d) - \mathbf{t}(k_n d)]/[1-\mathbf{R}(k_n, k_m)] - \delta_{mn}\mathbf{R}(k_n, k)k_m(1-d)/(\mathbf{n}_m\mathbf{n}_n)]\}\} \tag{29c}$$

in which $\phi_1'(kd) = \sinh kd/\mathbf{n}_1$, $\mathbf{Q}_m'\binom{q_m}{q_m d} = \sin\binom{q_m}{q_m d}/\mathbf{N}_m$, and δ_{mn} = Kronecker delta.

The dimensionless second-order hydrodynamic pressure moment about the wavemaker hinge, which is exerted on the fluid side only of the wavemaker may be determined form

$$_2\mathbf{M}(t) = w\varepsilon^2\int_{d-1}^{o}(y+1-d)\{_2\Phi_t)(o, y, t) - \frac{1}{2}[_1\Phi_x^2(o, y, t) + {_1\Phi_y^2}(o, y, t)] + {_2\mathbf{E}}\}dy$$

$$+\frac{w\varepsilon^2}{2}(1-d)\ {_1\Phi_t^2}(o, o, t)$$

$$= {_2M_p}\cos 2\omega t - {_2M_e}\sin 2\omega t - {_2M_{df}} \tag{30a}$$

$$= |{_2M}|\sin(2\omega t + \alpha_2) - {_2M_{df}} \tag{30b}$$

in which the dimensionless second-order moment components are given by the following integrals:

$$_2M_p = w\int_{d-1}^{o}(y+1-d)\{3H^2\omega^2\mathbf{C}(2k(y+1))/16\mathbf{S}^4(k) - 2\omega C_1\mathbf{Q}_1(q_1 y) + 2\omega\sum_{m=2}^{N_s}B_m\mathbf{Q}_m(q_m y)$$

$$-\frac{\mathbf{a}_1^2}{4}k^2[\phi_1^2(ky) - \phi_1'^2(ky)] + \frac{\mathbf{a}_1}{2}k\ \phi_1(ky)\sum_{m=2}^{L}\mathbf{a}_m k_m\phi_m(k_m y) - \frac{1}{4}\sum_{m=2}^{L}\sum_{n=2}^{L}\mathbf{a}_m\mathbf{a}_n$$

$$k_m k_n[\phi_m(k_m y)\phi_n(k_n y) + \phi_m'(k_m y)\phi_n'(k_n y)]\}dy$$

$$-\frac{wk_o}{4}(1-d)[\mathbf{a}_1^2\phi_1^2(k) - \sum_{m=2}^{L}\sum_{n=2}^{L}\mathbf{a}_m\mathbf{a}_n\phi_m(k)\phi_n(k)] \tag{31a}$$

$$_2M_e = -w\int_{d-1}^{o}(y+1-d)[-2\omega B_1\mathbf{Q}_1(q_1 y) - 2\omega\sum_{m=2}^{N_c}C_m\mathbf{Q}_m(q_m y)$$

$$-\frac{\mathbf{a}_1}{2}k\ \phi_1'(ky)\sum_{m=2}^{L}\mathbf{a}_m k_m\phi_m'(k_m y)]dy - \frac{wk_o}{2}(1-d)\mathbf{a}_1\phi_1(k)\sum_{m=2}^{L}\mathbf{a}_m\phi_m(k_m) \tag{31b}$$

$$ {}_2M_{af}=\frac{w}{4}\int_{d-1}^{0}(y+1-d)\{\mathbf{a}_1^2k^2[\phi_1^2(ky)+\phi_1'^2(ky)]-2\mathbf{a}_1k\,\phi_1(ky)\sum_{m=2}^{L}\mathbf{a}_mk_m\phi_m(ky) $$

$$ +\sum_{m=2}^{L}\sum_{n=2}^{L}\mathbf{a}_m\mathbf{a}_nk_mk_n[\phi_m(k_my)\phi_n(k_ny)+\phi_m'(k_my)\phi_n'(k_ny)] $$

$$ -H^2\omega^2/4\sinh^2k\}\,dy-\frac{wk_o}{4}(1-d)[\mathbf{a}_1^2\phi_1^2(k)+\sum_{m=2}^{L}\sum_{n=2}^{L}\mathbf{a}_m\mathbf{a}_n\phi_m(k)\phi_n(k)] \quad (31c) $$

6. DESIGN CURVES FOR NONLINEAR MOMENT

The dimensional design parameters which are most frequently specified for experimental work in wave flumes are the design wave height, H*, design water depth, h*, and design wave period, T*. Consequently, it is more desirable to nondimensionalize the previously derived dimensionless wavemaker variables by these three design parameters. The dimensionless design curve variables may be defined for the moment for the wave flume shown in Fig. 1 as $\mathbf{M}=\mathbf{M}^*/[1/2\,\rho^*g^*w^*H^*h^*(h^*-d^*)]$. The dimensionless amplitude of the linear hydrodynamic moment on the hinged wavemaker, Eq. (27), may now be rewritten as

$$ {}_1M_p=\frac{k_o\mathbf{D}(k)\ \mathrm{cosech}^2k}{k^3(1-d)} \quad (32a) $$

$$ {}_1M_e=\frac{k_ok^2\mathbf{n}_1^2}{(1-d)\mathbf{D}(k)}\sum_{m=2}^{L}\frac{\mathbf{D}_m^2(k_m)}{k_m^5\mathbf{n}_m^2} \quad (32b) $$

By substituting the values of the multiplicative constant coefficients given by Eqs. (7), (17), (18), (19), (20) into Eq. (31) and by utilizing the wavemaker gain function given by Eq. (8), the following expression for the dimensionless amplitude of the second-order hydrodynamic moment on the hinged wavemaker may be obtained:

$$ {}_2M_p=H\{3k_o[\mathbf{C}(2k)[2k(1-d)\mathbf{T}(2k)-1]+\mathbf{C}(2kd)]/[32k^2(1-d)\mathbf{S}^4(k) $$

$$ +[4k_ok^2/q_1^5(1-d)]\cdot $$

$$ [3\mathbf{Q}_1(q_1)\mathrm{cosech}\ 2k/2\Delta(2k,q_1)-k\mathbf{n}_1^3\Gamma_1(k,q_1)]/[8\mathbf{D}(k)\Delta^2(k,q_1)\mathbf{S}(k)]\cdot $$

$$ [\mathbf{Q}_1(q_1)[1-4k_o(1-d)-\mathbf{Q}_1(q_1d)] $$

$$ +[\frac{1}{8}-k_o(1-d)/16\,\mathbf{S}^2(k)-[k_o/2(1-d)](\mathbf{n}_1^2k^2/\mathbf{D}(k)]^2\cdot $$

$$ \sum_{m=2}^{N_S}\{[\mathbf{Q}_m(q_m)[1-4k_o(1-d)]-\mathbf{Q}_m(q_md)]/q_m^5\cdot $$

$$ \sum_{n=2}^{L}\mathbf{D}_n(k_n)\beta_{mn}(k_n,q_m)/\Delta^2(k_n,q_m)k_n\mathbf{n}_n\}+k_o\mathbf{n}_1^3\sigma_1(k,k_m)/[4(1-d)\mathbf{D}(k)\mathbf{S}(k)] $$

$$ -k_o(1-d)(n_1^2k^2/4\mathbf{D}(k)]^2\sigma_2(k,k_m)-k_o(\mathbf{n}_1^2k^2/\mathbf{D}(k))^2\sigma_3(k_m,k_n)/8(1-d) $$

$$ -k_o^2(\mathbf{n}_1^2k^2/\mathbf{D}(k))^2\sigma_4(k_m,k_n)/8\} \quad (33a) $$

$$ {}_2M_e=-H\{k_o(\mathbf{n}_1^2k^2/\mathbf{D}(k))^2[\mathbf{Q}_1(q_1)[4k_o(1-d)-1]/2q_1^5(1-d) $$

$$ +\mathbf{Q}_1(q_1d)]\sum_{m=2}^{L}\mathbf{D}_m(k_m)\beta_{m1}(k_m,q_1)/\Sigma^2(k_m,q_1)k_m\mathbf{n}_m $$

$$ +[4k_ok^2/(1-d)]\sum_{m=2}^{N_C}[\mathbf{Q}_m(q_m)[4k_o(1-d)-1]+\mathbf{Q}_m(q_md)][3\mathbf{Q}_m(q_m)/2\Sigma(2k,q_m)\mathbf{S}(2k) $$

$$-kn_1^3 \Gamma_m(k, q_m)/[8\mathbf{D}(k)\Sigma^2(2k, q_m\mathbf{S}(k)]/q_m^5 + [k_o k\, n_1^3/4(1-d)\mathbf{D}(k)\mathbf{S}(k) \sum_{m=2}^{L} \mathbf{D}_m(k_m)$$

$$\{\phi_1(k)\phi_m(k_m) \cdot [\Delta(k_m, k)\Omega^2(k) + 2\mathbf{R}^2(k_m, k) - 2k_o(1-d)\ \Sigma(k_m, k)]$$

$$+\phi_1(kd)\phi_m(k_m d)[\Delta(k_m, k)\mathbf{R}(k_m, k)\mathbf{T}(kd)\mathbf{t}(k_m d) - 2\mathbf{R}^2(k_m, k)]\}/[\Sigma^2(k_m, k)k_m^3\mathbf{n}_m]\} \quad (33b)$$

$$_2M_{af} = -H\{\frac{1}{8} - k_o[\mathbf{C}(2k)[2k(1-d)\mathbf{T}(2k) - 1] + \mathbf{C}(2kd)]/[32k^2(1-d)\mathbf{S}^2(k)]$$

$$+k_o(1-d)/16\mathbf{S}^2(k) + k_o\mathbf{n}_1^3\sigma_1(k, k_m)/[4(1-d)\mathbf{D}(k)\mathbf{S}(k)]$$

$$-k_o(1-d)(\mathbf{n}_1^2k^2/4\mathbf{D}(k))^2\sigma_2(k, k_m)$$

$$-k_o(\mathbf{n}_1^2k^2/\mathbf{D}(k))^2\sigma_3(k_m, k_n)/8(1-d) + k_o^2(\mathbf{n}_1^2k^2/\mathbf{D}(k))^2\sigma_4(k_m, k_n)/8\} \quad (33c)$$

in which

$$\sigma_1(k, k_m) = \sum_{m=2}^{L}\mathbf{D}_m(k_m)\{\phi_1(k)\phi_m(k_m)[\Delta(k_m, k) - 2\Omega^2(k)] - \phi_1(kd)\phi_m(k_m d) \cdot$$

$$[\Delta(k_m, k) + 2\mathbf{R}(k_m, k)\mathbf{T}(kd)\mathbf{t}(t_m d)]\}/[\Sigma^2(k_m, k)k_m^2\mathbf{n}_m] \quad (34a)$$

$$\sigma_2(k, k_m) = \sum_{m=2}^{L}(\mathbf{D}_m(k_m)/k_m^2\mathbf{n}_m^2)^2 \quad (34b)$$

$$\sigma_3(k_m, k_n) = \sum_{m=2}^{L}\sum_{n=2}^{L}(1-\delta_{mn})\mathbf{D}_m(k_m)\mathbf{D}_n(k_n)\mathbf{R}(k_m, k_n)\{\phi_m(k_m)\phi_n(k_n)[1+\mathbf{R}(k_o, k_m)$$

$$\mathbf{R}(k_o, k_n) + k_o[\mathbf{R}(k_m, k_n) + \mathbf{R}(k_n, k_m) - 2] - k_o d[1-\mathbf{R}(k_m, k_n)]^2]/\mathbf{R}(k_m, kn)$$

$$-\phi_m(k_m d)\phi_n(k_n d)[1+\mathbf{t}(k_m d)\mathbf{t}(k_n d)]\}/[\mathbf{n}_m\mathbf{n}_n(k_m k_n)^3[1-\mathbf{R}(k_m, k_n)]^2] \quad (34c)$$

$$\sigma_4(k_m, k_n) = \sum_{m=2}^{L}\sum_{n=2}^{L}\mathbf{D}_m(k_m)\mathbf{D}_n(k_n)\phi_m(k_m)\phi_n(k_n)/[\mathbf{n}_m\mathbf{n}_n(k_m k_n)^3] \quad (34d)$$

Note that the dimensinless second-order moment is proportional to the dimensionless design wave height H (= H*/h*) while the dimensionless first-order moment given by Eqs. (32) is independent of H.

The design curves for the linear moment as functions of the relative water depth (h^*/L_o^*) and hinge height (d) were given by Hudspeth and Chen (1981). Similar design tables and discussions were presented by Hyun (1976). The contributions of the evanescent eigenmodes to the total hydrodynamic moment were found to be more pronounced in relatively deep water for hinged wavemakers with deeper drafts.

The magnitude of the second-order dimensionless evanescent (sin $2\omega t$), propagating (cos $2\omega t$) and total hydrodynamic moments are shown in Fig. 2 as functions of relative water depth h^*/L_o^* for the dimensionless design wave height $H^*/h^* = 0.1$. The magnitude of the dimensionless hydrodynamic propagating mode moments, $_2M_p$, decreases as the relative water depth increases until a minimum is reached between $.27 < h^*/L_o^* < .31$. For relative water depths beyond these minimum values, $_2M_p$ increases. Since $_2M_p$ contributes the main portion of the total second-order hydrodynamic moment, $|_2M|$ shows a similar variation. The dimensionless hydrodynamic evanescent mode moments, $_2M_e$, demonstrate positive maxima

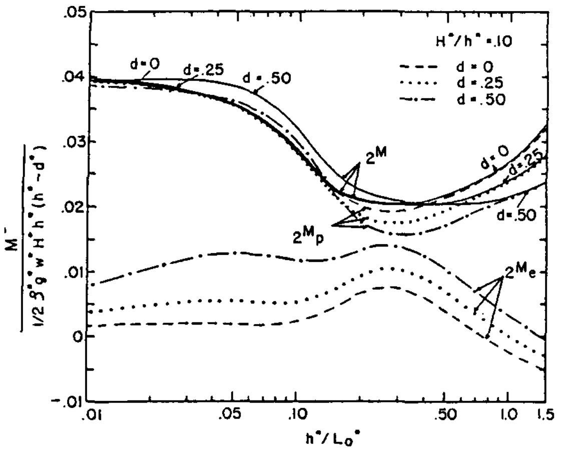

Fig. 2. Amplitude of the total (———), propagating and evanescent dimensionless second-order moment for dimensionless design wave eight H*/h* 0.1.

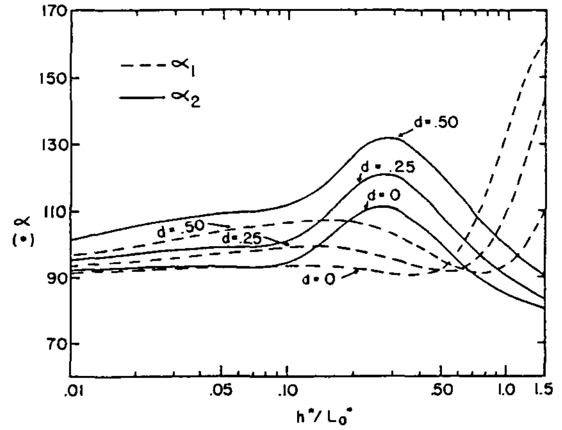

Fig. 3 Relative phase angle, α , between the total hydrodynamic moment and displacement of the wavemaker.

in relative water depth between $.25 < h^*/L_0^* < .27$. Consequently, their contributions to the total moments are most prominent in intermediate water depths. In addition, ${}_2M_e$ also contributes significantly to ${}_2M$ in deeper water ($h/L_0 > 1.0$).

The relative importance of the two hydrodynamic components of the total hydrodynamic pressure moment can also be seen from the phase angles between the hydrodynamic pressure moment and the displacement of the hinged wavemaker, α_2. Fig. 3 demonstrates the relative phase angles as functions of the relative water depth, h^*/L_0^*, for both the linear and

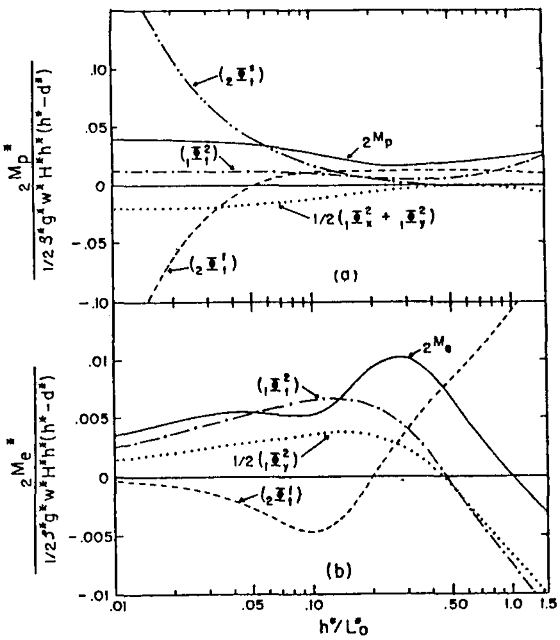

Fig. 4 Contribution of terms to the dimensionless second-order moment amplitude of a) cos $2\omega t$ and b) sin $2\omega t$ for dimensionless wavemaker draft $d = 0.25$ and dimensionless wave height $H^*/h^* = 0.1$.

second-order hydrodynamic moments. For the linear moments, Fig. 3 shows that, ${}_1M_e$ is relatively more important in deep water; especially when the hinge height, d, is small. For the second-order moments, however, ${}_2M_e$ is rather significant in intermediate water depth; especially for large d. This difference between the linear and the second-order moments is not surprising because ${}_2M_p$ do not physically represent the same effects as ${}_1M_p$ and ${}_1M_e$. For the linear solution, ${}_1M_p$ and ${}_1M_e$ represent the moments due to the propagating (or resistive) and the evanescent (or inertia) wave components, respectively. In classical naval hydrodynamics, ${}_1M_p$ and ${}_1M_e$ are associated with the damping coefficient and the added mass moment of inertia, respectively. In contrast, ${}_2M_p$ and ${}_2M_e$ result from both the propagating and the evanescent modes of the first-and the second-order wave components. They are termed the second-order "propagating" and "evanescent" mode moments simply for convenience and do not really represent the moments due to those two components of forced waves. Therefore, it should not be expected that the second-order moments would behave similar to the linear moments. It is, however, interesting to note that the ratio of $_2M_e$ to $_2M_p$ in Fig. 2 is similar to the ratio of $_1M_e$ to $_1M_p$ given in Fig. 4 of Hudspeth and Chen (1981).

The components of the second-order hydrodynamic moment acting on the wavemaker are obtained from the four terms in Eq. (30a); viz., 1) $_2\Phi_t$(o, y, t); 2) 1/2 [$_1\Phi_x^2$(o, y, t) + $_1\Phi_y^2$(o, y, t)]; 3) $_1\Phi_t^2$(o, o, t); and 4) $_2$**E**. We remark again that the second-order velocity potential $_2\Phi$ is composed of the Stoke's wave potential, $_2\Phi^s$, and a forced wave potential, $_2\Phi^f$, given by Eqs. (10) and (12), respectively. Fig. 4 shows the contribution by each of these four terms to $_2M_p$ and $_2M_e$ for the case of d = 0.25 and H*/h* = 0.10. For other values of d, similar graphs may be drawn. The hydrodynamic pressure component due to the Stoke's wave potential, $_2\Phi_t^s$, contributes most to $_2M_p$ in Fig. 4a for shallow water while its contribution diminishes reapidly as the water depth approaches deep water conditins. The hydrodynamic pressure component due to the second-order forced wave potential, $_2\Phi_t^f$, acts in opposition to the component due to $_2\Phi_t^s$ in Fig. 4a and cancels the major portion from $_2\Phi_t^s$ in shallow water. Its contribution to $_2M_p$ in intermediate and deep water is more significant. It is well known that the Stoke's perturbation expansion fails to converge in shallow water {Wehausen (1960)}. Figure 4a demonstrates this lack of convergence in ($_2\Phi_t^s$). However, $_2M_p$ remains bounded in Fig. 4a in shallow water region because of the cancellation between the forced wave ($_2\Phi_t^f$) and the Stoke's free wave ($_2\Phi_t^s$). The contribution from the quadratic first-order velocity component, $_1\Phi_x^2 + {}_1\Phi_y^2$, decreases gradually in Fig. 4a as relative water depth increases. The contribution from the first-order pressure component above still water level at the wavemaker, $_1\Phi_t^2$, is relatively constant over the entire relative water depth in Fig. 4a; but its relative contribution to $_2M_p$ increases in deep water.

For $_2M_e$ in Fig. 4b, the contribution from $_2\Phi_t^f$ is opposed by the contribution from $_1\Phi_y^2$; but it still contributes a significant portion to $_2M_e$ in deep water. The major portion of $_2M_e$ is, however, due to $_1\Phi_t^2$ over the whole relative water depth considered in

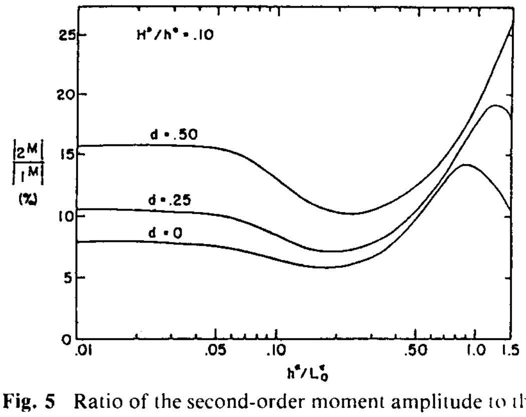

Fig. 5 Ratio of the second-order moment amplitude to the first-order moment amplitude for dimensionless wave height H*/h* = 0.1.

Fig. 4b. The quadratic first order horizontal velocity $_1\Phi_x^2$ does not contribute to $_2M$ in Fig. 4b.

Fig. 5 shows the ratio of the magnitude of the second-order total harmonic hydrodynamic moment, $|_2M|$, to the first-order values, $|_1M|$, for the dimensionless design wave hight H = 0.1. Nonlinear effects are more pronounced for shallow and deep water conditions, and increasing the draft, d, significantly enhances nolinear effects. Since the dimensionless second order hydrodynamic moment is proportional to the dimensionless wave height H, increasing the value of H will increase the nonlinear effect. However, for practical applications in most physical wave flumes, the value of H is limited to usually low values in shallow and deep water. For higher values of d, lower values of H are expected due to the high values of the wavemaker gain function, S/H. For example, the OUS-WRF studied by Hudspeth and Chen (1981) is capable of generating wave height up to 1.4 meters in intermediate water depth ($.18 < h^*/L_o^* < .40$) for a water depth h* = 4.4 meters and a hinge height d* = 0. The second-order total hydrodynamic pressure moments, $|_2M|$, for these largest waves represent an increase of up to 23% over the linear total moments, $|_1M|$. On the other hand, the largest wave heights generated in shallow ($h^*/L_o^* < .03$) and in deep water ($h^*/L_o^* > .8$) are about 0.5 meters and 0.4 meters, respectively. The ratio of $|_2M|/|_1M|$ for these two extreme conditions represent an increase of about 10% and 15%, respec-

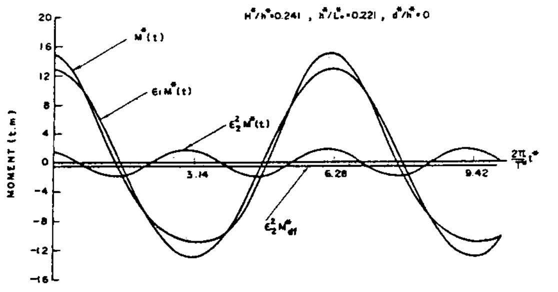

Fig. 6. Illustrative variations of the hydrodynamic pressure moment components acting on wave board over a wave cycle; $H^* = 1.07^m$, $T^* = 3.5^{seconds}$, $h^* = 4.4^m$, $d^* = 0$, $w^* = 3.66^m$.

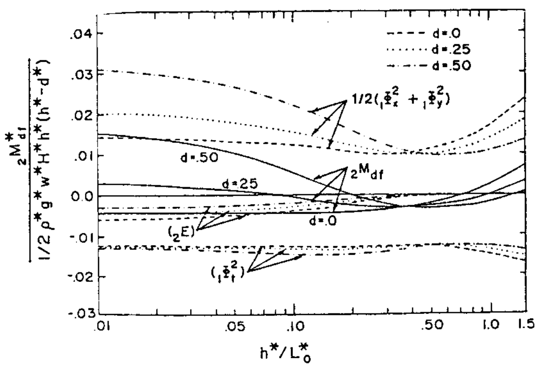

Fig. 7 Contribution of terms to dimensionless second-order time-independent moment for dimensionless wave height $H^*/h^* = 0.1$

tively. Note that $|_2M|$ is the magnitude of the second harmonic variation and does not include the time-independent second-order moment given by M_{df}.

Time varcations of the total moment including the linear and the second-order components in this facility for the case of $H^* = 1.07$ m, $T^* = 3.5$ sec., $h^* = 4.4$ m, $w^* = 3.66$ m, and $d^* = 0$ according to Eqs. (26c), (30c) are demonstrated in Fig. 6. In this case, the maximum second-order total hydrodynamic pressure moment, $M^*(t)$, including the time-independent second-order moment, $_2M^*_{df}$, shows an increase of 17% over the maximum linear moment, $\epsilon\ _1M^*(t)$.

The magnitude of the time-independent hydrodynamic moment is computed from both the time-independent components of the quadratic terms of the linear solution [i.e., $\frac{1}{2}(_1\Phi^2_x + {_1\Phi^2_y})$ and $_1\Phi^2_t$] as well as from the Bernoulli constant, $_2$**E**. Dimensionless time-independent moments about the hinge of the wavemaker are shown in Fig. 7 for a dimensionless design wave height $H^*/h^* = 0.1$. The larger portion of the component due to the quadratic first-order velocity is canceled by both the components due to the hydrodynamic pressure above still water level, $_1\Phi^2_t$, and the Bernoulli constant, $_2E$, but still contributes the major part to $_2M_{df}$ for higher values of d. The contribution of the Bernoulli constant component is almost negligible in deep water. The magnitude of $_2M_{df}$ is less than 25% of $|_2M|$ except for the case of $d = .50$ in shallow water ($.01 < h^*/L^*_0 < .06$) where it represents about 38% of $|_2M|$.

7. CONCLUSIONS

The dimensionless nonlinear hydrodynamic pressure force and moment on hinged wavemakers of variable-draft have been examined. Dimensionless design curves for the hydrodynamic moment at second-order are presented.

The hydrodynamic moment at second-order is found to be dependent on both the water depth and wavemaker draft in a similar manner as the linear dimensionless hydrodynamic moment shows.

The contribution of the second-order evanescent mode hydrodynamic moment (which is in phase with wavemaker acceleration) to the total hydrodynamic moment is significant in intermediate water depths. Nonlinear effects are pronounced in shallow and deep water for a given dimensionless wave hight, H^*/h^* and for higher values of the wavemaker draft. The second-order hydrodynamic moment is approximately between 5-25% of the linear values for the dimensionless design wave height $H^*/h^* = 0.1$. The magnitude of the time-independent hydrodynamic pressure moment at second-order was shown to be less than 25% of the amplitude of the harmonic second-order hydrodynamic pressure moment for most practical cases considered.

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APPENDIX. NOTATION

The following symbols are used in this paper:

A_o, A_n	: dimensionless multiplicative coefficients for time-independent second-order velocity potential. (All physical variables in nomenclature hereafter represent dimensionless quantities).
a_l, a_m	: multiplicative coefficients for the linear velocity potential.
B_l, C_l, B_n, C_n	: multiplicative coefficients for the second-order forced wave potential.
d	: height of wavemaker hinge above the bottom.
e	: height of wavemaker piston measured above wavemaker hinge.
E	: Bernoulli constant.
F, F_{df}, F_e, F_p	: total, time-independent, evanescent mode and propagating mode hydrodynamic pressure force, respectively.
g	: gravitational constant.
H	: deterministic wave height.

h : still water depth of wave channel.

$k\ (=\frac{2\pi}{L})\ k_m$: wave number for linear propagating and evanescent eigenmode, respectively.

$L, L_o (= g\ T/2\pi)$: wave length in finite depth and in deep water, respectively.

M, M_{df}, M_e, M_p : total, time-independent, evanescent mode and propagating mode hydrodynamic moment on the wavemaker, respectively.

$\mathbf{n}_m, \mathbf{N}_m$: normalizing constant for linear and second-order solution, respectively.

p : pressure.

$\mathbf{Q}_l, \mathbf{Q}_m$: orthonormal eigenfunctions for second-order solution.

q_l, q_m : second harmonic wave number for propagating and evanescenct mode, respectively.

S/2 : wavemaker stroke amplitude measured at the level of wavemaker piston.

T : wave period.

t : time variable.

U(•) : Heaviside step function

u,v : horizontal and vertical component of water particle velocity, respectively.

w : total width of wavemaker.

X : Stoke's material coordinate for wavemaker displacement.

x,y : horizontal and vertical Cartesian coordinate axis, respectively, with origin located at undisturbed water level at wavemaker.

Greek

α_1, α_2 : relative phase angle for total hydrodynamic moment at first-and second-order, respectively.

ε : small perturbation parameter equal to Hk/2.

η : instantaneous water surface elevation measured positive upwards from the still water level.

μ_n : nth eigenvalues for time-independent second-order velocity potential.

ν : truncation criterion for evanescent eigenmodes.

ξ : vertical dependence of the prescribed wavemaker displacement.

ρ : fluid density.

Φ : temporal and spatial velocity potential.

ϕ_l, ϕ_m : orthonormal eigenfunctions for linear solution.

Ψ : time-indenpendent second-order velocity potential.

ψ_n : orthonormal eigenfunction for time-independent second-order velocity potential.

$\omega\ (=2\pi T)$: radian frequency.

: prescribed instantaneous wavemaker displacement.

Superscripts

s : Stoke's second-order velocity potential.

f : forced second-order velocity potential.

* : dimensional variable.

Subscripts

p,e : propagating (m = 1) and evanescent eigenmode (m≥2).

o : deep water condition.

1(2) : linear (second-order).