論 文

Direction Finding Problem에서의 신호원 갯수 추정 신뢰도에 관한 AIC와 MDL의 비교

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Comparisons of AIC and MDL on Estimation Reliability of Number of Sources in Direction Finding Problem

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要 約 본 논문에서는 array processing에서, sensor array를 통해 들어오는 source signal들의 갯수를 결정하는 방법들을 판정의 정확도의 관점에서 연구 고찰한다. 첫번째 방법은 Akaike의 Akaike's Information Criterion (AIC)이고, 다른 하나는 Schwartz와 Rissanen의 Minimum Description Length(MDL)이다. 실용적인 측면에서 볼 때, 신호대잡음비 (S/N)가 매우 낮은 상태에서 얻어진 한정된 양의 data를 이용하여 제한된 갯수의 sensor들로 이루어진 array로 부터, 매우 근접해 있는 source signal들의 갯수를 예측해 내는 것은 대단히 중요한 일이다. 본 논문은 simulation 결과를 통하여, source signal들이 근접해 있을수록, array의 sensor 갯수가 줄어들수록, 이용할 data의 양이 한정될수록, 또 S/N가 낮아질수록, AIC이 MDL에 비해서 높은 신뢰도를 가심을 보여준다.

ABSTRACT In this paper, a couple of well-known methods for determination of the number of source signals impinging on sensor array in array processing are introduced and compared in terms of estimation accuracy. The one is the procedure issued by Akaike (Akaike's Information Criterion : AIC) and the other one by Schwartz and Rissanen (Minimum Description Length : MDL). This paper demonstrates, through computer simulation, that the AIC is more reliable than the MDL in such troublesome cases as very closely spaced source signals, very limited number of sensors in the array, finite data sequences and/or low Signal-to-Noise ratio (S/N).

I. Introduction

In sonar or radar array processing, it is important to estimate the direction angles of distant source signals using the recorded data sequences which are corrupted by additive noises on sensor array. One of the direction finding methods developed recently is the eigenstructure-based method such as standard Multiple Signal Classification (MUSIC) method[9][10], Coherent Signal-Subspace (CSS) processing

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method[13][14], spatial smoothing method[5][12], and nonlinear Second-Order Method (SOM) [2][6].

Even though the eigenstructure algorithm theoretically provides the information on the number of source signals by examining the identical minimum eigenvalues of the array covariance matrix, the eigenvalues of the array covariance matrix for a finite sampled data size practically result in differences of magnitude. Therefore, it is essential to estimate the number of source signals before using the eigenstructure based methods for direction finding problem.

In this paper two procedures introduced by Akaike (AIC)[1] and by Schwartz and Rissanen [7][11] for determining the number of closely located source signals are considered and compared, with respect to their estimation accuracies as a function of 1) closeness of source signals 2) number of sensors 3) number of data sequences obtained from sensor array 4) S/N ; because closely spaced source signals, limited number of sensors in array, finite data sequences from sensor array, and low S/N environments are frequently encountered in the real world.

Ⅱ. Problem Formulation

For the direction finding problem of multiple source signals incident to a sensor array in the presence of background additive noise, consider the M source signals impinging on the uniform linear array with Q sensors, as shown in Fig. 1., from directions $\{\theta_1, \theta_2, \cdots, \theta_M\}$.

Then the signals received at the ith sensor can be expressed as

$$r_i(t)=\sum_{m=1}^{M} s_m(t-(i-1)(D/c)\sin\theta_m)+x_i(t) \quad (1$$

where

$s_m(t)$=signal emitted by the mth source,
D=sensor spacing,
c=speed of wave propagation,
θ_m=direction angle of the mth source, and
$x_i(t)$=additive noise at the ith sensor, and
$x(1), x(2),\cdots,x(N)$ with N sampled data are independent and identically distributed (i. i. d.).

From Eq.(1), the sample covariance matrix **R** can be now obtained as

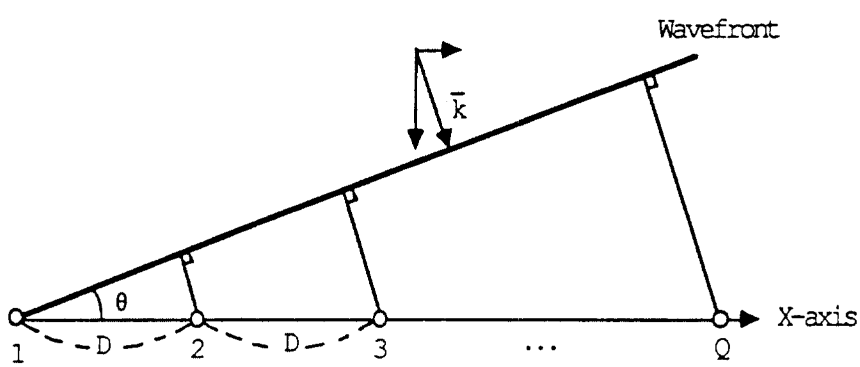

Fig.1 Configuration for a planewave impinging on the sensor array

$$\mathbf{R}=(1/N)\sum_{t=1}^{N} r(t)r^{*}(t) \qquad (2)$$

where

*denotes the complex conjugate transpose of a matrix or a vector, and

$$r^{T}(t)=[r_1(t),\ r_2(t),\cdots,r_Q(t)]$$

Assuming that $\varepsilon_1>\varepsilon_2>\cdots>\varepsilon_Q$ are the eigenvalues of the sample-covariance matrix $\mathbf{R}$, the AIC is then defined by

$$AIC(k)=-2L(k)+2\xi(k), \qquad (3)$$

where

$$L(k)=-2\log\left[\frac{\prod_{i=k+1}^{Q}\varepsilon_i}{\left[(1/(Q-k))\sum_{i=k+1}^{Q}\varepsilon_i\right]^{Q-k}}\right]^{N} \qquad (4)$$

is the maximum log-likelihood, and

$$\xi(k)=k(2Q-k)+1 \qquad (5)$$

is the number of free adjusted parameters within the model which provide the minimum AIC, i.e., $\xi(k)$=(total number of parameters) −(number of parameters due to the normalization of the eigenvectors) (number of parameters due to the mutual orthogonalization of the eigenvectors)= $(k+1+2Qk)-(2k)-2((1/2)k(k-1))$.

The number of source signals is now determined by selecting the minimum value of AIC(k), where $k=0, 1, 2,\cdots,Q-1$.

The other criterion to consider is the MDL, which is defined by

$$MDL(k)=-L(k)+\xi(k)\log(N/2). \qquad (6)$$

where

L(k) and $\xi(k)$ are given by Eqs. (4) and (5), respectively.

Note that according to Kashyap[4] the AIC has been found to be statistically inconsistent insofar as the probability of error in choosing the correct number of source signals does not tend to zero as N goes to infinity. Hence, for the large sample limit, the AIC tends to overestimte the true rank M. On the other hand, the MDL is known to yield a consistent estimate, i. e., the selection criterion converges to the true rank M in the large sample limit[3][8][15].

Most of comparisons between the AIC and MDL have been made by several authors [3][4][8][15], under the condition of reasonably high S/N. Practically, however, there may be confronted with very low S/N and limited data size situations. Therefore, it is important to compare the reliabilites of the MDL and AIC for estimating the number of closely spaced multiple source signals, provided very low S/N, limited number of sensors in an array, and/or finite data sequences.

Ⅲ. Computer Simulations

In this section, simulation results are presented to show the performances of the AIC and MDL for determining the number of distant closely spaced multiple source signals with relatively low S/N, in the case of finite sample size and limited number of sensors. Throughout the entire experiment, the source signals are assumed to be planewaves with uniformly distributed random phases on $(0, 2\pi)$ and the sensor spacing is assumed to be equal to half the wavelength of the impinging wavefront.

Four catagories are considered to observe the limitations of the detectability of the MDL and AIC. First catagory is provided to compare the detectability of the MDL and AIC in terms of S / N. Two examples for the first catagory are given. One example is for the closely spaced two sources, at $\theta_1=0°$ and $\theta_2=5°$, impinging on an sensor array of five sensors with 256 sampled data, as function of S / N. As shown in Table 1, both the MDL anAIC successfully detect the two closely spaced source signals at S / N=2 dB, even though the eigenvalues of the sample covariance matrix are all different, i. e., 11.44, 1.02, 0.71, 0.62, 0.56. However, when S / N decreases to 1 dB, the AIC minimum value can be correctly obtained for M=2, but the MDL minimum value is incorrectly detected for M=1. The lowest S / N for the AIC sufficient to determine precisely the number of source signals is −2 dB, while the MDL is satisfactory only if S / N is greater than or equal to 2 dB. The other example of the first catagory is for two sources, at $\theta_1=0°$ and $\theta_2=5°$, with four sensors and 512 sampled data, as a function of S / N. As shown in Table 2, both the MDL and AIC successfully detect the two closely spaced source signals at S / N=4dB. As S / N goes down to 3 dB, the MDL already shows incorrect number of source signals for M=1, while the AIC gives correct answer. Furthermore, Table 2 demonstrates that the AIC is reliable for the detection of right number of source signals until S / N is down to −3 dB.

Second catagory taken is to examine how well the MDL and AIC can detect the two closely located source signals. Two examples are also given for the second catagory. For the first example to compare the detectability of the MDL and AIC in terms of the closeness of the sources, as in Table 3, several source spacings in degree are taken with S / N=0 dB, Q=5, and N=256. In this example, the limitation of the detectability is 6° separation for the MDL, while 4°separation for the AIC. Second example is provided for two sources with S / N=2 dB, Q=6, and N=128. As shown in Talbe 4, the AIC could detect two sources of even 4° separation, but the MDL could do two sources of 6° separation.

Third catagory also with two example is provided for examining the limitation of the detectability of the MDL and AIC with respect to the number of sensors (Q) used. As shown in Table 5 for the first example, when the two closely spaced source signals, $\theta_1=0°$ and $\theta_2=5°$, are arriving at a sensor array with S / N =0 dB and N=256, the minimum number of sensors, required to achieve the right determination of the number of source signals using the MDL test, is seven. On the other hand, four sensors are enough to obtain right result for the first example using the AIC. Table 6 for the second example shows that in order to determine the number of source signals, $\theta_1=44°$ and $\theta_2=48°$, with S / N=1 dB and N=512, the minimum number of sensors required is seven for the MDL test. On the other hand, five sensors are enough to get right result for the first example using te AIC.

Finally, as a function of sampled data N, the limitations of the MDL and AIC are dealt with, by analyzing two examples. One example to be considered is for the two closely spaced source signals, $\theta_1=0°$, and $\theta_2=5°$, given S / N=0 dB and Q=5. As shown in Table 7, the MDL test fails to obtain correct number of source signals even with N=512. However, the AIC test successfully performs with just N=256. The other example is taken to com-

Table 1. The limitations of MDL and AIC for the two closely spaced source signals at $\theta_1=0$ and $\theta_2=5°$ with Q=5, N=256, in terms of S/N, where the value with superscript ♯ in each column represents the minimum value of MDL(k) or AIC(k).

	k	0	1	2	3	4
S/N=	MDL	1079.1	52.4	47.9 ♯	58.9	66.5
2 dB	AIC	2158.2	72.9	39.0 ♯	43.4	48.0
S/N=	MDL	931.1	46.1 ♯	47.9	58.9	66.5
1 dB	AIC	1862.1	60.3	39.0 ♯	43.3	48.0
S/N=	MDL	792.4	41.6 ♯	47.9	58.9	66.5
0 dB	AIC	1584.9	51.3	39.0 ♯	43.3	48.0
S/N=	MDL	549.7	36.3 ♯	47.9	58.8	66.5
−2dB	AIC	1099.1	40.7	39.0 ♯	43.2	48.0
S/N=	MDL	447.1	34.8 ♯	47.8	58.5	66.5
−3dB	AIC	894.1	37.7 ♯	39.0	43.1	48.0

Table 2. The limitations of MDL and AIC for the two closely spaced source signals at $\theta_1=13$ and $\theta_2=17°$ with Q=4, N=512, in terms of S/N, where the value with superscript ♯ in each column represents the minimum value of MDL(k) or AIC(k).

	k	0	1	2	3
S/N=	MDL	2090.2	41.5	38.4	46.8
4 dB	AIC	4180.5	53.4	25.9	30.0
S/N=	MDL	1832.0	37.5	38.4	46.8
3 dB	AIC	3664.0	45.2	26.0	30.0
S/N=	MDL	1137.8	30.7	38.5	46.8
0 dB	AIC	2275.7	31.8	26.1	30.3
S/N=	MDL	765.1	28.6	38.5	46.8
−2dB	AIC	1530.3	27.6	26.2	30.3
S/N=	MDL	610.4	27.9	38.5	46.8
−3dB	AIC	1220.7	26.2	26.1	30.0

Table 3. The limitations of MDL and AIC for the two closely spaced source signals at S/N=0 dB, Q=5, N=256, in terms of the closeness of surces, where the value with superscript ♯ in each column represents the minimum value of MDL(k) or AIC(k).

	k	0	1	2	3	4
$\theta_1=0$	MDL	763.7	57.8	47.9 ♯	59.0	66.5
$\theta_2=7$	AIC	1527.4	83.8	39.1 ♯	43.5	48.0
$\theta_1=0$	MDL	779.2	48.6	47.9 ♯	58.9	66.5
$\theta_2=6$	AIC	1558.4	65.3	39.0 ♯	43.4	48.0
$\theta_1=0$	MDL	792.4	41.6 ♯	47.9	58.9	66.5
$\theta_2=5$	AIC	1584.9	51.3	39.0 ♯	43.3	48.0
$\theta_1=0$	MDL	802.8	36.7 ♯	47.8	58.8	66.5
$\theta_2=4$	AIC	1605.6	41.5	39.0 ♯	43.1	48.0
$\theta_1=0$	MDL	809.5	33.7 ♯	47.8	58.7	66.5
$\theta_2=3$	AIC	1618.9	35.5 ♯	38.9	43.0	48.0

Table 4. The limitations of MDL and AIC for the two closely spaced source signals at S/N=2 dB, Q=6, N=128, in terms of the closeness of surces, where the value with superscript ♯ in each column represents the minimum value of MDL(k) or AIC(k).

	k	0	1	2	3	4	5
$\theta_1=25°$	MDL	649.3	56.9	53.8	67.2	78.5	84.9
$\theta_2=31°$	AIC	1298.7	82.5	50.6	57.5	65.7	70.0
$\theta_1=25°$	MDL	657.3	47.7	53.7	67.3	78.5	84.0
$\theta_2=30°$	AIC	1314.7	63.9	50.4	57.5	65.7	70.0
$\theta_1=25°$	MDL	664.6	41.4	53.5	67.3	78.6	84.9
$\theta_2=29°$	AIC	1329.2	51.5	49.9	57.6	65.9	70.0
$\theta_1=25°$	MDL	670.2	38.2	53.1	67.5	78.8	84.9
$\theta_2=28°$	AIC	1340.4	45.0	49.1	57.9	66.2	70.0

Table 5. The limitations of MDL and AIC for the two closely spaced source signals at $\theta_1=0°$ and $\theta_2=5°$ with S / N=0 dB, N=256, in terms of number of sensors Q, where the value with superscript # in each column represents the minimum value of MDL(k) or AIC(k).

	k	0	1	2	3	4	5	6
Q=7	MDL	1333.3	108.6	82.3 #	99.5	115.1	128.5	133.1
	AIC	2666.6	171.1	79.5 #	82.1	88.3	93.5	96.0
Q=6	MDL	1066.6	57.0 #	60.0	76.6	88.9	97.0	
	AIC	2133.2	75.1	49.1 #	57.1	64.3	70.0	
Q=5	MDL	792.1	41.6 #	47.9	58.9	66.5		
	AIC	1584.9	51.3	39.0 #	43.3	48.0		
Q=4	MDL	563.7	30.1 #	34.6	41.6			
	AIC	1127.4	35.3	26.7 #	30.0			
Q=3	MDL	356.3	14.1 #	22.2				
	AIC	712.5	10.5 #	16.0				

Table 6. The limitations of MDL and AIC for the two closely spaced source signals at $\theta_1=44°$ and $\theta_2=48°$ with S / N=1 dB, N=512, in terms of number of sensors Q, where the value with superscript # in each column represents the minimum value of MDL(k) or AIC(k).

	k	0	1	2	3	4	5	6
Q=7	MDL	3013.0	93.1	85.5	109.0	128.0	140.9	149.7
	AIC	6026.0	131.0	69.2	78.2	86.4	91.1	96.0
Q=6	MDL	2441.0	53.4	65.4	86.1	100.5	109.2	
	AIC	4881.8	60.2	46.0	57.8	65.4	70.0	
Q=5	MDL	1921.7	39.6	51.9	66.2	74.9		
	AIC	3849.5	41.1	36.0	43.3	48.0		
Q=4	MDL	1404.0	25.9	38.5	46.8			
	AIC	2808.5	22.1	26.1	30.0			

Table 7. The limitations of MDL and AIC for the two closely spaced source signals at $\theta_1=0°$ and $\theta_2=5°$ with S / N=0 dB, Q5, in terms of number of sampled data N, where the value with supersoript # in each column represents the minimum value of MDL(k) or AIC(k)

	k	0	1	2	3	4
N=512	MDL	1598.3	42.9 #	51.4	66.5	74.9
	AIC	3196.7	47.6	35.0 #	43.9	48.0
N=256	MDL	792.4	41.6 #	47.9	58.9	66.5
	AIC	1584.9	51.3	39.0 #	43.3	48.0
N=128	MDL	366.9	29.4 #	40.7	51.2	58.2
	AIC	733.8	33.1 #	35.7	42.4	48.0
N=64	MDL	111.6	26.6 #	34.9	44.2	49.9
	AIC	223.3	33.7 #	35.2	43.0	48.0

Table 8. The limitations of MDL and AIC for the two closely spaced source signals at $\theta_1=18°$ and $\theta_2=22°$ with S / N=3 dB, Q5, in terms of number of sampled data N, where the value with supersoript # in each column represents the minimum value of MDL(k) or AIC(k)

	k	0	1	2	3	4
N=512	MDL	2520.2	58.4	51.6	66.6	74.9
	AIC	5040.4	78.6	35.4	44.2	48.0
N=256	MDL	1263.9	40.1	46.1	59.3	66.5
	AIC	2527.9	48.2	35.4	44.2	48.0
N=128	MDL	573.1	33.5	40.7	51.2	58.2
	AIC	1146.1	41.3	35.7	42.5	48.0
N=64	MDL	227.7	26.3	35.1	44.2	49.9
	AIC	155.3	33.1	35.6	43.1	48.0

pare the reliability of the MDL and AIC for determining the number of the closely spaced source signals, $\theta_1=18°$ and $\theta_2=22°$, with S/N=3dB and Q=5, in terms of the number of sampled data N. From Talbe 8, it is obvious that the MDL test yileds correct number of source signals with more than or equal to 512 sampled data, while the AIC test successfully determine the number of source signals with just 128 sampled data.

Ⅳ. Conclusion

Recently developed eigenstructure-based methods for direction finding problem requires the prior knowledge on the number of source signals because the eigenvalues of the sample covariance matrix obtained from finite sampled data are all different in manitude. Therefore, it is of consequence to select an appropriate test procedure, for estimating the number of source signals, which could work well even in the environments of closely spaced sources, limited number of sensors, finite sampled data, and/or low S/N. Throughout the computer simulation, it is obvious to conclude that the AIC test offers more reliable result than the MDL test in determination of the number of closely spaced multiple source signals using the finite sampled data obtained from limited number of sensors in an array, with low S/N.

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