◉ Technical Paper

Cable Dynamics for Marine Applications
–Nonlinearities–

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(*Received November* 28, 1990)

해양 응용을 위한 케이블 동역학

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Key Words : Cable(케이블), Geometric Nonlinearities(기하학적 비선형성), Negative Tension (음의 동장력), Clipping Nonlinearity(클리핑 비선형성), Extreme Tension(극단장력)

초 록

해양 산업에서 심해로의 이동은 해양 구조물의 계류 장치의 중요성을 부각시켰고 그에 따른 기본적인 연구로서의 케이블 동력학에 대한 흥미를 일깨웠다. 거친 해상에서 케이블에 형성될 수 있는 큰 동장력과 기하학적 비선형성의 고려는 케이블의 비선형적 거동 해석에 주요 인자가 될 것이다. 또한 매우 큰 동장력 증폭에 의한 음의 큰 동장력은 케이블의 양의 정장력을 초과할 수 있고, 따라서 전체장력은 영 또는 음이 될 수 있다. 비선형 유체 항력을 포함한 모든 비선형성을 갖는 케이블의 이론적 해석 모델을 개발하고, 수치 결과와 기존의 실험 결과를 비교한다.

1. Introduction

Large dynamic tensions occurring in two typical offshore applications, such as deep water moorings and open sea towing, can result in slack-and-snapping phenomena of cable, and finally the cable fails. Also, a build-up of large dynamic tensile forces in long vertical cables, used in deep submergence vehicles for exploration and harvesting of natural resources in deeper water or employed in a deep diving system for salvage, may deteriorate the fatigue life of cables.

When the dynamic tension exceeds the static tension, for part of the cycle the cable is under high tension, and for another part of the cycle under no tension at all. This condition of slack in the cable may lead to a snap, that is, subsequent high build-up of tension leading to breakage or failure by fatigue.

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It is widely accepted in the offshore industry that the use of submerged buoys and inserting short synthetic-like segments are some of the remedies suggested.

In this paper, extreme tensions in snapping horizontal cables are analyzed in order to compare with experiments.

2. Dynamic Behavior of a Cable

2. 1 Nonlinear Governing Equations

The 2-dimensional, nonlinear equations of motion of a cable with coplanar static configuration (see Fig. 1), expressed along the local tangential and normal directions are.[5]

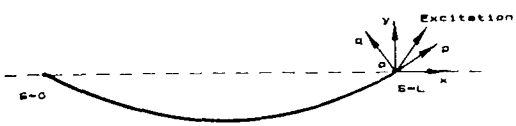


Fig. 1 Excitation and lagrangian coordinates

$$m\frac{\partial^2 p}{\partial t_2} = \frac{\partial T_1}{\partial s} - T_0\frac{d\phi_0}{ds}\phi_1 + F_p + m_a\frac{\partial^2 q}{\partial t^2}\phi_1 - T_1\frac{d\phi_0}{ds}\phi_1$$

$$M\frac{\partial^2 q}{\partial t^2} = \frac{dT_0}{ds}\phi_1 + T_0\frac{\partial \phi_1}{\partial s} + T_1\frac{d\phi_0}{ds} + F_q + \frac{\partial T_1}{\partial s}\phi_1 + T_1\frac{\partial \phi_1}{\partial s} \cdots (1)$$

$$\frac{\partial p}{\partial s} - q\frac{d\phi_0}{ds} = (1+e)\left(-\frac{\phi_1^2}{2}\right) + e_1$$

$$\frac{\partial q}{\partial s} + p\frac{d\phi_0}{ds} = \phi_1(1+e_0)$$

where

T_0 static effective tension

T_1 dynamic effective tension

p tangential displacement based on the static configuration

q normal displacement based on the static configuration

s Lagrangian coordinate

ϕ_0 static angle

ϕ_1 dynamic angle

e_0 static strain

e_1 dynamic strain($e = e_0 + e_1$)

m mass per unit length

m_a added mass per unit length($M = m + m_a$)

E Young's modulus

A cable sectional area

F_p tangential component of the fluid drag force

F_q normal component of the fluid drag force

The first two equations express the force equilibrium in the tangential and normal directions based on the static configuration respectively : the next two equations express compatibility of motion.

2. 2 Nonlinearities in Governing Equations

From equations(1), the following nonlinear terms can be identified.

$$m_a\frac{\partial^2 q}{\partial t^2}\phi_1 \quad \text{and} \quad T_1\frac{d\phi_0}{ds}\phi_1$$

$$\frac{\partial T_1}{\partial s}\phi_1 \quad \text{and} \quad T_1\frac{\partial \phi_1}{\partial s}$$

$$(1+e)\left(-\frac{\phi_1^2}{2}\right) \quad \cdots\cdots\cdots\cdots (2)$$

$$e\phi_1$$

F_p and F_q

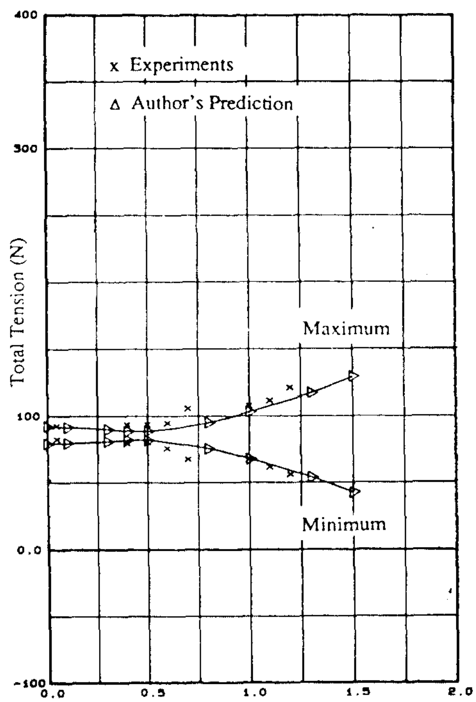
These terms are geometric nonlineartities, the product of large dynamic tension and dynamic angle, nonlinear inertia term, and nonlinear fluid drage forces, respectively.

2. 3 Simplified Cable Dynamic Equation

The fully nonlinear dynamic equation can be simplified, in order to use efficiently nonlinear numerical schemes, but care should be taken not to oversimplify and hence omit important nonlinear mechanisms.

We assume that the dynamic tension is almost uniformly distributed along the cable for frequencies which are not high enough to cause elastic waves;i. e., an quasi-static stretch assumption. Also, the axial motions of the cable becomes much smaller than the transverse motions when a horizontal cable is forced to move at the excitation of moderate frequencies given on the end of the cable.[1,2]

Then we simplify the formulation and construct a relatively simple model for a horizontal cable as follows.[6]



f(HZ)

Fig. 2 Comparison of extreme tensions : horizontal excitation, amplitude=2.5*cm*, static tension=88*N*

$$M\frac{\partial^2 q}{\partial t^2} = (T_0+T_1)\left(\frac{\partial^2 q}{\partial s^2} + \frac{d\phi_0}{ds}\right) + F_q - T_0\frac{d\phi_0}{ds} \quad \cdots\cdots\cdots\cdots (3)$$

with

$$T_1 = \frac{EA}{L}\left[p(L) - \int_0^L q\frac{d\phi}{ds}ds + \frac{1}{2}\int_0^L \left(\frac{\partial q}{\partial s}\right)^2 ds\right]$$

$$F_q = -\frac{1}{2}\rho C_D D \frac{\partial q}{\partial t}\left|\frac{\partial q}{\partial t}\right| \cdots (4)$$

where

ρ mass density of water

$p(L)$ tangential displacement imposed on the end of the cable

C_D drag coefficient

2. 4 Behavior of a Cable Subject to Negative Tension

Larger dynamic tension build-up in rough seas may cause the total tension to become negative in certain parts of the cable. This cannot be sustained by a cable or chain due to their low bending stiffness. The appearance of even a small negative overall tension sets in action a buckling mechanism very quickly. On top of the formation of a buckling mode, there occurs a free-falling of the cable opposed only by the action of the drag force, unlike a string with zero static curvature.

$$M\frac{\partial^2 q}{\partial t^2} = (T_0+T_1)\left(\frac{\partial \phi_1}{\partial s} + \frac{d\phi_0}{ds}\right) + \phi_1\frac{d}{ds}(T_0+T_1) - T_0\frac{d\phi_0}{ds} + F_q$$

$$\Rightarrow M\frac{\partial^2 q}{\partial t^2} = -T_0\frac{d\phi_0}{ds} + F_q \quad \text{as } T_0+T_1 \rightarrow 0$$

Since the transition to negative tension is continuous, the cable under the influence of its own weight has acquired a certain falling velocity by the time the cable goes slack.

It is reasonable to assume that, for moderately large frequencies, the effect of buckling is restricted to preventing the tension from becoming negative, while kinematically and dynamically contribute very small.

2. 5 Clipping-off Model

In order to get a model of a slack and then snapping cable, we assume that the buckling mechanism keeps the tension at near zero levels until a positive value is regained, while its dynamic behavior is governed by the balance of inertia and drag forces as soon as the total tension in an element of the cable reaches a negative value. Then we reformulate governing equations as follows :

$$M\frac{\partial^2 q}{\partial t^2} = -\frac{1}{2}\rho C_D D\frac{\partial q}{\partial t}\left|\frac{\partial q}{\partial t}\right| - T_0\frac{d\phi_0}{ds}$$

and set $T_1 = -T_0$

2. 6 Summary of Nonlinear Governing Equations

$$M\frac{\partial^2 q}{\partial t^2} = (T_0+T_1)\left(\frac{\partial^2 q}{\partial s^2} + \frac{d\phi_0}{ds}\right) - \frac{1}{2}\rho C_D D\frac{\partial q}{\partial t}\left|\frac{\partial q}{\partial t}\right| - T_0\frac{d\phi_0}{ds}$$

with

$$T_1 = \frac{EA}{L}\left[p(L) - \int_0^L q\frac{d\phi}{ds}ds + \frac{1}{2}\int_0^L\left(\frac{\partial q}{ds}\right)^2 ds\right]$$

If $T_0+T_1 < 0$,

$$M\frac{\partial^2 q}{\partial t^2} = -\frac{1}{2}\rho C_D D\frac{\partial q}{\partial t}\left|\frac{\partial q}{\partial t}\right| - T_0\frac{d\phi_0}{ds}$$

ana T_1 is set equal to $-T_0$.

3. Applications

For the numerical scheme, we employ an expansion of the response in terms of Chebyshev polynomials;a collocation method spatially and Newmark's method for time integration. Approximate solutions of the governing equations (3) and (4) are sought in the form of a trucated series.

$$F_N(s,\ t) = \sum_{n=1}^{N} f_n(t)b_n(s)$$

where

$F_N(s,\ t)$ approximate solution

$f_n(t)$ expansion coefficients of $F(s,t)$

$b_n(s)$ time independent orthogonal functions

Due to nonconstant terms in the governing equations of the cable, the collocation method is superior to Galerkin's method.[3]

The principal parameters of the horizontal cable used in the experiment of the Ship Research Institute of Norway are found in Table 1.[7]

Table 1 Cable used in the experiment of the Ship Research Institute of Norway[7]

$T_0 = 88N$	$M = 0.666kg/m$
$W = 5.05\ N/m$	$EA = 7,854,000N$
$L = 10.9774m$	$D = 0.01m$
$C_D = 1.5$	

For high frequencies, and after clipping-off sets in, numerically difficulties arise in the form of high frequency oscillations, that eventually lead to divergence. Also, smaller time steps must be used to ensure numerical accuracy.

4. Conclusion

In Figure 3, differences between experimental data and our numerical results above 1.5 *Hz* are shown. This is due to the delayed onset of zero

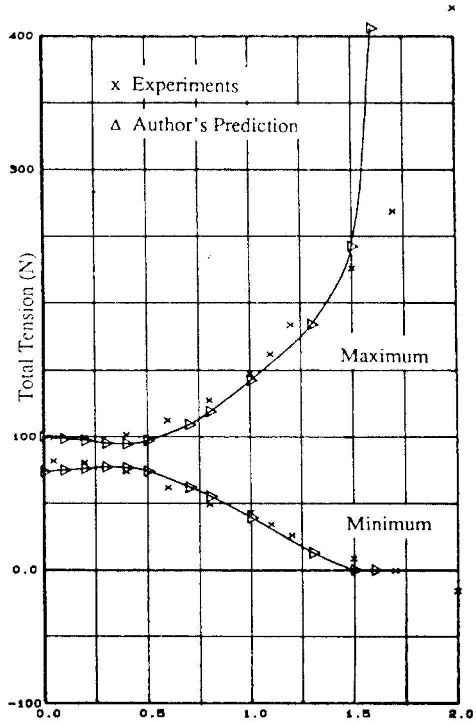


Fig. 3 Comparison of extreme tensions : horizontal excitation, Amplitude=5*cm*, static tension=88*N*

tension(since the bending stiffness is capable of supporting small negative tensions), relative to our numerical model predictions.

If the dynamic tension becomes negative and of amplitude equal to, or larger than the value of the static tension, forcing the total tension to become nonpositive, the small bending stiffness of the cable, which can be neglected under less extreme conditions, may become important due to the lack of any other restoring mechanisms.

In Figures 2, 4 and 5, the prediction of extreme tensions for the cable used in experiments showed good agreement in the maximum values of total tensions while there were discrepancies in the minimum values of them due to the delayed onset of zero tension, i.e., the bending siffness effect.

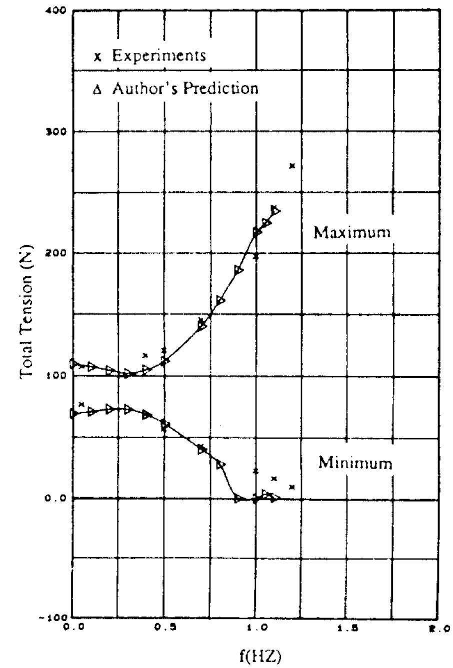


Fig. 4 Comparison of extreme tensions : horizontal excitation, amplitude=7.5*cm*, static tension=88*N*

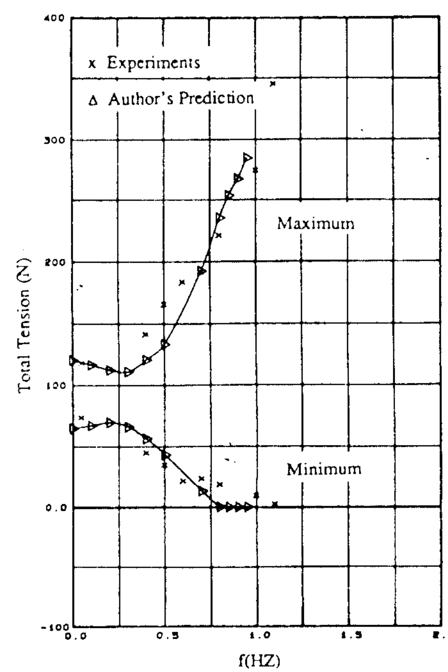


Fig. 5 Comparison of extreme tensions : horizontal excitation, amplitude=10*cm*, static tension=88*N*

후 기

본 연구는 한국과학재단의 1988년도 연구비 지원에 의하여 이루어졌으며 관계제위께 감사를 드린다.(과제번호 : 883-0902-002-2)

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