지수분포에 있어서 파손과 보수시간에 대한 베이지안 추정

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Bayesian Estimation of the Component Availability in the Exponential Failure/Repair Time Distributions

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서 론

본 연구의 목적은 파손과 보수비에 대한 미정보 사전분포와 공액 사전 분포의 제곱오차 손실 함수 하에서 Gaver와 Mazumder(1967)에 의하여 제기된 두개의 상태과정 즉 파손과 보수과정에 있어서 보수할 수 있는 성분에 대한 유용도를 어떤 베이지안으로서 실제의 값에 가까운 점 추정치를 구할 수 있는 수학적 모델을 만드는데 목적을 두며 앞으로 이를 선박기기 및 항해기기등의 보수정비에 관한 운용등에 적용할 수 있으리라 사료된다.

1. Introduction

Let us denote the distribution of failure time X and repair time Y by H(X) and G(Y) respectively. Then the repairable component with respect to these distribution can be determined by steady-state availablity $A=E(X)/(E(X)+E(Y))$, with is the probaility that the repairable component is in operation in the long fraction of time. Gaver and Mazumder(3) investigated the estimation of parameters in case that two probability distributions are specified on the two-state process in operation and under repair. By investigating the failure of previous try and the period of repairs, Nelson[4] determined the interval of prediction for the availability of repairable component.

Brender[1] worked on the Bayesian estimation of the steady-state availability by using failure rate and

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repair rate.

On the other hand, Thompson[5], [11] and Palicio[5] obtained a method to compute the Bayesian interval of the availability of a series or parallel system consisting of several statistically independent two-state subsystems having exponential of failure times and repair times.

The main purpose of this paper is to compute Bayesian estimation of component steady-state availability. In section 2, we will investigate Bayesian estimation of the steady-state availability for noninformative prior density function and in section 3, we will compute Bayesian estimation for conjugate prior density function

2. Bayes Estimation for Noninformative Prior Density Function.

Consider a repairable component for which the failure time X is distributed as H(θ) with exponential probability density function(pdf)

$$h(x|\theta)=\frac{1}{\theta},\exp(-\frac{x}{\theta}),\ x>0,\ \theta>0$$

and the repairable time Y is distributed as G(α, β) with Gamma-pdf

$$g(y|\alpha_\theta,\beta_\theta)=\frac{1}{\Gamma(\alpha_\theta)\beta_\theta^{\alpha_\theta}}\cdot y^{(\alpha_\theta-1)}\cdot\exp(-\frac{y}{\beta_\theta}),\ y>0,\ \alpha>0,\ \beta>0$$

where α (shape parameter) has a known value α_θ and β(scale parameter) is unknown value β_θ Assume that X, Y independent the steady-state availability is given by

$$A=\frac{E(X)}{E(X)+E(Y)},\ E(X)=0,\ \ E(Y)=\alpha_\theta\beta_\theta$$

Then, the likelihood function of q, T_x, T_y given

$$L(Q,T_x,T_y|\theta,\beta_\theta,\alpha_\theta)\ =\ \frac{1}{\theta^q}\exp(-\frac{T_x}{\theta})\left[\frac{1}{\Gamma(\alpha_\theta)}\right]^q(\frac{1}{\beta_\theta})^{q\alpha_\theta}(\prod_{i=1}^{q}y_i)^{(\alpha_\theta-1)}\exp\frac{T_y}{\beta_\theta}$$

(2-1)

Let's introduce some integration for calculation of the expectation,

$$\int_0^\infty z^{(p-1)}.\exp(-az)dz=\Gamma(p)a^{-p}$$

$$\int_0^\infty z^{(b-1)}(1-z)^{(c-b-1)}(1-tz)^{-a}dz=\frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)}\ {}_2B_1(a,b;c,t),$$

for $|t|<1,\ c>b>0$ and

$$ {}_2B_1(a,\ b;c,\ t)=\sum_{i=0}^{\infty}\frac{(a_i)\ (b_i)}{(c_i)\cdot i\,!}\ t^i $$

is confluent hypergeometric functions of Gauss form for $|t|<1$ with

$$ (a_i)=\frac{\Gamma(a+i)}{\Gamma(a)} $$

From Erdelyi[2], we have

$$ {}_2B_1(a,\ b;c;z)=(1-z)^{c-a-b)}\ {}_2B_1(c-a,\ c-b;c-z),\ \text{for}\ |z|<1; \qquad (2\text{-}3) $$

, where (a, b. c−a, c−b) is positive integer.

Assume that MTBF in the exponential failure time distributed and scale parameter in the Gamma repair time distribution has independent noninformative prior distribution given by

$$ f(\theta)\propto\frac{1}{\theta^{m_1}},\qquad m_1>0 \qquad (2\text{-}4) $$

$$ f(\beta)\propto\frac{1}{\beta_\theta^{m_2}},\qquad \beta_\theta>0,\ m_2>0 \qquad (2\text{-}5) $$

Then we have the following lemma.

LEMMA 2. 1 The joint posterior density function of θ and β is given by

$$ f(\theta,\beta_\theta|q,T_x,T_y;\alpha_o)=\frac{(T_x)^{(q+m_1-1)}(T_y)^{(q\alpha_o+m_2-1)}\exp[-(T_x/\theta)+(T_y/\beta_\theta)]}{\theta^{(q+m_1)}\beta_\theta^{(q\alpha_o+m_2)}\Gamma(q+m_1-1)\Gamma(q\alpha_\theta+m_2-1)} $$

where $\theta>0$, $\beta>0$ and $\Gamma(\cdot)$is a Gamma function.

PROOF. From(2-1), (2-2) and (2-3), the joint posterior distribution of θ, β_θ reduces to

$$ f(\theta,\beta_\theta|q,T_x,T_y;\alpha_\theta) \qquad (2\text{-}6) $$

$$ =\frac{L(q,T_x,T_y|\theta,\beta_\theta;\alpha_\theta)\cdot f_1(\theta)f_2(\beta_\theta)}{\int_\theta^\infty\int_\theta^\infty L(q,T_x,T_y|\theta,\beta_\theta;\alpha_\theta)\cdot f_1(\theta)f_2(\beta_\theta)d\beta_\theta d\theta} $$

$$ =\frac{e^{-q}\exp(-\frac{T_x}{\theta})\left[\frac{1}{\Gamma(\alpha_\theta)}\right]\cdot\beta^{-q\alpha_\theta}(\pi_1^q Y_1)^{(\alpha_o-1)}\exp(-\frac{T}{\beta_\theta})\,\theta^{-m_1}\theta^{-m_2}}{\int_\theta^\infty\int_\theta^\infty e^{-q}\exp(-\frac{T_x}{\theta})\left[\frac{1}{\Gamma(\alpha_\theta)}\right]^q\beta_\theta^{-q\alpha_o}(\pi_1^q Y_1)^{(\alpha_o-1)}\exp(-\frac{T_y}{\beta_\theta})\,\theta^{-m_1}\theta^{-m_2}d\beta_\theta d\theta} $$

By the cancellation, the denominator of (2-6) becomes

$$ f(\theta,\beta|q,T_x.T_y;\alpha) $$

$$= \int_0^\infty \int_0^\infty \theta^{-(q+m_1)} \beta^{-(q\alpha_o+m_2)} \exp\left[-\left(\frac{T_x}{\theta} + \frac{T_y}{\beta_\theta}\right)\right] d\beta_\theta d\theta$$

$$= \int_0^\infty \theta^{-(q+m_1)} \exp\left(-\frac{T_x}{\theta}\right)\left[\int_0^\infty \beta^{-(q\alpha_o+m_2)} \exp\left(-\frac{T_y}{\beta_\theta}\right) d\beta_\theta\right] d\theta.$$

Suppose $\beta_\theta = 1/\alpha$, then the inner integral is evaluated as

$$\int_0^\infty \beta_\theta^{-(q\alpha_o+m_2)} \exp\left(-\frac{T_y}{\beta_\theta}\right) d\beta_\theta = \int_0^\infty \alpha^{(q\alpha_o+m_2-2)} \cdot \exp(-T_y\alpha)\, d\alpha$$

$$= \frac{\Gamma(q\alpha_\theta+m_2-1)}{(T_y)^{(q\alpha_\theta+m_2-1)}}$$

Suppose $\theta = 1/\beta$, then the outer integral of (2-6) is evaluated as

$$\int_0^\infty \theta^{-(q+m)} \exp\left(-\frac{T_x}{\theta}\right) d\theta = \int_0^\infty \beta^{(q+m_1-2)} \exp(-T_x \cdot \beta)\, d\beta$$

$$= \frac{\Gamma(q+m_1-1)}{T_x^{(q+m_1-1)}} \quad \text{(Q.E.D.)}$$

LEMMA 2. 2 The posterior distribution of component steady-state availability

$A = 1/(1+\alpha_\theta\delta)$ is given by

$R(A \mid q, T_x, T_y; \alpha_\theta)$

$$= \left(\frac{\alpha_\theta T_y}{T_x}\right)^{(q\alpha_o+m_2-1)} A^{(q\alpha_o+m_2-2)} (1-A)^{(q+m_1-2)}$$

$$= B(q+m_1-1, q\alpha_o+m_2-1)\left[1-A\left(1-\frac{\alpha_\theta T_y}{T_x}\right)\right]^k, \quad (0<A<1)$$

where B(.,.) is Beta function, $k=(q+m_1+q\alpha_\theta+m_2-2)$ and $\delta=\beta_\theta/\theta$ is the service factor

PROOF. First, we find the posterior distribution of $\delta=\beta_\theta/\theta$. According to LEMMA 2. 1

We express that

$R_\delta(\delta \mid q, T_x, T_y; \alpha_\theta)\, \beta_\theta\, \delta^{-2} d\beta_\theta$

$$= \int_0^\infty S\left(\frac{\beta_\theta}{\delta}, \beta_\theta \mid q, T_x, T_y; \alpha_\theta\right) \beta_\theta \delta^{-2} d\beta_\theta$$

$$= \frac{(T_x)^{(q+m_1-1)} (T_y)^{(q\alpha_o+m_2-1)}}{\Gamma(q+m_1-1)\Gamma(q\alpha+m_2-1)} \int_0^\infty \left(\frac{\beta_\theta}{\delta}\right)^{-(q+m_1)} \beta^{-(q\alpha_o+m_2)} \exp$$

$$[-(\frac{\delta}{\beta_\theta} T_x + \frac{1}{\beta_\theta} T_y)] \beta_\theta \delta^{-2} d\beta_\theta$$

$$= \frac{(T_x)^{(q+m_1-1)} (T_y)^{(q\alpha_0+m_2-1)}}{\Gamma(q+m_1-1)\Gamma(q\alpha+m_2-1)} \int_0 \beta_\theta^{-(q+m_1+q\alpha_0+m_2-1)} \exp[-\frac{1}{\beta_\theta} (\delta T_x + T_y)] d\beta_\theta$$

Let $\beta_\theta = 1/\alpha$, then we have integration with respect to β_θ that

$$R_\delta(\delta \mid q, T_x, T_y; \alpha)$$

$$= \frac{(T_x)^{(q+m_1-1)} (T_y)^{(q\alpha_0+m_2-1)} \delta^{(q+m_1-2)} \Gamma(q+m_1+q\alpha_\theta+m_2-2)}{\Gamma(q+m_1-1)\Gamma(q\alpha+m_2-1)\cdot(\delta T_x+T_y)^{(q+m_1+q\alpha_\theta+m_2-2)}}$$

$$= \frac{(T_x)^{(q+m_2-1)} (T_y)^{(q\alpha_0+m_2-1)} \cdot \delta^{(q+m_2-2)}}{B(q+m_1-1, q\alpha_\theta+m_2-1)(\delta T_x+T_y)^{(q+m_1+q\alpha_0+m_2-2)}}, \quad 0 < \delta < \infty$$

Accordingly, the posterior distribution of component steady-state availability $A=1/(1+\alpha_\theta\delta)$ is given by

$$R(A \mid q, T_x, T_y; \alpha_\theta)$$

$$= R_\delta [\frac{(A^{-1}-1)}{\alpha_\theta} \mid q, T_x, T_y; \alpha_\theta] (\frac{A^{-2}}{\alpha_\theta})$$

$$= \frac{(T_x)^{(q+m_1+1)} (T_y)^{(q\alpha_0+m_1-1)} (1-A)^{(q+m_1-2)}}{B(q+m_1-1, q\alpha_\theta+m_2-1) [T_y + T_x (\frac{A^{-1}-1}{\alpha_\theta})]^{(q=m_1+q\alpha_0+m_2-2)}}$$

$$= \frac{(\frac{\alpha T_y}{T_x})^{(q\alpha_0+m_2-1)} A^{(q\alpha_0+m_2-2)} (1-A)^{(q+m_2-2)}}{B(q+m_1-1, q\alpha_\theta+m_2-1) [1-A (1-\frac{\alpha_\theta T_y}{T_x})]^{(q+m_1+q\alpha_0+m_2-2)}}$$

By using LEMMA 2.1 and LEMMA 2.2, we can prove the following theorem

THEOREM 2.1 Under a squarred-error loss function, Bayesian estimation of component steady-state availability A is given by

$$A_1^* = \frac{q\alpha_\theta+m_2-1}{q+m_1+q\alpha_\theta+m_2-2} \cdot {}_2B_1 (1, q+m_1-1; m_1+q\alpha_\theta+m_2-1; 1 - \frac{T_y\alpha_\theta}{T_x})$$

where $0 < \frac{\alpha_\theta T_y}{T_x} < 2$, ${}_2B_1(a, b; c; t)$ is a confluent hypergeometric function in Gauss form.

PROOF. Bayes estimation of component steady-state availability A is average of posterior distribution. From (2-2) and (2-3), we reduce as follows ;

$$A_1^* = \int_0^1 Ah(A \mid q, T_x, T_y; \alpha_\theta) dA$$

$$= \frac{\left(\frac{\alpha_\theta T_y}{T_x}\right)^{(q\alpha_\theta+m_2-1)}}{B(q+m_1-1,\ q\alpha_\theta+m_2-1)} \int_0^1 \frac{A^{(q\alpha_\theta+m_2-1)}(1-A)^{(q+m_1-2)}}{\left[1-A\left(1-\frac{\alpha_\theta T_y}{T_y}\right)\right]^{(q+m_1+q\alpha_\theta+m_2-2)}} \cdot dA$$

$$= \frac{\left(\frac{\alpha_\theta T_y}{T_x}\right)^{(q\alpha_\theta+m_2-1)} \Gamma(q+m_1+q\alpha_\theta+m_2-2)\Gamma(q\alpha_\theta+m_2)\Gamma(q+m_1-1)}{\Gamma(q+m_1-1)\Gamma(q\alpha_\theta+m_2-1)\Gamma(q+m_1+q\alpha_\theta+m_2-1)} \cdot Q_1$$

$$= \frac{\left(\frac{\alpha_\theta T_y}{T_x}\right)^{(q\alpha_\theta+m_2-1)} \cdot (q\alpha_\theta+m_2-1)}{q+m_1+q\alpha_\theta+m_2-2} \left(\frac{\alpha_\theta T_y}{T_x}\right)^{(q\alpha_\theta+m_2-1)} \cdot Q_1$$

$$= \frac{q\alpha_\theta+m_2-1}{q+m_1+q\alpha_\theta+m_2-2} \cdot Q_1$$

where $Q_1 = {}_2B_1(q+m_1+q\alpha_\theta+m_2-2\,;\ q\alpha_\theta+m_2\,;\ q+m_1+q\alpha_\theta+m_2-1\,;\ 1-(\alpha_\theta T_y/T_x))$ and $Q_2 = {}_2B_1(1,\ q+m_1-1\,;\ q+m_1+q\alpha_\theta+m_2-1\,;\ 1-(\alpha_\theta T_y/T_x))$.

(Q. E. D).

3. Bayes Estimation for Conjugate Prior Distribution

In this secetion, by employing the technique similar to that of the previous section, we shall obtain Bayes estimation of steady-state availability for conjugate prior distribution. Suppose MTBF in the exponential failure time distribution and the scale parameter in Gamma repair time distribution have independent conjugate density function ;

$$K_1(\theta) = \frac{b^a}{\Gamma(\theta)}\left(\frac{1}{\theta}\right)^{(a+1)} \exp\left(-\frac{b}{\theta}\right),\ \theta>0,\ a,\ b\geq 0$$

$$K_2(\theta) = \frac{d^c}{\Gamma(C)}\left(\frac{1}{\beta}\right)^{(c+1)} \exp\left(-\frac{d}{\beta_\theta}\right),\ \beta_\theta>0,\ c,\ d\geq 0.$$

Then, we have the following lemma.

LEMMA 3. 1 The Joint posterior density function of θ and β is given by

$$K(\theta,\beta_\theta|q,t_x,t_y;\alpha_\theta) = \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha_\theta+c)}\exp\{-(1/\theta)(T_x+b)-(1/\beta_\theta)(T_y+d)\}}{\theta^{(q+a+1)}\beta_\theta^{(q\alpha_\theta+c+1)}\cdot\Gamma(q+a)\Gamma(q\alpha_\theta+c)}$$

PROOF From (3-1) and (3-2), the joint distribution of θ and β_θ is as follows,

$$K(\theta,\beta_\theta|q,T_x,T_y;\alpha_\theta) = \frac{L(q,T_x,T_y|\theta,\beta_\theta;\alpha_\theta)k_1(\theta)k_2(\beta_\theta)}{\int_0^\infty\int_0^\infty L(q,T_x,T_y|\theta,\beta_\theta;\alpha_\theta)\cdot f_1(\theta)f_2(\beta_\theta)d\beta_\theta d\theta}$$

$$= \frac{e^{-q}\exp(-\frac{T}{\theta})\left[\frac{1}{\Gamma(\alpha_\theta)}\right]^{q}\cdot \beta_\theta^{-q\alpha_o}(\prod_{i=1}^{q} y_i)^{(\alpha_o-1)}\exp(-\frac{T_y}{\beta_\theta})\left[\left(\frac{b^c}{\Gamma(a)}a\right)\right]\theta^{-(a+1)}}{\int_0^\infty\int_0^\infty \theta^{-q}\exp(-\frac{T_x}{\theta})\left[\frac{1}{\Gamma(\alpha_\theta)}\right]^{q}\cdot\beta_\theta^{-q\alpha_o}(\prod_{i=1}y_i)^{(\alpha_o-1)}\exp(-\frac{T_y}{\beta_\theta})}$$

$$\cdot\frac{1}{\frac{T_y}{\Gamma(a)}\quad\beta^{-(a+1)}\exp(-\frac{b}{\beta})}$$

$$= \frac{\exp(-b/\theta)(d^c/\Gamma(c)\beta_\theta^{-(c+1)}\exp(-d/\beta_\theta)}{(d^c/\Gamma(c))\beta_\theta^{(c+1)}\exp(-d/\beta_\theta)d\beta_\theta d\theta}$$

By the cancellation,

$$\int_0^\infty\int_0^\infty \theta^{-(q+a+1)}\beta^{-(q\alpha_o+c+1)}\exp\left[-\frac{1}{\theta}(T_x+b)-\frac{1}{\beta_\theta}(T_y+d)\right]d\beta_\theta d\theta$$

$$=\int_0^\infty\theta^{-(q+a+1)}\exp\left[-\frac{1}{\theta}(T_x+b)\right]\int_0^\infty\beta_\theta^{-(q\alpha_o+c+1)}\exp\left\{-\frac{1}{\beta_\theta}(T_y+d)\right\}\cdot\ d\beta_\theta d\theta$$

Suppose $\beta_\theta=1/\alpha$, then the inner integral of (2−δ) as follows ;

$$\int_0^\infty\beta_\theta^{-(q\alpha_o+c+1)}\exp\left[-\frac{1}{\beta_\theta}(T_y+d)\right]d\beta_\theta = \int_0^\infty\alpha^{(q\alpha_o+c+1)}\exp\left[-(T_y+d)\right]d\alpha$$

$$= \frac{\Gamma(q\alpha_\theta+c)}{(T_y+d)^{(q\alpha_o+c)}}$$

and suppose $\theta=1/\alpha$, then the outer integral of (2-8) as

$$\int_0^\infty\theta^{-(q+a+1)}\exp\left[\{-\frac{1}{\theta}(T_y+b)\}\right]d\theta=\int_0^\infty\beta^{(q+a+1)}\exp\left[\{-\beta(T_y+d)\}\right]d\beta$$

$$= \frac{\beta^{(q+a)}}{(T_y+d)^{(q+a)}}\qquad (Q.\ E.\ D.)$$

LEMMA 3. 2 The posterior density function of component steady-state availability for $A=1/(1+\alpha:\delta)$ is given by

$$h(A|q,T_x,T_y:\alpha_\theta)$$

$$= \frac{(\alpha_\theta T_y/T_x)^{(q\alpha_o+m_1-1)}A^{(q\alpha_o+m_2-2)}(1-A)^{(q+m_1-2)}}{B(q+m_1-1,q\alpha_\theta+m_2-1)\{1-A(1-\alpha_\theta T_y/T_x)\}^{(q+m_1+q\alpha_o+m_2-2)}},$$

PROOF First, according to LEMMA 3. 1

$$S(\delta\mid q,T_x,T_y:\alpha_\theta)$$

$$= \int_0^\infty K\left(\frac{\beta_\theta}{\delta}, \beta_\theta \mid q, T_x, T_y; \alpha_\theta\right) \beta_\theta \delta^{-2} d\beta_\theta$$

$$= \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha_0+c)}}{\Gamma(q+a)\Gamma(q\alpha_\theta+c)} \int_0^\infty \left(\frac{\beta_\theta}{\delta}\right)^{-(q+a+1)} \beta_\theta^{-(q\alpha_0+c+1)}$$

$$\exp\left[-\frac{\delta}{\beta_\theta}\right](T_x+b)+\frac{1}{\beta_\theta}(T_y+d)\,]\,\beta_\theta \delta^{-2} d\beta_\theta$$

$$= \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha_0+c)}\delta^{(q+a-1)}}{\Gamma(q+a)\Gamma(q+c)} \int_0^\infty \beta_\theta^{-(q+a+q\alpha_\theta+c+1)}$$

$$\exp\left[-\frac{1}{\beta_\theta}\{\delta(T_x+b)+(T_y+d)\}\right] d\beta_\theta$$

, where $1/\delta=\theta/\beta_\theta$. suppose $\beta_\theta=1/\alpha$, then

$$S(\delta \mid q, T_x, T_y; \alpha_\theta)$$

$$= \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha_0+c)}\delta^{(q+a-1)}\Gamma(q+a+q\alpha_\theta+c)}{\Gamma(q+a)\Gamma(q\alpha_\theta+c)\{\delta(T_x+b)+(T_y+d)\}^{(q+a+q\alpha_0+c)}}$$

$$= \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha_0+c)}\delta^{(q+a-1)}}{B(q+a, q\alpha_\theta+c)[\delta(T_x+d)+(T_y+d)]^{(q+a+q\alpha_\theta+c)}}$$

Therefore, posterior distribution of steady-state availability is as follows

$$P(A \mid q, T_x, T_y; \alpha_\theta)$$

$$= S(A^{-1}-1/\alpha_\theta \mid T_x, T_y; \alpha_\theta)(A^{-2}/\alpha_\theta)$$

$$= \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha_0+c)}\{(A^{-1}-1)/\alpha_\theta\}^{(q+a-1)}(A^{-2}/\alpha_\theta)}{B(q+a,\ q\alpha_\theta+c)\{\delta(T_y+d)+(A^{-1}-1)(T_x+b)/\alpha_\theta\}^{(q+a+q\alpha_0+c)}}$$

$$= \frac{(T_x+b)^{(q+a)}(T_y+d)^{(q\alpha_0+c)}[(A^{-1}-1)/\alpha_\theta]^{(q+a-1)}A^{-2}/\alpha_\theta}{B(q+a, q\alpha_\theta+c)\{1-A[1-\{\alpha_\theta(T_y+d)/(T_x+b)\}]\}^{(q+a+q\alpha_0+c)}}$$

$$= \frac{[\{\alpha_\theta(T_y+d)/(T_x+b)]^{(q\alpha_0+c)}A^{(q\alpha_0+c-1)}(1-A)^{(q+a-1)}}{B(q+a, q\alpha_\theta+c)\{1-A[1-\{\alpha_\theta(T_y+d)/(T_x+b)\}]\}^{(q+a+q\alpha_0+c)}}$$

THEOREM 3. 1 Under the squarred-error loss function, Bayes estimation steady-state availability A is given by

$$A_2^* = \frac{q\alpha_\theta+c}{q+a+q\alpha_\theta+c}\ {}_2B_1\left[1, q+a; q+a+q\alpha_\theta+1; 1-\frac{\alpha_\theta(T_y+d)}{T_x+b}\right]$$

, where $0 < \frac{T_y+d}{T_x+b} < 2$, ${}_2B_1(a, b; c; t)$

PROOF. From LEMMA 3. 2 (2-2) and (2-3), we have following Bayes estimation of component steady-state availability A ;

$$A_2 = \int A \, P(A|q, Tx, Ty; \alpha_0)\, dA$$

$$= \frac{[\alpha_0 (T_y+d)/(T_x+b)]^{(q\alpha_0+c)}}{B(q+a, q\alpha_0+c)} \int \frac{A^{(q\alpha_0+c)}(1-A)^{(q+a-1)}}{\{1-A[1-\{\alpha_0 (T_y+d)/(T_x+b)\}]\}^{(q+a+q\alpha_0+c)}} dA$$

$$= \frac{[\alpha_0 (T_y+d)/(T_x+b)]^{(q\alpha_0+c)}\, \Gamma(q+a+q\alpha_0+c)\, \Gamma(q\alpha_0+c+1)\, \Gamma(q+a)}{\Gamma(q+a)\, \Gamma(q\alpha_0+c)\, \Gamma(q+a+q\alpha_0+c+1)} \cdot K_1$$

$$= \frac{q\alpha_0+c}{q+a+q\alpha_0+c} \cdot K_2$$

where $K_1 = {}_2B_1(q+a+q\alpha_0+c, q\alpha_0+c+1; q+a+q\alpha_0+c+1; 1-\alpha_0 (T_y+d)/(T_x+b))$,

and $K_2 = {}_2B_1[1, q+a; q+a+q\alpha_0+C+1; 1-\frac{\alpha_0 (T_y+d)}{T_x{}^b}]$

(Q.E.D.)

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