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R and S Arrays Approach for Transfer Function-Noise Model Identification +

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ABSTRACT

This paper proposes an approach to the identification of transfer functin models. A strategy for the identification of the model structure is based on R and S arrays constructed by the impulse response function of the model. Theoretical patterns of the arrays associated with the model are investigated, and the practical implementation method of the suggested approach is also discussed. Finally two published samples are employed to demonstrate the practicability of the approach.

1. Introduction

In recent years, there has been a growing interest in the problem of determining the order of the transfer function model. The importance of this problem is clear. Before the parameters of the model can be estimated, the order of model needs to be specified. There are two major approaches to this problem.

The first is that suggested by Box and Jenkins(1976) which involves not only the examination of auto and partial-correlation functions, but also that of cross-correlations between the pre-whitened input and output series. However skills are required to recognize the patterns of the correlation functions. The second approach is to choose the best model which minimizes a criterion(e.g. FPE, AIC), as advocated by Akaike(1969, 1974), Chan(1983) and Poskitt(1989) developed a computer package(TF SIFT) and a new criterion respectively for

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this approach. But, this approach is difficult if not time consuming.

This paper suggests a new approach which uniquely determines the order of the transfer function model and which is readily observable. Here we discuss only about the identification method of the model. Noise function identification, estimation and diagnostic checking for such model are traceable to the work of Box and Jenkins(1976). The approach uses "R and S arrays" which are originally adopted by Gray, Kelly, and McIntire(1978) for estimation of autoregressive moving-average(ARMA) process orders. For the construction of those arrays, we use the impulse response function associated with the model. The practicability of the approach is demonstrated by using published series and comparing the models obtained from the approach with those already published in the literature.

The content of this paper is as follows. Since R and S arrays are used in the suggested approach, a brief review of the definitions and theorem necessary to establish the arrays is given in Section 2. In Section 3, we develop the theoretical basis for the approach. The practical implementation of the approach described in Section 3 is presented in Section 4. In Section 5, the practicability of this approach and hence R and S arrays approach is demonstrated by using published series.

2. Definitions and Theorem

Definition 1. Let m be an integer, $h>0$, and let f be a real valued function. Also let $f_m = f(mh)$. Then Gray, Kelly, and McIntire(1978) define R and S array elemtns as following ratios of Hankel to bordered Hankel determinants;

$$R_n(f_m)=H_n[f_m]/H_n[1:f_m] \tag{2.1}$$

and

$$S_n(f_m)=H_{n+1}[1:f_m]/H_n[f_m], \tag{2.2}$$

where

$$H_n[f_m]=\begin{vmatrix} f_m & f_{m+1} & f_{m+n-1} \\ f_{m+1} & f_{m+2} & f_{m+n} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ f_{m+n-1} & f_{m+n} & f_{m+2n-2} \end{vmatrix}$$

$$H_0[f_m]=1, \tag{2.3}$$

and

$$H_{n+1}[1:f_m]=\begin{vmatrix} 1 & 1 & 1 \\ f_m & f_{m+1} & f_{m+n} \\ f_{m+1} & f_{m+2} & f_{m+n+1} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ f_{m+n-1} & f_{m+n} & f_{m+2n-1} \end{vmatrix}$$

Pye and Atchison(1973) have shown that a recursive calculation relations between (2.1) and (2.2) are

$$R_{n+1}(f_m)=R_n(f_{m+1})\ \{S_n(f_{m+1})/S_n(f_m)-1\} \tag{2.4}$$

and

$$S_n(f_m)=S_{n-1}(f_{m+1})\ \{R_n(f_{m+1})/R_n(f_m)-1\}\ , \tag{2.5}$$

with starting conditions

$$S_0(f_m)=1,\ m=0,\ \pm1,\ \pm2,\cdots\cdots,$$
$$R_1(f_m)=f_m,\ m=0,\ \pm1,\ \pm2,\cdots\cdots.$$

In future we only use (2.4) and (2.5) to define $R_{n+1}(f_m)$ and $S_n(f_m)$. In (2.4) and (2.5) we have tacitly assumed that $S_n(f_m)\neq0$ and $R_n(f_m)\neq0$. If $R_n(f_m)=R_n(f_{m+1})=0$, we leave $S_n(f_m)$ undefined. However, if $R_n(f_m)=0$ and $R_n(f_{m+1})\neq0$, we defined $S_n(f_m)=\pm\infty$. We used a similar definition for $R_{n+1}(f_m)$.

Definition 2. A function f will be said to be an element of $L(n, \Delta)$ over a set of integers $I=\{m_0, m_0+1,\cdots\cdots\}$ if there exists a smallest integer $n>0$ and a set of δ_i's such that f is a solution of the difference equation

$$Z_m-\delta_1Z_{m-1}\cdots\cdots-\delta_nZ_{m-n}=0 \text{ for } m\in I.$$

Gray, Houston, and Morgan(1978) derived following theorem which forms a basis for the development of several of the results to follow.

Theorem 1. (Gray, Houstons, and Morgan, 1978). Let $n>0$, and suppose $S_n(f_m)$ and $R_n(f_m)$ in (2.1) and (2.2) are defined: i.e. the denominators in (2.1) and (2.2) are not zero, and $S_n(f_m)\neq0$. Then $f_m\in L(n, \Delta)$ for $m\geq m_0+n$ if and only if $S_n(f_m)$ is constant, as a function of m for $m\geq m_0$.

3. Single Input Transfer Function Model: The Stationary Case

The model that is of immediate concern is defined below(See Box and Jenkins, 1976).

Definition 3. A bivariate stochastic process $\{(X_t, Y_t)\}$, $t=0, \pm 1, \pm 2, \cdots$, is said to be single input transfer function model of order (r, s) with the transfort delays b, if

$$Y_t = \delta^{-1}(B)\, W(B)\, X_{t-b}, \tag{3.1}$$

where

$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r,\ W(B) = w_0 - w_1 B - w_1 B^2 - \cdots - w_s B^s,$$

and B denotes the backward-shift operator.

Note that equation(3.1) is a parsimonious representation of linear filter transfer function

$$Y_t = V(B)\, X_t, \tag{3.2}$$

where $V(B) = (v_0 + v_1 B + v_2 B^2 + \cdots\cdots)$ and the weights $v_0, v_1, v_2, \cdots$. are called the impulse response function of the system.

In this section we show how Theorem 1 can be expanded in such a way as to establish a new criterion which uniquely determines r and s and which are readily observable.

Theorem 2. Let $\{(X_t, Y_t)\}$ be a stochastic process which follows a transfer function model of order (r, s) with the impulse response weights v_m and the transfort delays b. Suppose that $S_n(v_m)$ and $R_n(v_m)$ are defined, $n>0$ and $S_n(v_m) \neq 0$. Then for some integer m_0 and some constant $c_1 \neq 0$

$$S_n(v_m) = c_1, \quad m \geq m_0$$
$$S_n(v_m) \neq c_1, \quad m < m_0$$

if and only if $n=r$ and $m_0 = b+s-r+1$. Moreover

$$c_1 = (-1)^r \left(1 - \sum_{i=1}^{r} \delta_i\right), \tag{3.3}$$

where the δ_i's are coefficients defined in (3.1).

Proof. Since $\{(X_t, Y_t)\}$ follows a transfer function model of order (r, s), comparison of (3.1) with (3.2) gives tht identity

$$\delta(B)\, V(B) = W(B)\, B^b. \tag{3.4}$$

On equating coefficinets of B, we find a difference equation

$$\delta(B)\, v_m = 0,\ m \geq b+s+1. \tag{3.5}$$

This implies $v_m \in L(r, \Delta)$ for $m \geq b+s+1$. and hence the first part of the theorem follows from Theorem 1. From the first part of this theorem and the definition of $S_r(v_m)$, equation (3.3) follows by simple application of the Cramer law to (3.5). That is, the numerator $H_{r+1}[1{:}V_m]$ of $S_r(v_m)$ may be written

$$(-1)^r\ (1-\sum_{i=1}^{r} \delta_i)\ H_r[v_m] \text{ for } m \geq b+s-r+1.$$

This gives

$$S_r(v_m) = H_{r+1}[1{:}v_m]/H_r[v_m] = (-1)^r\ (1-\sum_{i=1}^{r} \delta_i) \text{ for } m \geq b+s-r+1.$$

In case, $s-r+1<0$, starting values v_m for $m<b$ will, of course, be zero, but even in this case first row of $H_r[v_m]$ contains at least one nonzero element, and hence the above result holds for every combination of r and s. Note that stationary condition of the transfer function model implies that $1-\sum_{i=1}^{r} \delta_i \neq 0$, and hence $c_1 \neq 0$.

Corollary 1. Let the conditions of Theorem 2 be satisfied with v_m replaced by $(-1)^m v_m$. Then for some integer m_0 and some constant $c_2 \neq 0$,

$$\begin{aligned} S_n\{(-1)^m v_m\} &= c_2, \quad m \geq m_0 \\ S_n\{(-1)^m v_m\} &\neq c_2, \quad m < m_0 \end{aligned} \tag{3.6}$$

if and only if $n=r$ and $m_0 = b+s-r+1$. Moreover,

$$c_2 = (-1)^r\ (1-\sum_{i=1}^{r} (-1)^i \delta_i\}. \tag{3.7}$$

Proof. From(3.5),

$(1-\Sigma\ \delta_i\ B^i)v_m = (1-\Sigma\ (-1)^i \delta_i\ B^i)(-1)^m v_m = 0$, for $m > b+s$,

so that $f_m = (-1)^m v_m \in L(r, \Delta)$ for $m<b+s$. Thus, the proof follows in the same manner as in Theorem 2.

Corollary 2. Under the assumption of Theorem 2 and Corollary 1

$$R_{r+1}(V_m) = R_{r+1}\{(-1)^m v_m\} = 0 \text{ for } m \geq b+s-r+1. \tag{3.8}$$

Proof. The result follows at once from the above theorems and the recursion rule(2.4).

Above results state that a process is stationary transfer function of order (r, s) with transfort delays b if and only if the associated R and S arrays are as in Table 1 and 2 for the impulse response weights $\{v_m\}$. Following tables display arrays of number referred to as "R and S arrays". They are computed via (2.4) and (2.5). Since v_m will be given in each case, we shorten the notation within the tabular to $S_n(v_m)=S_n(m)$ and $R_n(v_m)=R_n(m)$.

The S-arrays in Table 1 reveals constant behavior such that r-th column of the table has constant elements from (b+s−r+1)-th row. Thus, we can see that the S-arrays in Table 1 characterize order (r, s) of the transfer function model.

Table 1. S-arrays for $S_n(v_m)$, $r>b$

m \ n	1*	2	....	b-1	b	....	r**	r+1***
0	u	u	....	u	$\pm\infty$	....	$S_r(0)$	$S_{r+1}(0)$
1	u	u	....	$\pm\infty$	$S_b(1)$	....	$S_r(1)$	$S_{r+1}(1)$
2	u	u	....	$S_{b-1}(2)$	$S_b(2)$	....	$S_r(2)$	$S_{r+1}(2)$
⋮	⋮	⋮		⋮	⋮		⋮	⋮
b-2	u	$\pm\infty$	....	$S_{b-1}(b-2)$	$S_b(b-2)$	....	$S_r(b-2)$	$S_{r+1}(b-2)$
b-1	$\pm\infty$	$S_2(b-1)$	....	$S_{b-1}(b-1)$	$S_b(b-1)$	....	$S_r(b-1)$	$S_{r+1}(b-1)$
b	$S_1(b)$	$S_2(b)$	....	$S_{b-1}(b)$	$S_b(b)$	....	$S_r(b)$	$S_{r+1}(b)$
⋮	⋮	⋮		⋮	⋮		⋮	⋮
m_0-1	$S_1(m_0-1)$	$S_2(m_0-1)$		$S_{b-1}(m_0-1)$	$S_b(m_0-1)$		$S_r(m_0-1)$ −	c_1
m_0	$S_1(m_0)$	$S_2(m_0)$		$S_{b-1}(m_0)$	$S_b(m_0)$		c_1	$c_1[0/0-1]$
m_0+1	$S_1(m_0+1)$	$S_2(m_0+1)$		$S_{b-1}(m_0+1)$	$S_b(m_0+1)$		c_1	$c_1[0/0+1]$
⋮	⋮	⋮		⋮	⋮		⋮	⋮
⋮	⋮	⋮		⋮	⋮		⋮	⋮
⋮	⋮	⋮		⋮	⋮		⋮	⋮
j	$S_1(j)$	$S_2(j)$	....	$S_{b-1}(j)$	$S_b(j)$		c_1	$c_1[0/0-1]$

* Note undefined element stretches.

** Note constant stretches.

*** Note undefined stretches.

$m_0=b+s-r+1$.

Table 2. R-arryas for $R_n(v_m)$, $S-r\geqq 0$, $b<r+1$

m \ n	1*	2	....	b-1	b	b+1	r+1**
0	0	0		0	0	$R_{b+1}(0)$	Nonzero
1	0	0	....	0	$R_b(1)$	$R_{b+1}(1)$	Nonzero
2	0	0		$R_{b-1}(2)$	$R_b(2)$	$R_{b+1}(2)$	Nonzero
.	.	.		.	.	.	.
.	.	.		.	.	.	.
.	.	.		.	.	.	.
b-2	0	0	....	$R_{b-1}(b-2)$	$R_b(b-2)$	$R_{b+1}(b-2)$	Nonzero
b-1	0	$R_2(b-1)$	....	$R_{b-1}(b-1)$	$R_b(b-1)$	$R_{b+1}(b-1)$	Nonzero
b	v_b	$R_2(b)$	....	$R_{b-1}(b)$	$R_b(b)$	$R_{b+1}(b)$	Nonzero
.	.	.		.	.	.	.
.	.	.		.	.	.	.
.	.	.		.	.	.	.
m_0	v_{m_0}	$R_2(m_0)$	....	$R_{b-1}(m_0)$	$R_b(m_0)$	$R_{b+1}(m_0)$	0
m_0+1	v_{m_0+1}	$R_2(m_0+1)$	....	$R_{b-1}(m_0+1)$	$R_b(m_0+1)$	$R_{b+1}(m_0+1)$	0
.	.	.		.	.	.	.
.	.	.		.	.	.	.
.	.	.		.	.	.	.
j	v_j	$R_2(j)$		$R_{b-1}(j)$	$R_b(j)$	$R_{b+1}(j)$	0

* Note zero stretches.

** Note zero stretches.
$m_0=b+s-r+1$.

Table 2 shows that the first column has continuing zero behavior up to (b−1)-th row. Hence the behavior of the R-arryas is usually adequate for describing transfort delays b. Although zero sequence of (r+1)-th column in the R-arrays characterizes both r and s, their column behaviors are not as distinct as the S-arrays when $s-r<0$.

The arrays do not differ in their pattern if $f_m=(-1)^m v_m$. Of course, in that case first column in the R-arrays is made up of values of $(-1)^m v_m$. In the special case when $r=0$, (3.1) and (3.2) give $v_m=0$ for $m<b$ and $m>b+s$. Hence, without using R and S arrays, we can identify the order s directly from the pattern of the impulse response function.

In order to demonstrate the manner in which the above results can be utilized to identify a transfer function model of order (r, s), we now consider a simple example using a known transfer function model.

Example 1. Consider the R and S arrays for the process

$$(1-.57B)\ Y_t=-(.53+.37B+.51B^2)\ X_{t-3},$$

where $\{(X_t, Y_t)\}$, t=1, 2,⋯, is a bivariate stochastic process.

Table 3 and 4 present R and S arrays for $f_m=v_m$ and $f_m=(-1)^m v_m$, respectively. The impulse response weights $\{v_m\}$ are calculated from the identity relation in (3.4). Close inspection of those tables is worthwhile since all of the previous results are illustrated there and the remainder of this paper is predicated on those results.

We have presented theoretical R and S arrays for the single input transfer function model. These seem to be a satisfactory approach to the problem of identifying the orders (r, s) and the transfort delays b in the model. Since we can easily see that general incremental changes

$$\alpha_t=(1-B)^d\ (1-B^s)^D\ Y_t \text{ and } \beta_t=(1-B)^d\ (1-B^s)^D\ X_t$$

satisfy the same transfer function as do Y_t and X_t in (3.1). Identification of the prewhitened or the nonstationary transfer function model of a form

Table 3. R and S arrays at $f_m=v_m$

$R_1(m)$	$R_2(m)$	$R_3(m)$	m	$S_0(m)$	$S_1(m)$	$S_2(m)$
0	0	0	0	1.00	u	u
0	0	-.53	1	1.00	u	∞
0	.53	-1.9575	2	1.00	- ∞	-.3450
-.53	-.1522	-2.2464	3	1.00	.2681	-4.7822
-.6721	2.061	.000	4	1.00	.3288	.43
-.8931	.000	.000	5	1.00	-.43	u
-.5091	.000	.000	6	1.00	-.43	u
-.2902	.000	.000	7	1.00	-.43	u
-.1654	.000	.000	8	1.00	-.43	u
-.0943	.000	.000	9	1.00	-.43	u
-.0537	.000	.000	10	1.00	-.43	u
-.0306	.000	.000	11	1.00	-.43	u
-.0175	.000	.000	12	1.00	-.43	u
-.0100	.000	.000	13	1.00	-.43	u
-.0057	.000	.000	14	1.00	-.43	u
-.0032	.000	.000	15	1.00	-.43	u

Note: U denotes an undefined number.

Table 4. R and S arrays AT $f_m = (-1)^m v_m$

$R_1(m)$	$R_2(m)$	$R_3(m)$	m	$S_0(m)$	$S_1(m)$	$S_2(m)$
0	0	0	0	1.00	u	u
0	0	.53	1	1.00	u	∞
0	-.53	.3083	2	1.00	∞	2.1910
.53	-.0180	.3039	3	1.00	-2.2681	-35.3425
-.6721	-.2910	.000	4	1.00	-2.3288	1.570
.8931	.000	.000	5	1.00	-1.570	u
-.5091	.000	.000	6	1.00	-1.570	u
.2918	.000	.000	7	1.00	-1.570	u
-.1645	.000	.000	8	1.00	-1.570	u
.0943	.000	.000	9	1.00	-1.570	u
-.0537	.000	.000	10	1.00	-1.570	u
.0306	.000	.000	11	1.00	-1.570	u
-.0175	.000	.000	12	1.00	-1.570	u
.0100	.000	.000	13	1.00	-1.570	u
.0057	.000	.000	14	1.00	-1.570	u
.0032	.000	.000	15	1.00	-1.570	u

Note: U denotes an undefined number.

$$\alpha_t = \delta(B)^{-1} W(B) \beta_{t-b}$$

can be immediately accomplished by the same R and S arrays constructed in terms of the impulse response weights between α_t and β_t.

4. Transfer Function-Noise Model Identification

The model that is of immediate concern is shown below.

$$Y_t = \delta^{-1}(B) W(B) X_{t-b} + N_t. \tag{4.1}$$

This is a linear system corrupted by noise N_t, at the output and assumed to be generated by an ARIMA process which is statistically independent of the input X_t, and the other notations in (4.1) are defined as before.

As already stated, the problem of modelling the transfer fucntion-noise model is that of selecting order (r, s) and transfort delays b.

Two-stage least squares method described by Chan(1983) is adopted to estimate the impulse response function, related with (4.1), which will be utilized in calculating individual elements of R and S arrays. Let

$$Z_t = V(B)\ X_t,$$

where $V(B) = \delta^{-1}(B)\ w(B)\ B^b$. Then (4.1) can be rewritten as

$$Y_t = V(B)\ X_t + N_t \tag{4.2}$$

This is also known as the impulse response model. Since N_t is uncorrelated with X_t, V(B) can be estimated using linear least squares from (4.2). An estimate of Z_t can be calculated using the following equation

$$\hat{Z}_t = \hat{V}(B)\ X_t,$$

where $\hat{V}(B)$ represents an estimate. The order of $\hat{V}(B)$ is chosen using AIC. Using $\hat{Z}_t$ and X_t, the parameters of V(B) can be consistently esstimated(See Durbin. 1961), while transfort delays b can be initially estimated by counting how many non-siginificant parameters $\hat{V}(B)$ starts with.

Constructing and observing the patterns of R and S arryas(Table 1 and 2) based on the second least squares of the impulse response weights, we can identify order (r, s) of the model. Here we should remember that, in the presence of noise, the estimated impulse response weights would not give exact constant patterns as in Table 1 and 2, but show characteristics of them. A similar consideration of the $(r+1)-$th column in the S-array suggests that highly variable behavior should essentially continue from the $(b+s-r+1)$-th row, as later examples clarify. The reason is that the 0/0 quantity, appeared in the $(r+1)$-th column, will be practically replaced by the quotient of two samll numbers.

5. Applications

To demonstrate the practicability of the described procedure and hence of R and S arrays approach, published series are used and comparisons are made with models produced by the approach and those already published. Two examples, published in Box and Jenkins(1976), are shown here. The actual successive pairs of observations (X_t, Y_t) are labeled as Series J and Series M at the end of the volume.

Example 2. As a first illustration, consider the data on sales Y_t in relation to a leading indicator X_t, listed as Series M. Box and Jenkins(1976) showed that the data was well fitted by the nonstationary model of order (r=1, s=0) and transfort delays b=3:

$$\alpha_t = .035 + 4.82\,\beta_{t-3}/(1-.72B)+(1-.54B)a_t \tag{5.1}$$

with the notations α_t and β_t be the first difference of the series.

Table 5 shows the R and S arrays when $(-1)^m v_m$ is replaced by the two stage least squares estimates $(-1)^m \hat{v}_m$ obtained from the first difference series.

In observing arrays such as presented in Table 5 several approaches can be taken to identify the pattern presented. We believe the best approach is to inspect the S-arrays first. Choose the case which appears most distinctive, and identifiy r as the column having the constant behavior followed by a highly variable column. This behavior is clear in Table 5 where obviously r=1. That same table also makes clear that $m_0=b+s-r+1=3$ so that b+s=3. Next investigate the R-arrays. Recall that $(-1)^m \hat{v}_m$ appears in Column 1 of the arrays. This gives b=3, and hence s=0.

Table 5. R and S arrays at $f_m(-1)^m \hat{v}_m$

$R_1(m)$	$R_2(m)$	$R_3(m)$	m	$S_0(m)$	$S_1(m)$	$S_2(m)$
0.0000	0.0000	0.0000	0	1.0000	U	U
0.0000	0.0000	-4.7307	1	1.0000	U	∞
0.0000	4.7307	0.1082	2	1.0000	$-\infty$	1.7893
-4.7307	-0.1264	-0.2150	3	1.0000	-1.7428	0.2578
3.5138	-0.1070	0.2099	4	1.0000	-1.6801	0.7760
-2.3897	-0.0597	0.1395	5	1.0000	-1.7553	-1.9531
1.8049	-0.1283	-1.0778	6	1.0000	-1.6973	0.1698
-1.2585	-0.1167	-0.0514	7	1.0000	-1.8703	1.7386
1.0954	0.0047	-0.0579	8	1.0000	-1.6711	-17.2341
-0.7351	0.0536	0.0052	9	1.0000	-1.6604	1.0414
0.4854	0.0233	0.0340	10	1.0000	-1.8439	1.2751
-0.4097	0.0062	0.0329	11	1.0000	-1.7388	8.2317
0.3027	-0.0227		12	1.0000	-1.7746	-3.7367
-0.2344	-0.0662		13	1.0000	-1.9461	

* Note U denotes an undefined number.

** Note rectangulars in R and S arrays highlight the zero and the constant stretches, respectively.

As a result, the model of order($r=1$, $s=0$) and transfort delays $b=3$, identified by the R and S arrays approach, is consistent with that published.

Example 3. In this example we use the gas furnace data considered in Box and Jenkins(1976) and labeled "Series J". The data consists of 296 pairs of data points referred to as input gas rate(X_t) and CO_2 in outlet gas(Y_t). The published fitted model is the stationary model of order ($r=1$, $s=2$) and transfort delays $b=3$:

$$Y_t = -(.53+.37B+.51B^2)\, X_{t-3}/(1-.57B) + a_t/(1-1.53B+.63B^2). \qquad (5.2)$$

Table 6 shows the corresponding R and S arrays at $f_m = (-1)^m \hat{v}_m$.

The first column in the R-arrays gives the two stage least squares estimates of $(-1)^m \hat{v}_m$ with $b=3$. Again the S-arrays clearly show that $r=1$ and $s=2$. Note that both Column 2 and 3 of the S-arrays behave exactly as would be expected for a transfer function model of order ($r=1$, $s=2$). That is, Column 2 and Column 3 have the characteristic "constant behavior" and highly variable sequence respectively from $m_0=b+s-r+1=5$. Column 2 of the R-arrays also clearly gives $r=1$. Thus again the results from the R and S arrays

Table 6. R and S arrays at $f_m(-1)^m \hat{v}_m$

$R_1(m)$	$R_2(m)$	$R_3(m)$	m	$S_0(m)$	$S_1(m)$	$S_2(m)$
0.0000	0.0000	0.0000	0	1.0000	U	U
0.0000	0.0000	0.5396	1	1.0000	U	$-\infty$
0.0000	-0.5396	0.2213	2	1.0000	∞	2.5619
0.5396	0.0398	0.2352	3	1.0000	-2.3860	16.8083
-0.7478	-0.2564	0.0674	4	1.0000	-2.2590	1.3916
0.9415	-0.0393	0.0213	5	1.0000	-1.6437	-0.9934
-0.6061	-0.0617	0.4964	6	1.0000	-1.7504	-0.6502
0.4548	-0.0881	-0.0334	7	1.0000	-1.5131	3.0111
-0.2333	0.0392	0.6556	8	1.0000	-2.0847	0.4401
0.2531	0.0320	0.1164	9	1.0000	-2.4073	9.4515
-0.3562	-0.1061	0.1116	10	1.0000	-2.1910	-0.9200
0.4242	-0.1655	0.9586	11	1.0000	-1.6431	-0.2995
-0.2728	-0.1843		12	1.0000	-2.6398	1.3238
0.4474	-0.0271		13	1.0000	-1.5525	

* Note U denotes an undefined number.

** Note rectangulars in R and S arrays highlight the zero and the constant stretches, respectively.

approach are the same as those published.

6. Concluding Remarks

In this paper we have presented theoretical R and S arrays associated with the single input transfer function model. The practical implementation pertaining to the identification of the noise corrupted model via the R and S arrays is also discussed. Limited demonstration by two published series shows that the suggested approach is practicable and readily observable tool for identifying the order of the single input transfer function model.

This approach can be applied to the identification of multiple input transfer function models. Unfortunately, we have not found published multiple input samples for demonstration purpose. This needs a simulation study.

The generalization of the theorems of Section 3 for model identification to the nonstationary process is worthy to be considered. This will show how a transformation to stationarity may be obtained whenever the process is nonstationary due to roots, real or complex, on or within the unit circle.

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