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A Model for Measuring Market Efficiency of Rent Controls

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1. Introduction

The purpose of this paper is to investigate the market efficiency of rent controls. Most of the literature analyzed the market efficiency of rent controls have shown the inefficiency of rent controls. They insist that rent controls are unambiguously harmful. In recent years, however, there has been an increasing criticism of the traditional view of rent controls even among North American economists (see Arnott, 1988). One of them is the assumption that housing markets are perfectly competitive. While they certainly exhibit many of the distinguishing features of the perfectly competitive market, they also have significant non-competitive features. Most of the theoretical literature modeling rent controls ignored these non-competitive features of housing market.

It is evident that the housing market is far from a perfectly competitive market. Due to the heterogeneity of housing and the high transaction costs, landlords and tenants usually have some degree of market power. They influence rents. All agents are not perfectly well informed. Transactions cost — in this context, moving and search costs, both monetary and psychic — are significant. Obviously, the housing market does differ in important ways from a perfectly competitive market. As a result, in some societies and situations it may be possible for a government to impose rent controls to correct housing market imperfections as well as to achieve the social welfare goal of equity.

There has been a wide separation between the North American and European views of rent control. In North America (U.S. and Canada) housing markets are free in most cases. In these societies economists view rent controls as generally inefficient and as having adverse effects on housing quality and quantity. Also, because search and other transaction costs in a rent controlled market are in fact larger than in a free market, no apparent efficiency gain from rent controls is expected. In Europe (e.g., Sweden) rent controls are frequently associated with additional housing market regulation such as the centralized matching of households and landlords in controlled markets. Anecdotal evidence as well as economic reasoning suggests that such matching procedures can reduce overall transaction costs and thus result in efficiency gains. Many European economists apparently hold this view.

It is important to review the list of possible benefits to tenants (and landlords) from centralized matching as compared to personal free search: (a) a centralized matching procedure can consider a larger number of possible matches for each tenant or landlord than they could themselves in a free market. Thus, the quality of the ultimate match can be higher; (b) the rationing procedures of a central matching agency can screen out "bad tenants" and "bad landlords" and thus reduce uncertainty in the search process; (c) as in Sweden, centralized matching may apply only to publicly built housing which is more homogeneous in quality and (in that country) more reliably maintained. Tenants may be willing to "wait in line" for such housing than to search in a highly heterogeneous free market, if the costs of such search are high.

A question which arises is whether the

government can provide the benefits of centralized matching without controlling rents. The answer is different according to the type of benefit. For example, in Sweden the government can guarantee a higher quality dwelling by regulating maintenance and still allow private landlords and tenants to determine rents in a market process. However, if the benefit comes from imposed centralized matching of tenants and landlords, then private matches between landlords and tenants cannot be arranged and thus rents cannot be set in a market process. This argument provides the rationale for linking centralized matching with rent control.

As described above, there is a considerable distinction between the North American rent controlled housing market and the European rent controlled housing market. In North America, most of the housing units in rental housing markets are privately owned and the major rent control policy is a rent ceiling which allows a landlord to increase his rents by a certain percentage amount per year. Under this rent ceiling, tenants and landlords exchange their demand and supply as in a free market. Tenants can search housing units as in a free market to find a better housing unit. However, the variation of housing attributes such as housing quality between each housing unit in the controlled market can be larger than in a free market since landlords may cut the maintenance expenditure in order to reduce the losses of their profits. Under this situation, tenants will benefit from housing search in the market since the variation of housing attributes is relatively large. If a tenant chooses a housing unit randomly with no search, the probability that the housing unit will be an average dwelling in the market is very low. Thus, tenants face a very high risk that satisfactory housing units can be chosen without search.

In the European rental housing market, a major portion of the housing units in the controlled market are built and owned by municipal or local housing authorities. These public housing authorities control not only rent but also housing quality. Thus, the variation of housing attributes between each housing unit in the controlled market is much smaller than in the free market in the same countries. Under this situation, tenants in the controlled market can take a dwelling unit randomly with low risk since the variation of housing attributes among these housing units is very small. But tenants in the free market will search because of high risk involved in finding a satisfactory dwelling.

In this context, we can formulate two types of models. First, we may allow the household to search only in the free market. The household in the controlled market will be assigned a dwelling unit randomly with no search. But this assumption can be relaxed in the second type of the model so that households can search in any one of two submarkets, the free market and the controlled market. We may assume that if a household in the free market randomly chooses a dwelling unit with no search, then this will be very costly because of the high risk due to the large variance in the qualities of such units. However, the second type's model will not be presented in this paper due to the space limitation.

The paper consists of four sections. In section 2, a short-run economic equilibrium model with transaction costs is developed using the multinomial logit approach. Two types of search assumptions are also considered in the model: allowing search only in the free market. In section 3, a series of numerical simulations are conducted in order to examine the market efficiency using a hypothetical data set, under the fixed transaction costs assumptions. The changes in the total compensating variation are observed as a market efficiency measure associated with transaction costs and the size of the controlled market as well as the changes of housing quality values. The last part of the paper is devoted to the summary and conclusions.

2. The Model with Exogenous Transaction Costs

(1) The Housing Market and the Short-Side Rule

Let us consider two types of housing markets; a free market (uncontrolled market) and a controlled market in which the rent is controlled by the government. In both cases, for convenience, we are considering only rental housing markets. In the free market, the rent is determined by the competitive market mechanism and tenants can rent at the equilibrium rent without restrictions on transactions. There is no government intervention.

In the controlled market, all rents are controlled by the government and housing owners (landlord) can rent out only at the legally fixed rents. In the rent controlled housing market, usually the government limits the amount by which rents can increase. As a consequence, usually rents of controlled dwellings are set lower than they would be in the absence of rent control. However, for convenience, we will assume that the government just fixes the rent at any given level. Thus, the fixed rent can be higher than that of the equilibrium rent that would occur in this market in the absence of rent controls. Furthermore, it is assumed that dwellings in the controlled market are rationed by the short-side rule. Benassy (1982) described the "short" side of a market as that side where the aggregate volume of desired transactions is smallest. The "short" side is then the demand side if there is excess supply, or the supply side if there is excess demand. The other side is the "long" side.

In the controlled market, rents are fixed because of rent control. Each landlord can make his own decision on the supply side: renting out his dwelling or keeping it vacant at a fixed rent. Under the short-side rule, households or landlords on the short side of the market realize their demands or supplies, while households or landlords on the long side receive a level of transaction proportional to their demand or supply. However, in order to maintain an efficient rationing scheme, we have to assume that all demanders must in some way meet all suppliers. Thus, it is assumed that the market is centralized and the government provides information on housing such as vacancy lists, housing conditions, and so forth. Furthermore, it is assumed that landlords cannot directly rent out their dwellings to households. Instead, landlords who decide to rent out their dwellings entrust those dwellings to the government. The government then assigns the dwelling to a household by a rationing scheme. This may be a queueing, priority system, proportional rationing, or some other system. The landlord then receives the fixed rent, if the dwelling is rented out.

Let us assume that there is a fixed stock quantity of housing, S_i, in market i (where i = f, c, denotes the free market and the controlled market, respectively) during a certain time period. When the market is in the absence of rent control, the equilibrium rent, R_f^*, is determined by the intersection of the aggregate demand and supply schedules. And at this equilibrium rent, R^*_f, the quantity of housing supplied is determined at S^*_f and the same amount of housing is demanded. In such a market system, the notional demands and supplies are equal to the effective demands and supplies.

When rent control is imposed, the controlled rent, R_c, will be fixed at a certain level. At this given rent, R_c, the number of housing units supplied and the number of housing units demanded are determined at S_n and D_n, respectively, which represent the notional demands and supplies. Since the number of housing units demanded and the number of housing units supplied are not equal at the controlled rent, the government forces to equate the demand and the supply by rationing some portion of the number of housing units demanded or supplied according to the short-side rule. Since we assume that such rationing is determined by the short-side rule, the amount of actual transactions, Z°_{c}, is determined by

$$Z^{\circ}_{c} = D_n \cdot a = S_n \cdot g, \qquad (2.1)$$

where $a = \min [1, S_n / D_n]$ and $g = \min [1, d_n / S_n]$. which means that

$$Z^{\circ}_{c} = \min [D_n, S_n]. \qquad (2.2)$$

(2) Vacancy Supply Probabilities

On the supply side of the free market, landlords face the binary choice of renting out dwellings in the market or keeping them vacant. Suppose that the landlord's profit of renting out a dwelling in the free market is $\bar{\Pi}_f^o = \Pi_f^o + \varepsilon_f^0$ and the profit of keeping it vacant is $\bar{\Pi}_f^v = \Pi_f^v + \varepsilon_f^v$. Here the subscript f denotes the free market, Π_f^0 is the landlords' common profit when the dwelling unit is occupied and Π_f^v is the landlords' common profit when the dwelling unit is vacant, and ε_f^o, ε_f^v are independently, identically Gumbel distributed idiosyncratic costs. It may be assumed that the sources of these idiosyncracies come from the dispersion of maintenance cost because of exogenous variables such as building condition, tenants' characteristics and other factors, for instance. Then the probability that a landlord will rent out a dwelling unit in the free market, Q_f, is given by the binary logit model,

$$Q_f = \frac{\exp[\mu \Pi_f^o]}{\exp[\mu \Pi_f^o] + \exp[\mu \Pi_f^v]} \qquad (2.3)$$

where μ is a positive scale parameter which is inversely proportional to the variance of each idiosyncratic component across the population of landlords ($\mu = \pi / \sigma / 6$, where σ^2 is the variance). In this research, it is assumed that the common profit functions are

$$\Pi_f^o = R_f - C^o(q_f) + \bar{r} q_f \text{ and} \qquad (2.4a)$$

$$\Pi_f^v = -C^v(q_f), \qquad (2.4b)$$

where

R_f = the structure rent in the free market,
$C^o(q_f)$ = the maintenance cost of an occupied dwelling in the free market with q_f units of housing quality,
$C^v(q_f)$ = the maintenance cost of vacant dwelling in the free market with q_f units of housing quality,
q_f = the housing quality level in the free market,
$\bar{r}$ = the unit price of housing quality in the free market,
$\bar{r}q_f$ = the housing quality rent premium.

The treatment of housing quality needs brief comment. For simplicity, we assume that housing quality is a unidimensional input which can be purchased at a given unit market price, $\bar{r}$. We will show later, that when the price of quality inputs rises, landlords (in a free market) will reduce quality inputs and thus the quality level of the dwelling will go down. The opposite will happen when quality inputs become cheaper. We also assume that quality inputs do not have a cumulative effect on dwelling quality levels. This means that, the quality level must be decided anew each year because quality inputs are not durable. Thus, in this simple treatment, quality inputs may represent recurring short term maintenance such as painting, upkeep of furnace, lawncare etc. Not much is gained from treating durable quality inputs in this research, because the housing market models we developed not be dynamic ones in order to demonstrate the efficiency of rent controls.

The market rent, R_f, the landlord receives is determined by the market equilibrium and the maintenance cost, $C^o(q_f)$, the landlord has to pay is determined by the given cost function of the quality level q_f. The housing quality premium, $\bar{r}q_f$, means that the landlord will receive an additional rent premium if the housing unit is maintained at the level of quality q_f, when the housing unit is occupied.

Since the landlord must take the market rent, R_f, as given, he cannot influence the market rent. In order to maximize his profit, the landlord may determine the level of housing quality, q_f. it is assumed that the landlord's optimum level of housing quality is determined by maximizing his producer surplus. We will discuss this optimum housing quality problem after defining the producer surplus. For the moment, we consider the housing quality, q_f, and the value of housing quality, $\bar{r}$, as given.

The controlled rent, R_c, is fixed at a certain level by the government, regardless of the housing quality. The landlord must take the controlled rent as given. Even if the landlord maintains the housing unit at a higher quality level, he can not receive a quality premium as he could in the free market. Consequently, the landlord will reduce housing quality to the lowest possible level. Thus, there is no housing quality premium for the landlord in the controlled market, because the government controls the entire rent (equivalent to structure rent plus quality rent in the free market).

In the controlled market, a landlord first decides to rent out his dwelling in the market for the controlled rent or to keep it vacant. If he wants to rent it out, then the government assigns the dwelling to the tenant according to the short-side rule rationing scheme. If the demand side is the short side, there will be some landlords who cannot realize their decisions to rent out their dwellings. Hence, even if a landlord is willing to rent out his dwelling in the market at a given rent, it is not guaranteed that his decision will be realized.

We describe the expected profit functions of landlords in the controlled market as follows. Assuming the landlord's profit function of renting out the dwelling in the controlled market is $\bar{\Pi}_c^o = \Pi_c^o + \varepsilon_c^o$ and the profit of keeping it vacant is $\bar{\Pi}_c^v = \Pi_c^v + \varepsilon_c^v$, where the subscript c denotes the controlled market, Π_c^o and Π_c^v are the common profits and ε_c^o, ε_c^v are the independently, identically Gumbel distributed idiosyncratic costs. Then, the landlord's choice probability, Q_c, is given by the binary logit model,

$$Q_c = \frac{\exp[\mu \Pi_c^o]}{\exp[\mu \Pi_c^o] + [\mu \Pi_c^v]} \tag{2.5}$$

where μ is a positive scale parameter inversely proportional to the variance of the idiosyncratic costs. The expected profit functions are defined by,

$$\Pi_c^o = E[\Phi_c^o] + \gamma_s \log g \tag{2.6a}$$

$$E[\Phi_c^o] = g[R_c - C^o(q_c)] + [1 - g][-C^v(q_c)] \tag{2.6b}$$

$$\Pi_c^v = [-C^v(q_c)] \tag{2.6c}$$

where $E[\Phi_c^o]$ is the landlord's expected profit when a dwelling is offered for occupancy in the controlled market, g is defined by (2.1) and measures the probability that the offered dwelling will be assigned a tenant by the government. The last term of (2.6a) measures the landlords' common risk from the decision to offer the dwelling for occupancy in the controlled market. The scale factor, $\gamma_s > 0$, measures the marginal cost of risk. Since g is the minimum value out of one and D_n / S_n, g has a value between zero and one. If $g = 1$, the risk term becomes zero. It means that the dwelling will be assigned to a tenant with certainty. If $g = 0$, the risk term becomes negative infinity. The landlord's profit also goes to negative infinity. Thus, there is no chance that the dwelling will be assigned.

(3) Landlord's Optimum Housing Quality

When the market is free, the market structure rent, R_f, is determined by market equilibrium. The landlord maintains the housing quality at a certain level and receives a housing quality rent premium from the tenant. But he has to pay a maintenance cost for keeping quality at that level. The landlord, thus, has to decide what housing quality he must maintain in order to maximize his profit. It is assumed that the unit price of housing quality, exogenously given to each landlord.

To model the landlord's quality maintenance decision properly, we have to specify the timing relationship between this decision and the decision of whether to offer the dwelling for rent or not. We assume that the quality maintenance decision is made at the very beginning of the market period, before the idiosyncratic component of a landlord's profit function is realized. At such time the landlord knows the probability distribution of these idiosyncratic costs but does not yet know the realized values (for him) from the distribution. Thus, the landlord decides the quality level by maximizing his expected profit (or expected producer's surplus). Subsequent to this (early in the market period) the idiosyncratic costs ε_c^{o} and ε_c^{v} are revealed to each landlord and each makes the discrete choice decision of whether to offer his dwelling or whether to keep it vacant. Thus, because the quality maintenance decision comes first, the quality level of a dwelling is determined independently of whether it is occupied or vacant.

The landlord's expected profit maximization problem with respect to the housing quality q_f is:

$$\max_{q_f} \Omega_f = \frac{1}{\mu} S_f \{\log[\exp\{\mu\ [R_f - C^o(q_f) + C^v(q_f) + \bar{r} q_f]\} + 1]\} \quad (2.7)$$

where Ω_f is the producer's surplus when the market is free defined by (2.8).

Since the landlord cannot influence the structure rent, R_f, the above is equivalent to

$$\max_{q_f} \Pi = \bar{r} q_f - \{C^o(q_f) - C^v(q_f)\} \quad (2.8)$$

It means that the landlord will maximize his common and deterministic profit by deciding quality level q_f: That is, he will maximize the difference between revenue, $\bar{r} q_f$, and the costs, $C^o(q_f) - C^v(q_f)$, (the difference between the occupancy and vacancy maintenance costs). The monotonically increasing functions of the housing quality q_f. Thus, we assume that they are given by,

$$C^o(q_f) = d(q_f - A)^{\theta} + B, \quad (2.9a)$$

and,

$$C^v(q_f) = b q_f + D, \quad (2.9b)$$

where d, b > 0, B > D > 0, A $\geq$ 0 and θ > 1. Substituting (2.9a), (2.9b) into (2.8) and differentiating with respect to q_f, we obtain the first and the second order conditions as follows:

$$\frac{\partial \Pi}{\partial q_f} = \bar{r} - d\theta(q_f - A)^{\theta-1} + b \quad (2.10a)$$

$$\frac{\partial^2 \Pi}{\partial q_f^2} = -d\theta(\theta - 1)(q_f - A)^{\theta-2} < 0 \quad (2.10b)$$

as long as $q_f > A$, and zero if $q_f = A$.

From (2.10a), we can obtain the optimum housing quality q^*_f by solving:

$$\bar{r} - d\theta(q^*_f - A)^{\theta-1} + b = 0 \quad (2.11a)$$

This says that the landlord will improve quality to the point where the marginal cost of additional improvement just equals the marginal revenue. Solving,

$$q^*_f = [\frac{\bar{r} + b}{d\theta}]^{1/(\theta-1)} + A \quad (2.11b)$$

When the market is controlled, the landlord will not maintain the housing quality at q^*_f, because there is no housing quality premium. Thus, the landlord's expected profit maximization problem is written by

$$\max_{q_c} \Omega_c = \frac{1}{\mu} S_c \{\log[\exp\{\mu [g\ [R_c - C^o(q_c + C^v(q_c)] + \gamma_s \log g]\} + 1]\} \quad (2.12)$$

where Ω_c is the producer's surplus when the market is controlled defined by (2.31). We may obtain q^*_c simply by solving

$$\min_{q_c} C_c = C^o(q_c) - C^v(q_c) \quad (2.13a)$$

$$\text{where } C^o(q_c) = d(q_c - A)^{\theta} + B \quad (2.13b)$$

$$C^v(q_c) = b q_c + D \quad (2.13c)$$

Substituting (2.13b), (2.13c) into (2.13a) and solving for q^*_c we obtain the optimum housing quality by

$$q^*_c = [\frac{b}{d\theta}]^{1/(\theta-1)} + A \quad (2.14)$$

The landlord will maintain housing quality at q^*_c whenever the market is controlled regardless of the level of the controlled rent. since the unit price of housing quality, $\bar{r}$, must be positive, landlords will maintain higher quality in the free market than they will in the controlled market. This is implied by (2.11b) and (2.14).

(4) Households' Market Choice Probabilities

Suppose that there are N households in the market and the housing market is divided into the free and controlled markets. Suppose that there are S_i rental dwellings available for occupancy in each market i (where i = f, c, denotes the free and controlled markets, respectively). It is assumed that moving costs are zero. Suppose first that all dwelling units within the market are identical in all respects. Thus, each household faces a choice of housing market only. The total utility of households choosing housing market i, $\bar{U}_i$, can be expressed as

$$\bar{U}_i = U_i + \varepsilon_i, \text{ for all i,} \qquad (2.15)$$

where U_i is the common component of utility which varies by housing market alternative i, and ε_i is the idiosyncratic utility component for housing market alternative i. Assuming that the random utilities, ε_i, are independently and identically Gumbel distributed for each alternative i, the household's housing market choice probabilities are expressed as a multinomial logit model,

$$P_i = \frac{\exp[\lambda U_i]}{\Sigma_j \exp[\lambda U_j]} \qquad (2.16)$$

for j = 1,..., J, where λ is the taste heterogeneity coefficient, inversely proportional to the variance (σ^2) across households of each $\varepsilon_i(\lambda = \pi / \sigma /6)$.

Suppose next, that dwelling units within the same market i are distinct in their idiosyncratic components, so that each household faces a choice of market followed by a choice of dwelling. In such a situation, assuming that utilities are given by,

$$\bar{U}_{in} = U_i + \varepsilon_{in},$$

$$\text{for all i and n} = 1,\ldots, S_i \qquad (2.17)$$

with ε_{in}, the idiosyncratic component, independently and identically Gumbel distributed for each dwelling n and each market i gives housing market choice probabilities of the form:

$$P_i = \frac{S_i \exp[\lambda U_i]}{\Sigma_j S_j \exp[\lambda U_j]} \qquad (2.18)$$

for j = 1,..., J with λ the heterogeneity coefficient, as before, and S_i, i = 1,..., J the number of housing units in market i.

There is an important distinction between the two logit models (2.16) and (2.18). We will use (2.16) to express the choice probabilities in a controlled market, assuming (in this section) that households who choose the controlled market can not choose a specific dwelling in that submarket, but must accept being assigned (or rationed) a dwelling in that submarket. Assuming no bias in rationing the household will get the average dwelling in the submarket, thus $E[\varepsilon_{in}] = \varepsilon_i$ determines the choice among submarkets. We will use (2.18) to express the choice probabilities in a free market, assuming that a household can choose any one specific dwelling n = 1,..., S_i in a free market i.

Let us now define the common utility of each choice alternative. For our purposes, it is convenient to formulate the common utility as a function specifically of housing market attributes and of the household disposable income that remains after subtracting all housing-related expenses. Introducing a compensating variation and a rent dividend (to close the model), the common utility of a household that chooses the free market, U_f, or the controlled market, U_c, are respectively,

$$U_f = \alpha \log[y - R_f - \bar{r}q_f - M_f + D_{fc} - C] + \beta_t T_f + \beta_q q_f \qquad (2.19a)$$

$$U_c = \alpha \log[y - R_c - M_c + D_{fc} - C] + \beta_t T_c + \beta_q q_c \qquad (2.19b)$$

where

y = the household's income

R_f = the free market structure rent,
R_c = the controlled market rent,
$\bar{r}$ = the unit price of housing quality,
q_f, q_c = the housing quality level in the free market and in the controlled market, respectively,
M_f, M_c = the monetary search and transaction costs in the free market and the controlled market, respectively,
D_{fc} = the rent dividend when one market is free and the other market is controlled,
C = the compensating variation,
T_f, T_c = the search and transaction times in the free market and the controlled market, respectively.

Since there is a probability that the household will not be assigned a dwelling in the controlled market, the household's expected utility of choosing the controlled market (adjusted for risk) is represented by

$$\bar{U}_c = a[U_c + \varepsilon_c] + [1 - a] [U_f + \frac{1}{\lambda} \log S_f + \beta_t T_c + \varepsilon_f] + \gamma_d \log a + \eta \qquad (2.20)$$

where

a = the probability that the household will be assigned a dwelling in the controlled market,
γ_d = the coefficient of the common risk aversion penalty of applying to the controlled market ($\gamma_d > 0$),
$\gamma_d \log a$ = the common risk aversion penalty of applying to the controlled market,
η = the idiosyncratic risk aversion disutility.

A household that chooses the controlled market and is accepted is not allowed to search and to examine the dwelling units in the controlled market. But a household that is rejected from the controlled market can search and examine the dwelling units in the free market. In fact, there is search utility for the household that chooses the free market. We assume that this search utility is equivalent to the household's expected maximum utility in the free market, which is represented by the term $1/\lambda \log S_f$. In the second term of (2.20), adding the transaction time in the controlled market, T_c, is necessary, because the household that chooses the controlled market must wait some length of time to find out the outcome of the rationing decision. After this rationing process, the household can enter the free market if rejected from the controlled market.

The risk aversion penalty, $\gamma_d \log a$, is considered because uncertainty is involved in the rationing process. If the acceptance probability goes to zero ($a \to 0$), the risk aversion term approaches negative infinity. If the acceptance probability becomes one ($a \to 1$), the risk aversion term becomes zero. Thus, the risk disutility varies between negative infinity and zero. If the household is risk neutral the household's utility will be the same as its expected value, in which case the risk aversion term, $\gamma_d \log a$, is omitted ($\gamma_d = 0$).

The idiosyncratic risk disutility, η, is zero if there is no rationing process, thus, if $a = 1$, $\eta = 0$.

Under the assumptions that ε_c and ε_f are i.i.d. normal distributed with means zero and variance σ^2, while η is normal with mean zero and variance, we get the result that var $[\bar{U}_c] = \text{var}[\varepsilon_c] = \text{var}[\varepsilon_f] = \sigma^2$. This i.i.d. probit model is approximated by the multinomial logit choice probabilities which, in our case, take the form:

$$P_f = \frac{S_f \exp[\lambda U_f]}{S_f \exp[\lambda U_f] + \exp[\lambda U_c]} \qquad (2.21)$$

$$P_c = \frac{\exp[\lambda U_c]}{S_f \exp[\lambda U_f] + \exp[\lambda U_c]} \qquad (2.22)$$

where

$$U_c = aU_c + [1 - a] [U_f + \frac{1}{\lambda} \log S_f + \beta_t T_c] + \gamma_d \log a. \qquad (2.23)$$

(5) Market Efficiency Measures and Producer Surplus

Compensating variations are reasonable measures of the welfare effect of a price change. They can be used to evaluate the change in welfare between initial states and final states when prices change. The new price is used as the base and we ask what income change would be necessary to compensate the consumer for the price change (Varian, 1984). Suppose that the consumer faces two different policy regimes with the price p_0 as an initial (pre-policy) price and p_1 as an after-change (post-policy) price. The consumer has a constant income y in each situation. Since the compensating variation, C, is interpreted as the amount of money we have to pay (or tax) the consumer at price p_1 to make him just as happy as he was at price p_0, the compensating variation can be expressed as

$$V(p, y - C) = V(p_0, y), \tag{2.24}$$

where V is the consumer's indirect utility function. Hence we may use the compensating variation as a measurement of the market's efficiency.

The compensating variation can be either positive or negative. According to the above definition, a negative value of C is interpreted as the minimum amount of money we have to pay the consumer to compensate him for the change and a positive value of C is the maximum amount of money the consumer would be willing to pay for the policy change. Thus, if the compensating variation summed across all consumers is positive, we may say that there are some welfare gains resulting from the public policy change. And if the compensating variation summed across all consumers is negative, we may conclude that there are some welfare losses after the public policy change.

Suppose that a household has a twice differentiable, strictly quasi-concave utility function. The household maximizes its utility subject to the budget and non-negativity constraints. This maximization yields the ordinary demand functions. Substituting the demand functions into the utility functions, the household's indirect utility function is obtained, which is the maximum utility achievable at given prices and income. If household's preferences are characterized by random utilities on prices, income and other observable characteristics, the indirect utility function can be replaced by the expected maximum utility function. In this case, the expected maximum utility is identical to the consumer surplus measured in terms of utility (see, for example, Domencich and McFadden, 1975; Small and Rosen, 1981; Varian, 1984).

In order to evaluate the efficiency of rent controls, we will first find the equilibrium rents of a completely free market (assuming that both market are free), by solving the following free market aggregate excess demand system, E_{ff}:

$$E_{ff}(\bar{U}_{ff}) = D^{ff}(\bar{U}_{ff}) - S^{ff}(R_{ff}) = 0, \tag{2.25}$$

where

$\bar{U}_{ff} = U_{ff}[R_{ff}, Y_{ff} + D_{ff}]$, which is the households' common utility in the free market,

R_{ff} = rent of the free market,

D^{ff} = demand for the free market,

S^{ff} = supply for the free market,

Y_{ff} = household's disposable income,

D_{df} = rent dividend of the free market computed as the producer surplus (or expected maximized profit) divided by the number of households in the market, that is,

$$D_{ff} = \frac{1}{N}\Omega_{ff} = \frac{1}{N}\int_0^{R^*_{ff}} s^{ff}(R_{ff})dR_{ff} \tag{2.26}$$

N = number of households in the market,

R^*_{ff} = equilibrium rent in the free market satisfying (2.25),

Ω_{ff} = producer surplus of the free market.

The rent dividend, an equal share of

aggregate housing rent per household, is measured by the total producer surplus divided by the number of households (consumers) in the market. Following the tradition of standard urban microeconomic analysis, it is assumed that the producer surplus is equally shared by the households in the market, since all households in our model are identical.

Producer surplus is measured by the area to the left of the aggregate supply curve. It is normally obtained by integrating the inverse function of the supply function from zero to prices. Let Ω_{ff} be the total producer surplus when the both sub-markets are free markets. Using (2.3) and (2.25), the total producer surplus is computed by

$$\Omega_{ff} = \int_{o}^{R^{*}_{ff}} S^{ff}(R_{ff})dR_{ff}$$
$$= \frac{1}{\mu} S[\log\{\exp\ (\mu\ [R_{ff} - C^{o}(q_{ff}) + C^{v}(q_{ff} + \bar{r}q_{ff}]) + 1\}] \quad (2.27)$$

where S is the total number of housing units, and the landlord's supply probability, $Q_f(R_{ff})$, is defined by (2.3). Then, the rent dividend when the both sub-markets are free, D_{ff}, is obtained by

$$D_{ff} = \Omega_{ff} / N, \quad (2.28)$$

where N is the total number of households.

Suppose that rent controls are imposed on one submarket and the total stock of dwellings is divided into two submarkets: the free market and the controlled market. The producer surplus of the free market, Ω_f, when the other market is controlled, can be rewritten using (2.27):

$$\Omega_f = \int_{-\infty}^{R^{*}_{f}} S_f Q_f(R_f)dR_{ff}$$
$$= \frac{1}{\mu} S_f[\log\{\exp\ (\mu\ [R_f - C^{o}(q_f) + C^{v}(q_f) + \bar{r}q_f]) + 1\}] \quad (2.29)$$

where

R_f = the structure market rent of the free market,

S_f = the number of housing units in the free market,

Q_f = the landlord's supply probability in the free market,

$C^{o}(q_f)$ = the landlord's maintenance cost when the dwelling unit is occupied in the free market with q_f units of housing quality,

$C^{v}(q_d)$ = the landlord's maintenance cost when the dwelling unit is vacant in the free market with q_f units of housing quality,

$\bar{r}$ = the unit price of housing quality in the free market,

q_f = the housing quality level in the free market,

$\bar{r}q_f$ = the housing quality rent premium.

We now turn to the producer suplus of the controlled market which can obtained by integrating the "effective" supply function. This effective supply function is defined by

$$S_e = gS_cQ_c(g) \quad (2.30)$$

And the "effective" demand function is given by

$$D_e = NaP_c(R_f, R_c, a) \quad (2.31)$$

When the controlled rent, R_c, is fixed below the free market equilibrium rent, R^{*}_{f}, the effective supply curve remains the same as the supply curve in the free market. Since the supply is on the short side, the value of g becomes one. Thus, the producer surplus is measured by the area to the left of the supply curve, S_f. When the controlled rent, R_c, is fixed above the free market equilibrium rent, R^{*}_{f}, the demand is on the short side. Thus, the value of g becomes less than one and a = 1. The effective supply curve shifts upward to S'_e. Thus, the producer surplus is depicted as the area to the left of supply curve, S'_e.

The producer surplus of the controlled market, Ω_c, when the other market is free, is then:

$$\Omega_c = \int_{-\infty}^{R_c} gS_cQ_c(g)dR_c$$
$$= \frac{1}{\mu} S_c[\log\{\exp\ (\mu\ [g(R_c - C^{o}(q_c) + C^{v}(q_c)) + \gamma_s \log g]) + 1\}] \quad (2.32)$$

where

R_c = the controlled rent,

S_c = the number of housing units in the controlled market,

g = the landlord's housing unit assignment probability defined by (2.1),

Q_c = the landlord's supply probability in the controlled market,

$C^o(q_c)$ = the landlord's maintenance cost function when the dwelling unit is occupied in the controlled market,

$C^v(q_c)$ = the landlord's maintenance cost function when the dwelling unit is vacant in the controlled market,

γ_s = a positive scale factor measuring the landlord's risk aversion.

Thus, the producer surplus of the whole market, Ω_{fc}, is the sum of the producer surplus of each submarket. That is,

$$\begin{aligned}\Omega_{fc} &= \Omega_f + \Omega_c \\ &= \frac{1}{\mu} S_f[\log\{\exp(\mu [R_f - C^o(q_f) \\ &\quad + C^v(q_f) + \bar{r}q_f]) + 1\}] \\ &\quad + \frac{1}{\mu} S_c[\log\{\exp(\mu [g(R_c \\ &\quad - C^o(q_c) + C^v(q_c)) \\ &\quad + \gamma_s \log g]) + 1\}]\end{aligned} \tag{2.33}$$

The rent dividend of the whole market, D_{fc}, is therefore obtained by,

$$D_{fc} = \Omega_{fc} / N \tag{2.34}$$

Suppose that rent control is imposed on one market. Then rent is fixed and thus the market is not cleared by rent but cleared by the short-side rule. The variables, a and g serve as market adjustment variables in the system. The efficiency of the rent controlled market to the free market system is measured by the compensating variation, C, as

$$D^c(\bar{U}_c) - S^c(R_c, g) = 0 \tag{2.35a}$$

$$W_{ff}(\bar{U}_{ff}) - W_{fc}(\bar{U}_f, \bar{U}_c) = 0 \tag{2.35b}$$

where

$\bar{U}_c = U_c(R_c, a, Y_{fc} + D_{fc} - C)$, which is the households' common utility in the controlled market when the other market is free,

$\bar{U}_{ff} = U_{ff}(R_{ff}, Y_{ff} + D_{ff})$, which is the households' common utility when both markets are free

$\bar{U}_f = U_f(R_f, y_{fc} + D_{fc} - C)$, which is the households' common utility in the free market when the other market is controlled,

D^c = demand in the controlled market,

S^c = supply in the controlled market,

R_c = fixed rent in the controlled market,

R_f = market structure rent in the free market,

D_{fc} = rent dividend when one submarket is controlled,

Y = household's disposable income

C = compensating variation,

W_{ff} = household's expected maximum utility when both markets are free,

W_{fc} = household's expected maximum utility when one market is controlled and the other market is free.

The expected maximum utility is the utility of the best alternative in a subset of choices as a summary of the expected value of that subset to an individual. In a more general context the maximum utility of all alternatives in a choice set is a measure of the individual's expected utility associated with a choice situation. It is well known that the expected maximum utilities are equivalent to the consumer surplus measures in the case of discrete choice models (Williams, 1977; Small and Rosen, 1981; Khajavi, 1981; Ben-Akiva and Lerman, 1985).

If the household's common utility for all alternatives in the choice set is given, the expected maximum utility of the choice set, W, is:

$$W = \frac{1}{\lambda} \log \Sigma_k \exp[\lambda U_k] \tag{2.36}$$

where λ is a positive scale parameter and

U_k is the common utility of choice alternative k. We will utilize this specification in the next section.

(6) Market Equilibrium

Market equilibrium is determined by setting the demand by households for each submarket to equal the number of dwelling supplied to that submarket. Given that S and S_c are predetermined and given a controlled rent, R_c:

$$N\{P_f(R_f,\ a,\ C) + (1-a)P_c(R_f,\ a,\ C)\} - S_fQ_f(R_f) = 0 \qquad (2.37a)$$

$$\min\ \{1,\ S_n / D_n\} - a = 0 \qquad (2.37b)$$

$$\min\ \{1,\ D_n / S_n\} - g = 0 \qquad (2.37c)$$

$$W_{ff}(R_{ff}) - W_{fc}(R_f,\ a,\ C) = 0 \qquad (2.37d)$$

$$NP_{ff}(R_{ff}) - \frac{S}{2} Q_{ff}(R_{ff}) = \qquad (2.37e)$$

$$\text{where } S_n = S_cQ_c(g) \qquad (2.37f)$$

$$D_n = NP_c(R_f,\ a,\ C) \qquad (2.37g)$$

$$W_{ff}(R_{ff}) = \frac{1}{\lambda} \log \{S\ \exp[\lambda\ U_{ff}(R_{ff})]\} \qquad (2.37h)$$

$$W_{fc}(R_f,\ a,\ C) = \frac{1}{\lambda} \log \{S_f\ \exp[\lambda\, U_f(R_f,\ C)] + \exp[\lambda\, [aU_c(C) + (1-1)(U_f(R_f,\ C) + \frac{1}{\lambda} \log\ S_f + \beta_t T_c) + \gamma_d \log a]]\} \qquad (2.37i)$$

Equations (2.37a) is the market equilibrium condition of the free market and says that the demand for the free market is equal to the supply in that market. Equations (2.36b) and (2.37c) express the short-side rule. D_n and S_n represent the notional demand and the notional supply, respectively. For simplicity, in equation (2.37d) the expected maximum utility, W_{ff}, is computed for two identical submarkets which are free and of equal size. This is arbitrary, because of the form of the choice probabilities derived previously, any unequal partition of the stock would give the same rent. This will be proved below. And the expected maximum utility, W_{fc}, is computed when one market is controlled and the other market is free. Equation (2.37d) says that the household's expected maximum utility after rent control in the market in which rent control is imposed on one submarket must be the same as that before rent control in the market in which rents are not controlled. Equation (2.37e) is the market clearing condition of the free market when both markets are identical and of the same size. The endogenous variables are the rent of the free market, R_f, the household assignment probability, a, the landlord assignment probability, g, the compensating variation, C, and the equilibrium rent of the free market, R_{ff}, when both markets are free and of equal size.

3. Numerical Simulations

In this section, a series of simulations is done with two submarkets: the free market and the controlled market. At the initial stage, there is no rent control. The whole market is free. Rent control is imposed in part, at the very beginning, on a small number of housing units. The rest of the housing units are free. Then, the number of housing units that are rent controlled are gradually increased untill all housing units in the market are controlled. Changes in total compensating variations are examined for each given number of housing units in the controlled market.

The market equilibrium defined in the previous section exists only when every household in the market can obtain a housing unit either in the controlled market or in the free market since we are dealing with a closed market. If a household in the market can not obtain a housing unit in either market because of the shortage of housing supply, the market can not reach an equilibrium.

In order to achieve a market equilibrium, we have to assume that all households must in some way satisfy all their demands. This means that the total number of housing units available in both markets must be greater than or equal to the number of households in both markets. Otherwise, there will be no market equilibrium.

It is obvious that the controlled rent can not be fixed at any level. In other words, the controlled rents can not be fixed at zero nor near infinity. If the controlled rents are fixed much lower than the rent that would occur in the absence of rent control, the landlords will reduce the supply of housing units in the controlled market because of lower profit. If the controlled rents are fixed much higher than the rent that would occur in the absence of rent control, households cannot afford them because of the budget constraints. In this case, market equilibrium may exist, but it may not possible to obtain the solution numerically because of machine dependent computational overflow or underflow. A similar situation will happen if the unit price of housing quality is very high, because landlords cannot obtain the housing quality rent premium in the controlled market. They will reduce the housing supply in the controlled market, unless the high controlled rent compensates for the loss of the housing quality rent premium. Thus, there must be an upper and lower boundary that the controlled rent can be fixed in order to achieve a market equilibium and to obtain some reasonable solutions.

Since the landlord's probability that he will rent out his dwelling unit in each market is determined by the landlord's profit function defined by (2.4) and (2.6) respectively, the controlled rents must be determined with respect to the size of the controlled market, the unit price of housing quality, the housing quality in each market. It means that the market equilibrium may not exist at a certain level of the controlled rent given the size of the controlled market and level of housing quality, because all of these factors affect the housing supply in each market.

In order to conduct a series of numerical simulations, a set of values for exogenous variables and parameters are assumed as in Table 3.1. In the basic run, each of the two submarkets are free. All conditions in both markets are identical. Each market has the same quality housing stock. All housing units are rented at the equilibrium rent. The total number of dwelling units in the market are more than the total number of households in the market, so that there is no excess demand over stock at the rent that would hold in the absence of rent controls. Using the data given in Table 3.1, the coefficients of the model are calibrated.

Some qualitative comparative statics results from numerical simulations are presented in Table 3.2. It summarizes all the results and shows the qualitative effects in each exogenous variable on each endogenous variable. These results can be explained as follow. First, the total compensating variation can be increased or decreased by the level of controlled rent. And it increases as the gain in the costs of transaction in the controlled market increases, but decreases as the unit price of housing quality and the consumer's value of housing quality decrease. Thus, in order for rent control to be efficient, there must be some gains in the costs of transaction. But if the unit price of housing quality and / or the consumer's

Table 3.1. Data Set for the Basic Runs

Number of Households: $N = 9{,}500$
Number of Dwellings: $S = 10{,}000$
Household Income: $y = \$30{,}000$ / yr.
Monetary Transaction Costs: $M = \$200$ / yr.
Search and Transaction Time: $T_f = 100$ hrs.
Value of Time: $H = \$10$ / hr.
Rent Elasticity of Demand: $E^d = -0.5$
Rent Elasticity of Supply: $E^s = 0.3$
Heterogeneity Coefficient: $\lambda = 1.0$
Maintenance Cost Function Coefficients:
$d = 0.5$, $A = 0.0$, $\theta = 2.0$,
$b = 0.7$, $D = 200.0$
Unit Price of Housing Quality: $\gamma = 0.0$, 50.0
Value of Housing Quality: $\beta_q = 0.0$, 0.05
Equilibrium Rents:
1) market 1 (R_f) = \$7,500 / yr.
2) market 2 (R_f) = \$7,500 / yr.

Table 3.2. Some Qualitative Comparative Statics Results from Numerical Simulations

Endogenous Variables	Exogenous Variables				
	R_c	T_c	$\bar{r}$	β_q	S_c
R_f	?	+	+	+	+
TC	?	−	−	−	?
a	+	+	?	+	?
g	−	−	−	−	?
P_c	−	−	−	−	+
P_f	+	+	+	+	−
Q_c	+	−	−	−	+
Q_f	?	+	+	+	+
q_c	0	0	0	0	0
q_f	0	0	+	0	0
$C^o(q_c)$	0	0	0	0	0
$C^v(q_c)$	0	0	0	0	0
$C^o(q_f)$	0	0	+	0	0
$C^v(q_f)$	0	0	+	0	0
D_{fc}	?	+	?	+	?

value of housing quality is very high, rent control should not be imposed.

Second, the free market rent rises as transaction costs in the controlled market, the unit price of housing quality and the value of housing quality become high. They make the controlled market inefficient and also raises the probability that households will choose the free market. Third, rent dividend increases as the transaction costs in the controlled market and the consumer's value of housing quality become high, because they raise the free market rents. But the rent dividend can be increased or decreased by the level of the controlled rent and the unit price of housing quality, because of the free market rent and the high maintenance costs.

4. Summary and Conclusions

In this paper, the major hypotheses were: 1) the rent control can be efficient if the costs of transaction (search, matching, rationing) in the controlled market are lowered by centralized matching as is apparently the case in some European housing market: 2) rent controls become inefficient if utility of housing quality is important and landlords are not required to maintain quality.

In order to investigate these hypotheses, a multinomial logit based model of the housing market is developed. On the demand-side of the model, each household faces a housing market choice among the two alternatives of housing markets: the free market and the controlled market. On the supply-side of the model, each landlord decides whether he rents out his dwelling unit or not, responding the level of controlled rent and the housing quality premium. When the controlled rent is fixed below the equilibrium rent that would occur in the absence of rent controls, tenants who are accepted into the controlled market could have benefit because of less payment of rent. However, the landlord in the controlled market will cut his maintenance cost to offset losses. Thus, the housing quality in the controlled market will be lower than that of the free market.

In the model, households' search utility and search disutility are considered as well as waiting costs for processing. Two types of models are developed. The first is the model with search utility only in the free market and the second is the model with search utility in each sub-market: the free market and the controlled market.

The concept of compensating variation is used as a market efficiency measure. The variation in the transaction costs acts as a major factor of achieving the market efficiency in the model. The market equilibrium conditions are formulated by equating the demand for housing in the market with the number of housing units supplied in that market under the short-side rule.

A series of numerical simulations are conducted using a hypothetical data set. As a market efficiency measure, changes in the total compensating variation are observed for different values of such policy variables as the controlled rent level, the gain in the transaction costs, the unit price of housing quality, the consumer's value of housing quality, the number of housing units in the controlled market.

From a series of numerical simulations, the following results are obtained: First, rent control can be efficient if the government can reduce the transaction costs. However, if the unit price of housing quality and / or the consumer's value of housing quality becomes sufficiently high, the imposition of rent control is not efficient. The magnitude of the gain in the total compensating variation depend on the level of the controlled rent, the magnitude of gains in the transaction costs, the unit price of housing quality, the consumer's value of housing quality and utility gain from search in the controlled market.

Second, housing quality is one of the major factors which influence the market efficiency of rent controls. If the housing quality is regarded as very important attribute in the household's utility, rent control is not efficient unless the gains in the transaction costs are very high.

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