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# Analysis of Mechanical Face Seals for Design Purpose Geometric Effects on Sealing Gap

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# 설계목적을 위한 기계 평면 시일의 해석

제 1 보 : 시일링 간극에 대한 기하학적 영향에 관하여

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요약 — 시일의 접촉면이 경사지고, 정현파형이 존재하며 코닝이 있는 경우에 대한 해석을 하였다. 또한 시일링 간극에 존재하는 유체는 비압축성 점성유체이다. 이러한 요소들을 고려한 레이놀즈 방정식의 일반해를 구하기 위하여 멱급수 근사이론을 이용하여 압력분포를 구하였고, 이 결과를 이용하여 시일의 누설 유동량 및 마찰 토오크를 해석하였다. 계산 결과에 의하면 시일 접촉면의 표면파형이나 코닝에 비하여 경사정도가 시일의 성능에 커다란 영향을 주고 있음을 보여주고 있다.

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## Introduction

In the industries such as motor, power, and aerospace, mechanical face seals are widely used to prevent leakage of liquids or gases.

Theoretical and experimental works of mechanical seals have been reported in the literature by a large number of investigators [1-15]. The study of a misaligned face seal has been reported by Sneck [2-4], Hardt and Godet [5], Metcalfe [6], and Etsion [7-9]. These investigators focused on the effects of misalignment on seal performance; they reported that misalignment plays a vital role to axial force, tilting moment, torque and leakage flow rate.

Experiments done by Pape [10] and Stanghan-Batch and Iny [11] indicated the significance of surface waviness. Lebeck [12] examined the role of waviness on seal performance and developed an model for predicting the size of the sealing gap in face seals. He reported that an increases in waviness amplitude reduces friction, and increases leakage.

Coning of mating faces arises from the mechanical and thermal distortions, and wear. Etsion et al. [13-15] have shown that coning effects are very important to the separating force in the sealing gap.

In the present work, the combined geometric effects of mechanicial seals are analyzed for steady laminar and incompressible fluids using a temperature dependent viscosity. An analytical solution is obtained with a pressure distribution in polynomial form ; using this solution, the leakage flow rate is estimated.

## Analysis

In the seal leakage analysis with misalignment, surface waviness and coning included the following assumptions are made for short seals :

1. Steady axisymmetric flow ;
2. Incompressible viscous fluid ;
3. The axial flow is negligible compared with the small sealing gap ;
4. The temperature distribution with respect to the axial direction is symmetric ;
5. Body forces and inertia forces are negligible.

**Temperature Distribution.** We use the energy equation to determine the distribution of temperature within the sealing gap in mechanical seals. Considering the above assumptions, the reduced energy equation for an incompressible viscous fluid thus becomes

$$\rho c_v v_r \frac{\partial T}{\partial r} = \frac{K}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + K\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \eta\left[\left(\frac{\partial v_r}{\partial z}\right)^2 + \left(\frac{\partial v_\theta}{\partial z}\right)^2\right] \tag{1}$$

where the specific heat $c_v$ and the lubricant conductivity coefficient K have been assumed as constants. r is the radial coordinate and z denotes position across the sealing gap. $v_r$ and $v_\theta$ represent the radial and circumferential velocities, which are given by Eqs. (13) and (14).

Let's introduce the dimensionless variables to normalize the energy equation(1),

$$\bar{\eta} = \frac{\eta}{\eta_r},\ R = \frac{r - r_i}{r_0 - r_i},\ Z = \frac{z + \bar{h}}{2\bar{h}},\ V_r = \frac{v_r}{U},$$
$$V_\theta = \frac{v_\theta}{U},\ \bar{T} = \frac{T - T_u}{T_l - T_u} \tag{2}$$

By substituting the dimensionless variables of Eqs.(2)into Eq.(1) and rearranging with the Brinkman constant $[Br = \eta_r U^2/K(T_l - T_u)]$, the energy equation is rewritten as follows :

$$\frac{\partial^2 \bar{T}}{\partial Z^2} + \left(\frac{2\bar{h}}{r_0 - r_i}\right)^2 \frac{1}{R'}\frac{\partial}{\partial R}\left(R'\frac{\partial \bar{T}}{\partial R}\right) - \frac{(r_o - r_i)U}{\alpha}\left(\frac{2\bar{h}}{r_o - r_i}\right)^2 V_r \frac{\partial \bar{T}}{\partial R} = -\bar{\eta}\, Br\left[\left(\frac{\partial V_r}{\partial Z}\right)^2 + \left(\frac{\partial V_\theta}{\partial Z}\right)^2\right] \tag{3}$$

where $\alpha = K/\rho c_v$ is the thermal diffusivity. In Eq.(3), the rate at which heat can be conducted along the radial direction is small compared with the rate of conduction to the seal surfaces, since the leading coefficient, $(2\bar{h}/\bar{w})^2$ is very small. It is also clear that the convection term may be dropped for laminar flow and thin film thickness. For a laminar flow and increased temperature difference [16] the dissipation terms may be neglected. Then the energy equation can be simplified to

$$\frac{\partial^2 \bar{T}}{\partial Z^2} = 0 \tag{4}$$

The accuracy of Eq.(4) is increasing for the decreased Brinkman number. The boundary conditions for Eq.(4) are

$$\bar{T} = 0\ (T = T_u) \quad \text{at } Z = Z_u\ (z = z_u) \tag{5a}$$
$$\bar{T} = 1\ (T = T_l) \quad \text{at } Z = Z_l\ (z = z_l) \tag{5b}$$

Integrating Eq.(4) twice with respect to Z we have

$$\bar{T} = \frac{Z - Z_u}{Z_l - Z_u} \tag{6}$$

**Velocity Distributions**A dimensionless analysis of the order of magnitude of various terms which are included into the Navier-Stokes equation leads to the simplified equation of motion in the sealing gap. The mean sealing gap is assumed to be very small compared with the width of the face seal ; i.e., $2\bar{h} << \bar{w}$. When the fluid film of the sealing gap moves for the critical range of the Reynolds number, $Re = 2\bar{h}\rho_r U/\eta_r$, the influence of the inertia forces can be neglected ; i.e., $Re << \bar{w}/2\bar{h}$. Under these conditions, the simplified equations of motion for the steady flow with temperature dependent variable viscosity are given by

$$\frac{\partial}{\partial z}\left(\eta\frac{\partial v_r}{\partial z}\right)=\frac{\partial p}{\partial r} \tag{7}$$

$$\frac{\partial}{\partial z}\left(\eta\frac{\partial v_\theta}{\partial z}\right)=\frac{1}{r}\frac{\partial p}{\partial \theta} \tag{8}$$

The boundary conditions for Eq.(7) are

$$v_r=0 \quad \text{at } z=z_u \tag{9a}$$
$$v_r=0 \quad \text{at } z=z_l \tag{9b}$$

The boundary conditions for Eq.(8) are

$$v_\theta=U \quad \text{at } z=z_u \tag{10a}$$
$$v_\theta=0 \quad \text{at } z=z_l \tag{10b}$$

The viscosity of the lubricant is assumed to vary as a function of temperature according to the following equation :

$$\eta=\eta_r e^{\alpha^* T_r - T} \tag{11}$$

where $\alpha^*$ is the temperature-viscosity coefficient given by the reference [17]. The viscosity in dimensionless form can be rewritten by substituting Eq.(6) into Eq.(11) :

$$\bar{\eta}=e^{\alpha_1 - \frac{\alpha_2 z - z_l}{2h}} \tag{12}$$

where

$$\alpha_1=\alpha^* T_r(1-\Lambda_l), \quad \alpha_2=\alpha^* T_r \Lambda_l (\beta_\Lambda - 1)$$

Integration of Eqs.(7) and (8) gives the velocity distributions, we obtain :

$$v_r=\frac{4h^2}{\eta_r}\left(\frac{\partial p}{\partial r}\right)f(\beta) \tag{13}$$

$$v_\theta=\frac{4h^2}{r\eta_r}\left(\frac{\partial p}{\partial \theta}\right)f(\beta)+Ug(\beta) \tag{14}$$

where

$$f(\beta)=\frac{e^{\alpha^* T_r(\Lambda_l - 1)}}{\alpha_2(1-e^{-\alpha_2})}\left\{1-\left[1+\left(e^{-\alpha_2}-1\right)\frac{\beta-1}{\beta_\Lambda-1}\right]e^{\alpha^* T_r \Lambda_l(\beta-1)}\right\}$$

$$g(\beta)=\frac{1-e^{\alpha^* T_r \Lambda_l(\beta-1)}}{1-e^{\alpha_2}}$$

**Pressure Distribution.** Using the temperature and velocity distributions, the Reynolds equation for an incompressible fluid in polar coordinates is derived by integrating the continuity equation over the sealing gap.

$$\frac{\partial}{\partial r}\left[\Gamma_1 r(2h)^3\frac{\partial p}{\partial r}\right]+\frac{1}{r}\frac{\partial}{\partial \theta}\left[\Gamma_1 (2h)^3\frac{\partial p}{\partial \theta}\right]$$
$$=\eta_r U\frac{\partial}{\partial \theta}\left[\Gamma_2(2h)+z_u\right] \tag{15}$$

where

$$\Gamma_1=\frac{e^{\alpha^* T_r(\Lambda_l-1)}}{\alpha_2(1-e^{-\alpha_2})}\left[1+\frac{1}{\alpha_2^2}\left(1-e^{\alpha_2}\right)\left[1-e^{-\alpha_2}\right]\right]$$

$$\Gamma_2=-\left(\frac{1}{\alpha_2}+\frac{1}{1-e^{\alpha_2}}\right)$$

The dimensionless parameters are defined as follows :

$$P=\frac{(2\bar{h})^2 p}{\eta_r \omega (r_o-r_i)^2}, \quad R'=\frac{r}{r_o},$$

$$\hat{H}=\frac{2h}{2\bar{h}}, \; Z_u=\frac{z_u}{2\bar{h}} \tag{16}$$

Substituting the above parameters into Eq.(15) and employing the narrow seal approximation [8], a simplified Reynolds equation for the narrow seal width and the negligible seal curvature effect is given by

$$\frac{\partial}{\partial R}\left(\Gamma_1 \hat{H}^3\frac{\partial P}{\partial R}\right)=\frac{\partial}{\partial \theta}\left(\Gamma_2\hat{H}+Z_u\right) \tag{17}$$

To solve the equation, let the dimensionless sealing gap, $\hat{H}$ and P be expressed by the polynomial form

$$\hat{H}(R, \theta, t)=\sum_{i=0}^{n}\hat{H}_i R^i \quad (18)$$

$$P(R, \theta, t)=\sum_{i=0}^{\infty}P_i R^i \quad (19)$$

where $0 \le R \le 1$. Substitution of these dimensionless quantities into Eq.(17)

$$\sum_{m=0}^{\infty}\left\{\sum_{\ell=0}^{m}\sum_{k=0}^{\ell}\sum_{i=0}^{k}(m-e+\ell)\hat{H}_i\hat{H}_{k-i}\left[3(\ell-k+1)\hat{H}_{i-k+1}P_{m-\ell+1}+(m-\ell+2)\hat{H}_{i-k}P_{m-\ell+2}\right]-A_m\right\}R^m=0 \quad (20)$$

For Eq.(20) to be valid, all coefficients of all powers of R must vanish independently. This generates a recursive and infinite series of equations that involve the unknovn $P_n$. Solution of these equations leads to recursive formulae for $P_n$ given by

$$P_{n+2}=\frac{-S_n}{(n+1)(n+2)\hat{H}_0^3} \quad (21)$$

where

$$S_n=\sum_{i=0}^{n}\sum_{j=0}^{i+1}\sum_{k=0}^{j}(n-i)\hat{H}_k\hat{H}_{j-k}\left[3(i-j+2)\hat{H}_{i-j+2}P_{n-i}+(n-i+1)\hat{H}_{i-j+1}P_{n-i+1}\right]+3(n+1)\hat{H}_0^2\hat{H}_1P_{n+1}-A_n$$

The unknown parameters of Eq.(21) are given by

$$H_0=1+a_2\cos\theta+\hat{h}/(2\bar{h})$$

$$\hat{H}_1=\varepsilon\cos\theta+C_1$$

$$\hat{H}_j=C_j \quad \text{for } j\ge 2$$

$$A_0=\frac{\Gamma_2}{\Gamma_1}\left[-a_2\sin\theta-\zeta_u n_u\cos\left(n_u\theta-\Omega_u t\right)-\zeta_i n_i\cos\left(n_i\theta-\Omega_i t\right)\right]-\frac{\zeta_u n_u}{\Gamma_1}\cos\left(n_u\theta-\Omega_u t\right)$$

$$A_1=-\frac{\Gamma_2}{\Gamma_1}\varepsilon\sin\theta,\quad A_2=0,\quad A_3=0,\quad\cdots$$

where $a_2=\varepsilon\bar{R}/(1-\bar{R})$

The coefficients of the series (19) are determined by substituting the recursive formulae (21) and the pressure boundary conditions

$$P=P_{in} \quad \text{at } R=0 \quad (22a)$$

$$P=P_{ex} \quad \text{at } R=1 \quad (22b)$$

Substitution of these boundary conditions into Eq.(19) gives the result

$$P_0=P_{in} \quad \text{at } R=0 \quad (23a)$$

$$\sum_{n=0}^{\infty}P_n=P_{ex} \quad \text{at } R=1 \quad (23b)$$

The second coefficient $P_1$ of the polynomial form may be determined by substituting Eq.(21) for n =0, 1, 2,⋯ into Eq.(23b) and truncate the infinite series at some N that gives acceptable accuracy. The recursive formulae (21) generates $P_2$ or higher order coefficients, since $P_0$ and $P_1$ are solved. Thus the pressure distribution of the seals with the complicated geometry can be determined to any desired accuracy in R.

**Sealing gap Analysis.** Many of the boundary conditions involved in the fluids problem for seals are coupled to the determination of the sealing gap. The overall sealing gap 2h between the stator and the rotor described by a function of the form

$$2h(r, \theta, t)=2\bar{h}+h_m+\hat{h}+h_c. \quad (24)$$

Here $2\bar{h}$ is the mean film thickness, $h_m$ represents the sealing gap variation due to misaligned seal geometry, $\hat{h}$ denotes a random part of the film thickness due to surface waviness between the seal ring and the seal seat, and $h_c$ is the sealing gap change due to coning effects.

**1. Sealing gap due to angular misalignment.**

The geometry for a misaligned face seal is illustrated in Fig. 1. The tangential coordinate is chosen so that it starts through the point of maximum thickness. It can be assumed that the magnitude of the misalignment is small compared with the radius, which is acceptable for most practical seals. The film thickness variation $h_m$ for a misaligned seal is given by

$$h_m = 2\bar{h}\varepsilon\left(R + \frac{\bar{R}}{1-\bar{R}}\right)\cos\theta; \qquad (25)$$

where $\theta$ is the angular position measured from the point of maximum sealing gap and $\varepsilon$ is a tilt parameter restricted by the geometry of the misaligned disk ; i.e., $0 \leq \varepsilon \leq (1-\bar{R})$.

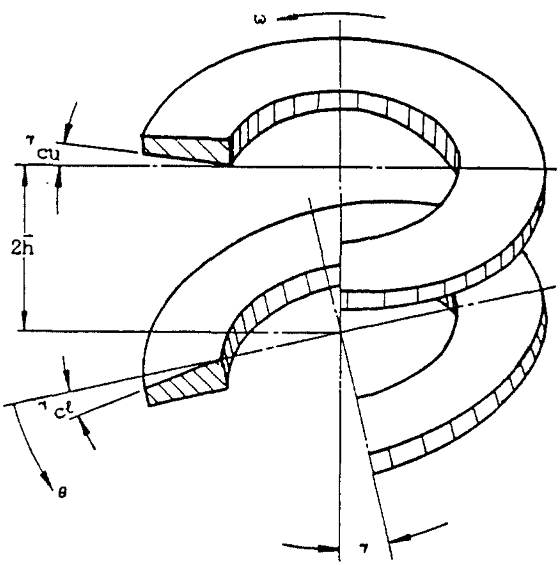


Fig. 1. Mechanical face seal with angular misalignment and linear coning.

**2. Sealing gap caused by sinusoidally wavy surfaces.**

The stator surface is tilted with respect to the rotor surface by the small angle $\gamma$. Both surfaces are wavy and project away from and toward each other. Since the tilt angle $\gamma$ between the two surface is small, the projection length of the waviness height of the stator onto the normal vector of the rotor is comparable to the composite waviness height ; i.e., $|\hat{h}_l|\cos\gamma \approx |\bar{h}_l|$. Also it will be assumed that the tangential film thickness varies sinusoidally around the seal circumference as shown in Fig. 2. The variations of the sealing gap $\hat{h}$ due to the wavy surfaces can be expressed as

$$\hat{h} = -|\hat{h}_u|\sin(n_u\theta - \Omega_u t) - \hat{h}_l|\sin(n_l\theta - \Omega_l t) \qquad (26)$$

where $|\hat{h}_l|$ and $|\hat{h}_u|$ are the amplitudes of surface waviness at the stator and rotor.

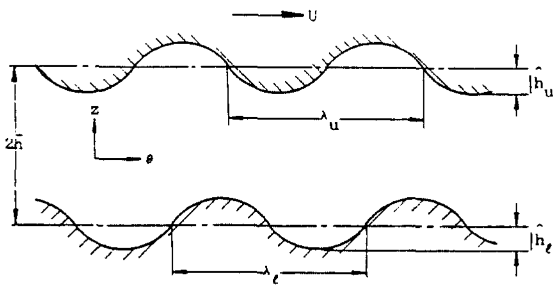


Fig. 2. Sinusoidal waviness in mechanical seals.

**3. Sealing gap due to coning effects.**

The composite surface profile $H_c$ of the upper and lower seals at the mating zone may be approximated in polynomial form

$$H_c = \sum_{i=0}^{\infty} C_i R^i \qquad (27)$$

where $C_0$ is zero because it will be grouped with the mean sealing gap $2\bar{h}$. The unknown constants $C_i$ $(i>1)$ will be determined by boundary conditions. If the coning model as shown in Fig. 1 is linear, the coning shape is obtained for the boundary conditions ; $H_c=0$ at $R=0$ and $H_c=\varepsilon_c$ at $R=1$

$$H_c = (\varepsilon_{cu} + \varepsilon_{cl})R \qquad (28)$$

The first term in Eq.(28) is a surface profile variation caused by coning at the upper edge and the second term corresponds to the lower edge of the mating zone.

**Volumetric Flow Rate.** Leakage takes place through the radial sealing gap formed by the two sliding surfaces. The radial leakage flow rate of an incompressible fluid at radius r is obtained from :

$$\dot{Q}=n\int_0^{\frac{2\pi}{n}}\int_{z_l}^{z_u} r v_r dz d\theta \tag{29}$$

By substituting Eq.(13) into Eq.(29) and integrating it with respect to z, the volumetric leakage flow rate is obtained.

$$\dot{Q}=\frac{n\bar{\gamma}}{\eta_r}\int_0^{\frac{2\pi}{n}}(2h)^3 r\left(\frac{\partial p}{\partial r}\right)d\theta \tag{30}$$

where

$$\bar{\gamma}=\frac{e^{\alpha^* T_r(\Lambda_l-1)}}{\alpha_2[1-e^{-\alpha_2}]}\left\{1+\frac{1-e^{\alpha_2}}{[\alpha^* T_r \Lambda_l(\beta_\Lambda-1)]^2}\left[1+\alpha_2+(1-\alpha_2)e^{\alpha_2}\right]\right\}$$

By substituting the dimensionless parameters of Eq.(16) and the series of Eqs.(18) and (19) into Eq.(30), the dimensionless flow factor $\bar{Q}$ is given by

$$\frac{\dot{Q}}{2\hbar\omega r_o(r_o-r_i)}=\bar{Q}=n\bar{\gamma}\int_0^{\frac{2\pi}{n}}(B_0+B_1R+\cdots)d\theta \tag{31}$$

where

$$B_0=\bar{R}P_1\hat{H}_0^3,\quad B_1=(1-\bar{R})P_1\hat{H}_0^3+\bar{R}\hat{H}_0^2(3P_1\hat{H}_1+2P_2\hat{H}_0),\ \cdots$$

Since the flow rate physically cannot vary with R, it can be simplified using R=0. Integrating Eq.(31) with respect to $\theta$, after substituting the expressions of $H_0$ and $B_0$ into Eq.(31), the dimensionless leakage flow rate is given by

where

$$\begin{aligned}\bar{Q}=\bar{\gamma}\bar{R}P_1\Big[&2\pi-3\zeta_u\zeta_l(\zeta_u K_{11}+\zeta_l K_{12})\\&+3a_2^2(\pi-\zeta_u K_{22}+\zeta_l K_{23})\\&+3\left\{\pi(\zeta_u^2+\zeta_l^2)+2\zeta_u\zeta_l K_{13}+2a_2\zeta_u\zeta_l K_{34}\right\}\\&-6a_2(\zeta_u K_{24}+\zeta_l K_{25})\Big]\end{aligned} \tag{32}$$

where

$$K_{11}=-\delta_{(2n_u)(n_l)}\frac{\pi}{2}\sin[(2\Omega_u-\Omega_l)t]$$

$$K_{12}=\delta_{(n_u)(2n_l)}\frac{\pi}{2}\sin[(\Omega_u-2\Omega_l)t]$$

$$K_{13}=\delta_{(n_u)(n_l)}\pi\cos[(\Omega_u-\Omega_l)t]$$

$$K_{22}=-\delta_{(n_u)(2)}\frac{\pi}{2}\sin(\Omega_u t)$$

$$K_{23}=-\delta_{(n_l)(2)}\frac{\pi}{2}\sin(\Omega_l t),$$

$$K_{24}=-\delta_{(n_u)(1)}\pi\sin(\Omega_u t)$$

$$K_{25}=-\delta_{(n_l)(1)}\pi\sin(\Omega_l t),$$

$$K_{34}=\delta(\delta_{(n_u)(n_l+1)}+\delta_{(n_u+1)(n_l)})\frac{\pi}{2}\cos[(\Omega_u-\Omega_l)t]$$

and a symbol, $\delta_{(m)(n)}$ is defined by

$$\delta_{(m)(n)}=\begin{cases}0, & \text{if } m\neq n\\1, & \text{if } m=n\end{cases}$$

**Friction Torque.** The torque caused by the viscous friction is defined as

$$T_f=\int_0^{2\pi}\int_{r_i}^{r_o}\eta\left(\frac{\partial v_\theta}{\partial z}\right)r^2 dr d\theta \tag{33}$$

For the negligible pressure gradient in the circumferential direction, the velocity gradient in the z direction is given by differentiating Eq.(14)

$$\frac{\partial v_\theta}{\partial z}=\frac{\alpha^* T_r \Lambda_i U e^{\alpha^* T_r \Lambda_i (\beta-1)}}{1-e^{\alpha_2}}\left(\frac{1-\beta_\Lambda}{2h}\right) \tag{34}$$

Substituting Eqs.(12) and (34) into (33), we obtain

$$T_i=\frac{\pi \eta_r \alpha^* T_r \Lambda_i \bar{w} U r_0^2 (1-\beta_\Lambda)}{\bar{h}(1-e^{\alpha_2})}\sum_{i=1}^{\infty}\frac{1}{i}\Big[\bar{R}^2 T_{i-1} + 2(1-\bar{R})\bar{R}T_{i-2}+(1-\bar{R})^2 T_{i-3}\Big] \tag{35}$$

where $T_{-2}=0$, $T_{-1}=0$, $T_0=1$, $T_i=-C_i$ for $i \geq 1$

## Results and Discussion

The problem was solved for 3 geometries such as misalignment, sinusoidal waviness of the mating surfaces and coning. The upper ring is a steel and the lower ring is Carbon-graphite. The following dimensions and operating conditions were selected :

| | |
|---|---|
| Seal inner radius, $r_i$ | 6.35 cm |
| Mean sealing gap, $2\bar{h}$ | 40 $\mu$m |
| Reference viscosity (MIL-L-7807), $\eta_r$ | $1.62 \times 10^{-3}$ pa-s |
| Thermal conductivity (MIL-L-7807), K | $9.66 \times 10^{-2}$ N/s-℃ |
| Temperature viscosity coefficient, $\alpha^*$ | $3.6 \times 10^{-2}$ 1/°K |
| Shaft speed, $\omega$ | 419 rad/s |
| Temperature differential, ΔT | 500℃ |
| Internal pressure, $P_{in}$ | $13.8 \times 10^4$ N/m² |
| External pressure, $P_{ex}$ | $10.1 \times 10^4$ N/m² |

Fig. 3 shows the convergence of pressure coefficient $P_i$ of Eq.(19) for the various R's, $2\bar{h}=40(\mu m)$, $\varepsilon = 0.05$, $t = 1.33 \times 10^{-6}$(s) and $U = 36$(m/s). It shows that four or five terms are sufficient to obtain good accuracy. In general, when the pressure coefficients converge very quickly, the accuracy of the pressure distribution was increased.

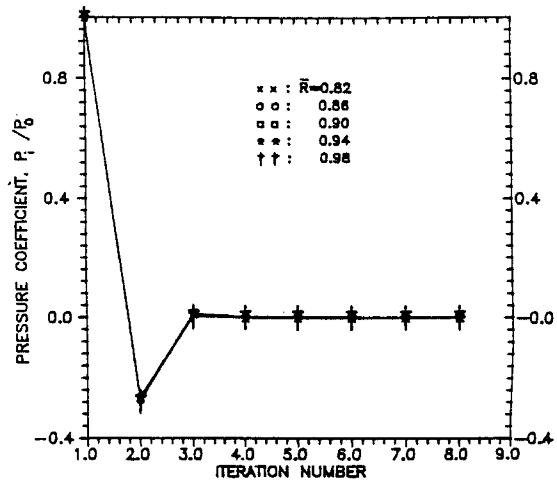


Fig. 3. Variations of pressure coefficient with iteration number.

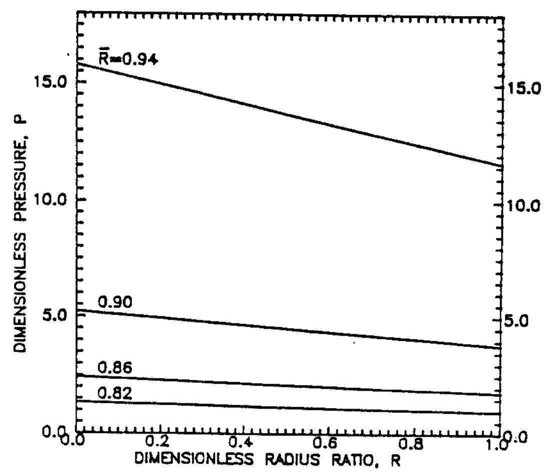


Fig. 4. Variations of dimensionless pressure as a function of dimensionless radius ratios.

Fig. 4 shows results for the pressure distributions along the radial direction for various radius ratios, $\bar{R}=r_i/r_o$. When the internal pressure is higher than the external pressure. As the dimensionless radius ratio, $R=(r-r_i)/(r_o-r_i)$ increases, the dimensionless pressure P monotonically decreasing for all curves. Bryant and Kim [19] showed

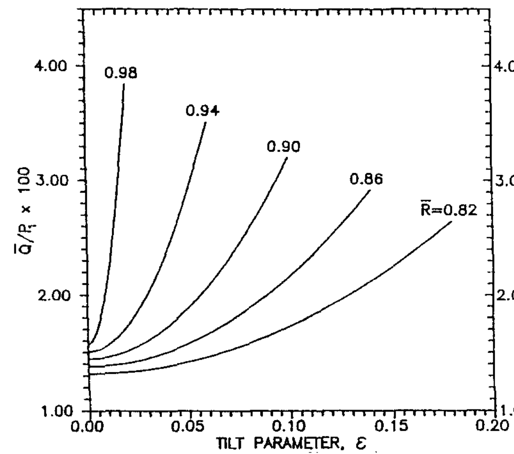


Fig. 5. Effect of tilt parameter on dimensionless leakage flow rate for various radius ratios and n=4000 rpm.

that an increase in radial distance decreases the pressure for a compressible fluid with a similar trend as shown in Fig. 4.

In Fig. 5, the volumetric flow rate due to the geometric configurations represented by the mean sealing gap, misalignment, wavy surfaces, and coning, presented for different ratios of radius ; $\bar{R}=0.82$ to 0.98 and for the whole range from $\varepsilon=0$ to $\varepsilon_{max}=(1-\bar{R})/2$ The curves show that an increase in the tilt parameter monotonically increases the leakage flow rate. The leakage rate is considerably sensitive for the misalignment at the higher radius ratio. The reduction of the radius ratios lowers considerably the volumetric leakage rate. Etsion [7] showed the similar curves of non-dimensional leakage flow as shown in Fig. 5.

In order to examine the behavior of the frictional torque, we have computed the solution for a case in which $2\bar{h}=40(\mu m)$, $\varepsilon=(1-\bar{R})/2$ for Figs. 6 and 7. As the mean sealing gap increases, the torque hyperbolically decreases for the wide range of dimensionless radius ratio. Fig. 7 shows the effect of sliding velocity on torque. As we expected, the frictional torque is decreased for the narrow seal width.

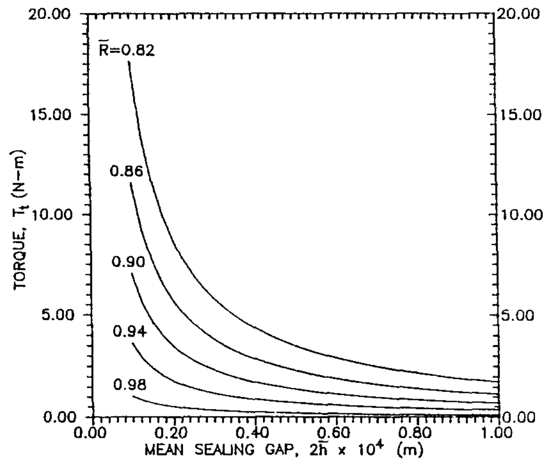


Fig. 6. Torque as a function of mean sealing gap for n=4000 rpm and $\varepsilon=(1-\bar{R})/2$.

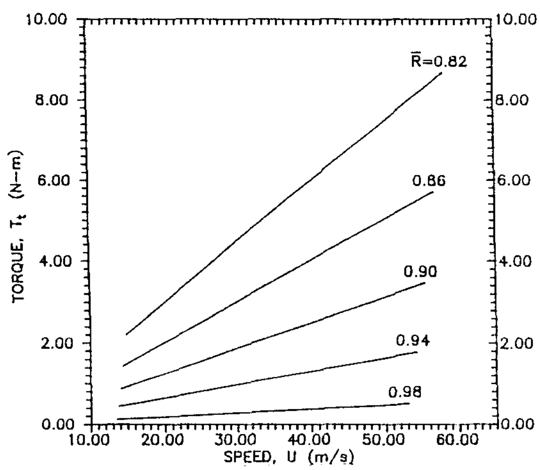


Fig. 7. Torque as a function of sliding velocity for $2\bar{h}=40\,\mu m$ and $\varepsilon=(1-\bar{R})/2$.

## Conclusions

A method to solve the Reynolds equation for an incompressible liquid with temperature dependent viscosity was developed using a power series technique. This approach may by powerful for a complex geometry. As demonstrated by this work, the pressure in polynomial form converges very rapidly. Sufficient accuracy may be obtained including only the first four or five terms for the various radius ratios and tilt parameters.

The calculated results seem to indicate that the misalignment term plays an important role compared with the surface waviness and coning in the volumetric flow rate. As the dimensionless radius ratio decreases, the torque is dramatically increased for small sealing gap and high speeds.

## Acknowledgements

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### Nomenclature

$2\bar{h}$ = mean sealing gap
n = number of waves
p = pressure
r = radius
$\bar{R}$ = $r_i/r_o$
R = $r/r_o$
t = time
T = temperature
U = velocity of the driving surface
$\overline{w}$ = $r_o$-$r_i$
$\hat{\beta}$ = $T_u/T_\ell$
$\varepsilon$ = $\gamma\,(r_o-r_i)/2\bar{h}$, tilt parameter
$\eta$ = viscosity
$\Lambda_\ell$ = $T_\ell/T_r$
$\zeta_\ell$ = $|\hat{h}_\ell|/2\bar{h}$
$\zeta_u$ = $|\hat{h}_u|/2\bar{h}$
$\rho$ = density
$\omega$ = angular speed of the moving surface

### Subscripts

c = coning
$i$ = inner radius
$\ell$ = lower surface
o = outer radius
r = reference conditions
u = upper surface

## References

1. Nau, B.S., "Review of the Mechanism of Hydrodynamic Lubrication in Face Seal", 3rd Int. Conf. on Fluid Sealing, Cranfield, 1967, Paper E., BHRA.
2. Sneck, H.J., "The Effects of Geometry and Inertia on Face Seal Performance-Laminar Flow", J. of Lub. Tech., April 1968, pp.333-341.
3. Sneck, H.J., "The Effects of Geometry and Inertia on Face Seal Performance-Turbulent Flow", J. of Lub. Tech., April 1968, pp.342-350.
4. Sneck, H.J., "The Misaligned, Eccentric Face Seal", J. of Lub. Tech., October 1969, pp.695-703.
5. Hardt, R. and Godet, M., "Axial Vibration of a Misaligned Radial Face Seal, Under a Constant Closure Force", ASLE Trans., 18, 1, 1974, pp. 55-61.
6. Metcalfe, H. J., "Dynamic Tracking of Angular Misalignment in Liquid-Lubricated End-Face Seals", ASLE Trans., 24, 42 pp.509-516.
7. Etsion, I., "Nonaxisymmetric Incompressible Hydrostatic Pressure Effects in Radial Face Seals", J. of Lub. Tech., Vol.100, July 1978, pp.379-385.
8. Etsion, I., "The Accuracy of the Narrow Seal Approximation in Analyzing Radial Face Seals", ASLE Trans., 22, 2, pp.208-216.
9. Etsion, I., "Radial Forces in a Misaligned Radial Face Seal", J. of Lub. Tech., Vol.101, Jan. 1979, pp.81-85.
10. Pape, J.G., "Fundamental Research on a Radial Face Seal", ASLE Trans.,11, 1968,

pp.302-309.

11. Stanghan-Batch, B. and Iny, E.H., "A Hydrodynamic Theory of Radial-Face Mechanical Seals", J. of Mech. Eng. Sci., Vol.15, No.1, 1973, pp.17-24.
12. Lebeck, A.O., "Prediction of Leakage Gap in Nonaxisymmetric Geometry Face Seals", J. of Lub. Tech., Vol.106, July. 1984, pp.329-337.
13. Etsion, I. and Sharoni, A., "Performance of End-Face Seals with Diametral Tilt and Coning-Hydrostatic Effects", ASLE Trans., Vol.23, pp.279-288.
14. Sharoni, A. and Etsion, I., "Performance of End-Face Seals with Diametral Tilt and Coning-Hydrodynamic Effets", ASLE Trans., 24, 1, pp.61-70.
15. Green, I. and Etsion, I., "Stability Threshold and Steady-State Response of Noncontacting Coned-Face Seals", ASLE Trans., 28, 4, pp.449-460.
16. Constantinescu, V.N., Smith, R.N., and Pascovici, M.D., "Note on the Influence of Unequal Surface Temperatures in Liquid Lubrication", Rev. Roum. Sci. Techn. Mech. Appl., Vol. 25, No.1, 1980, pp.113-124.
17. Gupta, P.K., Flamand, L., Berthe, D., and Godet, M., "On the Traction Behavior of Several Lubricants", J. of Lub. Tech., Vol.103, No.1, 1981, pp.55-64.
18. Kim, C.K. and Burton, R.A., "Thermoelastic Boundary Condition for Laminar Flows", N. C. State University, Aug. 1985.
19. Bryant, M.D. and Kim, C.K., "Estimating the Leakage of Compressible Gases Through Face seals Rotating at High Speeds, Part I : Geometric Effects on Sealing Gap", N. C. State University, 1987.