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〈단신〉

비뉴우톤성 유체에서의 Kolmogorov Microscales

전명식 · 최형진*

미국 Carnegie Mellon 대학교 화학공학과
*인하대학교 고분자공학과
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Kolmogorov Microscales for Non-Newtonian Fluid

Myung S.Jhon and Hyoung Jin Choi*

Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, U.S.A.
**Department of Polymer Science and Technology, Inha University, Incheon 402-751, Korea*
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요 약

n차 유체모델을 사용하여 비뉴우톤성 난류계에서 Kolmogorov microscales를 연구하였다. 뉴우톤성 유체와 2차 유체 양 모델에 대해서는 같은 미세척도 운동(small scale motion)의 dissipation energy 뿐만 아니고 Kolmogorov microscales도 같은 값으로 유도되었으나, 3차 유체모델을 사용하는 경우에는 비뉴우톤성 유체의 특성이 얻어졌다.

Abstract—When the n th-order fluid model is applied to investigate the Kolmogorov microscales for the non-Newtonian turbulence system, a same dissipation of energy of small scale motion is obtained for both Newtonian and second-order fluids. In addition, the Kolmogorov microscale is also derived to be the same for both Newtonian and second-order fluids. For the third-order fluid, the non-Newtonian fluid characteristics are developed.

Keywords: Kolmogorov microscale/Non-Newtonian fluid/Second-order fluid model/Third-order fluid model/Turbulence

INTRODUCTION

In studying turbulence, it is important to estimate eddy sizes. The largest eddies are as big as the width of the flow, and are relevant to the length scale in the analysis of the interaction of the turbulence with the mean flow. The largest eddies, in general, are not influenced by the nature of the fluid. However, the small scale motions are generally dependent on the constitutive relationships of the fluid.

In this study, we shall show a simple derivation of small scale fluctuations (Kolmogorov microscales) for the non-Newtonian constitutive relationship. The Kolmogorov microscales in Newtonian fluid are well established [1]; however, the analogous studies for the non-Newtonian fluid have not been examined yet.

We hope that the results shown in this study will be useful in studying the non-Newtonian turbulence problem. To simplify our analysis, We will study an n th-order fluid instead of a more general constitutive relationship

*To whom correspondence should be addressed.

DEVELOPMENT

Note that the n th-order fluid, stemming from the retarded motion expansion [2], is not sufficiently general to account for all polymer rheology. It is used here to illustrate the essence of non-Newtonian turbulence. The constitutive relationship, connecting stress tensor ($\underset{\approx}{\tau}$) and the rate of strain tensor ($\dot{\underset{\approx}{\gamma}}$), can be written for the n th-order fluid as follows:

$$\underset{\approx}{\tau} = -\mu\dot{\underset{\approx}{\gamma}} \leftarrow \text{Newtonian}$$
$$+\alpha_2\frac{\mathscr{D}}{\mathscr{D}t}\dot{\underset{\approx}{\gamma}} + \alpha_{11}\{\dot{\underset{\approx}{\gamma}}\cdot\dot{\underset{\approx}{\gamma}}\} \leftarrow \text{2nd-order} \quad (1)$$
$$-\alpha_3\frac{\mathscr{D}^2}{\mathscr{D}t^2}\dot{\underset{\approx}{\gamma}} + \alpha_{12}\{\dot{\gamma}\cdot\frac{\mathscr{D}}{\mathscr{D}t}\dot{\gamma} + \frac{\mathscr{D}}{\mathscr{D}t}\dot{\gamma}\cdot\dot{\gamma}\} - \alpha_{1,11}(\dot{\underset{\approx}{\gamma}}:\dot{\underset{\approx}{\gamma}})\dot{\underset{\approx}{\gamma}}$$
$$\text{3rd-order}$$

Here, $\mathscr{D}/\mathscr{D}t$ is the corotational (or Jaumann) derivative [2], defined by

$$\frac{\mathscr{D}}{\mathscr{D}t}\dot{\gamma} = \frac{D}{Dt}\dot{\gamma} + \frac{1}{2}(\omega\cdot\dot{\gamma} - \dot{\gamma}\cdot\omega) \quad (2)$$

Here, $D/Dt = d/dt + \mathbf{v}.\nabla$ is the Stokes derivative, $\underset{\approx}{\omega} = \nabla\mathbf{v} - (\nabla\mathbf{v})^T$ is the vortcity tensor, and $\underset{\approx}{\gamma} = \nabla\mathbf{v} + (\nabla\mathbf{v})^T$ is the rate of strain tensor.

According to Kolmogorov's universal equilibrium theory of small scale turbulence structure, one can calculate microscales of length (ζ), time (θ), and velocity (v) by the following considerations:

(1) The Reynolds number formed by ζ and v is equal to 1. That is, $\zeta v/\nu = 1$ (with μ/ρ as kinematic viscosity). This illustrates that the small scale motion is quite viscous and that the viscous dissipation adjusts itself to the energy supply by changing the length scale [3].

(2) The dissipation rate (per unit mass) of the small scale motion is to be calculated using the constitutive relationship of the fluid together with turbulent statistics. To simplify analysis, we assume Gaussian statistics for the fluctuating strain rate ($\dot{\underset{\approx}{\gamma}}^*$), that is:

$$<\dot{\underset{\approx}{\gamma}}^*> = 0,\ <\dot{\underset{\approx}{\gamma}}^*:\dot{\underset{\approx}{\gamma}}^*> = \Delta,\ <\dot{\underset{\approx}{\gamma}}^*\cdot\dot{\underset{\approx}{\gamma}}^*\cdot\dot{\underset{\approx}{\gamma}}^*> = 0$$

and

$$<(\dot{\underset{\approx}{\gamma}}^*:\dot{\underset{\approx}{\gamma}}^*)^2> = 3\Delta^2, \text{ and etc.} \quad (3)$$

Here, the angular bracket stands for the average over turbulent statistics. The dissipation of energy of small scale motion (ε) can be computed from the expression $\rho\varepsilon = -<\underset{\approx}{\tau}^*:\dot{\underset{\approx}{\gamma}}^*>$.

RESULT AND DISCUSSION

The results one obtains are

$$\varepsilon_0 = \varepsilon_2 = \nu\Delta$$

and

$$\varepsilon_3 = \nu\Delta + 3(\alpha_{1,11}/\rho)\Delta^2 \quad (4)$$

with the subscripts 0, 2, and 3 standing for Newtonian, second-order, and third-order fluids, respectively.

Therefore, from (1) and (2), and recognizing that $\Delta \simeq (\nu/\zeta)^2$, one can construct modified Kolmogrov microscales for the n th-order fluid:

Newtonian (zeroth-order) fluid

$$\zeta = (\nu^3/\varepsilon_0)^{1/4}, \text{ and } \theta_0 = (\nu/\varepsilon_0)^{1/2} \quad (5)$$

Second-order fluid

$$\zeta_2 = \zeta_0, \text{ and } \theta_2 = \theta_0 \quad (6)$$

(The same results as for Newtonian fluid)

Third-order fluid

$$\zeta_3 = \zeta_0\{(1+\sqrt{1+12\alpha_{1,11}/\mu\theta_0^2})/2\}^{1/4} > \zeta_0$$

and

$$\theta_3 = \theta_0\{(1+\sqrt{1+12\alpha_{1,11}/\mu\theta_0^2})/2\}^{1/2} > \theta_0$$

To obtain these results, we have assumed $\varepsilon_3 = \varepsilon_0$ (dissipation energy of small scale motion in non-Newtonian fluid is the same as that in Newtonian fluid) and $\alpha_{1,11} > 0$. In addition, the viscosity is assumed to be the same for the Newtonian, second-order, and third-order fluids. The assumption of equal viscosities is not critical to the development and can be easily done away with. However, the idea here is to keep the illustration as simple as possible.

Even though these results are derived from a simple dimensional argument, their consequences are interesting: the Kolmogorov microscale appears to be the same for both Newtonian and second-order fluids. However, for the third-order fluid, the time and length scales of the smallest eddies become larger than those of the Newtonian fluid. These characteristics are common to most of the constitutive relationships of non-Newtonian fluid. The results obtained in this study provided a complementary view of the molecular theory [4].

NOMENCLATURE

D/Dt : material time derivative (Stokes derivative)
$\mathscr{D}/\mathscr{D}t$: corotational (or Jaumann) derivative
t : time
v : fluid velocity
v : microscale of velocity

Greek Letters

α_2, α_{11}: coefficients in 2nd-order fluid model
α_3, α_{12}, $\alpha_{1,11}$: coefficients in 3rd-order fluid model
ρ : density
ν : kinematic viscosity
μ : viscosity
$\underset{\approx}{\dot{\gamma}}$: rate of strain tensor
$\tilde{\varepsilon}$: energy dissipation of small scale motion
θ : microscale of time
$\underset{\approx}{\dot{\gamma}}^*$: fluctuating strain rate tensor
$\underset{\approx}{\omega}$: vorticity tensor
ζ : microscale of length
$\tilde{\tau}^*$: fluctuating stress tensor
$\underset{\approx}{\tau}$: stress tensor

Superscript

T : Transpose

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