J. Astron. Space Sci., Vol. 7, No. 2, 125-139(1990)

MASS TO LUMINOSITY RATIOS OF SOME CLUSTERS IN THE LARGE MAGELLANIC CLOUD

Young-Jong Sohn and Mun-Suk Chun
Department of Astronomy and Atmospheric Science
Yonsei University
(Received November 5, 1990 ; Accepted November 22, 1990)

Abstract

Luminosity profiles and dynamical parameters of 12 globular clusters in the Large Magellanic Cloud(SB(s)m) are obtained from the concentric aperture photoelectric photometry of 3 different aged clusters and the collected photometric data of 9 clusters. The total masses of the globular clusters are calculated using the equation $M_{\oplus} = \widetilde{M}\widetilde{r}_{t}^{3}(4\widetilde{\Omega}^{2}-\widetilde{k}^{2})$, which is derived from the theoretical rotation curve for the exponential disk(Chun 1978). These masses lie between 0.3×10^{4} and 15.8×10^{4} $M_{\odot}$. From the determined total mass and luminosity, the mass-to-luminosity ratios are also derived. The M/L ratio of a cluster increases with the cluster age ; about 0.03 for the youngest clusters(SWB I) and about 0.24 for the oldest clusters(SWB VII). There is a difference in M/L by a factor of 10 between the galactic globular clusters and the old globular clusters in the LMC.

1. Introduction

The globular cluster system of the LMC differs markedly from that of the Galaxy. The LMC contains globular clusters with a wide range of ages between about 5×10^{6} and 10^{10} years(Frenk and Fall 1982, Hodge 1982). While in the Galaxy, they are all old objects with ages of about 10^{10} years. Therefore, the globular cluster of the LMC provide useful probes of the formation history of the galaxy.

According to the study of the globular clusters in the LMC(Freeman 1974, Chun 1978, Elson and Freeman 1985, Kontizas *et al.* 1987, Metaxa *et al.* 1988), there is a systematic difference between the globular clusters in the Galaxy and those in the LMC. The typical mass of the clusters in the LMC(less than 10^5 $M_\odot$) is smaller than the typical mass of the globular clusters in the Galaxy(greater than 10^5 $M_\odot$). And the mass-to-luminosity ratios(M/L) of the clusters in the LMC(0.1∼0.3) are also smaller than those of the galactic globular clusters with a mean value of 1.6(Illingworth 1976). A kinematical study of the globular clusters in the LMC by Freeman *et al.*(1983) has revealed the results that globular clusters in the LMC were found to be disk population and that there is no evidence for a kinematic halo population among the globular clusters in the LMC.

The aim of this paper is (i) to provide luminosity profiles, masses, and M/L for 3 globular clusters in the LMC, which have different ages each other : NGC1783(3×10^9 years), NGC1818 (2×10^7 years), and NGC 2210($\sim10^{10}$ years), and (ii) to verify whether the difference in M/L between globular clusters of the LMC and those of the Galaxy is real or not.

The procedure for deriving masses is to assume that the clusters are in circular orbits about the rotation center of the LMC. The tidal radius(r_t) of the clusters then depend on the LMC potential field through the relation(Chun 1978, Kontizas *et al.* 1987),

$$M_\oplus = (1/G)\, r_t^3\, (4\Omega^2 - k^2) \qquad (1)$$

where $M_\oplus$ is the mass of globular cluster, Ω and k are the angular velocity and epicyclic frequency at the cluster's location.

2. Observation

Photoelectric observation were made at Siding Observatory using 24-inch telescope. The scale in the focal plane of the telescope is 18.8 arcsec/㎜. The photometer is a single channel system with a dry ice cooled 1P21 photomultiplier and standard UBV filter/aperture box. The observations of 3 globular clusters were made through concentric aperture photometry with 7 diameters (0'.25, 0'.50, 0'.62, 0'.99, 1'.51, 2'.26, and 2'.97 respectiively).

The results of concentric aperture measures are summerized in Table 1. The columns give (i) the NGC number of the clusters ; (ii) the aperture diameter in arcmin. ; (iii) the V magnitude measured through each aperture ; (iv) the (B-V) color : and (v) the (U-B) color. The probable errors for V magnitude, (B-V), and (U-B) color are ±0.02, ±0.03, and ±0.07 magnitudes respectively.

Table 1. The results of concentric aperture measures for 3 clusters in the LMC

NGC	D(')	V	(B-V)	(U-B)
1783	0.25	13.58	0.75	0.29
	0.50	12.10	0.66	0.24
	0.62	11.71	0.61	0.23
	0.99	11.01	0.62	0.22
	1.51	10.52	0.65	0.23
	2.26	10.22	0.65	0.22
	2.97	10.13	0.65	0.22
1818	0.25	11.59	0.14	−0.45
	0.50	10.40	0.14	−0.43
	0.62	10.16	0.14	−0.44
	0.99	9.85	0.15	−0.43
	1.51	9.63	0.14	−0.44
	2.26	9.43	0.13	−0.44
	2.97	9.40	0.13	−0.44
2210	0.25	12.67	0.65	0.10
	0.50	11.37	0.63	0.10
	0.62	11.08	0.67	0.11
	0.99	10.81	0.71	0.15
	1.51	10.65	0.70	0.13
	2.26	10.46	0.69	0.11
	2.97	10.36	0.69	0.11

3. Surface Brightness Distribution

From the concentric aperture measures in Table 1 and the calculated areas for each annuls, the observed mean surface brightness f_{obs}(mag/□") was evaluated for a central circle and six annuli. A convenient point in the annulus is the radius bisecting the area of the annulus, so-called the effective radius(r_e), given by

$$r_e = [0.5(r_1^2 + r_2^2)]^{1/2} \quad \cdots\cdots (2)$$

where r_1 and r_2 are the inner and outer radii of the annulus, respectively. The correction required to give surface brightness on the effective radius($f(r_e)$) from f_{obs} was given by the following relation(Illingworth and Illingworth 1976) as,

$$f(r_e)=f_{obs}\frac{2(x_2-x_1)}{1+x_1+x_2}\left[\ell n(\frac{1+x_2}{1+x_1})\right]^{-1} \quad \cdots\cdots(3)$$

where $x_i=(r_i/r_e)$, i=1, 2.

Table 2 gives the surface brightness distributions of clusters from the aperture measures. The columns give (i) the NGC number for each globular cluster ; (ii) $\log(r_e)$, the logarithmic effective radius in arcmin. ; (iii) log f_{obs} the logarithmic observed mean surface brightness for each annulus; (iv) δ, the logarithm of the corrrection between f_{obs} and $f(r_e)$; (v) $\log f(r_e)$, the logarithmic value of the local surface brightness at r_e in units of V=10.00mag./□" : and (vi) mean probable errors in log $f(r_e)$.

Table 2. Surface brightness distributon of 3 clusters in the LMC

NGC	$\log(r_e)$	$\log f_{obs}$	δ	$\log f(r_e)$	p.e.
1783	−0.987	−3.679	−0.005	−3.684	0.024
	−0.704	−3.693	−0.016	−3.709	0.040
	−0.550	−3.784	−0.003	−3.787	0.060
	−0.384	−3.954	−0.021	−3.975	0.020
	−0.195	−4.213	−0.021	−4.234	0.016
	−0.017	−4.608	−0.021	−4.629	0.016
	0.120	−5.173	−0.010	−5.183	0.032
1818	−0.987	−2.883	−0.040	−2.923	0.080
	−0.704	−3.061	−0.041	−3.101	0.044
	−0.550	−3.346	−0.005	−3.351	0.040
	−0.384	−3.711	−0.027	−3.798	0.012
	−0.195	−4.154	−0.024	−4.178	0.008
	−0.017	−4.449	−0.022	−4.471	0.004
	0.120	−5.326	−0.011	−5.337	0.004
2210	−0.987	−3.315	−0.016	−3.331	0.048
	−0.794	−3.429	−0.024	−3.453	0.080
	−0.550	−3.642	−0.012	−3.654	0.076
	−0.384	−4.190	−0.017	−4.207	0.004
	−0.195	−4.715	−0.024	−4.739	0.004
	−0.017	−4.882	−0.023	−4.965	0.004
	0.120	−5.221	−0.014	−5.235	0.004

The results of the surface brightness for the clusters were plotted in Figure la, b, c. The theoretical surface density distribution curves(King 1966) were fitted to obtaine the length pa-

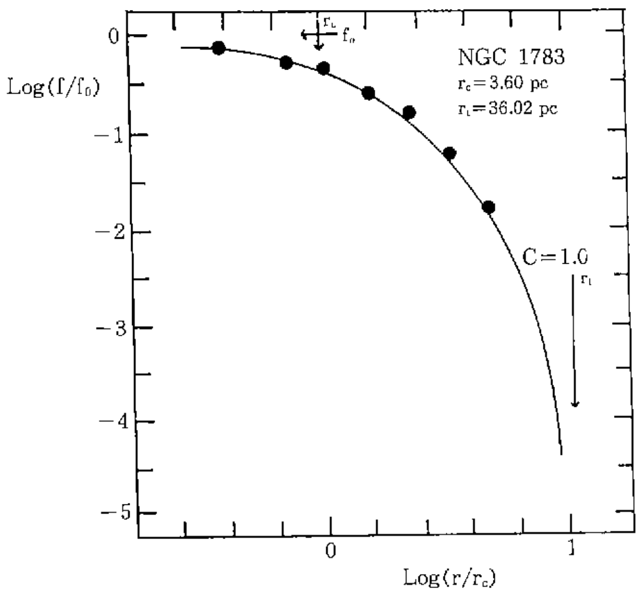

Figure 1a. Surface brightness profile of the NGC 1783.

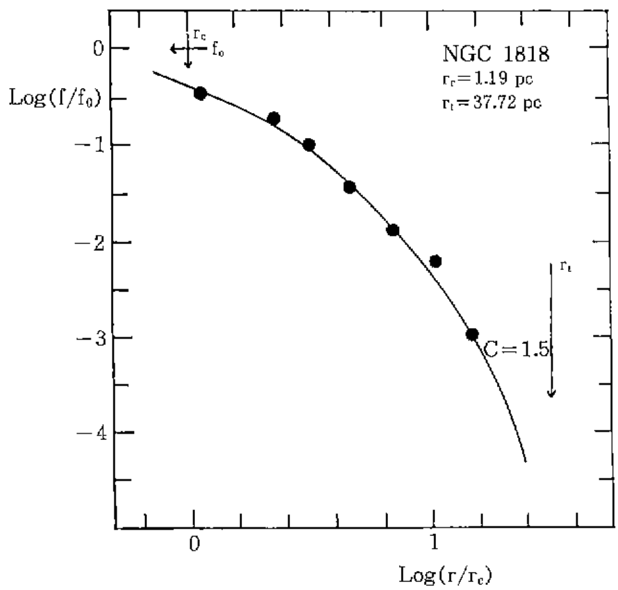

Figure 1b. Surface brightness profile of the NGC 1818.

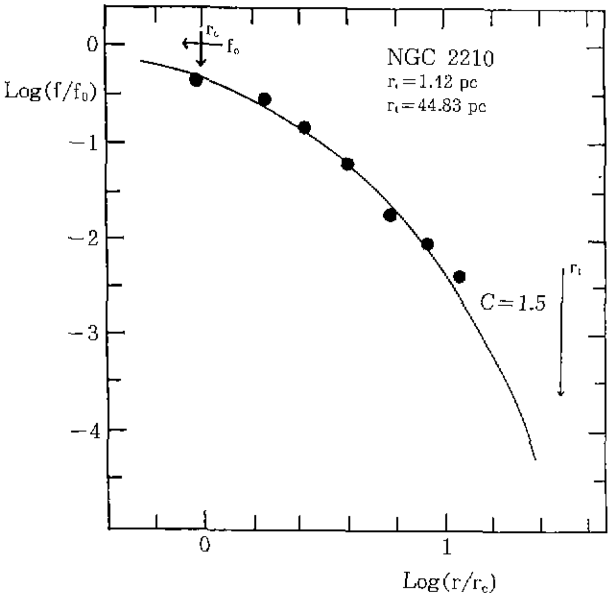

Figure 1c. Surface brightness profile of the NGC 2210.

rameters which we need to calculate the concentric parameters $c=\log(r_t/r_c)$ and the masses of the globular clusters. We used the distance of the LMC as 46 kpc adopted from the Table 13 of Sohn(1987). Table 3 gives the determined concentric parameters (c) and the length parameters (r_t, r_c) for the clusters.

Concentric aperture photometry data for additional 9 clusters in the LMC were available from Bernad and Bigay(1974) and Gordon and Kron(1983). The surface brightness distribution for these clusters were also derived using the same method described above. Table 3 also gives the 3 dynamical parameters (c, r_t, r_c) for the clusters.

4. *Masses and Luminosities*

(a) Masses

From the theoretical rotation curve for the exponential disk(Freeman 1970), the cluster masses($M_\odot$) can be calculated using the following relation,

Table 3. The concentric parameters and the length parameters for the clusters in the LMC

NGC	$\log(r_t/r_c)$	r_c(arcsec)	r_c(pc)	r_t(arcmin)	r_t(pc)
1783	1.0	16.1	3.60	2.96	36.0
1818	1.5	5.3	1.19	2.82	37.7
2210	1.5	6.4	1.42	3.35	44.8
1755[1]	0.75	13.6	3.02	1.27	17.0
2004[1]	1.5	4.4	0.99	2.34	31.3
2019[1]	1.5	4.0	0.88	2.09	27.9
1856[2]	1.3	6.7	1.50	2.24	29.9
1866[2]	1.3	10.7	2.38	3.55	47.5
1916[2]	1.8	3.9	1.84	3.98	53.3
1978[2]	0.8	28.1	6.28	2.96	39.6
2107[2]	1.3	4.3	0.95	1.42	18.9
2134[2]	1.3	5.0	1.12	1.68	35.5

1 : Results were determined from the data of Bernard and Bigay(1974)

2 : Results were determined from the data of Gordon and Kron(1983)

$$M_{\mathfrak{D}} = (1/G)\ r_t^{\,3}\ (4\Omega^2 - k^2),$$

where Ω(R) and κ(R) are the angular velocity and epicyclic frequency,

$$k^2 = 4\Omega^2\{1 + R/(2\Omega)\ (d\Omega/dR)\} \quad \cdots\cdots (4)$$

at radius R in the disk of a galaxy. This relation requires the assumption that the cluster has circular orbit and lies in the plane of the parent galaxy. The theoretical rotation curve gives the dimensionless quantities $\tilde{\Omega}(\tilde{R})$, $\tilde{k}(\tilde{R})$, where $\tilde{\Omega}=\Omega/(GM\alpha^3)^{1/2}$ and $\tilde{k}=k/(GM\alpha^3)^{1/2}$. M is the mass of the exponential disk and $\tilde{R}(=\alpha R)$ is the dimensionless radius. Here α is the length scale derived from the surface photometry. We used $\alpha=0.010$/arcmin. from de Vaucouleurs(1960). Then we obtained the mass of exponential disk (M) is $5\times10^9\ M_{\odot}$. Now if we write $\tilde{r}_t=\alpha r_t$, then

$$\tilde{M}_{\mathfrak{D}} = \tilde{M}\ \tilde{r}_t^{\,3}(4\tilde{\Omega}^2 - \tilde{k}^2) \quad \cdots\cdots (5)$$

Generally, the center of rotation is slightly different from the optical center of the Magellanic type irregular galaxies (de Vaucouleurs and Freeman 1973). Here we used the rotatioin center of the LMC(Freeman *et al.* 1983) as

R.A.$=5^h\ 20^m .9$
Dec$=-69°\ 17' .6$ (1975)

Now we must estimate the radial distance(R) of each cluster from the rotation center of the LMC. We used the position of the clusters given by Sulentic and Tifft(1973). Since R is the corrected value for the projection effect, we need the inclination(i) and position angle(p.a.) of the line of node for the LMC. Table 4 is the collected inclinations and position angles of the LMC. We adopted the mean value i$=27°$ and p.a.$=171°$ for the LMC plane.

Table 4. Collected inclinations and position angles of the LMC

authors	methods	i(°)	p.a.(°)
de Vaucouleurs(1957)	Red isophotes	27±3	170±5
Westerlund(1964)	Outlying clusters	45	187±10
McGee and Milton(1966)	HI density	29±9	171
Feitzinger *et al.*(1966)	R and B isophotes	33±3	168±4
Gascoign and Shobbrook(1979)	Cepheid	27±10	(170)
de Vaucouleurs(1980a)	Cepheid	27±2	(171)
	adopted	27	171

Finally, we need the function $4\Omega^2-k^2$ for the exponential disk. Figure 2 shows the run of $4\tilde{\Omega}^2-\tilde{k}^2$ with $\tilde{R}$ calculated from the theoritical rotation curve for the exponential disk. Table 5 gives the resulting masses of the clusters in the LMC. The columns give, (i) the NGC number of the clusters ; (ii) R, radial distance of clusters in kpc ; (iii) $4\tilde{\Omega}^2-\tilde{k}^2$ for each cluster ; (iv) r_t, tidal radius of the clusters in pc ; (v) M, the determined masses in solar mass unit ($M_\odot$). If the very old clusters in the LMC have not circular orbit, we can derive an upper limit on their masses($M_{\oplus lim}$)

$$M_{\oplus lim}=0.50\ M\ r_t^3 \quad \cdots\cdots(6)$$

Because the luminosity profile of the LMC is very closely exponential and the quantity $4\tilde{\Omega}^2-\tilde{k}^2$ is less than 0.5 everywhere on the exponential disk(Figure 2), we can use the equation (6) for the upper limit of cluster masses.

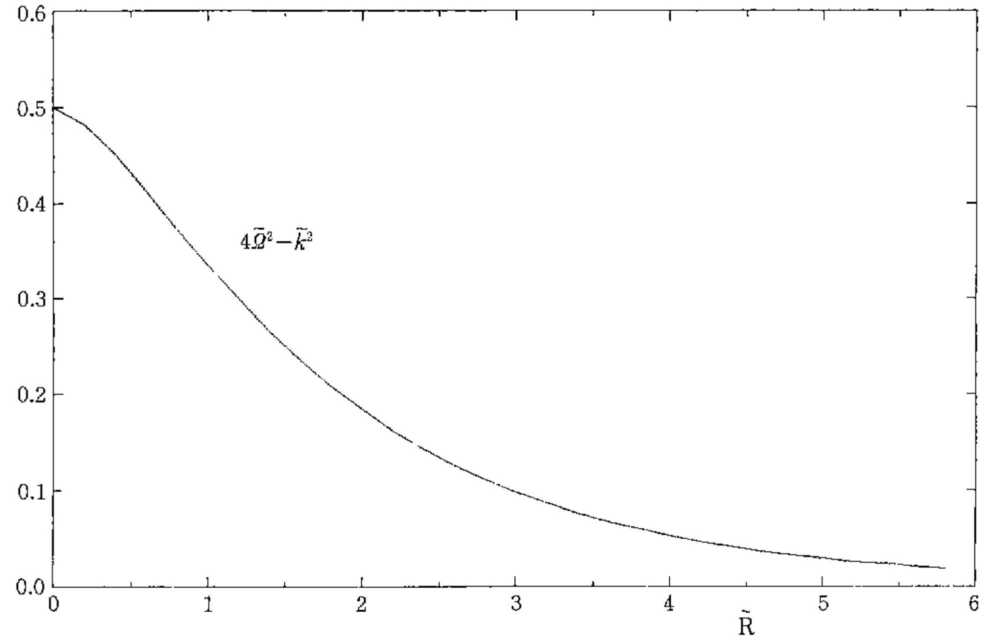

Figure 2. The dimensionless quantity $4\tilde{\Omega}^2-\tilde{k}^2$ for the exponential disk from the theoretical rotation curve(Freeman 1970).

Table 5. Masses of clusters in the LMC

NGC	R(kpc)	$4\tilde{\Omega}^2-\tilde{k}^2$	r_t(pc)	M(10^4M⊙)	
1783	3.260	0.1399	36.0	1.4	
1818	2.714	0.1808	37.7	2.0	
2210	4.055	0.0962	44.8	1.8	9.4*
1755	2.296	0.2194	17.0	0.3	
2004	1.839	0.2700	31.3	1.7	
2019	1.084	0.3705	27.9	1.7	2.3*
1856	0.923	0.3940	30.0	2.2	
1866	3.123	0.1492	47.5	3.3	
1916	0.281	0.4810	53.3	15.2	15.8*
1978	2.561	0.1941	39.6	3.5	
2107	1.967	0.2541	18.9	0.4	
2134	2.553	0.1948	25.5	0.7	

* : upper limit

(b) Total Luminosities

The total luminosities of the clusters were derived by extrapolating the observed surface brightness profiles. As a check on the zero point, we determined the luminosity within a 60 arcsec circle for 3 clusters to compare with the photoelectric V_{60} values given in van den Bergh and Hagen(1967). The difference was in no case lager than $m_v = 0.15$. Table 6 gives the total luminosities for the globular clusters in the LMC.

Table 6. Total luminosities for clusters in the LMC

NGC	V_{60}	V_t	E(B-V)	M_v	L(10^5Lo)
1783	10.95	9.79	0.02	−8.58	2.3
1818	9.83	9.39	0.25	−9.67	6.3
2210	10.80	10.35	0.08	−8.20	1.6
1755	—	0.83	0.02	−8.54	2.2
2004	—	9.90	0.30	−9.31	4.5
2019	—	10.94	0.06	−7.55	0.9
1856	—	9.59	0.08	−8.96	3.3
1866	—	8.98	0.06	−9.51	5.5
1916	—	9.88	0.06	−8.61	2.4
1978	—	9.81	0.17	−9.06	3.4
2107	—	11.02	0.10	−7.95	1.3
2134	—	10.51	0.02	−7.86	1.2

5. Age-Mass/Luminosity Relation

We can expect that the M/L for the young and old clusters will differ considerably because their stellar contents may be so different each other. If there is a clear systematic difference in the M/L between globular clusters in the Galaxy and those in the LMC, it will be very difficult to use them as secondary distance indicators.

(a) Ages

It is very difficult to calculate ages of the clusters in the LMC from their color magnitude diagrams. Because of the LMC distance, photometry down to the main sequence turn-off of the old clusters is very difficult. Therefore the age can only be estmated indirectly. Searle *at al.*(1980) have constructed two reddening free color indicies(Q(ugr), Q(vgr)) for the 61 clusters in the LMC. According to their Q-Q diagram, clusters in the LMC can be divided by 7 age groups(I , II ,···VII) one dimensionally from the youngest and metal rich(SWB I) to the oldest and metal poor clusters(SWB VII). Freeman *et al.*(1983) plotted SWB clusters in the (U-B)/(B-V) plane. They found the distribution of the clusters in their color-color plane is very similer to the Q-Q diagram of Searle *et al.*(1980). From the (U-B)/(B-V) plane for the clusters in the LMC, we determined the NGC 1783, NGC 1818 and NGC 2210 belong to SWB V, SWB I, and SWB VII respectively.

Frenk and Fall(1982) and Hodge(1983) determined the absolute range of the ages for the SWB groups. Table 7 gives the range of the ages for the SWB groups in 10^9 years unit.

Table 7. The range of the ages for the SWB groups(X 10^9yrs)

SWB Class	Frenk and Fall(1982)			Hodge(1983)		
I	0.005	–	0.03	0.007	–	0.018
II	0.03	–	0.1	0.013	–	0.04
III	0.1	–	1	0.03	–	0.3
IV	1	–	3	0.07	–	2
V	3	–	6	0.2	–	5
VI	5	–	10	0.7	–	9
VII	8	–	14	3	–	13

(b) Age-M/L Relation

The young and old clusters in the LMC have a distinct difference on their color-magnitude diagrams and on the integrated colors. Therefore the M/L ratios of the clusters contain information about stellar contents and hence conditions of formation and evolution of the LMC.

Table 8 gives M/L for the clusters with their corresponding SWB group. The last columns of Table 8 show the upper limits of M/L for the the oldest clusters in the LMC. The former M/L determination concerning clusters in the LMC using the same method in this paper are displayed in Table 9 with their SWB group. Now, we can see the calculated mean values of the M/L of clusters with respect to the SWB groups on the Table 10 and Figure 3. For the youngest clusters(SWB I) the mean M/L is about 0.03, and for the oldest clusters(SWB VII) about 0.24.

Some globular clusters in our Galaxy have been studied so far with King-Michie multi-mass anisotropic dynamical models(Gunn and Griffin 1979, Lupton *et al.* 1985, Pryor *et al.* 1986,

Table 8. Mass-to-Luminosity ratios for the clusters with their corresponding SWB group

NGC	SWB	M(10^4M⊙)	L(10^5L⊙)	M/L	
1783	V	1.4	2.3	0.06	
1818	I	2.0	6.3	0.03	
2210	VII	1.8	1.6	0.11	0.60*
1755	II	0.3	2.2	0.01	
2004	I	1.7	4.5	0.04	
2019	VII	1.7	0.9	0.19	0.25*
1856	IV	2.2	3.3	0.07	
1866	III	3.3	5.5	0.06	
1916	VII	15.2	2.4	0.63	0.66*
1978	VI	3.5	3.4	0.10	
2107	IV	0.4	1.3	0.03	
2134	III	0.7	1.2	0.06	

* : upper limit

Table 9. Collected M/Ls for several clusters in the LMC with their SWB group

NGC	SWB	M(10^4M⊙)	L(10^5L⊙)	M/L	References
1831	V	2		0.08	Freeman(1970)
1866	III	6		0.08	
1835	VII	4.5	2.1	0.2	Chun and
2210	VII	3.2	1.5	0.2	Freeman(1972)
1818	I	0.95		0.02	Freeman(1974)
2157	III	0.38		0.02	
1866	III	5.6		0.08	
1831	V	1.8		0.08	
1835	VII	4.5		0.2	
1818	I	3.5	5.4	0.06	Chun(1978)
1835	VII	4.4	3.7	0.12	
2157	III	1.1	3.5	0.03	
2210	VII	2.7	2.0	0.14	
1835	VII	7.3	4.02	0.18	Elson and
2210	VII	1.9	1.85	0.11	Freeman(1985)
2257	VII	3.7	0.66	0.56	

Table 10. Mean values of the M/L of clusters in the LMC with respect to the SWB group

SWB	I	II	III	IV	V	VI	VII
M/L	0.03	0.01	0.05	0.05	0.08	0.10	0.24
p.e.	±0.01		±0.02	±0.02	±0.01		±0.12

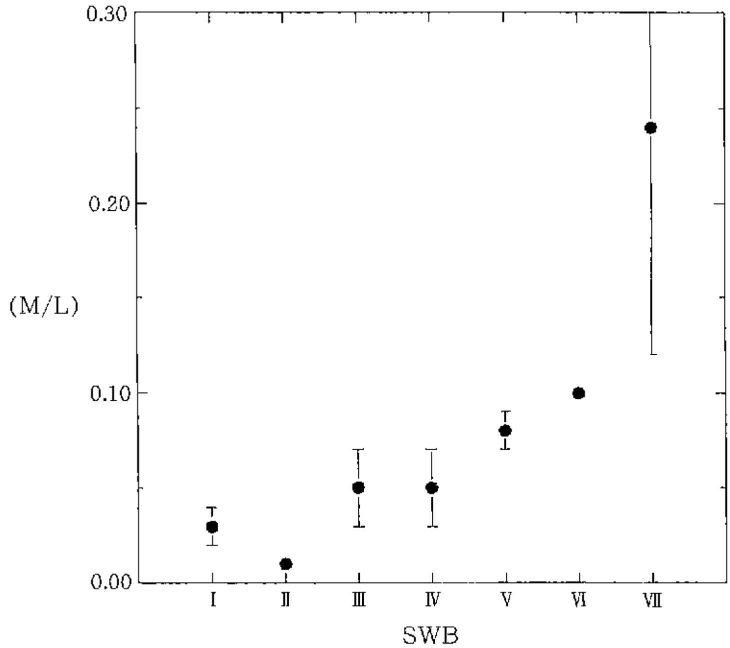

Figure 3. Mean values of the M/L of clusters in the LMC with respect to the SWB group.

Lupton *et al.* 1987, and Meylan 1987, 1988). According to these models, the total masses of the globular clusters in the Galaxy range from 0.4 to $1.1\times10^{6}M_{\odot}$ and M/Ls are located between about 2 and 3 in solar unit. Compared with the M/L for the oldest clusters in the LMC($\sim$0.24), there is a clear difference in M/L by nearly a factor of 10. If this systematic difference between the globular clusters in the LMC and those in the Galaxy is real, it is very dangerous to use them as a secondary distance indicators.

On the other hand, Meylan(1988) calculated the M/L for the oldest cluster NGC 1835 using the King-Michie multi-mass anisotropiic dynamical model with the observational constraints as the surface brightness profile and the central velocity dispersion. The results concerning the total mass of NGC 1835 were larger than the previous determinations from the method based on the assumtion of circular orbit and in the transformation of the tidal radius into mass. And the M/L for this old LMC globular clusters was similar to those obtained for galactic globular clusters.

If the difference of M/L comes from the different method to derive mass of the clusters, we need more observations for the LMC globular clusters with the same observational constraints, surface brightness profiles and central velocity dispersions, as galactic globular clusters.

6. Conclusion

The results of this paper are summerized below.

1. The young and old cluster in the LMC have luminosity profile that are well represented by King's(1966) relaxed models. Therefore, the young clusters in the LMC as well as the old clusters can be concerned dynamically well rellaxed system.
2. On the assumption that clusters in the LMC have a circular orbit and lie in the plane of the parent galaxy, we derived their masses in the range 0.3 to $15.8 \times 10^4 M_\odot$.
3. The M/L values for the clusters in the LMC are increasing with their ages. The mean M/L is about 0.03 for the youngest clusters (SWB I), and about 0.24 for the oldest clusters (SWB VII) in the LMC. There is a difference in M/L by nearly a factor of 10 between galactic globular clusters and the old globular clusters in the LMC.

We need more observations for the LMC globular clusters not only for the surface profiles but also the central velosity dispersions.

References

Bernard, A. and Bigay, J. H. 1974. *Astron. Astrophys.*, **33**, 123.

Chun, M. S. 1978, *Astron. J.*, **83**, 1062.

de Vaucouleurs, G. 1960. *Astrophys. J.*, **131**, 574.

de Vaucouleurs, G. and Freeman, K. C. 1973, *Vistas in Astronomy*, **14**, 265.

Elson, R. A. W. and Freeman, K. C. 1985, *Astrophys. J.*, **288**, 521.

Freeman, K. C. 1970, *Astrophys. J.*. **160**, 811.

Freeman, K. C. 1974, ESO/SRC/CERN *Conference on Research Programs for New Large Tele-*

scopes, Geneva, 21–31, May, 177.

Freeman, K. C. Illingworth, G. and Oemler, A. Jr. 1983, *Astrophys. J.*, **272**, 488.

Frenk, C. S. and Fall, S. H. 1982, *M.N.R.A.S.*, **199**, 565.

Gordon, K. C. and Kron, G. E. 1983, *Publ. A.S.P.*, **95**, 461.

Gunn, J. E. and Griffin, R. F. 1979, *Astron. J.*, **84**, 752.

Hodge, P.W. 1983, *Astrophys.J.*, **264**, 470.

Illingworth, G. 1976, *Astrophys. J.*, **204**, 73.

King, I. R. 1966, *Astron. J.*, 71, 276.

Kontizas, M., Chrysovergies, M. and Kontizas, E. 1987, *Astron. Astrophys. Suppl. Ser.*, **68**, 147.

Lupton, R., Gunn, J. E. and Griffin, R. F. 1985, *Dynamics of Star Clusters IAU Symp.*. **113**, ed. Goodman, J. and Hut, P., 19.

Lupton, R., Gunn, J. E. and Griffin, R. F. 1987, *Astron. J.*, **93**, 1114.

Metaxa, M., Kontizas, E. 1988, *Astron. Astrophys. Suppl. Ser.*, **73**, 373.

Meylan, G. 1987, *Astron. Astrophys.*, **184**, 144.

Meylan, G. 1988, *Astron. Astrophys.*, **191**, 215.

Pryor, C., McClure, R. D., Fletcher, J. M., Hartwick, F. D. A. and Kormendy, J. 1986, *Astron. J.*, **91**, 546.

Searle, L., Wilkinson, A. and Bagnuolo, W. G. 1980, *Astrophys. J.*, **239**, 803.

Sohn, Y. J. 1987, Yonsei Univ. Ms. thesis.

Sulentic, J. W. and Tifft, W. G. 1973, *The Revised New General Catalogue of Nonstellar Astronomical Object*(The Univ. of Arizona Press : tucson).

van den Bergh, S. and Hagen, G. L. 1968, *Astron. J.*, **73**, 569.