A Channel Flood Routing by the Analytical Diffusion Model

Yoon, Yong Nam* · Yoo, Chul Sang**

ABSTRACT/The analytical diffusion model is first formulated and its characteristics are critically reviewed. The flood events during the 1985–1986 flood seasons in the IHP Pyungchang Representative Basin are routed by this model and are compared with those routed by the kinematic wave model. The present model is proven to be an excellent means of taking the backwater effects due to lateral inflow or downstream river stage variations into consideration in channel routing of flood flows. It also requires much less effort and computing time at a desired station compared to any other reliable flood routing methods.

1. Introduction

The hydraulic flood routing is known as a method of computing runoffs by solving simultaneously the continuity and momentum equations for the unsteady open channel flows with appropriate initial and boundary conditions. It enables the runoff computations with high level of accuracies even for the channel reaches with backwater effects, which is not possible in the case of hydrologic routing. However, the method requires considerable amounts of channel section data within the routing reach as well as relatively long computer time in order to numerically solve the system of unsteady flow equations(Amein and Fang, 1970 ; Price, 1974 ; Amein and Chu, 1975 ; Morris and Woolhiser, 1980 ; Joliffe, 1984). Therefore, a necessity for a flood routing method arises which makes it possible to compute the discharge(or depth)variations with respect to time at a particular section of the channel reach.

In the present study an effort is made to apply the analytical diffusion model to route the flood

* Prof., Dept. of Civil Eng., Korea Univ.

** Research Assistant, Dept. of Civil Eng., Korea Univ.

hydrographs for the reach with due consideration to the backwater effects caused by the stage fluctuations and the lateral inflows from the tributaries. The characteristics of the analytical diffusion model is first reviewed and the model is then applied to the Janpyung–Banglim reach of the Pyungchang River which has been operated as a Representative Basin under the International Hydrological Program. For the purpose of comparison, the analytical solution based on the kinematic wave approximation is also obtained and compared with that by the present method as well as with the observed.

2. Theoretical Background

2.1 Development of Analytical Diffusion Model

The complete Saint Venant equations for unsteady open channel flows, the so–called dynamic wave model, can be expressed as follows :

$$V\frac{\partial A}{\partial x}+\frac{\partial V}{\partial x}+\frac{\partial A}{\partial t}=Q_L \tag{1}$$

$$S_f=S_o-\frac{\partial y}{\partial x}-\frac{V}{g}\frac{\partial V}{\partial x}-\frac{1}{g}\frac{\partial V}{\partial t}+\frac{Q_L(V-U_l)}{gA} \tag{2}$$

where x denotes the distance from the upper boundary along the flow direction ; V, the mean flow velocity ; A, the corss sectional area ; t, the time ; Q_L, the lateral inflow rate per unit width of the channel ; U_1, the longitudinal velocity component of the lateral inflow ; S_f, the friction slope ; S_o, the channel slope: y, the flow depth and g, the gravitational acceleration.

Because of the complexities involved in solving the complete Saint Venant equations, simplifications to the momentum equation(Eq.2) are often made(Ponce and Simons, 1978 ; Vieira, 1983 ; Tingsanchali and Manandhar, 1985).

In the kinematic wave model the terms representing acceleration and pressure are assumed to be negligible compared to the friction and gravity terms. And, in the diffusion wave model the pressure term is considered along with the friction and gravity terms.(Akan and Yen, 1981 ; Anderson and Burt, 1985 ; Chow, Maidement and Mays, 1988 ; Hromadka and Devries, 1988).

According to the basic assumptions and the procedure for the model formulation, three categories of analytical diffusion models have been so far proposed. Hayami(1951) developed the following equation for the unsteady flow in a rectagular channel without lateral inflow incorporating the diffusivity, k, to take care of channel irregularities(Thomas and Wormleaton, 1970 ; Gonwa and Kavvas, 1986).

$$\frac{\partial y}{\partial t}+C_w\frac{\partial y}{\partial x}=K\frac{\partial^2 y}{\partial x^2} \tag{3}$$

where

$$K=\frac{y}{2}\frac{V}{(S_o-\frac{\partial y}{\partial x})}+k$$

$$C_w=\frac{3}{2}V$$

Eq.3 is a linearized partial differential equation of diffusion–type, but it can not take care of the backwater effects caused in the channel networks.

Sutherland and Barnett(1972) derived a more general diffusion wave equation with due consideration to the lateral inflow and channel irregularities.

$$\frac{\partial y}{\partial t}+C_o\frac{\partial y}{\partial x}=K_o\frac{\partial^2 y}{\partial x^2} \tag{4}$$

where

$$C_o=(1+\frac{m}{k})V$$

$$K_o=(jVy\cos\phi)/(k\cdot S_f)$$

where m and j are constants depending on the flow characteristics, and ϕ is the channel slope angle. However, this equation is applicable only for a triangular or a wide–rectangular channel and can not take the backwater effects into consideration.

2.2 Derivation of Analytical Diffusion Model

Recently, Tingsanchali and Manandhar(1985) developed analytical procedure to solve Eq. 3 assuming C_w, K as constants, and taking backwater effects and boundary conditions. Including the lateral inflow Q_L, Eq.3 can be expressed as

$$\frac{\partial y}{\partial t}+\frac{3}{2}V\frac{\partial y}{\partial x}=[k+\frac{y\cdot V}{2(S_o-\frac{\partial y}{\partial x})}]\frac{\partial^2 y}{\partial t^2}+Q_L \tag{5}$$

where

$$V=C[y(S_o-\frac{\partial y}{\partial x})]^{\frac{1}{2}}$$

where C represents the Chezy roughness coefficient.

Uniform depth H in the channel is considered as the initial condition, i., e., x,0) = H

boundary conditions for $t>0$ are taken as $y(0, t)=H+U(t)$ and $y(\ell, t)=H+D(t)$, in which U(t) and D(t) are the water level variations above the initial depth H at the upstream and downstream ends ; and ℓ is the length of the channel reach.

The tributaries and distributaries are treated as the sources and sinks distributed along the main channel. The contribution of lateral flows to momentum along the main channel have been neglected. The preceding nonlinear diffusion equation is linearized to obtain an analytical solution by expressing the solution in the functional series(Hayami ; 1951) as in the following:

$$y(x,t)=(H+h)[1+\frac{\phi_0}{(H+h)}+\frac{\phi'_0}{(H+h)}+\cdots] \qquad (6)$$

in which h is the average height of the water level above H.
Let $\phi_o(x,t) = \phi(x,t) + \phi_3(x,t)$ and then by substituting Eq. 6 into Eq. 5, and taking the first approximate solution, it yields:

$$y(x,t)=H+h+\phi(x,t)+\phi_3(x,t) \qquad (7)$$

in which $\phi(x,t)$ is the solution of

$$\frac{\partial\phi}{\partial t}+W\frac{\partial\phi}{\partial x}=\mu\frac{\partial^2\phi}{\partial x^2} \qquad (8)$$

for the boundary conditions

$$\phi(o,t)=U(t)-k \qquad (8-a)$$
$$\phi(\ell,t)=D(t)-k$$

and the initial conditon

$$\phi(x,o)=-k \qquad (8-b)$$

and $\phi_3(x,t)$ is the solution of the equation

$$\frac{\partial\phi_3}{\partial t}+W\frac{\partial\phi_3}{\partial t}=\mu\frac{\partial^2\phi_3}{\partial x^2}+Q_L \qquad (9)$$

for the conditions

$$\phi_3(0,t)=0 \qquad (9-a)$$
$$\phi_3(0,t)=0 \qquad (9-b)$$
$$\phi_3(x,0)=0$$

$$\mu=k+(H+k)\frac{y_o}{2S_o}$$

$$W=\frac{3}{2}V_o$$

$$V_o=C[(H+h)S_o]^{\frac{1}{2}}$$

Using a Laplace transformation technique, the solution of Eq. 8 for the corresponding conditions can be obtained as(Waston, 1981 ; Manandhar, 1982 ; Tingsanchali and Manadhar, 1985)

$$\phi(x,t)=\phi_1(x,t)+\phi_2(x,t)-h \tag{10}$$

$$\phi_1(x,t)=u_1R_1(x,t)+\sum_{j=1}^{m1-1}R_1(x,t-j)(u_{j+1}-u_j) \tag{11}$$

$$\phi_2(x,t)=d_1R_2(x,t)+\sum_{j=1}^{m2-1}R_2(x,t-j)(d_{j+1}-d_j) \tag{12}$$

where

$$R_1(x,t)=EXP(\frac{wx}{2\mu})\cdot B(x,t) \tag{11-1}$$

$$R_2(x,t)=EXP(-\frac{wx'}{2\mu})\cdot B(x',t) \tag{12-1}$$

in which

$$B(p,t)=\sum_{n=0}^{\infty}\{S(P,\ \frac{2n\ell}{P}+1,\ t)-S[P,\ \frac{2(n+1)\ell}{P}-1,\ t]\}$$

$$S(P,\ b,\ t)=EXP(-\frac{wPb}{2\mu})-\frac{2b}{\sqrt{\pi}}\int_{o}^{P/2\sqrt{\mu t}}EXP(-b^2\xi^2-\frac{w^2P^2}{16\mu^2\xi^2})d\xi$$

In the above equations, $x' = \ell - x$ represents the distance measured from the downstream end in the upstream direction.

When the water level variations at the upstream and downstream ends, U(t) and D(t), are approximated by histograms, in which u_j and d_j are the changes in stepped water levels at the upstream and downstream ends and m_1, m_2 are the corresponding flood durations.

The dimensionless coefficient, $R_1(x,t)$ in Eq. 11–1 represents the upstream effect at a location, x, due to a unit rise in water level at the upstream end of a long channel in which no lateral flow exists ; and the flow at the downstream end is uniform with the depth of H.

The same description applies for the dimensionless downstream effect, $R_2(x,t)$ for a unit rise at the downstream end when the upstream end is uniform with the depth, H. Using a similar approach for the solution of Eq. 9 with corresponding conditions yields.

$$\phi_3(x,t)=q_{L1}\cdot R_3(x,t)+\sum_{j=1}^{m3-1}R_3(x,t-j)(q_{Lj+1}-q_{Lj}) \tag{13}$$

in which

$$R_3(x,t)=\frac{EXP(\frac{wx}{2\mu})}{2\mu\sqrt{\pi}}\int_0^{\sqrt{\pi t}}[EXP(-\frac{w^2P^2}{16\mu^2})\times\int_{x1}^{x2}I(x,\ \xi,\ P)d\xi]dP$$

$$I(x,\ \xi,\ P)=EXP(-\frac{w\xi}{2\mu})\sum_{n=0}^{\infty}\{I'(2n\ell+x-\xi,\ P)-I'(2n\ell+x+\xi,\ P)+I'\langle 2(n+1)\ell-x+\xi,\ P\rangle-I'\langle 2(n+1)\ell-x-\xi,\ P\rangle\}$$

$$I'(b,P)=EXP(-\frac{b^2}{P^2})$$

Where q_{Lj} is the stepped lateral dischare per unit width of the tributary and per unit width of the main channel, over its initial value, during the time period$(j-1) < t \leq j$; x_1 and x_2 are the positions of two banks of the tributary ; and m_3 is the duration of lateral dischage variation.

The coefficient $R_3(x,t)$ has a dimension of time and represents the effect of lateral folw at the location, x, due to a unit rise in the lateral discharge per unit width of the tributary and per unit width of the main channel, when the two ends of the channel are uniform according to the given initial condition.

Summarizing, the water level at a location x at the time t, with the effects of uptream–downstream boundaries and the lateral inflows taken into consideration, can be represented as follows ;

$$y(x,t)=H+\phi_1(x,t)+\phi_2(x,t)+\phi_3(x,t) \tag{14}$$

2.3 Response Characteristics of $\Phi_1(x,t)$, $\Phi_2(x,t)$ $\Phi_3(x,t)$

To analyze the response characteristics of $\phi_1(x,t)$, $\phi_2(x,t)$ and $\phi_3(x,t)$ which consist the solution of diffusion equation(Eq. 5) the variations of dimensionless coefficients $R_1(x,t)$, $R_2(x,t)$, $R_3(x,t)$ are first investigated for assumed values of W and μ . For a hypothetical reach length $\ell = 50$ ㎞ with the tributary joining at x = 25 ㎞ W = 1 m/ sec, μ = 10,000 ㎡/ sec are assumed and the variations of $R_j(x,t)$ at the locations x = 10, 20, 30 and 40 ㎞ are computed as shown in Fig. 2.1.

The variations of ϕ_i (x,t) for the same conditions are computed as shown in Fig.2.2 under the assumption that an increase of 1m per unit time occurs in the upstream, downstream and tributary water levels.

3. Data

The river reach taken in the present study is the Jangpyung–Bamlim reach of the Pyungchang River Basin as shown in Fig. 3.1. This reach has been properly gauged by five stage gauges so that the applicability of analytical diffusion model can fully be investigated in view of the downstream water level variation and the backwater effects caused by the lateral inflows from

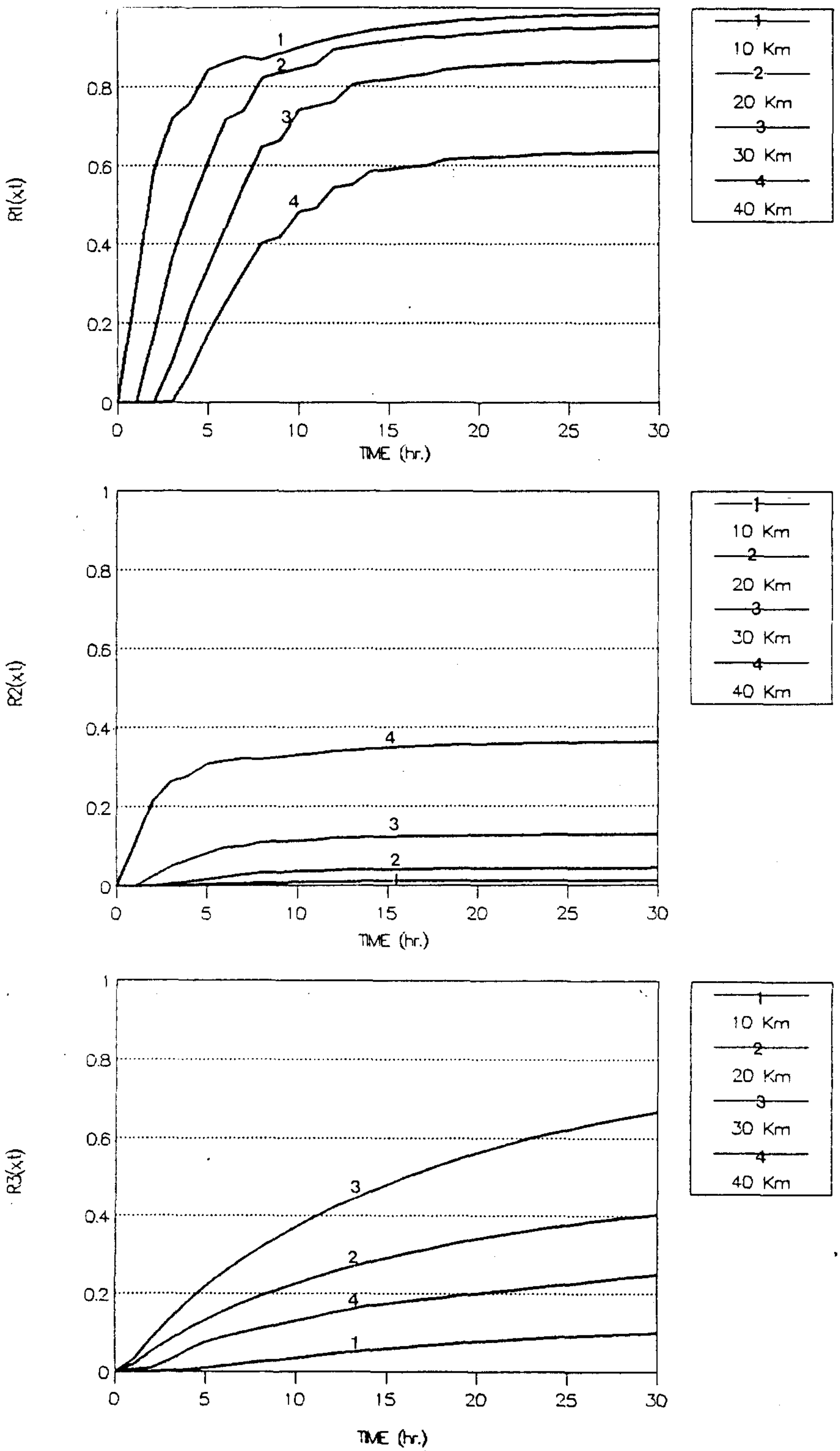

Fig. 2.1 Variations of the coefficients $R_1(x,t)$, $R_2(x,t)$, and $R_3(x,t)$ with time

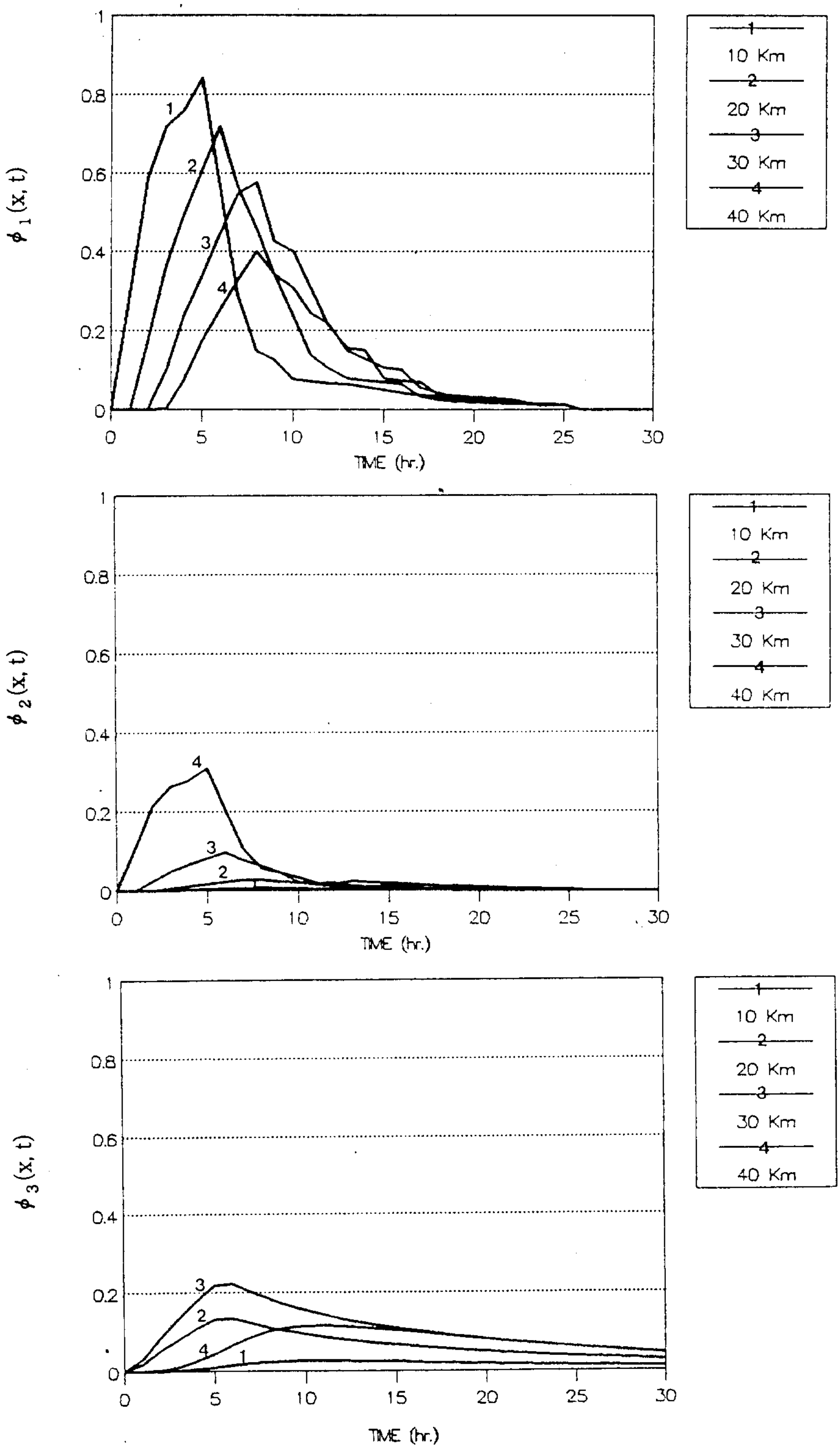

Fig. 2.2 Variations of the functions $\phi_1(x,t)$, $\phi_2(x,t)$ and $\phi_3(x,t)$ with time

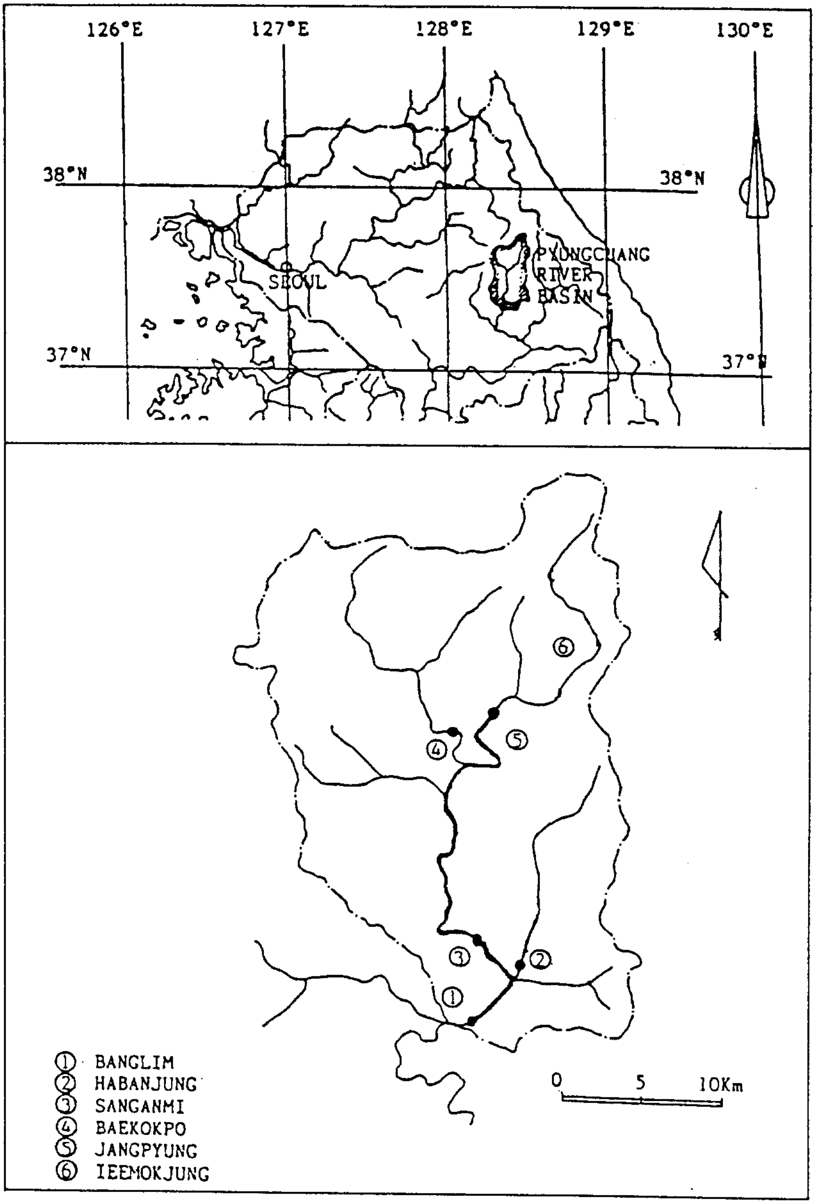

Fig. 3.1 Location map of the flood routing reach

tributaries. The Jangpyung, Sanganmi and Banglim stage gauging stations are on the main reach and the Backokpo, Habanjung stations are on the tributaries.

The water level data at the five stage gauging stations have been well recorded by automatic recording stage gauges since 1983 and the flood level records of Aug. 9–31, 1986 are taken for the present analysis. The basic channel characteristics data required in the present model, the average cross section and the bed slope, are obtained from the report on the IHP Representative Basins in Korea(1987).

4. Model Application to Natural Channel

4.1 Determination of Average Initial Depth and Channel Characteristic Parameters

The average initial depth(H) is considered as the relatively constant depth along the reach maintained prior to the flood and hence, taken as the steady gradually varied flow which prevailed right before flood arrival at the upstream end.

The channel characteristic parameters, the Chezy's C and diffusivity k, are determined through the process of model calibrations using the flood events of June 23–31, 1986. The diffusivity k was first set to zero and the value of C was consecutively assumed to fit the computed stage hydrograh by the model with the observed. The result showed that the C values of 130–210 gave reasonable agreements between the computed and observed hydrographs. Hence, C was assumed as 150. With C = 150, the diffusivity k was varied so that the best fit of the computed hydrograph with the observed could be obtained. For k values of 11,000–13,000 a fair agreement was attainable and hence k = 12,000 was adopted, and with it the appropriateness of C = 150 was again checked. The final values of C and k so determined are 150 and 12,000, respectively.

4.2 Results by the Analytical Diffusion Model

With the parameter value of C = 150, k = 12,000 the flood event of July 9–31, 1986 was routed through the reach with the conditions of lateral inflows, the upstream, and the downstream as shown in Fig. 4.1.

The routed hydrograph by the present model is compared with the observed in Fig. 4.2. As can

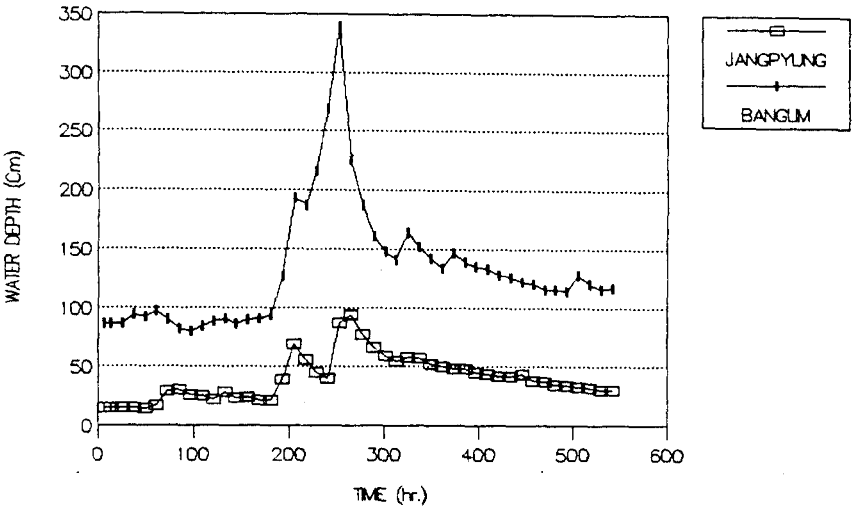

Fig. 4.1 Boundary conditions for the lateral inflows, the upstream and downstream ends(flood event of July 9–31, 1986)

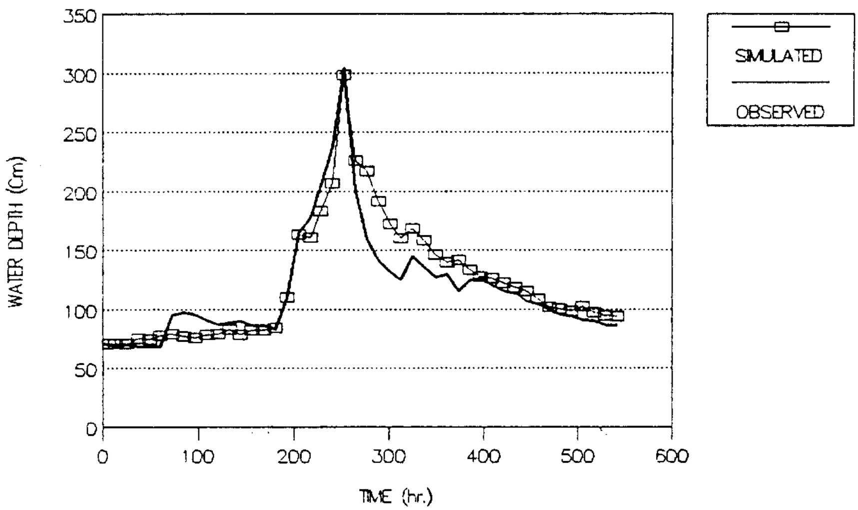

Fig. 4.2 Comparison of the simulated and the observed hydrographs(Flood events of July 9–31, 1986)

be seen in Fig. 4.2 the model assumes a good agreement both in the overall shape and the peak of the hydrographs.

4.3 Comparison with the Results by the Kinematic Wave Model

The flood routing result by the kinematic wave model(Li, Simons and Stevens, 1975) is compared with that by the analytical diffusion model and with the observed for the flood event of July 9–31, 1986 as shown in Fig. 4.3. It can be seen in Fig. 4.3 that the outflow hydrograph computed by the analytical diffusion model fits the observed much better than that by the kinematic wave

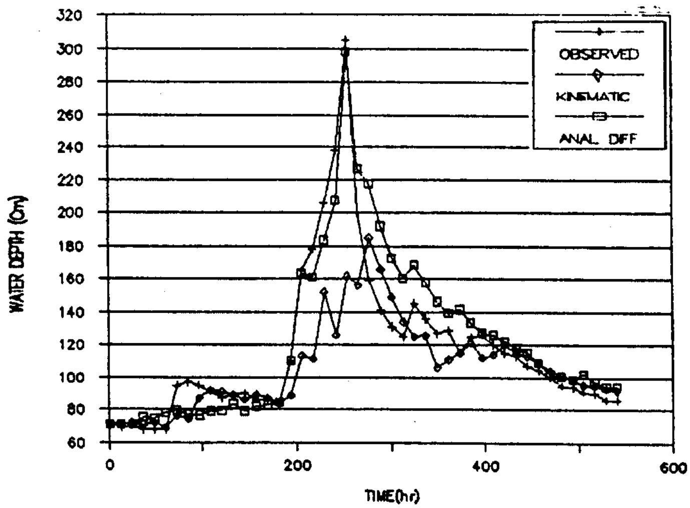

Fig. 4.3 Comparison of the flood routing results by the analitical diffusion model and the kinematic wave model(Flood event of July 9–31, 1986)

model both in the flood peak and volume. In the kinematic wave model the backwater effect caused by the downstream water level variations and lateral inflows can not be taken into account in the routing as previously mentioned. In the case of July 9–31, 1986 flood the water level variation at the upstream end of the reach was relatively much smaller than that at the downstream end and hence the backwater effects due to the downsteam water level variation could not be accounted by the kinematic wave model.

Moreover, the lateral inflows along the reach except through the Backokpo station could not be considered in the flood computation by the kinematic wave model, which is considered as a part of the reasons for the discrepancy between the computed and the observed.

5. Summary and Conclusion

An analytical diffusion model for flood routing with backwater effects and lateral flows is developed. The basic diffusion equation is linearized about an average depth of $(H + h)$, and is solved using the boundary conditons which take into account the effects of backwater and lateral flows. The solutons of the water depth is expressed in a form that can be conveniently applied to boundary conditions which are approximated by a series of histograms. The model is applied to the flood event of July 9–31, 1986 in the Jangpyung–Banglim reach of the Pyungchang IHP Basin, and the result is compared with that by the kinematic wave model. The following coclusions are made with respect to the present study:

1) The anaytical diffusion model is superior to the dynamic wave model in view of the data requirments and computer time when the flood computation is requred at a specified location of the reach.

2) Among the simplified models for the complete unsteady flow equations, the analytical diffusion model is more accurate than the kinematic wave model since the former can take into account the backwater effects caused by the downstream watr level variations and lateral inflows.

3) A proper determination of the parameters C, k which represent the channel characteristics is the key for the success of the present model. The average values of C, k determined through calibrations are 150 and 12,000, respectively for the reach considered in the present study.

4) It is speculated that a better determiantion of C and k is possible in case of the finite difference numerical solution of the diffusion equations because the channel characteristic parameters can be more accurately determined for the small intervals of the reach under consideration, and hence, better routing results are expected.

References

1. Akan, A. and Yen, B. C.(1981) Diffusion Wave Flood Routing in Channel Network, J. of

Hydraulics Div., ASCE, Vol. 107, No. HY 6

2. Amein, M. and Fang, C. S.(1970) Implicit Flood Routing in Natural Channel, J. of Hydraulics Div., ASCE, Vol. 96, No. HY 12
3. Amein, M. and Chu, H. L.(1975) Implicit Numerical Modeling of Unsteady Flows, J. of Hydraulics Div., ASCE, Vol. 101, No. HY 6
4. Anderson, M. G. and Burt, T. P.(1985) Hydrological Forecasting, John Wiley & Sons, New York, pp.437–503
5. Chow, V. T., Maidment, D. and Mays, L.(1988) Applied Hydrology, McGraw–Hill, New York, pp.279–290
6. Gonwa, W. and Kavvas, L.(1986) A Modified Diffusion Equation for Flood Propagation in Trapezoidal Channels, J. of Hydrology, 83, pp.43–58
7. Hayami, S.(1951) On the Propagation of Flood Waves, Bulletin of Disaster Prevention Research Institute, Vol. 1, No. 1, Kyoto, Japan, pp.1–16
8. Hromadka , T. V. and Devries, J, J.(1988) Kinematic Wave Routing and Computational Error, J. of Hydraulic Engineering, Div. ASCE, Vol. 114, No. 2
9. Jollife, I. B.(1984) Computation of Dynamic Waves in Channel Networks, J. of Hydraulic Engineering, Div. ASCE, Vol. 110, No. 10
10. Li, R. M., Simons, D. B. and Stevens, M. A.(1975) Nonlinear Kinematic Wave Approximation for Water Routing, Water Resources Research, AGU, Vol. 11, No. 2
11. Mananhar, S. K.(1982) Analytical Diffusion model for Flood Routing with Downstream Effect, Thesis presented to the Asian Institude of Technology in partical fulfillment of the requirements for the degree of Master of Engineering, Bangkok, Thailand
12. Ministry of Construction, Korea(1987) Representative Basin Study Report, International Hydrological Program
13. Morris, E. and Woolhiser, D.(1980) Unsteady One–Dimensional Flow over a Plane:Partial Equilibrium and Recession Hydrograph, Water Resources Research, Vol. 16, No. 2, pp.335–360
14. Ponce, V. and Simoms, D. B.(1978) Applicability of Kinematic and Diffusion Models, J. of Hydraulics Div., ASCE, Vol. 104, No. HY 3
15. Price, R. K.(1974) Comparison of Four Numerical Methods for Flood Routing, J. of Hydraulics Div., ASCE, Vol. 100, No. HY 7
16. Sutherland, A. and Barnett, A(1972) Diffusion Solution to Flows with Upstream Control, J. of Hydraulics, Div., ASCE, Vol 98, No. HY 11
17. Thomas, I. E. and Wormleaton, R. R.(1970) Flood Routing Using a Convective Diffusion Model, Civil Engineering and Public Works Reivew, pp.257–259
18. Tingsanchali, T. and Manadhar, S. K.(1985) Analytical Diffusion Model for Flood Routing, J. of Hydraulic Engineering Div. ASCE, Vol. 111, No. 3
19. Vieira, D.(1983) Conditions Govering the Use of Approximations for the Saint Venant

Equations for Shallow Surface Water Flow, J. of Hydrology, 60 pp.44–58

20. Watson, E. J. (1981) Laplace Transforms and Applications, Van Nostrand Reinhold, pp. 108–150