Energy and Mass Balance of Snowpack
—Rapid snowmelt during Föhn events in the Takada plain—

Kim, Sun Joo *, Shinichi Takami**
**College of Agriculture, Kon-kuk University*
***Hokuriku National Experiment Station, Inada, Joetsu-shi, Japan*

Abstract □ Several models physically based to predict the evolution of the snowpack have been proposed. Validity of these models for hourly estimation is, however, questionable, since they have been tested only on a daily basis. A computational model to predict the amount of snowpack on an hourly basis in terms of snowload from a set of meterological measurements was developed and investigated the rapid snowmelt conditions during Föhn events in the Takada plain.

Keywords □ Ablation of a snow cover, Snow water equivalent, Mass and energy balance, Föhn event, Energy equation, Radiation, Sensible heat flux, Latent heat of vaporization, Maximum water holding capacity of snowpack, Snow core sampler.

I. INTRODUCTION

The ablation of a snow cover or the net volumetric decrease in its snow water equivalent (Gray et al[3]) is governed by the energy exchange processes at the upper and lower layer of snow surface as well as the storage and hydraulic properties of the snowpack and its underlying ground. Better understanding of this physical process is important for the improvement of forecasting the time of snowmelt, the quantity and rate of water released, the volume of water entering the soil, the amount of evaporation, and the design of measures against the mechanical damages by snow.

Mathematical models to predict changes of snowpack, based on mass and energy balance, have existed for some time. Although these models only predict daily chages of snow load or water equivalent from the meteorological records with reasonable accuracy, their applicablity on hourly basis is questionable for several reasons.

The objective of this study was to develop a computational model to predict the amount of snowpack on hourly basis in terms of snowload from a set of meteorological measurements. The rapid snowmelt conditions during Föhn events in the Takada plain, Japan were investigated.

II. METHODS AND PROCEDURES

1. Computational model

a. Energy balance of snowlayer

The amount of energy avaliable for melting the snow surface layer of a snowpack is determined from the following energy equation.

$$dU/dt=(1-a)I+L_d-L_u-S-lE-G-M \quad (1)$$

Where

dU/dt : Rate of change of internal energy per unit area of snow cover(W/m^2)
I : Short wave radiation on the snow surface (W/m^2)
a : Albedo
L_d : Downward longwave radiation(W/m^2)
L_u : Upward longwave radiation(W/m^2)
S : Sensible heat flux to the air at the snow-air interface
l : Latent heat of vaporization(J/g)
E : Evaporation rate(g/m^2s)
G : Soil heat flux
M : Energy flux available for melting or freezing of snow

Eq. (1) reduces to

$$Rn=S+lE+M \quad (2)$$

If we assume that the steady state conditions exists, the rate of change of internal energy dU/dt is zero, and the temperature gradient within a snowpack is neglegible, that is, $G=0$.

In eq.(2), net radiation Rn is defined by
$Rn=(1-a)I+Ld-Lu$

The short wave radiation and upward longwave flux can be measured by instrumentation. In eq.(2), sensible heat flux is calculated by the following equation.

$$S=Cv(Ts-Ta)/Ra \qquad (3)$$

Where

Cv : volumetric heat capacity of air(J/m^3 deg)
Ts : temperature of snow surface(℃)
Ta : air temperature(℃)
Ra : boundary layer resistance

In eq.(2), latent heat exchange through evaporation of water from the snow surface is calculated by the following equation

$$lE=(Cv/\gamma)[\{e_s(Ts)-e_a(Ta)\}/Ra] \qquad (4)$$

Where

γ : psychrometric constant(mb/°k)
e_s : saturation vapor pressure(mb)
e_a : actual vapor pressure(mb)

In eq.(4), saturation vapor pressure is a function of the temperature of the snow surface. Therefore it can be expressed as follow ;

$$\begin{aligned} e_s(Ts)&=f(Ts) \\ &\fallingdotseq e_s(Ta)+\Delta(Ts-Ta) \qquad (5) \end{aligned}$$

where

Δ : gradient between the temperature curve and staturated vapor pressure

If we introduce $e_s(Ta)$ in eq.(4), it can be rewritten by adding and subtracting $e_s(Ta)$ from the right hand side as follow

$$lE=(Cv/\gamma)(1/Ra)[e_s(Ts)-e_s(Ta)+e_s(Ta)-e_a(Ta)] \qquad (6)$$

From the eq. (2) and eq. (3), Ts-Ta can be expressed as ;

$$Ts-Ta=(Ra/Cv)[(Rn-M)-lE] \qquad (7)$$

And using eq.(5), substitute Δ (Ts−Ta) for $[e_s(Ts)-e_s(Ta)]$. Also put $\delta e=e_s(Ta)-e_a(Ta)$, thus we obtain ;

$$lE=(Cv/\gamma)(1/Ra)[\Delta(Ts-Ta)+\delta e] \qquad (8)$$

Now replace Ts-Ta by eq.(7) ;

$$lE=(Cv/\gamma)(1/Ra)[\Delta\{(Ra/Cv)(Rn-M)-(Ra/Cv)lE\}+\delta e] \qquad (9)$$

if we tidy up eq.(9),

$$lE+lE[(Cv/\gamma)(1/Ra)\Delta(Ra/Cv)]=(Cv/\gamma)(1/Ra)[\Delta(Rn/Cv)(Rn-M)+\delta e]$$
$$lE(1+\Delta/\gamma)=(\Delta/\gamma)(Rn-M)+(Cv/\gamma)(\delta e/Ra)$$
$$\therefore\ lE=[(\Delta/\gamma)(Rn-M)+(Cv/\gamma)(\delta e/Ra)]/(1+\Delta/\gamma) \qquad (10)$$

From eq.(2), the sensible heat flux is calculated as follows ;

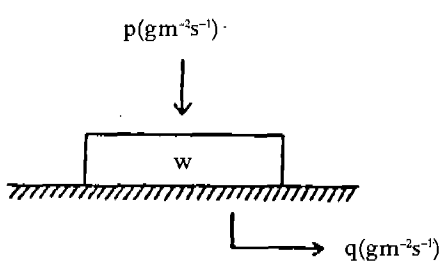
$$\begin{aligned} S&=(Rn-M)-lE \qquad (11) \\ &=(Rn-M)-[(\Delta/\gamma)(Rn-M)+(Cv/\gamma)(\delta e/Ra)]/(1+\Delta/\gamma) \end{aligned}$$

and if we combine eq.(11) & eq.(3).

$$Cv\frac{Ts-Ta}{Ra}=(Rn-M)\times \frac{(\Delta/\gamma)(Rn-M)+(Cv/\gamma)(\delta e/Ra)}{1+(\Delta/\gamma)} \qquad (12)$$

$$Ts=Ta+\frac{Cv}{Ra}[(Rn-M)\times \frac{(\Delta/\gamma)(Rn-M)+(Cv/\gamma)(\delta e/Ra)}{1+(\Delta/\gamma)}] \qquad (13)$$

Temperature of the snow surface is calculated using eq. (13). So the sensible heat flux (S) in eq. (3) and the saturated vapor pressure $[e_s(Ts)]$ in eq.(5) also can be calcualted using the above obtained Ts.



(unit area : 1m^2)

Fig. 1. Imagined snowlayer of a unit ground area

b. Mass balance of snowlayer

When we imagine a snowlayer of unit ground area as Fig. 1, the weight of a snowlayer of unit ground area is as follows.

Where

p : precipitation rate($gm^{-2}s^{-1}$)
q : drainage rate($gm^{-2}s^{-1}$)

$$W = Ws + Ww \tag{14}$$

Where

W : weight of snowlayer(g/m^2)
Ws : weight of snowgrains(g/m^2)
Ww : weight of water in the snowlayer (g/m^2)

When there is a precipitation on the snowlayer and the melted snow is drained from the layer in a unit time (Δt), the change of weight of snowlayer in a unit time (ΔW) is expressed as the following eq. (15)

$$\Delta W = (p - q)\Delta t \tag{15}$$

Assuming that the drainage takes place only if the amount of water exceeds the maximum water holding capacity of snowpack, the amount of drainage can be calculated from

$$q\Delta t = Ww + (r + mM)\Delta t - Wmax \tag{16}$$

Where

r : rainfall rate($g/m^2 \cdot s$)
m : latent heat of fusion(J/g)
M : heat flux available for fusion(W/m^2)
Wmax : maximum water holding capacity of snowpack(g/m^2)

So the change of weight of the snowlayer in a unit time (ΔW) in eq. (15) can be calculated using the calculated value of q Δt.

Similarily, from the mass balance for the liquid water of the snow, we have the following equations.

$$\text{No drainage}: \Delta Ww = (r + mM)\Delta t \tag{17}$$
$$\text{Drainage}: \Delta Ww = Wmax - Ww \tag{18}$$

From the given meteorological variables and Ts, ΔW can be calculated through the above mentioned equations.

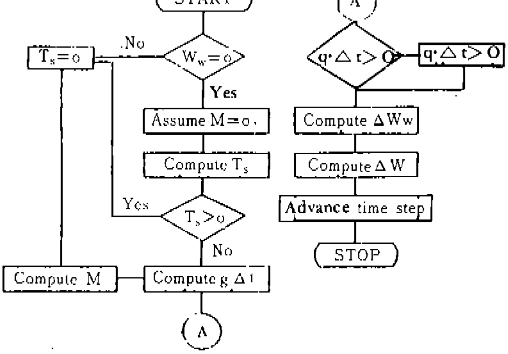


Fig. 2. Flow chart of the computational procedures

c. Computational procedures

The energy and mass balance is computed by the procedures shown in Fig. 2

As a first step, we should make a decision whether the snow is wet ($W_w > 0$) or dry ($W_w = 0$). If it is wet, compute the energy flux available for melting/ freezing ofsnow (M) from eq. (13) with the condition that the surface temperature of snow (Ts) is zero. The temperature of the ice and water mixture can be assumed zero. If the snow is dry, we assume equals zero and compute Ts from eq. (13). If computed Ts is positive, let it be zero and reestimate M as for wet snow.

Then compute $q \cdot \Delta t$ from eq. (16). After that decide whether the computed $q \cdot \Delta t$ is negative or not. If not, compute ΔWw and ΔW, respectively from eq. (17), eq. (18) and eq. (15). If negative, let $q \cdot \Delta t$ to be zero and compute ΔWw and ΔW as before.

2. Experiment

a. Measurements of output

The final output of the developed model is ΔW. Since precipitation (p) is a measured input, ΔW is basically equivalent to q Δt. Also equivalent to this quantity is ΔWw and this is a more direct output. Another output of the computational model is Ts. Each of these factors are instrumented as follows.

a. ΔW :
1) gravimetry using a snow core sampler
2) Weighing using a snow pillow

b. $q \cdot \Delta t$: two lysimeters installed on the ground

c. Ts : thermistors installed at the height of 0, 10 and 40cm above ground

b. Measurements of input

Required input data for the model was measured from January through March 1990 at the weather station of Hokuriku Agricultural Experiment Station, Joetsu-city, Niigata, Japan.

Net radiation(Rn) is measured by the radiometers. To estimate r, it is assumed that the precipitation is rainfall if the air temperature (Ta) is above 3.5℃.

III. RESULTS AND DISCUSSION

c. Additional measurements
- a. Incident solar radiation
- b. Reflected solar radiation
- c. Soil heat flux
- d. Soil temperature(0, −10 and −30cm)
- e. Height of snowcover
- f. Wind direction
- g. Duration of sunshine hours

1. Test of the developed model

Rapid snowmelt occurred three times during the winter of 1990 as shown in Fig.3, of which we used data from two days, January 10 and February 11, for analysis. Additional two other days, January 27 and February 7, were chosen for the comparison. Simulation started from midnight for each day and extended over a whole day. Initial conditions for total water content were estimated from the density and depth of snowpack of the experimental plot, whereas those for liquid water content were assumed values that the snowpack was fully saturated at the beginning of the day as shown in Table-1. Diurnal values of the meteorological inputs illustrate the days of Fohn event, Fig.4–a, b, respectively, a cold cloudy day (Fig. 4–c) and a warm sunny day(Fig.4–d).

On January 10, a strong wind, above 5m/sec, started to blow just before noon(Fig.4–a). It was associated with a marked upward revision of air temperature and a marked drop in relative humidity, and continued for the rest of the day. The day was generally cloudy, with light rain at night. In response to such conditions, a rapid snowmelt occurred and 55mm of snow had disappeared by 23 hour as hown in Fig.5–a.

On February 11, a similar event started at the beginning of the day and continued untill noon as shown in Fig.5–b. Wind weakened for several hours in the daytime during which intermittent rainfall took place. Strong wind resumed after 21hour, again with a marked drop in relative humidity. Snowmelt proceeded for the whole day, and 68mm of snow had disappeared by 23 hour(Fig.5–b).

To test the model, we compared the measured and simulated cumulative drainages for these two days(Fig.5). The model underestimated 20% to 30% compare to the measured values at the time when all the snow melted. This is partly due to the fact that 5mm and 10mm of rainfall occurred on January 10 and February 11, respectively. Prediction of this model could be more effectively improved if the rainfall was considered in calculation.

Ideally, any model should be tested with all inputs that are specifically measured or determined for the system concerned. In this study, however, we used the values of the system parameters A (maximum fraction of liquid water LWmax/SW) and C(transport coefficient)

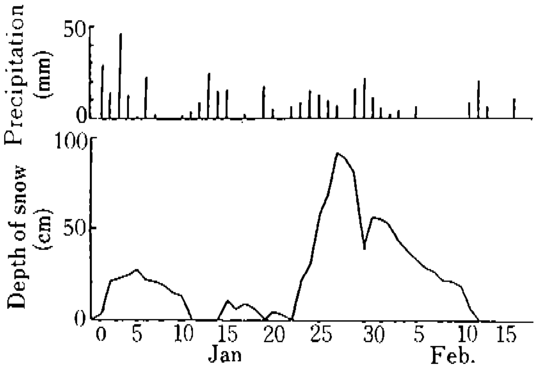


Fig. 3. Precipitation and depth of snowpack from Jan. 1 to Feb. 15, 1990

Table 1. Properties and initial values of snowpack used for the simulation

Date	Density (g/cm³)	Depth of snow pack (m)	SW (mm)	LW (mm)	Remarks
Jan. 10	0.388	0.14	54.3	3.80	Fohn day
Jan. 27	0.164	0.81	133.0	9.30	Cold & cloudy
Feb. 7	0.443	0.28	124.0	8.68	Warm & sunny
Feb. 11	0.450	0.15	67.5	4.73	Fohn day

which are 0.05 and 0.002, respectively, and assumed the initial value of the liquid water content arbitrarily. Furthermore, Ld is not measured directly, but estimated from Ta, e_s(Ta) and I. Thus the errors in any of these variables could result in the error in the model outputs.

1. Measured around 09 hour on the day, except 11th of February (measured on 8th of February)
2. SW (total water content Density × Snow depth × 1000)
3. LW (liquid water content SW×0.07)

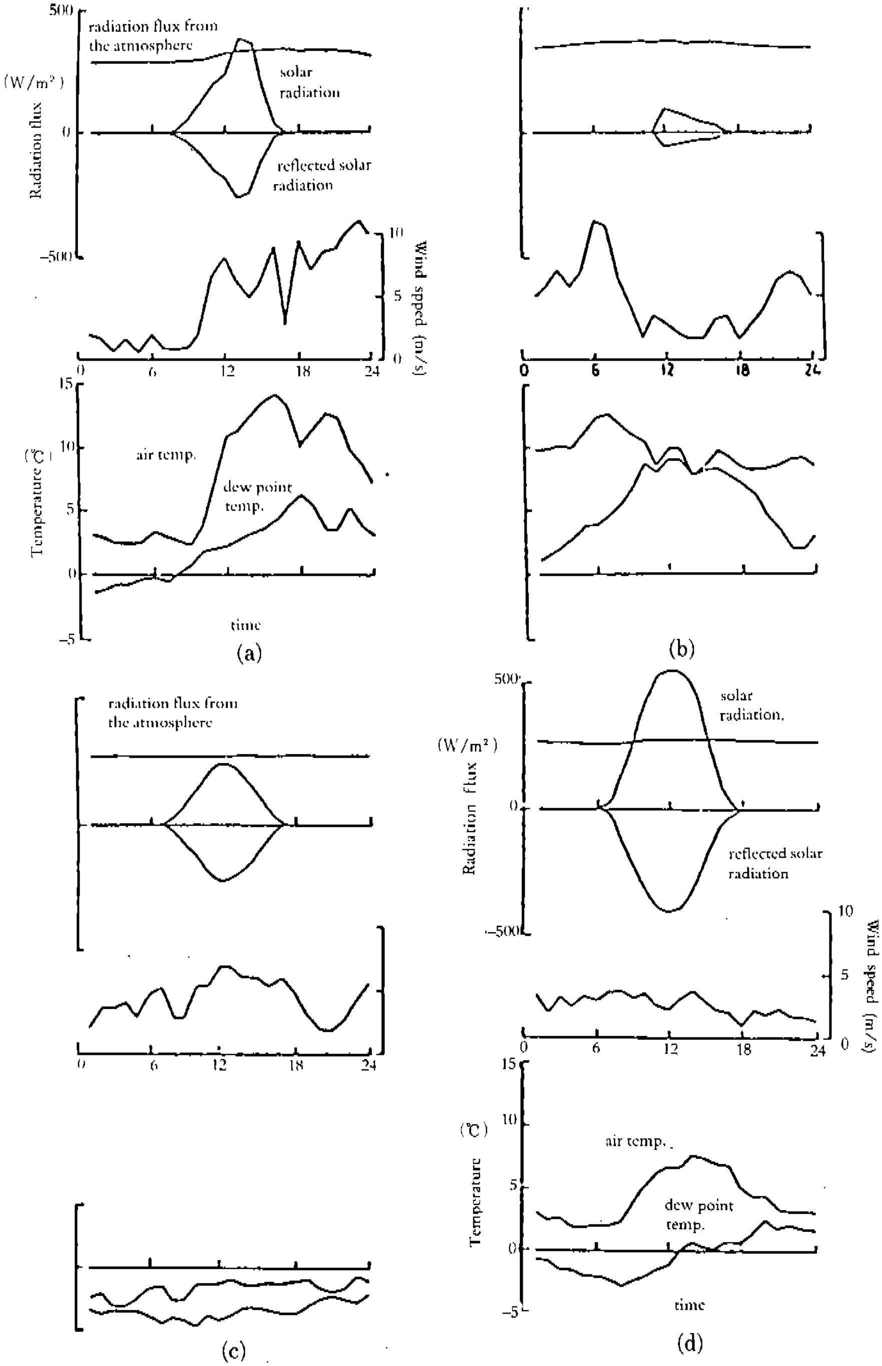
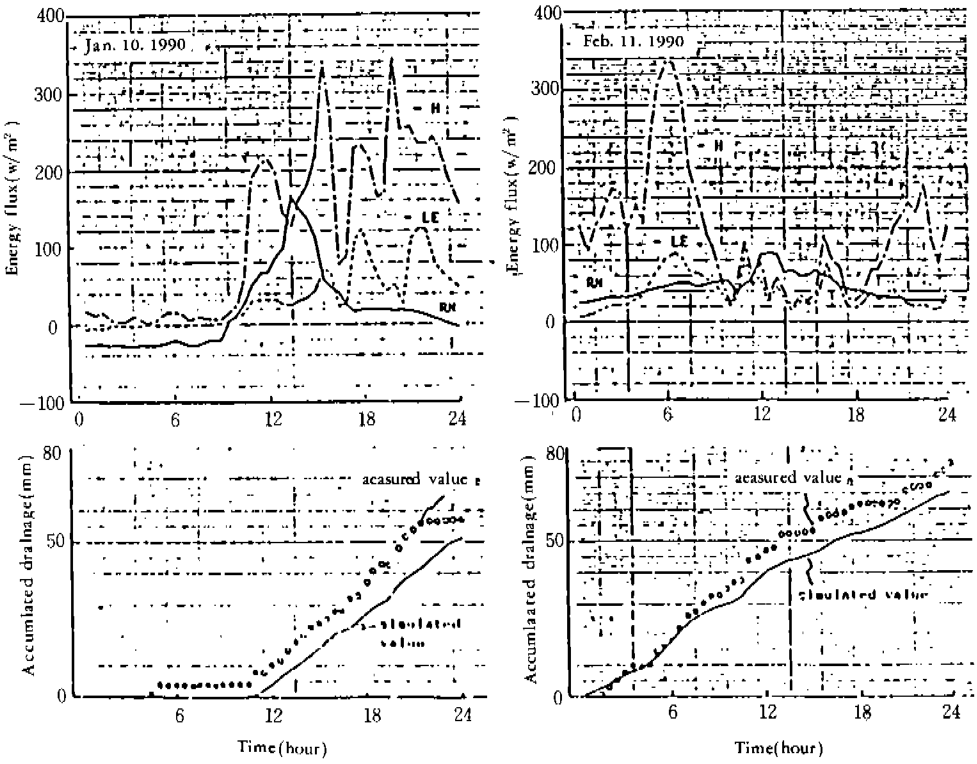


Fig. 4. Diurnal values of the meteorological inputs



(a) (b)

Fig. 5. Energy flux and accumulated drainage amount

2. Comparison of Föhn and Föhn free days

Given above results and circumstances, we assumed that the model predictions were adequate enough to analyse the snow melt processes in a Föhn event as compared to the Föhn free days. Followings are the important outcomes from this analysis.

a. There are substantial differences between Föhn days and föhn free weather both in the absolute and ralative magnitude of energy components that contributed to the snowmelt. Radiation, sensible and latent heat, contributed to snowment in a Fohn event, whereas only sensible heat was supplied to the snow surface in fohn Free days.
b. In a Föhn environment, sensible heat was the most important energy source for snowmelt, occupying 60% to 80% of the total energy supply. On the other hand, latent heat was not insignificant, its magnitude being comparable to the radiation energy. Significant energy supply by condensation appears at a first glance to contradict the perception of the Föhn as a dry wind. But driness merely means the drop in relative humidity that reflects the rise in temperature. The dew point temperature in a Föhn event was well above zero degree centigrade, indicating vapor flux toward the snow surface.
c. Positive net radiation at the snow surface in a Föhn event was primarily due to to the larger longwave irradiance from the sky. Because of its higher temperature and absolute humidity, a Föhn dominated atmosphere was estimated to emit 15% to 30% greater thermal radiation than that of the Föhn free atmosphere.
d. Because of the higher value of Ld, snow melt was greater in February 11 than January 10.
e. Among the componetns of radiation balance, we found that the Ld was the most important factor in a Föhn environment. Shortwave radiation is of less significance since most of it was reflceted and the absorbed energy was

only 10% of total radiation load on to the snow surface.

f. Linear approximation of eq.(5), as used in any analytical combination formula, is totally inadequate for the estimate of latent heat flux LE in a Föhn environment, since LE is overestimated by 100%. The reason is obvious : air temperature rises above 10℃ while the snow surface temperature is maintained at 0℃.

IV. SUMMARY AND CONCLUSIONS

Several models physically based to predict the evolution of the snowpack have been proposed. Validity of these models for hourly estimate is, however, questionable, since they have been tested only on a daily basis. An hourly or even shorter time test should be done for any mechanistic models, since longer time comparisons may produce fortuitous agreement by a balancing of errors.

A computational model predicts the amount of snowpack on an hourly basis in terms of snowload from a set of meteorological measurements was developed and investigated the rapid snowmelt conditions during Föhn event in the takada plain.

The model developed in this study appears to predict the snowmelt accurately on an hourly basis although a more rigorous test should be performed to confirm this conclusion. Föhn events enhance snowmelt by providing not only a large amount of sensible heat but a significant amount of latent heat by condensation. Furthermore, because of its higher temperature and absolute humidity, a Föhn dominated atmosphere provides more radiant energy for snowmelt than a Föhn free environment.

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