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Dynamic Adjustment of Noncooperative Games Where Informations are Given at Discrete Time Intervals

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Abstract

This paper concerns the analyses of dynamic adjustments in noncooperative games where the market informations are given at discrete time intervals. During the game period, the inventories initially stored by players are to be released one day based to the completely competitive market so as to maximize each palyer's revenue, where players' parameters are unknown one another. Game results have shown that the continuous dynamic adjustment does not necessarily assure the better revenue, and if a player thinks that his parameter is underestimated by his opponent, then he is better overestimate his opponent's parameter.

Keywords : Dynamic adjustment, incomplete information, noncooperative game, adjustment coefficient, Hamiltonian, simultaneous differential equations.

1. Introduction

The complexity of noncooperative game model is heavily dependent upon the situations whether players choose a single act or multi-act game during the given game period. The model becomes even more complicated in case that the players have incomplete informations about their opponents' parameters.

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The study of noncooperative game theory is originated from the Nash's paper which states that in any noncooperative game, there is at least one equilibrium point in mixed strategies[7]. But he has not presented the methodology of finding them.

Harsanyi[3] has introduced an idea in his three consecutive papers that noncooperative game with incomplete information can be converted into the game with complete information by inserting a subjective probability distribution to the game with incomplete information. However his paper was restricted in dealing only with discrete strategies and was not extended to the case of dynamic situation either.

The technique of finding Nash equilibrium points in systematic way was first attempted by Kreps and Wilson[4]. They introduced a method of "sequential equilibrium". But they presented quite a subjective concept of "belief" in their method, and again the method presented is still left open for further objective justification.

Feichtinger[1] solved a research efforts allocation problem for obtaining better rewards. Two noncooperative research teams should finish the study ahead of their opponent where the time of completing project was probabilistically known to each other. However his focal point was to determine only the optimal amount of initial efforts to be fixed at the beginning of the project. He has not mentioned about the "in-process control", which might be required when new informations are provided.

He also presented a two-person nonzero-sum differential game model of competition between a thief and the police[2]. He quoted, in his paper, quite a variety of literature survey. But the various utility functions such as utility of police occurring at the instant the thief is caught, and salvage utility of the thief being not yet caught at the specific time etc. were expected to be given from the game start. Moreover, in deriving control functions, he ignored the presence of punishment rate of the thief; psychological or otherwise, and suggested the numerical methods to obtain the qualitative insights in the behavior of the Nash solutions.

H. Oh[8] introduced a mathematical representation of solving a differential game with incomplete information. Two noncooperative suppliers should determine the optimal daily amount of inventories for better revenues to be earned from the market at the end of the game period. Since suppliers are uncertain about the opponents' parameters, they should be careful of allocating the inventories to each day of game period. But he has not extended his methodology to the case where model should continuously be ajusted by the new informations provided as time passes.

Namatame and Tse〔6〕 presented a dynamic adjustment model. Their assertion is that if a player's behavior with incomplete information over his opponent's strategies was mathematically representable, and his opponent's actual strategy function was known during the entire time horizon, it was possible to provide the criteria for the system stability. However the above-mentioned mathematical representability seems unrealistic and moreover the assumption of knowing the mathematical formula for the opponent's behavior from the game start is hardly acceptable.

This paper presents an extension model considered by Oh〔8〕. The new model is applicable to situations where market price informations are provided at the discrete time intervals and the noncooperative game players are required to continuously adjust their decision making model based on the new informations updated.

2. Environments for Model Building

Noncooperative players are interested in obtaining the maximum revenues during the given game period by properly releasing the daily amount of warehouse inventories to the market. Revenue is determined by the market price which is assumed inversely proportional to the total amount released by competitive players, and inventory carrying cost which reduces revenues proportional to the current inventory level. Therefore, in case that market price informations are provided at the discrete time intervals, each player should pay much attentions not only on the market price informations, but also the informations about his opponents' parameters which may be predictable from the market information, because each player's decisions are to be made based on these data and his own parameter value.

Throughout the game period, it is assumed that each player knows each day's market price at the time he decides the amount of inventory to be released for the next day, but he has no informations about his opponents' parameters, e. g., his opponents' inventory carrying costs. Therefore what he has to do is to determine the optimal daily supply amount out of his current inventories by use of daily-informed market price and his opponents' inventory carrying costs to be estimated one day based.

The diagram of Figure 2. 1 shows the environments for model building. Noncooperative and competitive players release certain amount of inventories on t-th day. Then each supplier or warehouse manager is informed of the exact market price on t+1st day. And on the same day each player is also able to know the total sum of amount released on the previous day if the market price is expected to be determined by certain mathematical formula'. If the total sum is different from the expected value, as will be shown mathematically in the next chapter, the warehouse manager should revise his "model" in the direction of better revenue to determine the t+1st day's portion.

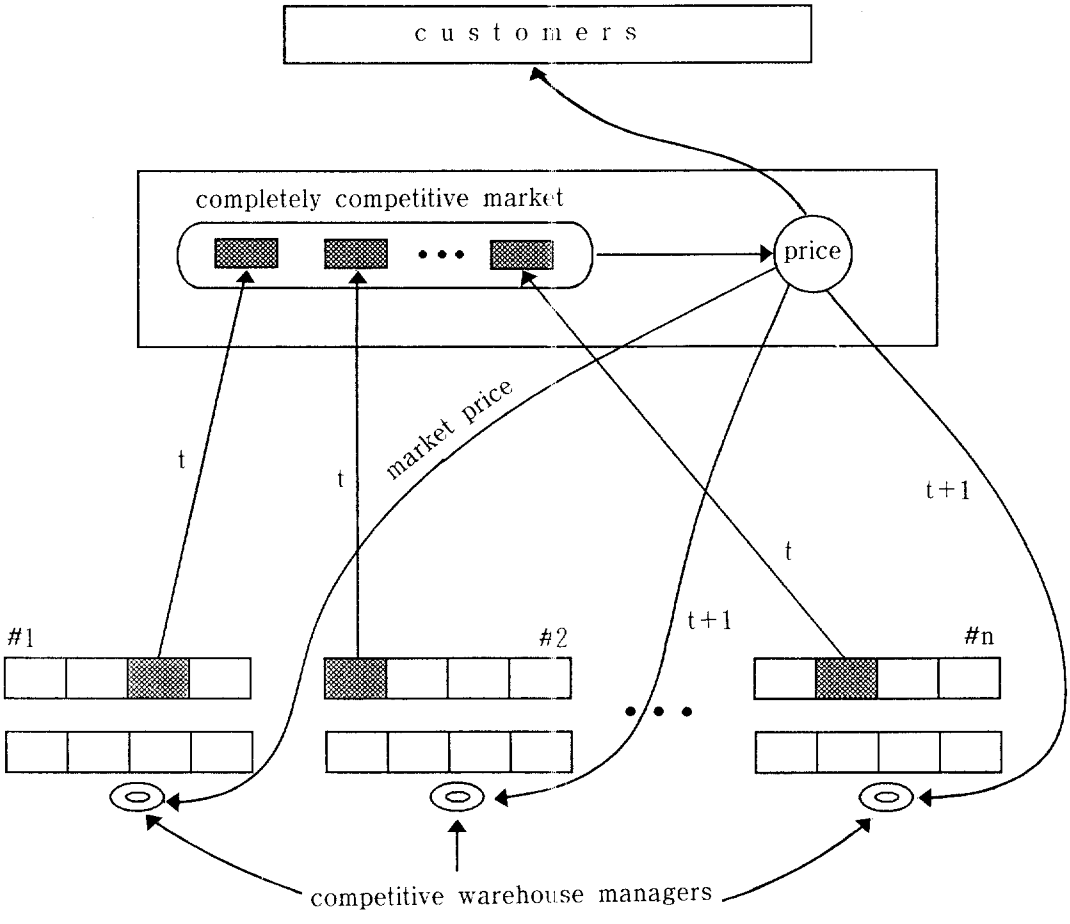

Figure 2. 1 Environments for Model Building

In this paper, it is assumed that market price is determinedy by certain mathematical formula, but actually market price is affected by many exogenous variables.

3. Dynamic Model with Complete Information

In dynamic model with complete information, it is assumed, as was mentioned in chapter two, that each player is informed of daily market price from the completely competitive market and also has complete information about his opponent's parameter. In this model the game is two person game and the unknown parameter is confined to the inventory carrying cost.

Model is constructed in such a way that each player decides his daily supply amount at any specific instant as best as he can, given that his opponent behaves as the same way he does. In case of dual game, each player's strategy selection function so derived represents a Nash equilibrium expressed in the form of continous function[5].

Suppose that market price is determined as

$$p(t) = M - q_1(t) - q_2(t) \tag{3.1}$$

where, $p(t)$: market price

$q_i(t)$: daily amount to be released by player i at time t

M : positive constant.

Then player i's revenue at time t is represented as

$$rev_i(t) = q_i(t)\,(p(t) - h_i(t)) \tag{3.2}$$

where, $rev_i(t)$: player i's revenue at time t

h_i : inventory carrying cost per unit per period for player i.

Since each player is interested in maximizing his own revenue to be made during the game period, player i, for example, should find his optimal supply fuction, $q_i(t)$, given that his opponent's supply function is $q_j(t)$, and the same rule is applied to player j. Therfore a mathematical model with complete information can be constructed as follows;

$$\underset{q_i|q_j}{\mathrm{Max}} \int_0^T rev_i(t)\ dt$$

subject to (3. 3)

$$\int_0^T q_i(t)\ dt = R_i, \quad i=1,2, \quad j \neq i$$

where, T : game period

R_i : player i's initial inventories.

To derive the strategic functions, let us consider the Hamiltonian functional H associated with (3. 3) ;

$$H^i = q_i(t)\,(p(t) - h_i t) + \lambda_i q_i(t). \tag{3. 4}$$

By differentiating (3. 4) with respect to $q_i(t)$, we obtain

$$\frac{\partial H^i}{\partial q_i} = M - 2q_i(t) - q_j(t) - h_i t + \lambda_i = 0. \tag{3. 5}$$

Adjoint system of H^i is represented as

$$\dot{\lambda}_i = -\frac{\partial H^i}{\partial x_i} = 0, \text{ hence } \lambda_i = \text{constant}. \tag{3. 6}$$

From (3. 5) and (3. 6), we have the following simultaneous differential equations,

$$2\dot{q}_i(t) + \dot{q}_j(t) = -h_i, \quad i=1,2, \quad j \neq i. \tag{3. 7}$$

From (3. 3) and (3. 7), $q_1(t)$ and $q_2(t)$ are written respectively in the forms of

$$q_1(t) = R_1/T - (T/6)\,(h_2 - 2h_1) + (h_2 - 2h_1)\,t/3$$

$$q_2(t) = R_2/T - (T/6)\,(h_1 - 2h_2) + (h_2 - 2h_1)\,t/3$$

where $t \in (0, T)$.

4. Dynamic Model with Incomplete Information

The game situation of the dynamic model with incomplete informations presented in this paper is that each player should adjust his model continuously in the direction of maximizing his total revenue, where each player has incomplete information about his opponent's parameter but the complete informations about the market prices provided at every stage of discrete time intervals.

In this paper, players are assumed to be sufficiently wise enough that once they are given the comlplete informations about their opponents' parameters, they are able to obtain the maximum revenues i, e, the revenues resulted by the Nash solutions. However the problem arises from the fact that since they are uncertain about their opponents' parameters, the market prices daily generated are not in general coincident with the values which are expected by each player. Therefore players should revise his strategy function generating model, based upon the newly informed market price toward the better revenue. This process is continued until the end of the game.

In case that the real-time based complete informations are provided at discrete time intervals, the assumptions of mathematical representability about the player's behavior from the game start seems almost meaningless, because the regularly informed new informations may distort each plater's preset strategic functions if the new informations are not coincident with each player's expected values which may be ascribed to the estimation error of unknown parameters.

Therefore, for the construction of dynamic adjustment model with incomplete informations it is required to use the specific formulas in the model, such as formulas for market price and revenues etc., otherwise each player is not able to revise his model at every stage of discrete time intervals. The applications of (3. 1) through (3. 3) in chapter 3 expressed in terms of specific formulas respectively is for this reason. Although (3. 1) is too much simplified, it can be a good start for further generalization. If market price is affected by many exogenous factors, then some market price generating scenario may be applied.

Below are presented a dynamic adjustment model (DAM) algorithm and illustrative examples. For the exploitation of concrete behavior function of each player, several restrictions and rules are applied in DAM algorithm as described in 4. 1.

4. 1 DAM Algorithm

Restrictions and rules applied to DAM are as follows;

a. Market price is determined strictly according to (3. 1) and known to each player on daily base,

b. Each player applies (3. 8) as his supply function,

c. Each player calculates his and his opponent's slopes of supply functions by use of previous two consecutive days' supply amounts, and applies (3. 7) to derive new inventory carrying costs (icc) for the next day. In that case, each player should revise not only his opponent's icc value which he estimated previously, but also his own value according to the results of (3. 7 even though he knows his one exactly.

By use of the statements a, b, and c above, DAM algorithm is constructed as follows, and the diagramatic representation of DAM algorithm is given in figure 4. 1. For the writing simplicity, the term "player i" is suppressed in the statements of algorithm and DAM diagram.

〈 DAM Algorithm 〉

Step 1. Estimates his opponent's true icc value h_j as $\bar{h}_j$.

Step 2. Releases his daily supply amount calculated by (3. 8) where $\bar{h}_j$ is applied in place of h_j. The same rule is applied to his opponent.

Step 3. Estimates $\dot{q}_i(t+1, \bar{h}_j)$, the slope of $q_i(t+1)$ where $\bar{h}_j$ is used for h_j using the previous two consecutive days' data obtained in step 2.

Step 4. Drives $\dot{q}_i(t+1, \tilde{\mathbf{h}})$ by use of (3. 7) where $\tilde{\mathbf{h}} = (\tilde{h}_1, \tilde{h}_2)$, revised vector of $\bar{\mathbf{h}} = (\bar{h}_1, \bar{h}_2)$.

Step 5. Applies adjustment coefficient to $\dot{q}_i(t+1, \bar{\mathbf{h}})$ and $q_i(t+1, \tilde{\mathbf{h}})$ to determine $\dot{q}_i(t+1)$ and decides $q_i(t+1)$ with restrictions of the remaining inventory level and game period. Go to step 2 if game is not finished.

4. 2 An Illustrative Example

As an illustrative example, consider a noncooperative game conducted between player 1 and player 2. They are playing game for 40 days, and their initial inventory levels are 300 for player 1, and 650 for player 2 respectively. Market mechanism is completely competitive, therefore the market price is strictly determinded by the formula (3. 1). The true inventory carrying costs are unknown to each other throughout the game period. Game is conducted for the various $\bar{h}_1$, and $\bar{h}_2$ values where M and true inventory carrying costs h_1 and h_2 values are given as follows;

case 1 : $M=50$, $h_1=1$, $h_2=1$

case 2 : $M=60$, $h_1=1$, $h_2=2$.

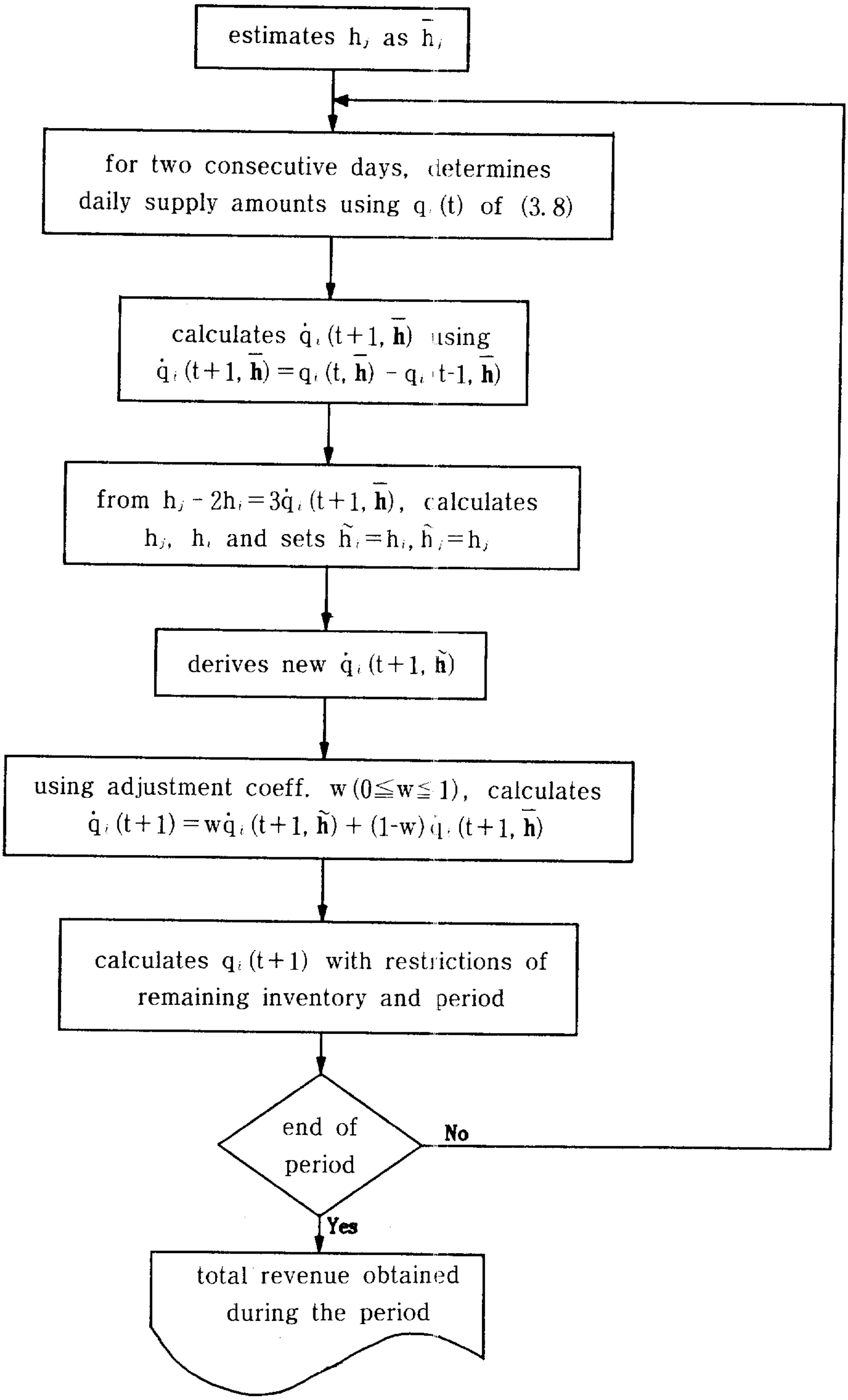

Figure 4.1 Diagramatic Representation of DAM Algorithm

Figure 4.2 shows the results obtained in case 1 where h_1, and h_2 go from .6 to 1.4 with interval of .2, and the adjustment coefficient, w, has the values of .0, .5, and 1.0. If w is zero, player is called conservative in his decision behavior and rather unwilling to change his initial estimate about his opponent's parameter, while if it is one, the situation becomes in reverse. Figure 4.3 shows the results in case 2.

upper : w= .0
middle : w= .5
lower : w=1.0

P_2

P_1

$\bar{h}_2$ \ $\bar{h}_1$	.6	.8	1.0	1.2	1.4
.6	3984 4298 4157 1968 2283 2141	4138 4328 4245 2065 2290 2216	4291 4358 4315 2119 2291 2268	4445 4387 4365 2128 2286 2294	4598 4417 4396 2094 2275 2296
.8	4082 4306 4233 2122 2312 2229	4213 4332 4299 2197 2317 2283	4344 4359 4346 2228 2315 2312	4476 4385 4375 2216 2306 2317	4607 4412 4384 2159 2291 2296
1.0	4135 4307 4284 2275 2342 2299	4244 4330 4328 2329 2342 2331	4353 2338	4463 4377 4360 2304 2326 2321	4572 4400 4347 2225 2308 2278
1.2	4144 4302 4310 2429 2372 2349	4231 4322 4332 2469 2370 2359	4319 4342 4336 2448 2361 2345	4406 4362 4320 2391 2346 2305	4493 4382 4285 2291 2325 2241
1.4	4109 4290 4311 2582 2401 2380	4174 4307 4312 2592 2396 2369	4240 4324 4293 2557 2384 2332	4305 4340 4256 2479 2366 2271	4371 4357 4199 2356 2342 2185

Figure 4.2 Total Revenue for Case 1

P_2

	$\bar{h}_2$ $\bar{h}_1$	1. 5	2. 0	2. 5
	. 5	1210 2090 1885 4485 4691 4457	2170 2282 2201 4729 4727 4668	3131 2475 2399 4698 4722 4724
P_1	1. 0	1420 2071 2064 4621 4713 4654	2243 4728	3067 2416 2304 4560 4704 4048
	1. 5	1356 2012 2088 4758 4734 4732	2042 2164 2131 4728 4730 4669	2728 2317 2054 4422 4685 4452

Figure 4. 3 Total Revenue for Case 2

5. Conclusions

From the results shown at figures 4. 2 and 4. 3, several conclusions are at least predictable even though the number of trials for sensitivity analyses with various true inventory carrying costs, adjustment coefficients, and estimated inventory carrying costs etc. are not big enough. Several selected conclusions are as follows;

a. When players estimate their opponents' parameters correctly, then each player's total revenue is independent of the adjustment coefficient value.

b. Player's adjusting behavior expressed in terms of adjustment coefficient does not necessarily assure the better revenue depending upon the game situations.

c. If a player thinks that his opponent overestimates his parameter, then he is better underestimate his opponent's one in the way of making the sum of two estimates equal to that of two parameters.

d. Sometimes the application of mutually misestimated parameters results better revenues. In other words, properly-combined cooperative game sometimes yields better revenue against noncooperative game.

Conclusion a. doesn't seem to need any explanations from figures 4. 2 and 4. 3. As far as b. is concerned, figure 4. 2 shows, in case that player 1 estimates $\bar{h}_2$ as 1. 4 and player 2 does $\bar{h}_1$ as . 6, player 1 has better revenue 2296 when w is one, while player 2 has 4598 when w is zero.

Conclusion c. is made by examining figure 4. 2 and 4. 3. Consider in figure 4. 2 the case when the value of $\bar{h}_1+\bar{h}_2$ is 2 and equal to h_1+h_2, i, e, the sum of estimated parameter values is equal to that of two true values. Suppose player 1 under-estimates player 2's parameter as . 8 and player 2 overestimates player 1's as 1. 2 in the way of making sum of two estimates being 2. Then, with w being zero, player 1's revenue is 2469 which is better than any choice other than . 8, given that player 2's decision is 1. 2, while player 2's revenue is 4332 with w being 1 which is better than any choice, given that player 1's decision is . 8.

Figure 4. 3 shows the case that the sum of two estimates is 3. If player 1 thinks that his opponent over-estimates his parameter 1. 5, then by the same token mentioned above he is better underestimate player 2's one as 1. 5 in the way of making the sum of two estimates equal to 3. In that case, player 1 receives 4578 with w being zero which is better than any value obtainable otherwise. The same is true for player 2.

As far as the conclusion d. is concerned, figuyre 4. 3 shows that player 1 is not bad as long as he estimates his opponent's parameter correctly, while, player 2 receives 2282 which is bigger than 2243 by under-estimating player 1's parameter as . 5.

As was mentioned in chapter 4, it is required to build a model by use of specific formulas e. g., formulas for market price and revenues etc. Otherwise the continuous dynamic adjustment work is impossible under the situations that the informations are given at the disscrete time intervals as time passes.

This paper has assumed that the market price is strictly determined by (3. 1). However, if (3. 1) is only used as the reference formula and therefore it is determined also by many exogenous factors, some market price generating scenario may be applied. These studies are left for further research areas.

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