Journal of the Korean
OR/MS Society
Vol.15, No.1, June, 1990

Optimal Scheduling in Power-Generation Systems with Thermal and Pumped-Storage Hydroelectric Units

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Abstract

This paper is concerned with the unit commitment problem in an electric power system with both thermal and pumped-storage hydroelectric units. This is a mixed integer programming problem and the Lagrangean relaxation method is used. We show that the relaxed problem decomposes into two kinds of subproblems : a shortest-path problem for each thermal unit and a minimum cost flow problem for each pumped-storage hydroelectric unit. A method of obtaining an incumbent solution from the solution of a relaxed problem is presented. The Lagrangean multipliers are updated using both subgradient and incremental cost. The algorithm is applied to a real Korean power generation system and its computational results are reported and compaired with other works.

Key words : Energy, Electric power, Scheduling, Lagrange multipliers, Networks.

1. Introduction

Electricity load pattern usually exhibits extreme variation between peak and off-peak hours. If sufficient generation to meet the peak is kept on the line throughout the planning

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horizon (24-168 hours), it is probable that some units will be operating near their minimum generation levels during the off-peak period. This implies that the thermal efficiency could be very low. The problem of determining a schedule of which units should be taken off and for how long is called the unit commitment problem. This problem has been the subject of considerable discussion in the power system literature.

An electric generating system has three types of units - thermal, hydro and pumped-storage hydroelectric units. A pumped-storage hydroelectric unit is used principally for storing electric enery. The normal operation of a pumped-storage unit consists of pumping water into the upper reservoir during off-peak periods, such as nights and on weekends, when the incremental cost of fossil energy is low. During periods of peak load, water is released from the upper reservoir to generate energy, thus replacing fossil energy of higher incremental cost.

Our main objective in this paper is to provide an efficient algorithm for the unit commitment problem of a power system with both thermal units and pumped-storage hydroelectric units. Hydro units are not considered.

The unit commitment problem is a combinatorial problem. Branch and bound method (Ohuchi and Kaji [1975], Cohen and Yoshimura [1983]), dynamic programming method (Lowery [1966], Zurn and Quintanna [1975], Pang and Chen [1976]) and Benders' partitioning method (Turgeon [1978]) have been used to solve this problem. These methods can provide an accurate solution for a small system but they cannot handle a system with a large number of units.

Recently, Muckstadt and Koenig [1977] applied the Langrangean relaxation method to a system with thermal units. They showed that the relaxed problem decomposes into single-generator subproblems and each subproblem becomes a shortest-path problem on an acyclic network. Merlin and Sandrin [1983] improved the method of updating the Lagrangean multipliers in the subgradient optimization procedure. Bertsekas et al [1983] used the penalty function method in updating the Lagrangean multipliers. All these works did not considered pumped-storage hydroelectric units.

A pumped-storage hydroelectric unit uses electricity from thermal units during off-peak periods and generates electricity during peak periods. The resulting electricity load pattern for thermal unit system will then have lower peak load and higher off-peak load than the original load pattern. This will change the optimal unit commitment schedule of the thermal units and the inclusion of pumped-storage hydroelectric units into the unit commitment problem is essential. Dynamic programming methods (Joy and Jenkins [1970], Cobian [1971]), linear programming methods (Ko et al [1982]) and a heuristic method (Le et al [1983]) have been used to solve this problem.

We extend the work of Muckstadt and Koenig to a system with thermal and pumped-storage hydroelectiric units. We apply the Lagrangean relaxation method and show that the relaxed problem decomposes into two kinds of subproblems : a shortest-path problem for each thermal unit and a minimum cost flow problem for each pumped-storage hydroelectric unit. Both are simple network problems and their efficient algorithms have been well developed.

The Lagrangean relaxation method is much more powerful if it is possibel to find a feasible solution of the original problem from the solution of the relaxed problem. In this case we can find an incumbent solution and upper bound as well as lower bound. The unit commitment problem has two kinds of coupling constraints - demand constraints and spinning reserve constraints. If the solution of the relaxed problem satisfies the spinning reserve constraints then we can easily find a feasible solution which also satisfies the demand constraints. Our computational experience for a real-case test problem shows that about 10 iteration steps out of 28 iterations provide incumbent solutions.

We shall begin by formulating the unit commitment problem as a mixed integer programming problem in Section 2. In Section 3, we apply the Lagrangean relaxation method and show that the relaxed problem decomposes into simple network problems. Section 4 provides the method of obtaining an incumbent solution from the solution of the relaxed problem and the method of updating the Lagrangean multipliers. In the last section the computational results of our algorithm, applied to the real Korean power generating system, is reported and compaired with others.

2. Model formulation

For thermal units, we follow the assumptions and notations used in Muckstadt and Koenig. When a thermal unit is committed to operating, the unit's output must be between its minimum and maximum operating capacities. Let m_i and M_i respectively represent these minimum and maximum operating capacities, measured in mege-watts, for $i=1, \cdots, I$, where I denotes the number of thermal units in the system. A production cost curve is defined on the feasible operating range for each unit. This curve is assumed to be a convex, piecewise linear function. The slope of each segment corresponds to the incremental production cost. Let K_i be the number of linear segments in the production cost curve for unit i, and let M_{ik} be the maximum amount of power that generator i can produce at the k-th incremental production cost, g_{ik}, for $k=i, \cdots, K_i$ and $i=1, \cdots, I$.

An integer variable x_{it} is used to indicate whether a specified thermal unit is operating during a period. In particular, $x_{it}=1$ if thermal unit i is operating in period t and $x_{it}=0$ otherwise, where $i=1, \cdots, I$ and $t=1, \cdots, T$. The continuous variable y_{ikt} represents the proportion of the available capacity M_{ik} that is actually used throughout period t, where $k=1, \cdots, K_i$.

The total energy output from unit i in period t is

$$m_i x_{it} + \sum_{k=1}^{K_i} M_{ik} y_{ikt},$$

assuming $0 \leq y_{ikt} \leq x_{ik}$. Let $w_{it}=1$ if unit i is started in period t and $w_{it}=0$ otherwise; $z_{it}=1$ if unit i is shut down in period t and $z_{it}=0$ otherwise; c_i be the start-up cost for generator i ; d_i be the shut down cost for generator i; g_i be the cost of operating generator i at its minimum capacity.

Assuming the planning horizon is divided into one hour interval, the unit commitment problem for a thermal system is formulated as follows :

(PT)

$$\min \sum_{i=1}^{I} \{\sum_{t=1}^{T} \{c_i w_{it} + d_i z_{it} + g_i x_{it} + \sum_{k=1}^{K_i} M_{ik} g_i\ y_{ikt}\}\}$$

$$\text{s. t. } \sum_{i=1}^{I} (m_i x_{it} + \sum_{k=1}^{K_i} M_{it} y_{ikt}) \geq D_t, \text{ for all } t, \tag{1}$$

$$\sum_{i=1}^{I} M_i x_{it} \geq R_t, \text{ for all } t, \tag{2}$$

$$\left.\begin{array}{l} 0 \le y_{ikt} \le x_{it},\ \ w_{it} \ge x_{it} - x_{i,t-1},\ \ z_{it} \ge x_{i,t-1} - x_{it}, \\ w_{it} \ge 0,\ \ z_{it} \ge 0,\ \ x_{it} \in \{0, 1\},\ \text{for all i, k and t.} \end{array}\right\} \quad (3)$$

The demand constraint (1) implies that the total generated power should be greater than or equal to the electricity demand, D_t, and the spinning reserve constraint (2) implies that the total maximum capacity of thermal units which are on the line should be greater than a specified minimum R_t. For a detailed explanation of this model, see Muckstadt and Koening (p. 392).

Now let us consider pumped-storage hydroelectric units. A pumped-storage hydroelectric unit has two reservoirs - an upper reservoir and a lower reservoir. The upper reservoir has a finite capacity while the lower reservoir has unlimited capacity. In a given time period the unit may pump water from lower resservoir to upper reservoir or may generate electricity by releasing water in the upper reservoir. The state equation for j-th (j=1,⋯, J) pumped-storage hydroelectric unit can be stated as follows :

$s_{jt} = s_{j,t-1} + p_{jt} - q_{jt}$, for $t = 1, \cdots, T$ and

$s_{jo} = s^*_{jo}$,

where s_{jt} is the amount of water in the upper reservoir of the unit j at the end of period t, p_{jt} is the amount of water pumped to the upper reservoir of unit j in period t, q_{jt} is the amount of water released from the upper reservoir to generate electricity at unit j in period t and s^*_{jo} is the amount of water in the upper reservoir at the beginning of the planning horizon. Since it is not possible to simultaneously pump and generate at a single pumped-storage hydroelectric unit, we need a constraint $p_{jt}q_{jt} = 0$ for all j and t. Later, it will be shown that this constraint can be eliminated.

The generated power in a hydroelectric unit is a function of the outflow and the head. But in a pumped-storage hydroelectric unit the head variation due to the volume of outflow is not critical since the head is significantly greater than the water level in the upper reservoir. Thus we can assume that the generated power is a function of the outflow only. Let a_j be the electric power generated by releasing one unit of water from the upper reservoir and b_j be the electric power consumed to pump one unit of water at j-th pumped-storage hydroelectric unit. The ratio a_j/b_j is called the efficiency of the pumped-storage hydroelectric unit j. From the second law of thermal physics, $a_j/b_j < 1$.

This number is, in most cases, around 70%.

The unit commitment problem for a system with thermal and pumped-storage hydroelectric units is now formulated as follows :

(PTS)

$$\min \sum_{i=1}^{I} \{ \sum_{t=1}^{T} (c_i w_{it} + d_i z_{it} + g_i x_{it} + \sum_{k=1}^{K_i} M_{ik} g_{ik} y_{ikt}) \}$$

$$\text{s. t.} \ \sum_{i=1}^{I} (m_i x_{it} + \sum_{k=1}^{K_i} M_{ik} y_{ikt}) + \sum_{j=1}^{J} a_j q_{jt} - \sum_{j=1}^{J} b_j p_{jt} \ge D_t, \ \text{for all } t, \quad (4)$$

$$\sum_{i=1}^{I} M_i x_{it} + \sum_{j=1}^{J} b_j P_j \ge R_t, \ \text{for all } t, \quad (5)$$

$$\left. \begin{array}{l} 0 \le y_{ikt} \le x_{it}, \ w_{it} \ge x_{it} - x_{i,t-1}, \ z_{it} \ge x_{i,t-1} - x_{it}, \\ w_{it} \ge 0, \ z_{it} \ge 0, \ x_{it} \in \{0, 1\}, \ \text{for all i, k and t,} \end{array} \right\} \quad (6)$$

$$s_{jt} = s_{j,t-1} - q_{jt} + p_{jt}, \ \text{for all j and t,} \quad (7)$$

$$0 \le s_{jt} \le S_j, \ \text{for all j and t,} \quad (8)$$

$$0 \le q_{jt} \le Q_j, \ \text{for all j and t,} \quad (9)$$

$$0 \le p_{jt} \le P_j, \ \text{for all j and t,} \quad (10)$$

$$s_{jo} = s^*_{jo}, \ s_{jT} = s^*_{jT}, \ \text{for all j,} \quad (11)$$

where S_j is the capacity of the upper reservoir of the pumped-storage hydroelectric unit j, Q_j and P_j are respectively the maximum generating capacity and pumping capacity of the unit j, and s^*_{jo} and s^*_{jT} are respectively the initial and the final values for s_{jt}. In the demand constraints (4) the net effect of pumped-storage hydroelectric units is added. In the spinning reserve constraints (5) the maximum generation capacity of pumped-storage hydroelectric units is added to the spinning reserve since a pumped-storage hydroelectric unit can generate electricity within a minute.

The feasible operation condition $q_{jt}p_{jt}=0$ is guaranteed at an optimal solution of the problem (PTS) from the following theorem :

(Theorem 1) At an optimal solution of (PTS) , for any time period t if

$y_{ikt} > 0$ for some i and k then $q_{jt}p_{jt}=0$ for any j.

(Proof) Suppose at an optimal solution $q_{jt} > 0$, $p_{jt} > 0$ and $y_{ikt} > 0$ for some j, t, i and k. Find k* such that $y_{ik^*t} > 0$ and $y_{ikt}=0$ for all $k > k^*$. Then for a sufficiently small $\varepsilon > 0$, a solution constructed by replacing q_{jt}, p_{jt} and y_{jk^*t} respectively by $(q_{jt}-\varepsilon)$, $(p_{jt}-\varepsilon)$ and $(y_{ik^*t}-(b_j-a_j)\varepsilon/M_{ik^*})$ is a feasible solution and has a strictly less objective function. This is a contradiction.

(Note 1) If $y_{ikt}=0$ for all i and k then it is possible to have an optimal solution with $q_{jt}>0$ and $p_{jt}>0$. In this case we can always construct an another optimal solution with $q_{jt}p_{jt}=0$ by simply reducing q_{jt} and p_{jt} until either one becomes zero.

In a real operation, we need to consider the minimum down-time, minimum up-time, time-dependent start-up cost and ramping hours (the start-up response time of a thermal unit). All these constraints are not coupling constraints of different units. Inclusion of these constraints does not change the Lagrangean relaxation and decomposition procedure. These constraints will only changes the subproblem for a thermal unit. In the relaxation and decomposition procedure explained in the next section we do not consider these constraints for simplicity of presentation. In section 4 we present computational results of our algorithm for both case with and the case without these constraints.

3. Relaxation and Decomposition of the Model

In applying the Lagrangean relaxation method, the constraints (4) and (5) are selected to be incorporated into the objective function. Let u_t and v_t be the nonnegative Lagrangean multipliers corresponding to constraints (4) and (5), respectively. Then the resulting relaxed porblem is

(PR)

$$L(u, v) = \min \sum_{i=1}^{I} \{ \sum_{t=1}^{T} \{c_i w_{it} + d_i z_{it} + g_i x_{it} + \sum_{k=1}^{K_i} M_{ik} g_{ik} y_{ikt}\}\}$$
$$+ \sum_{t=1}^{T} u_t (D_t - \sum_{i=1}^{I} m_i x_{it} - \sum_{i=1}^{I} \sum_{k=1}^{K_i} M_{ik} y_{ikt} - \sum_{j=1}^{J} a_j q_{jt} + \sum_{j=1}^{J} b_j p_{jt})$$
$$+ \sum_{t=1}^{T} v_t (R_t - \sum_{j=1}^{J} b_j P_j - \sum_{i=1}^{I} M_i x_{it})$$

subject to constraints (6)-(11).

This problem decomposes into I single thermal unit subproblems and J single pumped-storage hydroelectric unit subproblems of the following form : For $i=1, \cdots, I$,

(SPTi)

$$\min \sum_{t=1}^{T} \{c_i w_{it} + d_i z_{it} + (g_i - v_t M_i - u_t m_i) x_{it} + \sum_{k=1}^{K} M_{ik} (g_{ik} - u_t) y_{ikt}\}$$

subject to constraints (6) for the given i, and for $j=1, \cdots, J$,

(SPPj)

$$\min \sum_{t=1}^{T} \{u_t b_j p_{jt} - u_t a_j q_{jt}\}$$

subject to constraints (7)-(11) for the given j.

Muckstadt and Koenig showed that the problem (SPTi) can be converted into a shortest-path problem on an acyclic network (If minimum down-time, minimum up-time, time-dependent start-up cost and ramping hours are included then this problem can be solved using dynamic programming.) The subproblem for a pumped-storage unit j, (SPPj), is a linear programming problem and can be directly solved using the simplex method. But this problem has a special structure and can be converted to a well-known minimum cost flow problem in a capacitated directed network. Figure 1 shows the network (subscript j is deleted).

Node 0 denotes the lower reservoir and node t $(t=1, \cdots, T)$ denotes the upper reservoir at period t. The first term on each arc is the variable representing the flow on the arc. The first and second terms in the parenthesis on each arc denote the cost per unit flow and the arc's capacity. Then the problem (SPPj) is equivalent to finding a minimum cost flow of this capacitated directed network such that the net inflow at node (1) is s_0^*, the net outflow at node T is s_T^* and the net inflow at node (0) is $(s_T^*-s_0^*)$ (outfow, if negative). In the remaining part of this paper we will assume that $s_T^*=s_0^*=s^*$ to reduce the problem into a minimum cost flow problem with a single source and a single sink (This is a quite reasonable assumption in a practical planning. Even if $s_T^* \neq s_0^*$, we can convert this problem into a minimum cost flow problem with a single source and a single sink by simply introducing an artificial node.) In this case s^* is called the value of a flow. Out-of-Kilter algorithm (Minty[1960], Fulkerson[1961]), dual method (Busacker and Gowen[1961]) and primal method (Klein[1971]) are developed for the minimal cost flow problem. The algorithm suggested in Lawler [1976, pp. 129-133] which is a combination of the dual method and the primal method has been used here.

The subproblem (SPPj) also generates an optimal solution satisfying the operational constraints, $q_{jt}p_{jt}=0$.

(Theorem 2) If $u_t > 0$ then $q_{jt}p_{jt}=0$ at an optimal solution of (SPPj).

(Proof) If $q_{jt} > 0$, $p_{jt} > 0$ and $u_t > 0$ for some j and t then we can find another feasible solution to (SPPj) with strictly less objective function by reducing both q_{jt} and p_{jt} by a sufficiently small amount.

(Not 2) If $u_t = 0$ then it is possible to have optimal solution with $q_{jt} > 0$ and $p_{jt} > 0$. In this case we can construct another optimal solution with $q_{jt}p_{jt} = 0$ by simply reducing them until ether one becomes zero.

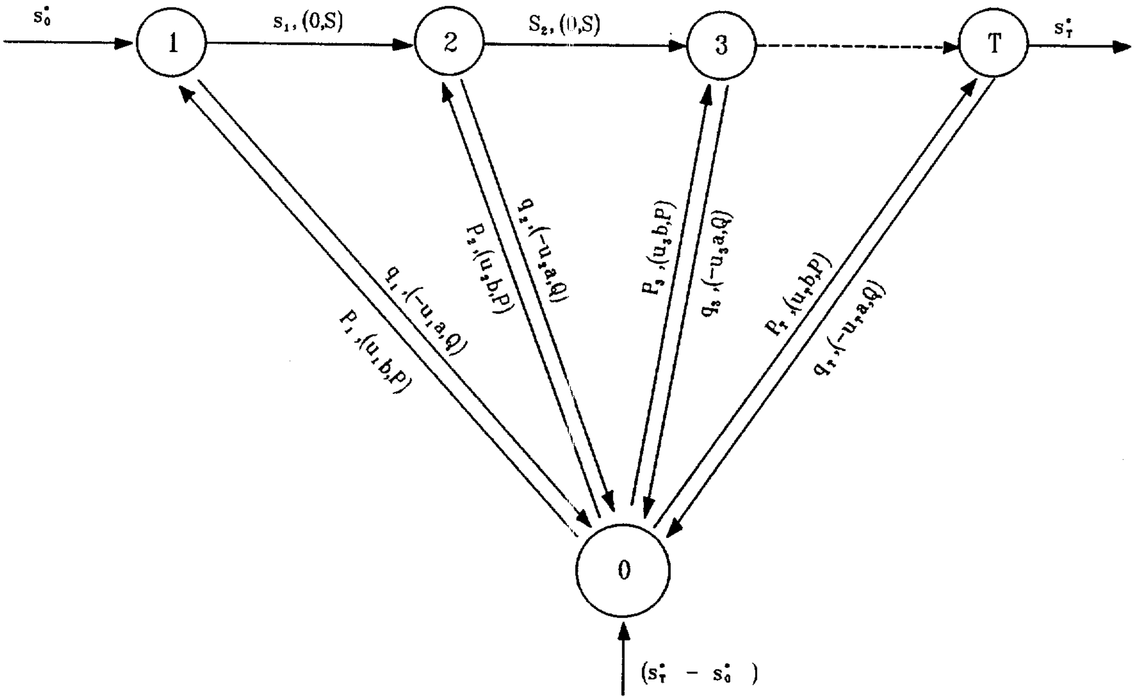

Figure 1. Network representing the minimum cost flow problem

4. Algorithm

The Lagrangean relaxation method is much more powerful if it is possible to find a feasible solution of the original problem from the solution of the relaxed problem. Then the Lagrangean L(u, v) provides a lower bound while the feasible solution provides an incumbent solution and an upper bound. In the unit commitment problem this is not always possible. If the solution of the relaxed problem satisfies the spinning reserve constraints (5) then we can always construct a solution which also satisfies the demand constraints (4). Furthermore, this feasible solution provides the incremental cost of the thermal system which can be used in improving the Lagrangean multiplier updating procedure. Our computational experience for a real-case test problem shows that about 10 itearations out of a total of 28 iterations provide incumbent solutions.

For given Lagrangean multipliers $\{u_t\}$ and $\{v_t\}$, let $\{x^*, w^*, z^*, y^*, p^*, q^*, s^*\}$ be the solution of the subproblems (SPTi) and (SPPi). There are three possile cases.

(Case 1) The solution of the subproblems satisfies both the demand constraints (4) and the spinning reserve constraints (5).

(Case 2) The solution of the subproblems satisfies only the spinning reserve constraints (5).

(Case 3) The solution of the subproblems does not satisfy the spinning reserve constrinats (5).

Finding a feasible solution

(Case 1) The solution of the subproblems is a feasible solution of the original problem. This is a candidate for a new incumbent solution. If the multipliers, u_t and v_t, and constraints (4) and (5) satisfy the complementarity condition (Geoffrion[1974]), then this is an optimal solution. Otherwise a better feasible solution can be found by using the procedure explained in (Case 2).

(Case 2) In this case there are enough spinning reserve to cover demand and we can find a good feasible solution of y_{ikt} for the given values of $\{x^*, w^*, z^*, p^*, q^*, s^*\}$ by simply solving the following economic dispatching problem for each time period t : Let $I_t^* = \{i : x_{it}^* = 1\}$.

$$\text{(ED)} \qquad \min \sum_{i \in I_t^*} (g_i + \sum_{k=1}^{K_i} M_{ik} g_{ik} y_{ikt})$$

$$\text{s.t.} \sum_{i \in I_t^*} (m_i + \sum_{k=1}^{K_i} M_{ik} y_{ikt}) \geq D_t^*,$$

$$0 \leq y_{ikt} \leq 1, \text{ for all } i \text{ and } k,$$

where $D_t^* = D_t - \sum_{j=1}^{J} a_j q_{jt}^* + \sum_{j=1}^{J} b_j p_{jt}^*$.

This problem can be easily solved in the following greedy method.

(Algorithm for (ED))

Step 0 Let $L = D_t^* - \sum_{i \in I_t^*} m_i$.

$A = \{(i, k) \mid i \in I_t^*, \ k = 1, \cdots, K_i\}$.

Step 1 Let $g_{i^*k^*} = \min_{(i,k) \in A} g_{ik}$.

If $M_{i^*k^*} \leq L$, then let $y_{i^*k^*t}=1$ and go to Step 2.

Otherwise, let $y_{i^*k^*t}=M_{i^*k^*}/L$, $g_t^*=g_{i^*k^*}$ and Stop.

Step 2 Let $L=L-M_{i^*k^*}$ and $A=A-(i^*, k^*)$.

Go to Step 1.

The value g_t^* is the incremental cost of the thermal system in time period t.

(Case 3) In this case we can not currently find a feasible solution.

The Lagrangean multipliers are updated in the following method :

Updating Lagrangean multipliers u_t and v_t

(Case 1 and Case 2) The Lagrangean multiplier v_t corresponding to the spinning reserve constraint is updated using the usual subgradient optimization procedure (See Held et al [1974] for classical exposition of the method and its implementation). The Lagrangean multiplier u_t corresponding to the demand constraint is updated using both subgradient and incremental cost. Suppose the subgradient method updates u_t to u_t^*. Then a convex combination of u_t^* and the incremental cost obtained from problem (ED), g_t^*, is used as the new update value of u_t.

(Case 3) The conventional subgradient method is used to update both u_t and v_t.

In order to start the algorithm we need to choose an initial value of u_t and v_t. To choose good initial values we use the following method :

Initialization

(Initialization of u_t) A heuristic unit commitment method based on a priority list is used to find a feasible unit commitment solution. A priority list for thermal units is constructed based on the efficiency at the minimum operation level, g_i/m_i. Then for each time period t, we find i_t^* such that $\sum_{i=1}^{i_t^*} M_i + \sum_{j=1}^{J} b_j P_j \geq R_t$. Let $x_{it}=1$ if $i \leq i_t^*$ and 0 otherwise. This is a feasible unit commitment solution. Then we solve the economic dispatching problem (ED) and find the incremental cost g_t^*. This incremental cost is used as the initial value for u_t.

(Initialization of v_t) We use the following formula for the initial value of v_t :

$v_t = (g_{i_t^*+1} - g_t^* M_{i_t^*+1}) / M_{i_t^*+1}$.

The numerator and the denominator are, respectively, approximations of the net increases of the fuel cost and the spinning reserve induced by introducing the next unit.

Termination criterion

The algorithm terminates when the incumbent solution is within a prespecified tolerance of the best lower bound from Lagrangean.

Figure 2 shows the flow chart of the whole algorithm.

4. Computational Results

Our algorithm has been programmed in FORTRAN IV. The example is a real Korean power generating system comprising 39 thermal units (unclear, fossil fueled, gas turbine) and 1 pumped-storage hydroelectric unit.

Most of the works about unit commitment with thermal units considered only one or two days of planning horizon (See Table 1). But the unit commitment problem with pumped storage units should be considered in a longer planning horizon. Figure 3 shows a significant error which is frequently occurred when one considers the problem for a one day planning horizon. Suppose the load pattern for two days is given as shown in Figure 3. If we perform two separate one-day planning then it is optimal to pump at

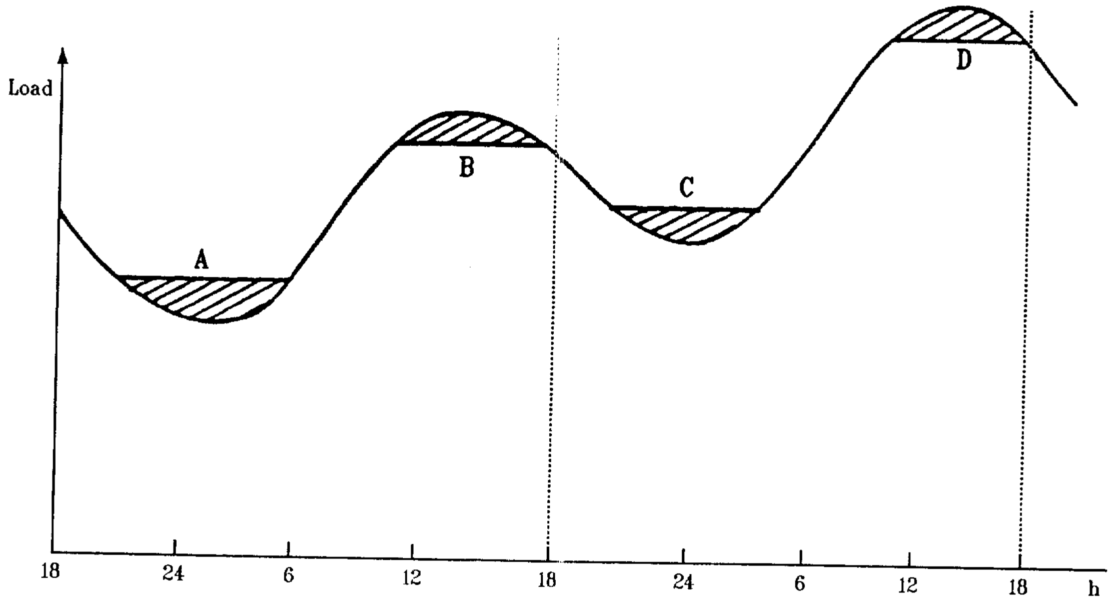

Figure 3. A possible significant error of single-day planning

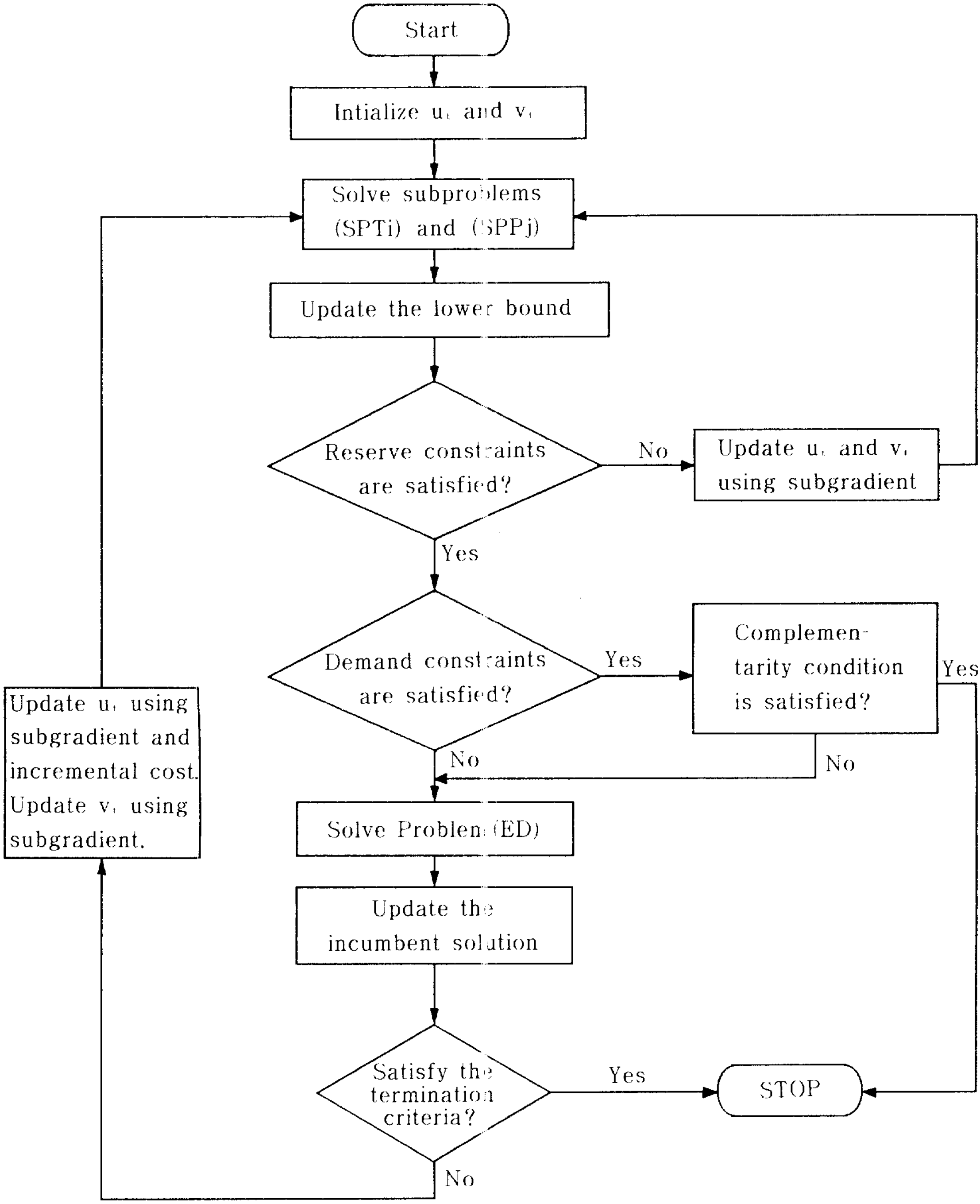

Figure 2. Flow chart of the algorithm

A and C and generate at B and D from pumped-storage units. However, if we perform two-day planning then it is optimal to pump only at A and generate only at D. Hence in the two single-day planning an inefficient operations B and C are generated. In our test problem we considered 168 hours (one week). Our computational results are summarized and compaired with others in Table 1.

5. Summary and extensions

In this paper, we have presented an algorithm for the unit commitment problem with thermal and pumped-storage hydroelectric units and shown that it finds very accurate solutions (within 0.5 percent of optimal) very quickly. Muckstadt and Koenig used a branch-and-bound method to find an optimal solution. This was the reason why they could not solve a large-scale problem. But we could find an incumbent solution within .5% of optimal without using branch-and-bound method.

Hydro units are usually hydraulically coupled and it is not possible to decompose into separate hydro subproblems unless we introduce more Lagrangean multipliers. Inclusion of hydro units is under investigation.

Table 1. Computational Results

	Number of thermal Units	Number of pumed-storage Units	Number of Planning periods	Error Tolerance (%)	CPU time (seconds)
Ours[1] Case 1	39	1	168	.27[4]	201[6]
Case 2	39	2	168	.32[4]	193[6]
Case 3	39	1	168	.47[4]	497[6]
Muckstadt, et al.[2] (largest two cases)	15	0	12	1.[5]	27.19[7]
	15	0	12	.5[5]	>60.0[7]
Bertsekas, et al. (largest three cases)	200	0	24	.27[4]	709[8]
	200	0	24	.09[4]	732[8]
	200	0	24	.15[4]	588[8]
Merlin et al.[3]	172	0	48	.42[4]	120[9]

(1) Piecewise linear cost function is used.

Case 1 : Time independent start-up cost, instantaneous start-up without minimum down-time and minimum up-time.

Case 2 : Same as the Case 1 except that one artificial pumped-storage hydroelectric unit is added.

Case 3 : Time dependent start-up cost, with ramping hours, minimum down-time and minimum up-time.

(2) Piecewise linear cost function is used. Branch-and-bound method is used to find an incumbent solution.

(3) Linear cost function is used.

(4) Realized error bound.

(5) Pre-specified error tolerance.

(6) CDC CYBER 170/845

(7) IBM 370/168

(8) VAX-11/780

(9) IBM 3081

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