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Optimal Reporting Strategy of an Insured — Dynamic Programming Approach —

Jae Hyung Min*

Abstract

We consider an insured who wishes to determine his optimal reporting strategy over a given planning horizon, when he has the option of reporting or not reporting his at-fault accidents. Assuming that the premium in future period is continually adjusted by the insured's loss experience, the insured would not report every loss incurred. Rather, considering the benefits and costs of each decision, the insured may want to seek a way of optimizing his interests over the planning horizon. The situation is modeled as a dynamic programming problem. We consider an insured's discounted expected cost minimization problem, where the premium increase in future period is affected by the size of the current claim. More specifically, we examine two cases : (1) the premium increase in the next period is a linear function (a constant fraction) of the current claim size; (2) the premium increase in the next period is a concave function of the current claim size. In each case, we derive the insured's optimal reporting strategy.

* Department of Industrial Engineering, Kwandong University

1. Introduction

In real life, people, as insureds, sometimes do not report their fault related accidents or losses (accidents or losses caused by the insured's fault) to the insurance company. This behavior happens because they want to maintain good "credentials" and hence prevent a possible increase in their premiums during a later period. One example of this case is automobile insurance, in which the individual insured's premium is based on his past driving record. In this paper, we attempt to answer the following question : what is the optimal reporting strategy when an insured has the choice of reporting or not reporting his at-fault accidents to the insurance company so as to maintain a good "record" and prevent an increase in future premiums because of his loss experience?[1] This particular problem reflects the common hesitancy of an insured person to report a small loss, and relaxes the assumption that an individual always report his loss to the insurer in order to restore his wealth position.

Suppose you, as an insured, cause an accident during a period. Facing a financial loss from the accident that could be covered by the insurance, you may take one of two possible actions. The first action is that you report the loss to the insurance company for reimbursement. The second is simply that you do not claim your loss. By taking the second action, your wealth position is reduced by the amount of the loss incurred since you pay the loss yourself. Obviously, if you decide to report your loss, then you do not have to worry about the loss incurred since the company will pay it to restore your wealth position, assuming that the loss is fully insured. However, you may be concerned about a possible premium increase in a future period. On the other hand, if you decide not to report, you will experience a financial loss; but in this case, you do not have to worry about your premium being increased because of your claim. Considering the benefits and costs of each decision, you may want to seek a way of optimizing your interests.

Reflecfting this human behavior, the goal of this paper is to identify the insured's optimal reporting strategy each period which minimizes his discounted expected costs during a given planning horizon.[2] To achieve this goal, we present two dynamic programming models. In these models, it is assumed that the amount of the increase in future premium depends on the severity of the loss which is claimed by the insured during a current period. That is, the insurer adjusts the premium each period to reflect the insured's current claim size, where the size of claim is random. The state variables of each model are the current premium and current size of loss. More specifically, in the first model, the amount of premium increase in the next period is assumed to be a linear function (a constant fraction) of the size of the current loss which is reported, and in the second one, it is assumed to be a concave function of the current loss size which is reported. From each of these two models, conditions are derived to determine the insured's optimal reporting strategy each period.

2. Dynamic Programming Model where Premium Increase is a Linear Function of the Claim Size

In this section and the next section, we examine an insured's optimal reporting strategy which minimizes his discounted expected costs during a given planning horizon. To do this, we now assume that the amount of premium increase depends solely on the amount of the current loss which is reported. Also, we assume that a deductible is zero (i. e., we consider a full-cover policy). Assume that the loss amounts are i. i. d. random variables $X_1, X_2, \cdots$ from some distribution F[3]. The premium can change (increase) only when a loss occurs and it is reported. Suppose r and x denote the current premium and current loss, repectively. If the insured reports x to the insurance company in order to restore his wealth position during a current period, then the premium at the beginning of the next period will be $r+kx$ where $0 < k < 1$. That is, the premium increse is a fraction of the current loss which is reported.

Let $V_n(r, x)$ be the insured's minimum discounted expected costs when the current premium and the current loss are r and x, respectively, and n periods remain. If we assume that the costs incurred from the next period are discounted by factor β $(0 < \beta < 1)$, then the dynamic programming recursion for V_n, considering the insured's two decisions : "not report" or "report", will be

$$V_n(r,x)=\min\begin{cases} r+x+\beta EV_{n-1}(r,X) & \text{(not report)} \\ r+\beta EV_{n-1}(r+kx,X) & \text{(report)} \end{cases}$$

$$=r+\min[x+\beta EV_{n-1}(r,X),\ \beta EV_{n-1}(r+kx,X)] \quad (1)$$

where E denotes expectation operator and random variable X is the size of the next loss. That is, if the insured has loss x, but he decides not to report it, then his costs incurred during a current period are premium r plus current loss x. From such a decision, the premium in the next period will not increase. On the other hand, if the insured decides to report the loss incurred, then his current cost is just the current premium r since the company will reimburse his current loss x. However, his premium in the next period will increase by a fraction of the current loss, written kx. Considering benefits and costs of each decision, the insured wishes to identify the optimal reporting strategy each period in order to minimize the sum of the current costs and the discounted expected costs in future periods.

Before identifying the insured's optimal reporting strategy for the above general n-period model, we first look at the insured's reporting behavior for two, three, and four period models.

Using (1), and assuming $V_0=0$, we have, when $n=1$,

$$V_1(r,x)=r+\min[x,0]=r$$

which says that the optimal decision for $n=1$ is "report", and

$$V_1(r,x)=r$$

When $n=2$,

$$\begin{aligned} V_2(r,x) &= r+\min[x+\beta EV_1(r,X),\ \beta EV_1(r+kx,X)] \\ &= r+\min[x+\beta E(r),\ \beta E(r+kx)] \\ &= r+\min[x+\beta r,\ \beta r+\beta kx] \\ &= r+\beta r+\min[x,\ \beta kx] \end{aligned}$$

Since the insured's discounted expected costs from both decisions are linear in x with the same intercept term $(r+\beta r)$, we just need to compare the slope of x for each decision. However, since βk is clearly less than 1, the optimal decision for $n=2$ is "report", and

$$V_2(r, x) = \beta kx + r + \beta r$$

Now, when n=3, we have

$$V_3(r, x) = r + \min[x + \beta EV_2(r, X), \beta EV_2(r + kx, X)]$$
$$= r + \min[x + \beta E(\beta kX + r + \beta r), \beta E(\beta kX + (r + kx) + \beta(r + kx))]$$
$$= r + \min[x + \beta^2 kEX + \beta r + \beta^2 r, \beta^2 kEX + \beta r + \beta kx + \beta^2 r + \beta^2 kx]$$
$$= r + \beta r + \beta^2 r + \beta^2 kEX + \min[x, (\beta k + \beta^2 k)x]$$

From this equation for $V_3(r, x)$, we see that the optimal reporting strategy depends on the slope of x for each decision. That is,
if $1 > \beta k + \beta^2 k$, then the optimal decision for n=3 is "report", and

$$V_3(r, x) = (\beta k + \beta^2 k)x + r + \beta r + \beta^2 r + C_3$$

Otherwise, the optimal decision for n=3 is "not report", and

$$V_3(r, x) = x + r + \beta r + \beta^2 r + C_3$$

where $C_3 = \beta^2 kEX$ does not depend on r or x.

Now, to calculate $V_4(r, x)$, we consider 2 cases.

(i) Suppose $V_3(r, x) = (\beta k + \beta^2 k)x + r + \beta r + \beta^2 r + C_3$ (i. e., $\beta k + \beta^2 k < 1$). Then,

$$V_4(r, x) = r + \min[x + \beta EV_3(r, X), \beta EV_3(r + kx, X)]$$
$$= r + \min[x + \beta E((\beta k + \beta^2 k)X + r + \beta r + \beta^2 r + C_3),$$
$$\beta E((\beta k + \beta^2 k)X + (r + kx) + \beta(r + kx) + \beta^2(r + kx) + C_3)]$$
$$= r + \min[x + \beta^2 kEX + \beta^3 kEX + \beta r + \beta^2 r + \beta^3 r + \beta C_3,$$
$$\beta^2 kEX + \beta^3 kEX + \beta r + \beta kx + \beta^2 r + \beta^2 kx + \beta^3 r + \beta^3 kx + \beta C_3]$$
$$= r + \beta r + \beta^2 r + \beta^3 r + \beta^2 kEX + \beta^3 KEX + \beta C_3 + \min[x, (\beta k + \beta^2 k + \beta^3 k)x]$$

(ii) Suppose $V_3(r, x) = x + r + \beta r + \beta^2 r + C_3$ (i. e., $\beta k + \beta^2 k > 1$). Then,

$$V_4(r, x) = r + \min[x + \beta EV_3(r, X), \beta EV_3(r + kx, X)]$$
$$= r + \min[x + \beta E(X + r + \beta r + \beta^2 r + C_3),$$
$$\beta E(X + (r + kx) + \beta(r + kx) + \beta^2(r + kx) + C_3)]$$
$$= r + \min[x + \beta EX + \beta r + \beta^2 r + \beta^3 r + \beta C_3,$$
$$\beta EX + \beta r + \beta kx + \beta^2 r + \beta^2 kx + \beta^3 r + \beta^3 kx + \beta C_3]$$
$$= r + \beta r + \beta^2 r + \beta^3 r + \beta EX + \beta C_3 + \min[x, (\beta k + \beta^2 k + \beta^3 k)x]$$

Hence, from (i) and (ii), we see that the optimal reporting strategy again depends on the slope of x for each decision. That is,
if $1 > \beta k+\beta^2 k+\beta^3 k$, then the optimal decision for n=4 is "report", and

$$V_4(r, x) = (\beta k+\beta^2 k+\beta^3 k)x+r+\beta r+\beta^2 r+\beta^3 r+C_4$$

Otherwise, the optimal decision for n=4 is "not report", and

$$V_4(r, x) = x+r+\beta r+\beta^2 r+\beta^3 r+C_4$$

where C_4 denotes a constant which does not depend on r or x, when 4 periods remain.

By this fashion, we conjecture that

$$V_n(r, x) = \min[x, (\beta k+\beta^2 k+\beta^3 k+\cdots+\beta^{n-1}k)x]+r+\beta r+\beta^2 r+\beta^3 r+\cdots+\beta^{n-1}r+C_n$$

where C_n is a constant which does not depend on r or x, when n periods remain. If $V_n(r, x)$ has the form we conjecture, then the optimal reporting strategy for the n-period model depends on the magnitude of $\beta k+\beta^2 k+\beta^3 k+\cdots+\beta^{n-1}k$. The conjecture is formalized by the following proposition.

PROPOSITION 1 : Suppose the premium increase in the next period depends solely on the amount of current loss x which is reported, and the amount of increase is a fraction of loss x, written $kx(0 < k < 1)$. Then, for a given planning horizon, say $n(\geq 2)$ periods, $V_n(r, x)$ has the following form

$$V_n(r, x) = \min[x, (\beta k+\beta^2 k+\beta^3 k+\cdots+\beta^{n-1}k)x]+r+\beta r+\beta^2 r+\beta^3 r+\cdots+\beta^{n-1}r+C_n$$

where C_n is a contant which does not depend on r or x, when n periods remain. If $1 > \beta k+\beta^2 k+\beta^3 k+\cdots+\beta^{n-1}k$, then the insured should report his loss; otherwise he should not.

PROOF : The proof is by backward induction on n. When n=2, 3, 4, the result is true. Now we assume that the result is valid for n-1. That is, we can write

$$V_{n-1}(r, x) = \min[x, (\beta k+\beta^2 k+\beta^3 k+\cdots+\beta^{n-2}k)x]+r+\beta r+\beta^2 r+\beta^3 r+\cdots+\beta^{n-2}r+C_{n-1}$$

where C_{n-1} is a constant which does not depend on r or x, when (n-1) periods remain. To prove the result, we consider 2 cases.

(i) Suppose $V_{n-1}(r, x) = x+r+\beta r+\beta^2 r+\beta^3 r+\cdots+\beta^{n-2}r+C_{n-1}$ (i. e., $\beta k+\beta^2 k+\beta^3 k+\cdots+\beta^{n-2}k > 1$). Then, by (1),

$$V_n(r, x) = r + \min[x + \beta EV_{n-1}(r, X), \beta EV_{n-1}(r+kx, X)]$$
$$= r + \min[x + \beta E(X + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}),$$
$$\beta E(X + (r+kx) + \beta(r+kx) + \beta^2(r+kx) + \beta^3(r+kx) + \cdots + \beta^{n-2}(r+kx) + C_{n-1})]$$
$$= r + \min[x + \beta EX + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + \beta C_{n-1},$$
$$\beta EX + \beta r + \beta kx + \beta^2 r + \beta^2 kx + \beta^3 r + \beta^3 kx + \cdots + \beta^{n-1} r + \beta^{n-1} kx + \beta C_{n-1}]$$
$$= r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + \beta EX + \beta C_{n-1} + \min[x, (\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-1} k) x]$$

(ii) Suppose $V_{n-1}(r, x) = (\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-2} k) x + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}$ (i. e., $\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-2} k < 1$). Then,

$$V_n(r, x) = r + \min[x + \beta EV_{n-1}(r, X), \beta EV_{n-1}(r+kx, X)]$$
$$= r + \min[x + \beta E((\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-2} k) X + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}),$$
$$\beta E((\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-2} k) X + (r+kx) + \beta(r+kx) + \beta^2(r+kx) +$$
$$\beta^3(r+kx) + \cdots + \beta^{n-2}(r+kx) + C_{n-1})]$$
$$= r + \min[x + (\beta^2 k + \beta^3 k + \cdots + \beta^{n-1} k) EX + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + \beta C_{n-1},$$
$$(\beta^2 k + \beta^3 k + \cdots + \beta^{n-1} k) EX + \beta r + \beta kx + \beta^2 r + \beta^2 kx + \beta^3 r + \beta^3 kx + \cdots + \beta^{n-1} r$$
$$+ \beta^{n-1} kx + \beta C_{n-1}]$$
$$= r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + (\beta^2 k + \beta^3 k + \cdots + \beta^{n-1} k) EX + \beta C_{n-1}$$
$$+ \min[x, (\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-1} k) x]$$

From (i) and (ii), we now see that $V_n(r, x)$ has the conjectured form. Hence, the result (the insured would report his loss each period until $\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-1} k \geqq 1$) follows. This completes the proof. □

The interesting thing from the result is that when we assume the amount of premium increase is a fraction of the current loss which is reported, the insured's decision of "reporting" or "not reporting" the loss does not depend on the amount of loss. Rather it depends on discount factor β, fractional coefficient k, and the number of periods which remain. Also, since the optimal reporting strategy for n-period model depends on the magnitude of $\beta k + \beta^2 k + \beta^3 k + \cdots + \beta^{n-1} k$, it is clear that the insured is less likely to report his loss as β, k, and n increase.

The next section describes the insured's optimal reporting strategy when the premium increase in the next period is a concave funtion of the current loss which is reported.

3. Dynamic Programming Model where Premium Increase is a Concave Function of the Claim Size

In this section, we again examine an insured's optimal reporting strategy which minimizes his discounted expected costs during a given planning horizon. The model description is mostly same as in the previous section. However, in this section we assume that the premium increase is a concave function of the current reported loss rather than a fraction of it. As before, let r and x denote the current premium and current loss, respectively. If the insured reports x to the insurance company in order to restore his wealth position during a current period, then the premium at the beginning of the next period will increase by $\triangle(x)$, where $\triangle(x)$ is a increasing, concave function of x with boundary condition $\triangle(0)=0$.

Let $V_n(r, x)$ be the insured's minimum discounted expected costs when the current premium and the current loss are r and x, respectively, and n periods remain. If we assume that the costs incurred from the next period are discounted by factor β $(0<\beta<1)$, then the dynamic programming recursion for V_n is

$$V_n(r, x)=\min\begin{cases} r+x+\beta EV_{n-1}(r, X) & \text{(not report)} \\ r+\beta EV_{n-1}(r+\triangle(x), X). & \text{(report)} \end{cases}$$
$$=r+\min[x+\beta EV_{n-1}(r, X), \beta EV_{n-1}(r+\triangle(x), X)] \tag{2}$$

As in the previous model, the insured's goal is to identify the optimal reporting strategy each period in order to minimize the sum of the current costs and the discounted expected costs in future periods which are affected by his current reporting decision.

Using (2), and assuming $V_0=0$, we have, when n=1,

$$V_1(r, x)=r+\min[x, 0]=r$$

which says that the optimal decision for n=1 is "report", and

$$V_1(r, x)=r$$

When n=2, we have

$$V_2(r, x)=r+\min[x+\beta EV_1(r, X), \beta EV_1(r+\triangle(x), X)]$$

$$= r + \min[x + \beta E(r), \beta E(r + \triangle(x))]$$
$$= r + \min[x + \beta r, \beta r + \beta\triangle(x)]$$
$$= r + \beta r + \min[x + \beta\triangle(x)]$$

Since $\triangle(0) = 0$, the insured's discounted expected costs from both decisions have the same intercept $(r+\beta r)$. Also, $\triangle(x)$ is increasing and concave in $x(\geqq 0)$. So, it is clear that linear function x and concave function $\triangle(x)$ will intersect at a certain point x, which is unique and greater than or equal to 0. Let $x_2^*(\geqq 0)$ be this point.[4] Then, the optimal decision for n=2 depends on the size of the current loss x. That is, with a critical point, x_2^*, the optimal reporting strategy for n=2 is characterized as follows :

if $x < x_2^*$, then the optimal decision for n=2 is "not report", and

$$V_2(r, x) = x + r + \beta r$$

Otherwise, the optimal decision for n=2 is "report", and

$$V_2(r, x) = \beta\triangle(x) + r + \beta r$$

Now, when n=3, we consider 2 cases.

(i) Suppose $V_2(r, x) = x + r + \beta r$. Then, by (2),

$$V_3(r, x) = r + \min[x + \beta EV_2(r, X), \beta EV_2(r + \triangle(x), X)]$$
$$= r + \min[x + \beta E(X + r + \beta r), \beta E(X + (r + \triangle(x)) + \beta(r + \triangle(x)))]$$
$$= r + \min[x + \beta EX + \beta r + \beta^2 r, \beta EX + \beta r + \beta\triangle(x) + \beta^2 r + \beta^2\triangle(x)]$$
$$= r + \beta r + \beta^2 r + \beta EX + \min[x, (\beta + \beta^2)\triangle(x)]$$

(ii) Suppose $V_2(r, x) = \beta\triangle(x) + r + \beta r$. Then,

$$V_3(r, x) = r + \min[x + \beta EV_2(r, X), \beta EV_2(r + \triangle(x), X)]$$
$$= r + \min[x + \beta E(\beta\triangle(X) + r + \beta r), \beta E(\beta\triangle(X) + (r + \triangle(x)) + \beta(r + \triangle(x)))]$$
$$= r + \min[x + \beta^2 E\triangle(X) + \beta r + \beta^2 r, \beta^2 E\triangle(X) + \beta r + \beta\triangle(x) + \beta^2 r + \beta^2\triangle(x)]$$
$$= r + \beta r + \beta^2 r + \beta^2 E\triangle(X) + \min[x, (\beta + \beta^2)\triangle(x)]$$

From (i) and (ii), we see that the optimal decision for n=3 also depends on the size of the loss x. That is, for n=3, there also exists a unique critical point of x, written $x_3^*(\geqq 0)$, at which linear function x and concave function $(\beta+\beta^2)\triangle(x)$ intersect. Hence, the optimal reporting strategy for n=3 is described as follows :

if $x < x_3^*$, then the optimal decision for $n=3$ is "not report", and

$$V_3(r, x) = x + r + \beta r + \beta^2 r + C_3$$

Otherwise, the optimal decision for $n=3$ is "report", and

$$V_3(r, x) = (\beta + \beta^2) \triangle(x) + r + \beta r + \beta^2 r + C_3$$

where C_3 represents a constant which does not depend on r or x, when 3 periods remain.

In this fashion, we conjecture that

$$V_n(r, x) = \min[x, (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-1}) \triangle(x)] + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + C_n$$

where C_n is a constant which does not depend on r or x, when n periods remain. If the conjecture is true, then the optimal reporting strategy for the general n-period model depends on the size of loss (i. e., whether or not the size of current loss is greater than the critical point of x at which linear function x and concave function $(\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-1}) \triangle(x)$ intersect). The formal argument is described in Proposition 2.

PROPOSITION 2 : Suppose the premium increase depends solely on the amount of the current reported loss x, and the amount of increase is a concave function of x, written $\triangle(x)$. Then, for a given planning horizon, say $n(\geq 2)$ periods, $V_n(r, x)$ has the following form

$$V_n(r, x) = \min[x, (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-1}) \triangle(x)] + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + C_n$$

where C_n is a constant which does not depend on r or x, when n periods remain, and there exists a critical point of x, written x_n^*, such that $x_n^* = (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-1}) \triangle(x_n^*)$ (i. e., x_n^* is the point at which the insured would be indifferent between reporting and not reporting). If $x > x_n^*$, then the insured should report his current loss; otherwise, he should not report it.

PROOF : The proof is by backward induction on n. When $n=2, 3$, the result is true. Now we assume that the result is valid for n-1. That is, we can write

$$V_{n-1}(r, x) = \min[x, (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-2}) \triangle(x)] + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}$$

where C_{n-1} is a constant which does not depend on r or x, when (n-1) periods remain. Also, let x_{n-1}^* denote the critical point such that $x_{n-1}^* = (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-2}) \triangle(x_{n-1}^*)$. Then, to prove the result, we consider 2 cases.

(i) Suppose $V_{n-1}(r, x) = x + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}$ (i. e., $x < x^*_{n-1}$). Then, by (2),

$$\begin{aligned}
V_n(r, x) &= r + \min[x + \beta EV_{n-1}(r, X),\ \beta EV_{n-1}(r + \triangle(x), X)] \\
&= r + \min[x + \beta E(X + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}), \\
&\qquad \beta E(X + (r + \triangle(x)) + \beta(r + \triangle(x)) + \beta^2(r + \triangle(x)) + \beta^3(r + \triangle(x)) \\
&\qquad + \cdots + \beta^{n-2}(r + \triangle(x)) + C_{n-1})] \\
&= r + \min[x + \beta EX + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + C_{n-1}, \\
&\qquad \beta EX + \beta r + \beta\triangle(x) + \beta^2 r + \beta^2\triangle(x) + \beta^3 r + \beta^3\triangle(x) + \cdots + \beta^{n-1} r + \beta^{n-1}\triangle(x) + \beta C_{n-1}] \\
&= r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + \beta EX + \beta C_{n-1} + \min[x,\ (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-1})\triangle(x)]
\end{aligned}$$

(ii) Suppose $V_{n-1}(r, x) = (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-2})\triangle(x) + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}$ (i. e., $x > x^*_{n-1}$). Then,

$$\begin{aligned}
V_n(r, x) &= r + \min[x + \beta EV_{n-1}(r, X),\ \beta EV_{n-1}(r + \triangle(x),\ X)] \\
&= r + \min[x + \beta E((\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-2})\triangle(X) + r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-2} r + C_{n-1}), \\
&\qquad \beta E((\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-2})\triangle(X) + (r + \triangle(x)) + \beta(r + \triangle(x)) + \beta^2(r + \triangle(x)) \\
&\qquad + \beta^3(r + \triangle(x)) + \cdots + \beta^{n-2}(r + \triangle(x)) + C_{n-1})] \\
&= r + \min[x + (\beta^2 + \beta^3 + \cdots + \beta^{n-1})E\triangle(X) + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + \beta C_{n-1}, \\
&\qquad (\beta^2 + \beta^3 + \cdots + \beta^{n-1})E\triangle(X) + \beta r + \beta\triangle(x) + \beta^2 r + \beta^2\triangle(x) \\
&\qquad + \beta^3 r + \beta^3\triangle(x) + \cdots + \beta^{n-1} r + \beta^{n-1}\triangle(x) + \beta C_{n-1}] \\
&= r + \beta r + \beta^2 r + \beta^3 r + \cdots + \beta^{n-1} r + (\beta^2 + \beta^3 + \cdots + \beta^{n-1})E\triangle(X) + \beta C_{n-1} \\
&\qquad + \min[x,\ (\beta + \beta^2 + \beta^3 + \cdots + \beta^{n-1})\triangle(x)]
\end{aligned}$$

From (i) and (ii), we have shown that $V_n(r, x)$ has the conjectured form. Hence, the result follows. This completes the proof. □

This result, which depends on the concavity of $\triangle(x)$, is quite intuitive. If the insured has a small loss, then he should not report it since the premium increase (if he reports it) is relatively large compared to his loss. On the other hand, if the amount of loss is quite large, then the insured should report it since the premium increase is relatively small compared to the loss. That is, the insured would rather submit to the premium increase than suffer a large loss. The reasoning behind this intuition lies in the concavity of $\triangle(x)$. Also, this intuition explains the reason why we do not consider the case where $\triangle(x)$ is a convex function of x. If $\triangle(x)$ is convex, then the insured's reporting strategy will be the opposite. That is, the insured will report a small loss, and he will not report a large loss. However, this argument leads to unrealistic results. After all, the one "big" reason why we have insurance is that we want to protect ourselves from a possible disaster, not from a petty loss.

The next result from the model is that the critical point of the size of loss increases as the number of periods which remain increases. Let x_n^* be the critical point when the current premium and loss are r and x, respectively, and n periods remain. Then by Proposition 2, $x_n^* = (\beta+\beta^2+\beta^3+\cdots+\beta^{n-1})\triangle(x_n^*)$. With this definition, we prove the following corollary.

COROLLARY 1 : $x_2^* \leqq x_3^* \leqq \cdots \leqq x_{n-1}^* \leqq x_n^*$.

PROOF : It suffices to prove that when $n \geqq 3$,

$$x_n^* \geqq (\beta+\beta^2+\beta^3+\cdots+\beta^{n-2})\triangle(x_n^*) \tag{3}$$

Now, by definition, $x_n^* = (\beta+\beta^2+\beta^3+\cdots+\beta^{n-1})\triangle(x_n^*)$

$$= (\beta+\beta^2+\beta^3+\cdots+\beta^{n-2})\triangle(x_n^*) + \beta^{n-1}\triangle(x_n^*)$$

However, $\beta^{n-1}\triangle(x_n^*) \geqq 0$ since $\triangle(\cdot)$ is increasing with boundary condition $\triangle(0)=0$. Therefore, (3) follows. This completes the proof. □

The result implies that the longer the planning horizon is, the less likely the insured is to report a small loss, as the critical point of the size of loss increases.

4. Summary

We considered an individual insured who wishes to seek a way of determining his optimal reporting strategy each period, when he has the option of reporting or not reporting his fault-related accidents. Assuming that future premium is continually adjusted by the insured's loss experience, the insured would not report every loss incurred. Dynamic programming was employed to model the problem over time. The goal of each model is to determine the insured's optimal reporting strategy each period which minimizes his discounted expected costs over a finite planning horizon. In each dynamic programming model, it was assumed that the premium each period is adjusted according to the size of the reported loss during the previous period. We examined two cases.

In the first model, the amount of premium increase in future period was assumed to be a fraction (a linear function) of the size of the insured's claim during a current period. The following results were derived from the model.

1. The insured's optimal reporting strategy each period does not depend on the size of the reported loss. Rather, it depends on the discount factor (β), fractional coefficient (k), and the number of remaining periods in his planning horizon (n). If $\beta k+\beta^2 k+\beta^3 k+\cdots+\beta^{n-1}k<1$, then the insured should report his loss; otherwise he should not.
2. The insured is less likely to claim his loss as the above parameters (β, k, and n) increase.

In the second model, we considered the case where the amount of premium increase in future period is a concave function of the size of the current claim. The results from the model are as follows.

1. Let x be the size of the current loss and $\triangle(x)$ be the amount of premium increase if the insured reports x. Then at each period, there exists a critical number x_n^* such that $x_n^*=(\beta+\beta^2+\beta^3+\cdots+\beta^{n-1})\triangle(x_n^*)$, where β and n denote the discount factor and the number of remaining periods, respectively. If the current loss is greater than this critical number, then the insured should claim the loss; otherwise, he should not.
2. The critical number x_n^* increases with the number of periods remaining in the insured's planning horizon. This result implies that the insured is less likely to report a small loss as he has more periods to go in his planning horizon.

5. Concluding Remarks

As a pioneering work, we attempted to identify the insured's optimal reporting strategy of his at-fault accidents over a given planning horizon. Unfortunately, there is no existing study which is concerned with this type of problem. Therefore, we hope that this study would be a stepping stone to further research of this kind. Of course, this study has limitations, as most of others do, which we would like to mention. In this paper, we have neglected a deductible in the model formulation. In reality, however, many insurance policies entail a deductible. This fact is also warranted by several studies [2, 5, 6, 7, 8]. Therefore, it would be a logical extension of the problem to include a deductible in the model formulation and examine how the insured's optimal reporting strategy is affected by the existence of a deductible. Next, it was assumed that the premium each period is adjusted according to the size of the insured's claim (reported loss) during the previous period. A more complete modeling, however, would be the one such that the premium each period is adjusted to reflect both the frequency and the size of the insured's claims. Further, if we restrict our assumption such that the current premium is affected by the number of claims and the size of claims only in a specific length of time (e.g., past 3 or 5 years), the modeling would be much more complicated, even though it is more realistic. Such a model might be mathematically intractable; however, it will be the promising problem to think about in order to reduce the "gap" between theoretical modeling and practice.

Footnotes

1. Our analysis is delimited to those types of insurance that are "experience rated." The "experience rating" is an insurance pricing system in which the insured's loss experience determines the premium for the current protection.
2. It is true that many decision making problems deal with an individual's expected utility of wealth and they are to maximize his expected utility (or equivalently minimize his expected disutility) over a planning horizon. This paper, however, deals with an individual's expected costs rather than his expected utility. The reason for this is that one is sometimes more sensitive to his wealth decrease than to utility of it. The results we will derive in this paper would be the same for both cases.
3. It is assumed that F has a mass of probability at 0 (i.e., no loss).
4. If $\beta\triangle'(x) < 1$ for $x=0$, then $x_2^*=0$. We allow this case.

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