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A Numerical Solution Method of the Boundary Integral Equation

—Axisymmetric Flow—

by

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경계적분방정식의 수치해법

—축대칭 유동—

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Abstract

A numerical solution method of the boundary integral equation for axisymmetric potential flows is presented. Those are represented by ring source and ring vorticity distribution. Strengths of ring source and ring vorticity are approximated by linear functions of a parameter ζ on a segment. The geometry of the body is represented by a cubic B-spline.

Limiting integral expressions as the field point tends to the surface having ring source and ring vorticity distribution are derived upto the order of $\zeta\ln\zeta$. In numerical calculations, the principal value integrals over the adjacent segments cancel each other exactly. Thus the singular part proportional to $\left(\frac{1}{\zeta}\right)$ can be subtracted off in the calculation of the induced velocity by singularities. And the terms proportional to $\ln\zeta$ and $\zeta\ln\zeta$ can be integrated analytically. Thus those are subtracted off in the numerical calculations and the numerical value obtained from the analytic integrations for $\ln\zeta$ and $\zeta\ln\zeta$ are added to the induced velocity. The four point Gaussian Quadrature formula was used to evaluate the higher order terms than $\zeta\ln\zeta$ in the integration over the adjacent segments to the field points and the integral over the segments off the field points.

The root mean square errors, E_2, are examined as a function of the number of nodes to determine convergence rates. The convergence rate of this method approaches 2.

요 약

본 보에서는 축대칭포텐시얼유동에 대한 경계적분방정식의 해법이 제시된다. 이 문제는 고리용출점과 고리보오텍스에 의해서 표시되는데 이들의 세기는 한 구간내에서 매개변수 ζ의 선형함수로 근사된다. 물체의 형상은 3차 B-spline으로 표시된다.

속도가 계산되는 점이 고리용출점이나 고리보오텍스에 접근할 때의 극한표현식이 $\zeta\ln\zeta$항까지 유

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도된다. 수치계산에서 양 옆구간에 의한 주치적분은 정확하게 서로 상쇄되기 때문에 특이점에 의한 유기속도중 $\left(\frac{1}{\zeta}\right)$에 비례하는 항은 계산에서 제외된다. 그리고 $\ln\zeta$에 비례하는 항과 $\zeta\ln\zeta$에 비례하는 항은 해석적으로 적분이 가능하기 때문에 수치계산에서 이에 비례하는 항을 빼고 계산한 후 해석적으로 계산한 값을 더해 준다. 기타 수치적분은 4점 Gaussian Quadrature 공식에 의해서 수행되었다.

수렴률을 정하기 위하여 구간의 개수에 따른 평균자승근오차를 조사하였으며, 이 방법의 수렴률은 2에 접근함이 밝혀졌다.

1. Introduction

Boundary Integral Equation methods are used to solve potential flows like ship wave problem, foil problem, etc. Evaluation of Cauchy Principal Value Integral is the most important part to reduce numerical errors. Numerical error depends on the evaluation method of Cauchy Principal Value Integral. Considerable literature exists for solving such problems [9, 10].

When the boundary integral equations are solved, the numerical evaluation of Cauchy Principal Value Integral affects the accuracy of the solution. While the Principal Value Integral can be treated analytically in the lowest order method (i.e. constant strength over a panel), any accurate method to evaluate the Principal Value Integral is not known without using higher order numerical quadrature in the higher order method.

In this paper, a numerical solution method of the boundary integral equation for axisymmetric potential flows is presented. Those are represented by ring source and ring vorticity distribution. Strengths of ring source and ring vorticity are approximated by linear functions of a parameter ζ on a segment. The geometry of the body is represented by a cubic B-spline (Barsky and Greenberg [2]).

Limiting integral expressions as the field point tends to the surface having ring source and ring vorticity distribution are derived upto the order of $\zeta\ln\zeta$. In numerical calculations, the principal value integrals over the adjacent segments cancel each other exactly. Thus the singular part proportional to $\left(\frac{1}{\zeta}\right)$ can be subtracted off in the calculation of the induced velocity by singularities. And the terms proportional to $\ln\zeta$ and $\zeta\ln\zeta$ can be integrated analytically. Thus those are subtracted off in the numerical calculations and the numerical values obtained from the analytic intergrations for $\ln\zeta$ and $\zeta\ln\zeta$ are added to the induced velocity. The four point Gaussian Quadrature formula(Ferziger [8], Abramowitz & Stegun [1]. was used to evaluate the higher order terms than $\zeta\ln\zeta$ in the integration over the adjacent segments to the field points and the integral over the segments off the field points. This method can be extended to two-dimensional and three-dimensional problem.

2. Mathematical Formulation

Consider an ideal fluid which is assumed to be inviscid and incompressible. The flow is assumed to be irrotational. The fluid domain is bounded with the following surfaces, the body, S_B, and the surfaces at infinity, S_∞. The surfaces, taken as a whole, will be denoted as S. The governing equation and the boundary conditions are as follows: Laplace equation:

$$\nabla^2\phi=0 \qquad \text{in the fluid domain} \tag{1}$$

Body boundary condition:

$$\nabla\phi\cdot\underline{n}(\underline{x},t)=\underline{V}\cdot\underline{n} \qquad \text{on } B(\underline{x},t)=0 \tag{2}$$

where $\underline{x}(x,y,z)$ is a right-handed coordinate system with z positive upwards, $\underline{V}$ includes both translational and rotational velocities, and $B(\underline{x},t)=0$ is the function representing the body surface geometry at time t.

The Green function, $G(\underline{x};\underline{y})$, satisfies the following equation.

$$\nabla^2G(\underline{x};\underline{y})=-\delta(\underline{x}-\underline{y}) \tag{3}$$

where $\underline{x}$ is the vector to the field point, $\underline{y}$ is the vector to the source point, and $\delta(\underline{x}-\underline{y})$ is the Dirac delta function. Through the application of Green's second identity in the fluid domain, the potential is given as.

$$\alpha(\underline{x},t)\phi(\underline{x},t)=\iint_S\left[\frac{\partial\phi}{\partial n}-\phi\frac{\partial}{\partial n}\right]GdS \tag{4}$$

where α is an included solid angle at $\underline{x}$ and S is the body surface.

The Green function that satisfies Eq. (3) is

$$G(\underline{x},\underline{y})=\frac{1}{R}=\frac{1}{|\underline{x}-\underline{y}|} \tag{5}$$

where $\underline{x}$ is the position vector of a field point and $\underline{y}$ is that of a source point. ϕ_n on the body is known from the body boundary condition, Eq. (2).

For axisymmetric bodies, Eq. (4) can be reduced as follows (Newman, [12]):

$$\alpha(\underline{x},t)\phi(\underline{x},t)=\int_s r'\left[\frac{\partial\phi}{\partial n'}-\phi\frac{\partial}{\partial n'}\right]G^a ds \tag{6}$$

where

$$G^a=\int_0^{2\pi}\frac{d\theta}{R}=\frac{4}{\rho_1}K(m)$$

$$\frac{\partial G^a}{\partial n'}=\int_0^{2\pi}\frac{\partial}{\partial n'}\left(\frac{1}{R}\right)d\theta$$

$$=\frac{4(z-z')}{\rho_1\rho^2}E(m)n_{z'}+\left[\frac{4(r-r')}{\rho_1\rho^2}E(m)+\frac{2}{\rho_1 r'}(E(m)-K(m))\right]n_{r'}$$

$$\rho^2=(r-r')^2+(z-z')^2$$

$$\varphi_1^2=(r+r')+(z-z')^2$$

$$m=1-\rho^2/\varphi_1^2$$

K and E are the complete elliptic integral of the first kind and the second kind, (r,z) is a field point, (r',z') is a source point in the polar coordinate system, and s is the line representing the body surface in the polar coordinate system. A different formulation can be derived in order to get the tangential velocities, ϕ_s and ϕ_b, on the body directly. The normal velocity, ϕ_n, on the body is given from the body boundary condition, Eq. (2). Taking the gradient of Eq. (4) yields,

$$\alpha(\underline{x},t)\nabla\phi(\underline{x},t)=\iint_S\left[\frac{\partial\phi}{\partial n}\nabla G-\phi\nabla\frac{\partial G}{\partial n}\right]dS \tag{7}$$

The second term of the right-handed side in the above equation can be represented as follows (Brockett, [5]):

$$\alpha\nabla\phi=\iint_S\left[\phi_n\nabla G-\underline{\gamma}\times\nabla G\right]dS \tag{8}$$

where

$$\underline{\gamma}=-\underline{n}\times(\nabla\phi)$$

$$=-\underline{n}\times(\phi_n\underline{n}+\phi_s\underline{s}+\phi_b\underline{b})$$

$$=-\phi_s\underline{b}+\phi_b\underline{s},$$

and $\underline{s}$ is a tangential vector on the suraface, $\underline{b}$ is a bi-normal vector, and $\underline{n}$ is a normal vector all of which satisfy the relation, $\underline{b}=\underline{n}\times\underline{s}$. Taking dot products of $\underline{s}$, $\underline{b}$, and $\underline{n}$, Fredholm integral equations of the second kind for ϕ_s, ϕ_b, and ϕ_n can be obtained respectively:

$$\alpha\underline{s}\cdot\nabla\phi=\iint_S[\phi_n\underline{s}\cdot\nabla G-\underline{s}\cdot(-\phi_s\underline{b}'+\phi_b\underline{s}')\times\nabla G]dS \quad \text{on } S_B$$

$$\alpha\underline{b}\cdot\nabla\phi=\iint_S[\phi_n\underline{b}\cdot\nabla G-\underline{b}\cdot(\phi_s\underline{b}'+\phi_b\underline{s}')\times\nabla G]dS \quad \text{on } S_B \tag{9}$$

$$\alpha\underline{n}\cdot\nabla\phi=\iint_S[\phi_n\underline{n}\cdot\nabla G-\underline{n}\cdot(-\phi_s\underline{b}'+\phi_b\underline{s}')\times\nabla G]dS \quad \text{on } S_B$$

The domain of integration, S, for the source points includes both S_B and S_∞.

For axisymmetric bodies, Eq. (8) can be reduced as follows:

$$\alpha\nabla\phi=\int_s r'\left[\frac{\partial\phi}{\partial n'}(u_r^s\underline{e}_r+u_z^s\underline{e}_z)-\frac{\partial\phi}{\partial s'}(u_r^v\underline{e}_r+u_z^v\underline{e}_z)\right]ds \tag{10}$$

where u_r^s and u_z^s are ring source r-directional and z-directional induced velocities respectively, u_r^v and u_z^v are ring vortex r-directional and z-directional induced velocities respectively, and $\underline{e}_r$ and $\underline{e}_z$ are r-directional and z-directional unit vectors respectively. The induced velocity by a ring vortex can be derived from the stream function given in Batchelor [4]. When r is not zero,

$$u_s{}^r=\frac{4(r'-r)}{\rho_1\rho^2}E(m)+\frac{2}{\rho_1 r}(E(m)-K(m))$$

$$u_s{}^z=-\frac{4(z-z')}{\rho_1\rho^2}E(m)$$

$$u_v{}^r=-\frac{2(z-z')}{\rho_1}\left[\frac{1}{r'r}(K(m)-E(m))-\frac{2}{\rho^2}E(m)\right]$$

$$u_v{}^z=\frac{2}{r'\rho_1}(K(m)-E(m))+\frac{4(r'-r)}{\rho_1\rho^2}E(m) \tag{11}$$

When $r=0$,

$$u_r^s=u_r^v=0,$$

$$u_z^s = -\frac{2\pi(z-z')}{[r'^2+(z-z')^2]^{3/2}},$$

$$u_z^v = -\frac{2\pi r'}{[r'^2+(z-z')^2]^{3/2}}. \qquad (12)$$

Taking dot products of s for Eq. (10),

$$\alpha s\cdot\nabla\phi = \int_S r'\Big[\frac{\partial\phi}{\partial n'}\, s\cdot(u_r^s e_r + u_z^s e_z) - \frac{\partial\phi}{\partial s'}\, s\cdot(u_r^v e_r + u_z^v e_z)\Big] ds \text{ on } s_B \qquad (13)$$

Eq.(13) is a Fredholm integral equation of the second kind for ϕ_s on s_B.

3. Numerical Implementation

The boundary, s_B, is discretized into small segments in order to solve the integral equations numerically. ϕ_s and ϕ_n are approximated by linear functions of the parameter ζ on a given segment. In particular.

$$\phi_{sj}(\zeta) = (1-\zeta)\phi_{sj} + \zeta\phi_{sj+1} \qquad \text{for } 0\le\zeta\le 1$$

and

$$\phi_{nj}(\zeta) = (1-\zeta)\phi_{nj} + \zeta\phi_{nj+1} \qquad \text{for } 0\le\zeta\le 1.$$

The geometry of the body is represented by a cubic B-spline (Barsky and Greenberg[2]), or

$$r_j(\zeta) = \sum_{s=-2}^{1} a_s(\zeta) V_{j+s}^r \text{ and } z_j(\zeta) = \sum_{s=-2}^{1} b_s(\zeta) V_{j+s}^z \qquad (14)$$

where $a_s(\zeta)$ and $b_s(\zeta)$ are the uniform cubic B-spline basis functions and V_j are vertices.

The end condition should be imposed to get a complete B-spline approximation. There are several methods to impose end conditions according to the geometrical characteristics (Barsky[3]). The derivative of B-spline interpolation at the end is set to get the tangent of the given geometry if the tangent is known. If the tangent is not known, the derivative at the end is set to be the slope between two vertices at the end obtained by using B-spline algorithm.

To evaluate the integrals over the segments the four point Gaussian Quadrature formula was used (Ferziger [8]), Abramowitz & Stegun[1]). The integrands in Eq. (13) have $\left(\frac{1}{r}\right)$ type singularities as the field point approaches the source point. When the principal value integrals are evaluated, it is convenient to subtract off $\left(\frac{1}{r}\right)$ type singularities (Brockett, et. al [6]). The geometry is represented by a cubic B-spline curve, i.e.

$$r' = c_0 + c_1\zeta + c_2\zeta^2 + c_3\zeta^3 \qquad z' = d_0 + d_1\zeta + d_2\zeta^2 + d_3\zeta^3$$

Since the control points are the ends of the segments, there are $\left(\frac{1}{\zeta}\right)$ singularities in the induced velocities when $r=c_0$ and $z=d_0$ and when $r=c_0+c_1+c_2+c_3$ and $z=d_0+d_1+d_2+d_3$. Consider the r-directional induced velocity by ring source distribution excluding the small segment ϵ from a control point,

$$\int_{\Delta s-\epsilon} u_r^s r' ds = I_1 + I_2$$

with

$$I_1 = \int_{\zeta_\epsilon}^1 \frac{4(r'-r)}{\rho_1\rho^2} E(m) r' J(\zeta) d\zeta$$

$$= \int_{\zeta_\epsilon}^1 \frac{4(c_1\zeta+\cdots)(1-\delta_1 m_1 \ln(m_1)+\cdots)(c_0+\cdots)\sqrt{(c_1+\cdots)^2+(d_1+\cdots)^2}}{\sqrt{(2c_0+c_1\zeta+\cdots)^2+(d_1\zeta+\cdots)^2}[(c_1\zeta+\cdots)^2+(d_1\zeta+\cdots)^2]} d\zeta$$

$$= \frac{2c_1}{\sqrt{c_1^2+d_1^2}}\int_{\zeta_\epsilon}^1 \frac{d\zeta}{\zeta} - \frac{c_1\sqrt{c_1^2+d_1^2}}{c_0^2}\delta_1\int_0^1 \zeta\ln\zeta d\zeta + O\Big(\int_0^1 \zeta^2\ln\zeta d\zeta\Big) \qquad (15)$$

where $m_1 = 1-m$, $J = ds/d\zeta$, $\zeta_\epsilon = \frac{\epsilon}{\sqrt{c_1^2+d_1^2}}$ for small ϵ, and $\delta_1 = 0.24998368310$ from Abramowitz & Stegun [1]. Similarly,

$$I_2 = \int_0^1 \frac{2}{\rho_1 r}(E(m)-K(m)) r' J d\zeta$$

$$= \frac{2\beta_0\sqrt{c_1^2+d_1^2}}{c_0}\Big[\int_0^1 \ln\zeta d\zeta + \Big(\frac{c_1}{2c_0} + \frac{2(c_1c_2+d_1d_2)}{c_1^2+d_1^2}\Big)\int_0^1 \zeta\ln\zeta d\zeta\Big] + O\Big(\int_0^1 \zeta^2\ln\zeta d\zeta\Big) \qquad (16)$$

where $\beta_0 = 0.5$ from Abramowitz & Stegun [1].

When $\zeta\to 1$,

$$I_1' = \int_0^{1-\zeta'_\epsilon} \frac{4(r'-r)}{\rho_1\rho^2} E(m) r' J(\zeta) d\zeta$$

$$=\frac{2C_1}{\sqrt{C_1^2+D_1^2}}\int_{\zeta_\epsilon'}^1\frac{d\zeta'}{\zeta'}-\frac{C_1\sqrt{C_1^2+D_1^2}}{C_0^2}\delta_1\int_0^1\zeta'\ln\zeta' d\zeta'+O\left(\int_0^1\zeta'^2\ln\zeta' d\zeta'\right) \tag{17}$$

and

$$I_2'=\int_0^1\frac{2}{\rho_1 r}(E(m)-K(m))r'Jd\zeta$$

$$=\frac{2\beta_0\sqrt{C_1^2+D_1^2}}{C_0}\left[\int_0^1\ln\zeta' d\zeta'+\left(\frac{C_1}{2C_0}+\frac{2(C_1C_2+D_1D_2)}{C_1^2+D_1^2}\right)\int_0^1\zeta'\ln\zeta' d\zeta'\right]+O\left(\int_0^1\zeta'^2\ln\zeta' d\zeta'\right) \tag{18}$$

where $\zeta'=1-\zeta$ and $\zeta_\epsilon'=\frac{\epsilon}{\sqrt{C_1^2+D_1^2}}$. Since $C_0(i)=c_0(i+1)$, $C_1(i)=-c_1(i+1)$, $C_2(i)=c_2(i+1)$, and $C_3(i)=-c_3(i)$, where i and $i+1$ represent i-th and $(i+1)$-th segments of the surface, by using the characteristics of the cubic B-spline, singular parts of the integrals I_1+I_1' for the same ϵ cancel each other exactly. Thus the singular part $\left(\frac{1}{\zeta}\right)$ can be subtracted off in the calculation of the induced velocity by ring sources on a segment. This statement can be applied to the case for the calculation of the induced velocity by ring vortices on a segment. Consider now the vertical component of the induced velocity by the ring source distribution:

$$\int_{\Delta s-\epsilon}u_z^s r' ds=I_3$$

with

$$I_3=\int_{\zeta_\epsilon}^1\frac{4(z'-z)}{\rho_1\rho^2}E(m)r'J(\zeta)d\zeta$$

$$=\frac{2d_1}{\sqrt{c_1^2+d_1^2}}\int_{\zeta_\epsilon}^1\frac{d\zeta}{\zeta}-\frac{d_1\sqrt{c_1^2+d_1^2}}{c_0^2}\delta_1\int_0^1\zeta\ln\zeta d\zeta+O\left(\int_0^1\zeta^2\ln\zeta d\zeta\right). \tag{19}$$

Similarly, the induced velocities at the ends of the segment by ring vortices on a segment has $\left(\frac{1}{\zeta}\right)$ singularities. The radial component of the induced velocity due to the vorticity distribution are:

$$\int_{\Delta s-\epsilon}u_r^v r' ds=I_4+I_5$$

with

$$I_4=\int_0^1\frac{2(z-z')}{\rho_1 r}(E(m)-K(m))Jd\zeta=-\frac{2\beta_0 d_1\sqrt{c_1^2+d_1^2}}{c_0^2}\int_0^1\ln\zeta d\zeta+O\left(\int_0^1\zeta^2\ln\zeta d\zeta\right) \tag{20}$$

and

$$I_5=-I_3$$

$$=-\int_{\zeta_\epsilon}^1\frac{4(z'-z)}{\rho_1\rho^2}E(m)r'J(\zeta)d\zeta$$

$$=-\frac{2d_1}{\sqrt{c_1^2+d_1^2}}\int_{\zeta_\epsilon}^1\frac{d\zeta}{\zeta}-\frac{d_1\sqrt{c_1^2+d_1^2}}{c_0^2}\delta_1\int_0^1\zeta\ln\zeta d\zeta+O\left(\int_0^1\zeta^2\ln\zeta d\zeta\right). \tag{21}$$

Consider the vertical component of the induced velocity due to the vorticity distribution:

$$\int_{\Delta s-\epsilon}u_z^v r' ds=I_6+I_7$$

with

$$I_6=-\int_0^1\frac{2}{\rho_1}(E(m)-K(m))Jd\zeta$$

$$=-\frac{2\beta_0\sqrt{c_1^2+d_1^2}}{c_0}\left[\int_0^1\ln\zeta d\zeta+\left(-\frac{c_1}{2c_0}+\frac{2(c_1c_2+d_1d_2)}{c_1^2+d_1^2}\right)\int_0^1\zeta\ln\zeta\, d\zeta\right]+O\left(\int_0^1\zeta^2\ln\zeta\, d\zeta\right). \tag{22}$$

and

$$I_7=I_1$$

$$=\int_{\zeta_\epsilon}^1\frac{4(r'-r)}{\rho_1\rho^2}E(m)r'J(\zeta)d\zeta=\frac{2c_1}{\sqrt{c_1^2+d_1^2}}\int_{\zeta_\epsilon}^1\frac{d\zeta}{\zeta}-\frac{c_1\sqrt{c_1^2+d_1^2}}{c_0^2}\delta_1\int_0^1\zeta\ln\zeta\, d\zeta+O\left(\int_0^1\zeta^2\ln\zeta\, d\zeta\right). \tag{23}$$

The logarithmic function is integrable and can be integrated by numerical quadrature. But since the accurate integration of the logarithmic function requires a higher order quadrature formula, the method following Ferziger [8] and Dommermuth & Yue [7] can be used. The integral can be factored into the sum of the logarithmic singular part up to $\zeta\ln\zeta$ which is integrable analytically and the non-singular part which requires numerical quadrature (Ferziger [8]).

In numerical calculations, the principal value integrals over the adjacent segments cancel each other xactly. Thus the singular part proportional to $\left(\frac{1}{\zeta}\right)$ can be subtracted off in the calculation of the induced velocity by singularities. And the terms proportional to $\ln\zeta$ and $\zeta\ln\zeta$ can be integrated analytically. Thus those are subtracted off in the numerical calculations and the numerical values obtained from the analytic integrations for $\ln\zeta$ and $\zeta\ln\zeta$ are added to the induced velocity. For example,

$$I_7-\int_{\zeta_\epsilon}^{1}\frac{2c_1}{\sqrt{c_1^2+d_1^2}}\frac{d\zeta}{\zeta}$$
$$=\int_{\zeta_\epsilon}^{1}\left(\frac{4(r'-r)}{r_1\rho^2}E(m)r'J(\zeta)-\frac{2c_1}{\sqrt{c_1^2+d_1^2}}\frac{1}{\zeta}+\frac{c_1\sqrt{c_1^2+d_1^2}}{c_0^2}\delta_1\zeta\ln\zeta\right)d\zeta-\frac{c_1\sqrt{c_1^2+d_1^2}}{c_0^2}\delta_1\left(-\frac{1}{4}\right). \tag{24}$$

The integral in Eq. (24) includes all the higher order trems than $\zeta\ln\zeta$ in the integration over the adjacent segments to the field points. The four point Gaussian Quadrature formula (Abramowitz & Stegun [1]) was used to evaluate the integral in Eq. (24) and the integral over the segments off the field points.

The above statements are valid for a smooth surface. To get a finite induced velocity at a corner, which means that the principal value integral at the corner has a finite value, the following condition should be satisfied (Kang [11]):

$$(\phi_n)_F=-(\phi_n)_B\cos\theta+(\phi_s)_B\sin\theta$$
$$(\phi_s)_F=-(\phi_n)_B\sin\theta-(\phi_s)_B\cos\theta \tag{25}$$

where θ is the intersection angle at the corner and the subscript F means the other boundary intersected at thc corner with the angle θ. The above condition requires the continuity of the fluid velocity at the corner when the fluid is approaching the corner along the body surface and the free surface. The fluid particle has finite velocity at the corner if the above condition is satisfied. The cubic B-spline is not continuous through the intersection point. There are some finite contributions from the singular part $\left(\frac{1}{\zeta}\right)$ as explained in Appendix 5.A1 of the reference (Kang [11]).

4. Numerical Caiculation

The present method has been applied to the flow around a sphere and a prolate speroid (minor axis to major axis ratio, $b/a=0.5$). The velocity of the body, w, is -1. All the quantities are calculated with a double-precision program. The root mean square errors, E_2, are examined as a functions of the number of nodes to determine convergence rates. Fig. 1 shows comparison of the numerical and the analytic solution for a sphere(radius=1, $N=64$) where N is the number of nodes. Fig. 2 shows E_2 errors in log-log scale for various number of nodes. The convergence rate is defined as the slope of $(1/E_2)$ in log-log scale. The convergence rate of this method approaches 2. Fig. 3 and Fig. 4, which are the results for the prolate spheroid, show the trends are similar to those for the sphere.

5. Conclusion

A numerical solution method of the boundary integral equation for axisymmetric potenoial flows are presented. Those are represented by ring source and ring voricity distribution. Strengths of ring source and ring vorticity are approximated by linear functions of a parameter ζ on a segment. The geometry of the body is represented by a cubic B-spline.

Limiting integral expressions as the field point tends to the surface having ring source and ring vorticity distribution were derived upto the order of $\zeta\ln\zeta$. By using those, an efficient numerical method was proposed and justified.

The root mean square errors, E_2, were examined

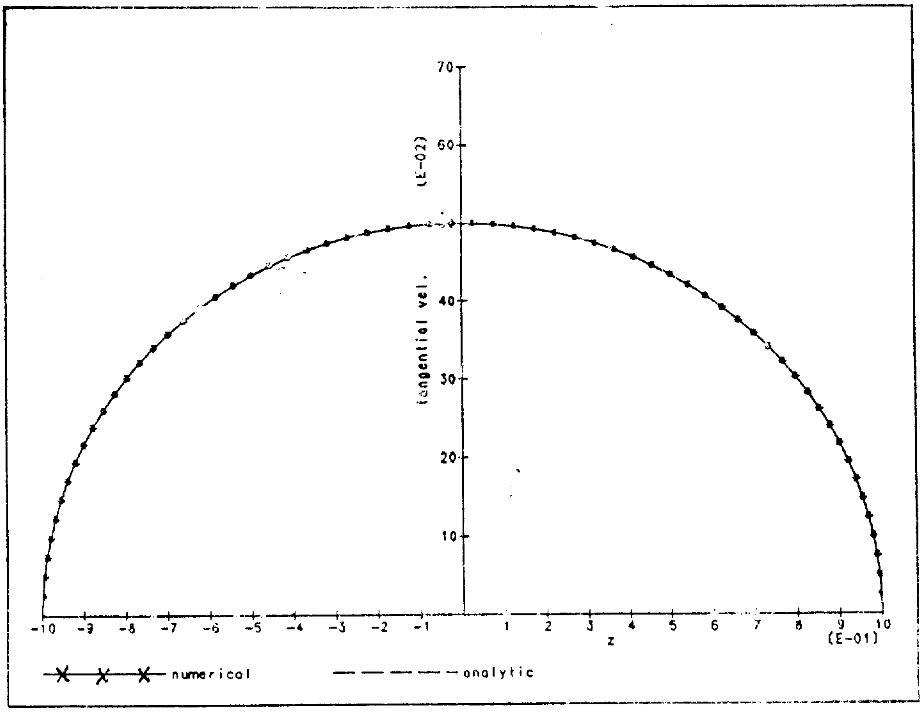

Fig. 1 Comparison of the numerical and analytic solution for the sphere (r=1, N=64, w−1)

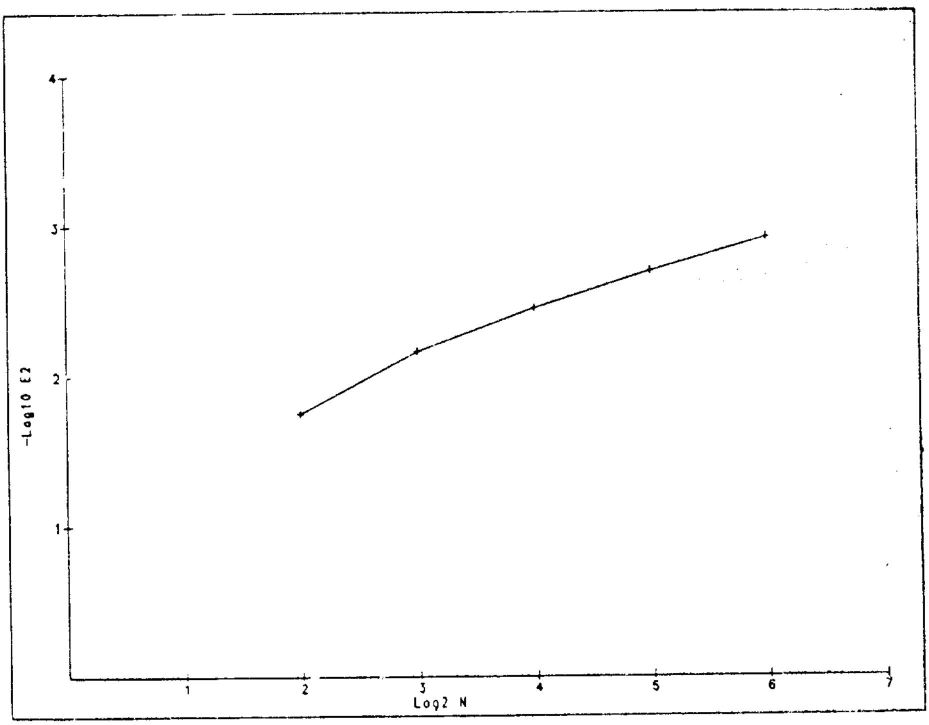

Fig. 2 E_2 Errors for various number of nodes for the sphere (r=1, w=−1)

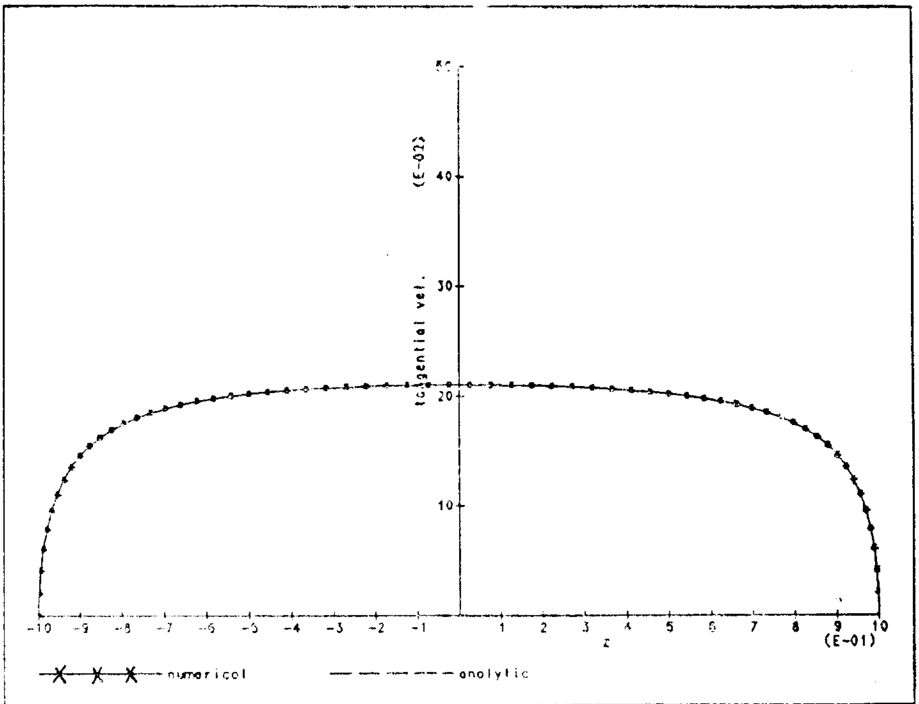

Fig. 3 Comparison ot the numerical and analytic solution for the prolate spheroid (b/a=0.5, N=64, w−1)

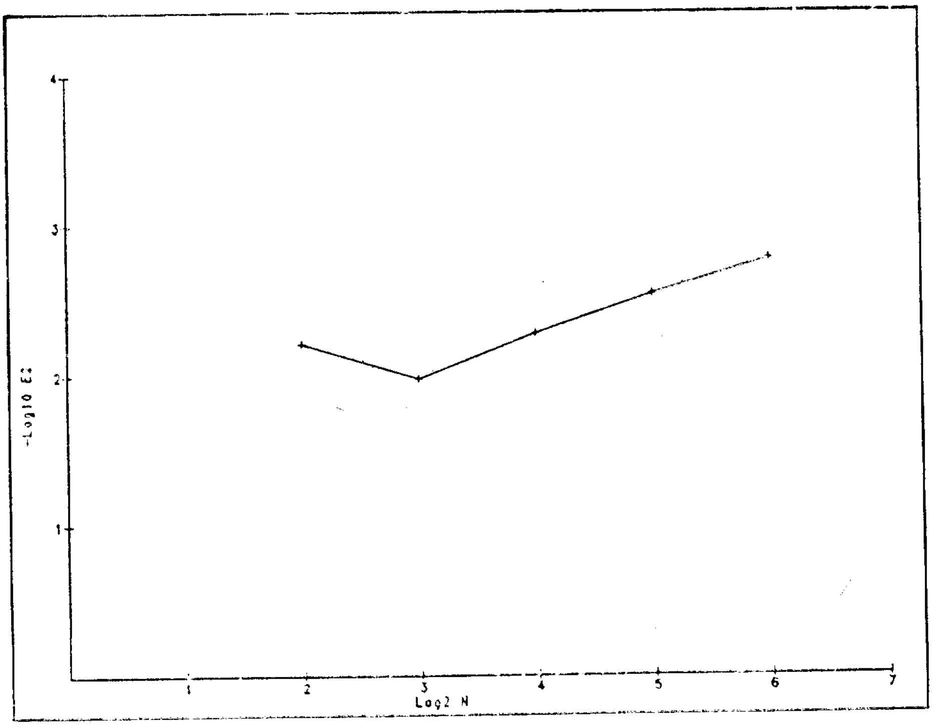

Fig. 4. E_2 Errors for various number of nodes for the prolate spheroid(b/a=0.5, w=−1) as a functions of the number of nodes to determine convergence rates. The convergence rate of this method approaches 2. Thus this method is useful for the computation of local flow like the leading edge flow.

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