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A Uniform Expansion for Ship Waves

by

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선박에 기인한 파의 해석을 위한 Uniform Expansion에 관하여

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Abstract

The Kelvin's method of stationary phase breaks down at the cusp point and yields a nonuniform expansion near it. By making use of the nonuniform expansion, we derive a uniform expansion near the cusp point.

요 약

Kelvin의 principle of stationary phase는 cusp point에서는 사용불능이며 이 cusp point근처에서 nonuniform expansion을 준다. 이 nonuniform expansion을 이용해서 cusp point 근처에서 유용한 uniform expansion을 도출하는데 본 논문의 목적이 있다.

1. Introduction

When integrals for ship waves are asymptotically approximated for a large parameter, one gets the so called Kelvin wave which is expressed as the sum of the two waves called the transverse and divergent waves, respectively. The amplitudes of the transverse and divergent waves become infinite at the cusp point. Hence, the Kelvin wave obtained from the application of the principle of stationary phase is represented by a nonuniform expansion near the cusp point. In particular, the Kelvin wave is in a simple form, but it is hardly useful. In the present paper, the authors attempt to derive a practically useful and valid expansion for the Kelvin wave near the cusp point. In doing so, the so called linear interpolation is introduced and the non-uniformity in the Kelvin wave is eliminated.

We apply the present method to the Kelvin wave induced by the moving pressure point over the free surface in the negative x-direction and compute the wave elevation behind the pressure point. Then, the free surface configurations are graphically illustrated.

2. Linear interpoation and its application

When the integrals for ship waves are asymptotically approximated for a large parameter, the res-

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ulting expansion has the same phase as that of the Kelvin wave induced by the pressure point moving with constant velocity over the free surface. Hence, we consider the Kelvin wave induced by the pressure point instead of the ship waves

When a concentrated pressure point moves in the negative x-direction at a small speed U, the wave elevation induced by the moving pressure point is given by

$$h(r,\theta)\sim A(k_0)\int_{-\pi/2}^{\pi/2} e^{k_0 z\sec^2\sigma}\sec^4\sigma\, e^{ik_0 r\psi(\theta,\sigma)}$$

for large k_0 and $(x,y)\neq(0,0)$ (1)

where $k_0=g/U^2$, $\psi(\theta,\sigma)=\sec^2\sigma\cos(\theta+\sigma)$, and $A(k_0)=-p_0k_0^2/2\pi\rho g$ if the pressure $p(x,y)$ is expressed as $p(x,y)=p_0\delta(x)\delta(y)$ in terms of the δ-function for the concentrated pressure point. If k_0 is large, equation $\psi_\sigma(\theta,\sigma)=0$ yields stationary points. There are two stationary points σ_1 and σ_2 if $-\theta^*<\theta<\theta^*$ where $\theta^*=\tan^{-1}2^{-3/2}$ or $\theta^*=19.47$ degrees. The two stationary points become equal at $|\theta|=^*\theta$, i.e., $\sigma_1=\sigma_2=\pm\sigma^*$ at $|\theta|=\theta^*$ where $\sigma^*=\tan^{-1}2^{-1/2}$. But there is no stationary point if $|\theta|>\theta^*$.

We now apply the principle of stationary phase to (1) for large k_0 and get the following nonuniform expansion for the Kelvin wave

$$h(r,\theta)\sim A_1(r,\theta)\sin\left[k_0r\psi(\theta,\sigma_1)+\frac{\pi}{4}\right]+A_2(r,\theta)\sin\left[k_0r\psi(\theta,\sigma_2)-\frac{\pi}{4}\right]\quad\text{for } -\theta^*<\theta<\theta^* \tag{2}$$

where

$$A_1(r,\theta)=A(k_0)\left[\frac{2\pi}{k_0r|\psi_{\sigma\sigma}(\theta,\sigma_1)|}\right]^{1/2}\sec^4\sigma_1,$$

$$A_2(r,\theta)=A(k_0)\left[\frac{2\pi}{k_0r|\psi_{\sigma\sigma}(\theta,\sigma_2)|}\right]^{1/2}\sec^4\sigma_2 \tag{3}$$

The first term on the right side of (2) is called the transverse wave and the second is called the divergent wave. Hence, the Kelvin wave consists of the transverse and divergent waves. The expansion in (2) is nonuniform because the principle of stationary phase breaks down at $|\theta|=\theta^*$. That means that the amplitudes $A_1(r,\theta)$ and $A_2(r,\theta)$ become infinite at $|\theta|=\theta^*$ because $\psi_{\sigma\sigma}(\theta,\sigma_1)$ and $\psi_{\sigma\sigma}(\theta,\sigma_2)$ become zero there. The values of $A_1(r,\theta)$ and $A_2(r,\theta)$ are not valid near the cusp point. Therefore, the Kelvin wave given by (2) is in simple form but we cannot use it for computations.

In order to circumvent the nonuniformity and make use of (2), we proceed to find a treatment of $A_1(r,\theta)$ and $A_2(r,\theta)$ near the cusp point. Because the Kelvin wave is symmetrical about the x-axis, we consider the Kelvin wave only in the upper half plane from now on. The wave elevation $h(r,\theta)$ at the cusp point is given by

$$h(r,\theta)\sim\frac{1}{\sqrt{3}}\Gamma(1/3)A(k_0)\left[\frac{6}{k_0r|\psi_{\sigma\sigma\sigma}(\theta^*,\sigma^*)|}\right]^{1/3}\sec^4\sigma^*\sin[k_0r\psi(\theta^*,\sigma^*)]\quad\text{at } \theta=\theta^* \tag{4}$$

where $\Gamma(x)$ is the Gamma function [1, pp.241-242]. The Kelvin wave is confined inside a V-shaped region bounded by two radial lines at $\theta=\pm\theta^*$. The waves in the region for $|\theta|>\theta^*$ are ignored because $h(r,\theta)$ is an exponentially decaying function of k_0r. (4) is further written as the sum of the transverse and divergent waves

$$\left.\begin{aligned}h(r,\theta)\sim&\frac{\sqrt{6}}{6}\Gamma(1/3)A(k_0)\left[\frac{6}{k_0r|\psi_{\sigma\sigma\sigma}(\theta^*,\sigma^*)|}\right]^{1/3}\sec^4\sigma^*\sin\left[k_0r\psi(\theta^*,\sigma^*)+\frac{\pi}{4}\right]\\&+\frac{\sqrt{6}}{6}\Gamma(1/3)A(k_0)\left[\frac{6}{k_0r|\psi_{\sigma\sigma\sigma}(\theta^*,\sigma^*)|}\right]^{1/3}\sec^4\sigma^*\sin\left[k_0r\psi(\theta^*,\sigma^*)-\frac{\pi}{4}\right]\end{aligned}\right\}\text{at }\theta=\theta^* \tag{5}$$

The first term on the right side of (5) is the transverse wave at the cusp point and the second is the divergent wave. Therefore, the amplitudes $A_1(r,\theta)$ and $A_2(r,\theta)$ at $\theta=\theta^*$ are defined as

$$A_1(r,\theta)=A_2(r,\theta)=\frac{\sqrt{6}}{6}A(k_0)\Gamma(1/3)\left[\frac{6}{k_0r|\psi_{\sigma\sigma\sigma}(\theta^*,\sigma)|}\right]^{1/3}\sec^4\sigma^*\quad\text{at }\theta=\theta^* \tag{6}$$

Since the Kelvin wave in (5) is expressed as the sum of the transverse and divergent waves, $h(r,\theta)$ in (5) is in the same form as that of $h(r,\theta)$ in (2). It follows from (2), (5), and (6) that $h(r,\theta)$ near the cusp point can be obtained from linear interpolation. We write $h(r,\theta)$ near the cusp point in the following form:

$$h(r,\theta)\sim\alpha_1(r,\theta)A(k_0)\sec^4\sigma_1\sin\left[k_0r\psi(\theta,\sigma_1)+\frac{\pi}{4}\right]+\alpha_2(r,\theta)A(k_0)\sec^4\sigma_2\sin\left[k_0r\psi(\theta,\sigma_2)-\frac{\pi}{4}\right]\quad\text{for }\theta_0\leq\theta\leq\theta^* \tag{7}$$

where θ_0 is some θ bounded away from $\theta=\theta^*$ and $\alpha_1(r,\theta)$ and $\alpha_2(r,\theta)$ are to be determined from the following linear interpolation:

$$\alpha_i(r,\theta)=\alpha_i^1(r,\theta)+\frac{\theta-\theta_0}{\theta^*-\theta_0}[\alpha_i^2(r,\theta^*)-\alpha_i^1(r,\theta_0)]$$

with r fixed for $i=1,2$ (8)

where

$$\alpha_i^1(r,\theta_0)=\left[\frac{2\pi}{k_0r|\psi_{\sigma\sigma}(\theta_0,\sigma_i)|}\right]^{1/2},$$

$$\alpha_i^2(r,\theta^*)=\frac{\sqrt{6}}{6}\Gamma(1/3)\left[\frac{6}{k_0r|\psi_{\sigma\sigma\sigma}(\theta^*,\sigma^*)|}\right]^{1/3} \quad (9)$$

If r is fixed, $\alpha_1(r,\theta)$ and $\alpha_2(r,\theta)$ depend on θ only. Hence, we get $\alpha_1(r,\theta)$ and $\alpha_2(r,\theta)$ from the linear interpolation given by (8). We compute $h(r,\theta)$ from (2), (5), and (7) on a series of circular arcs for $0<\theta\leq\theta^*$ in the upper half plane where the origin is the center of the arcs. On each circular arc where r is fixed, (2) is used to compute $h(r,\theta)$ for $0<\theta<\theta_0$ and (7) is used for $h(r,\theta)$ for $\theta_0\leq\theta\leq\theta^*$ in the upper half plane. θ_0 is taken to be 17 degrees in subsequent computations. The computed free surface configurations in two different viewing directions are given in Figs. 1 through 6.

The same treatment is applied to the waves induced by the Havelock source moving horizontally in the negative x-direction with a constant velocity below

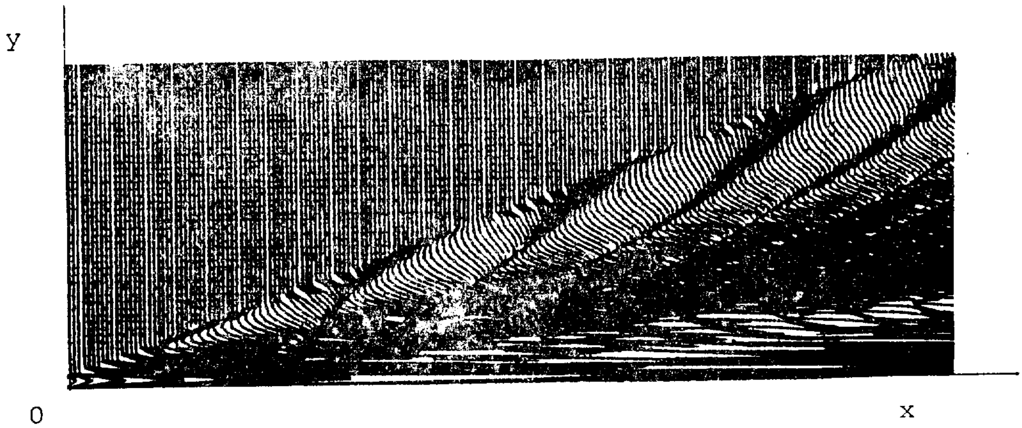

Fig. 1 Free surface configuration for $A(k_0)=1$ and $k_0=100$. The viewing direction is that of vector (1, 0, −3)

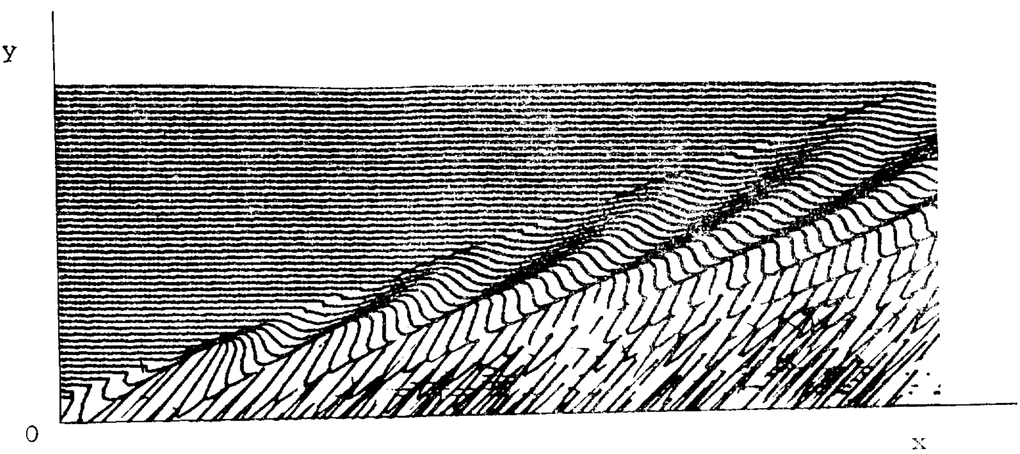

Fig. 2 Free surface configuration for $A(k_0)=1$ and $k_0=100$. The viewing direction is that of vector (−1, −1, −7)

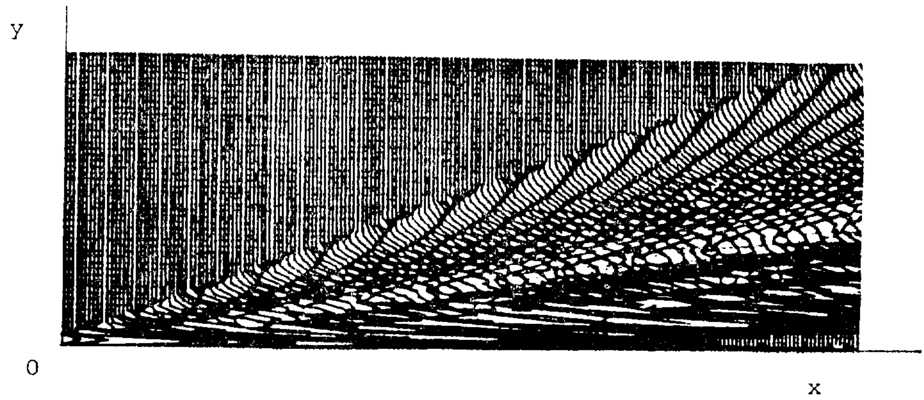

Fig. 3 Free surface configuration for $A(k_0)=1$ and $k_0=300$. The viewing direction is that of vector (1, 0, −3)

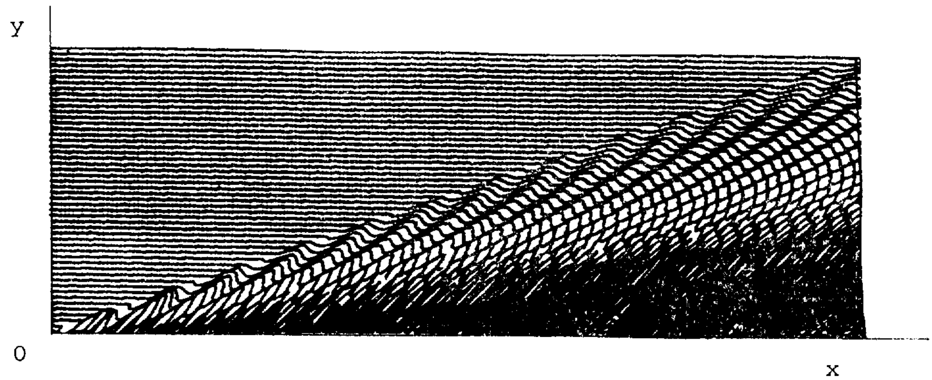

Fig. 4 Free surface configuration for $A(k_0)=1$ and $k_0=300$. The viewing direction is that of vector (−1, −1, −7)

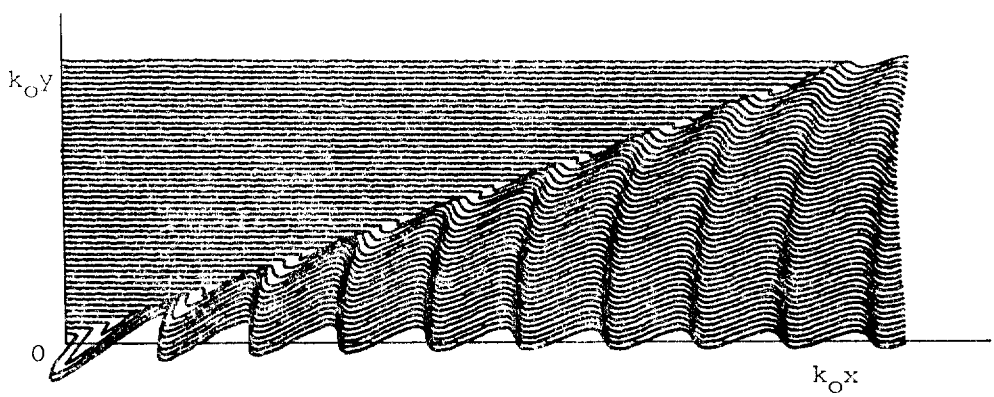

Fig. 5 Free surface configuration for $k_0\zeta_0=-3.0$. The viewing direction is that of vector (−1, −1, −7)

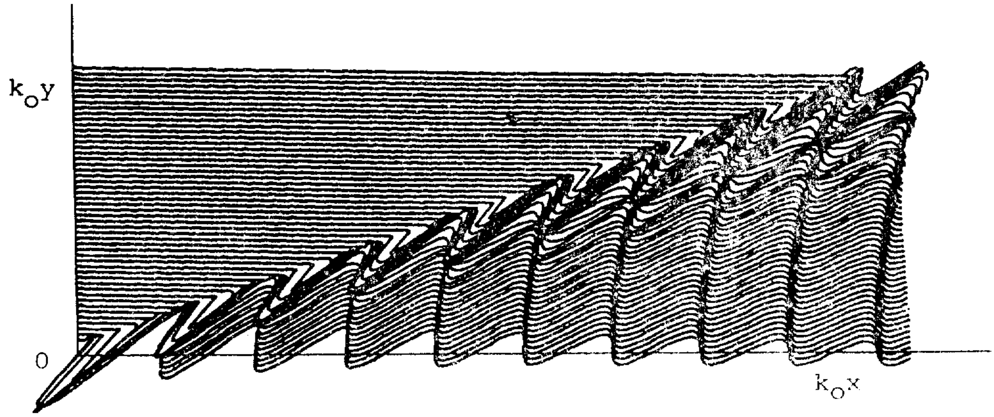

Fig. 6 Free surface configuration for $k_0\zeta_0=-1.0$. The viewing direction is that of vector (−1, −1, −7)

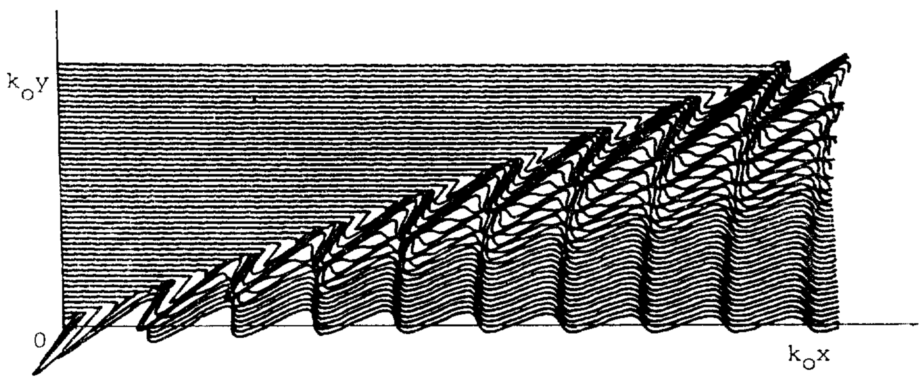

Fig. 7 Free surface configuration for $k_0\zeta_0=-0.5$. The viewing direction is that of vector (−1, −1, −7)

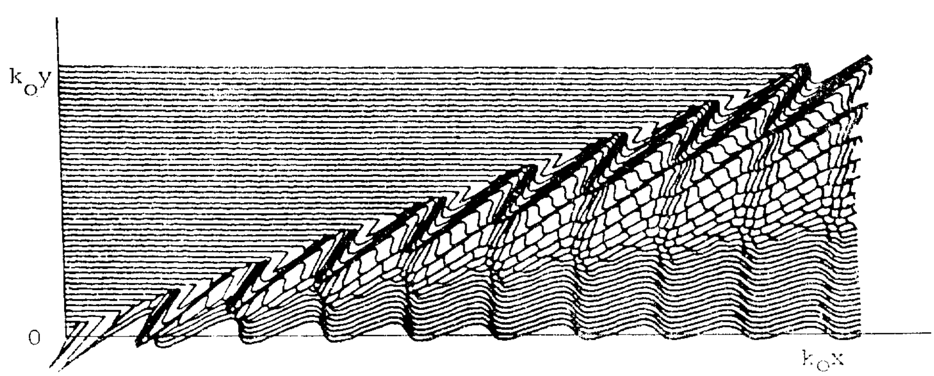

Fig. 8 Free surface configuration for $k_0\zeta_0=-0.25$. The viewing direction is that of vector (−1, −1, −7)

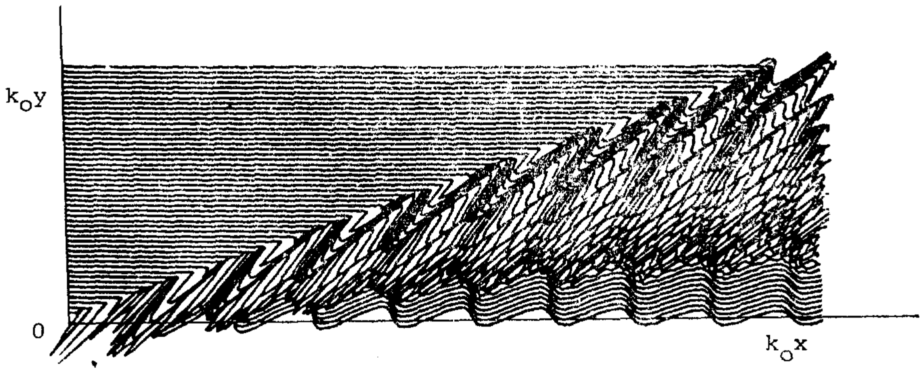

Fig. 9 Free surface configuration for $k_0\zeta_0=-0.10$. The viewing direction is that of vector (−1, −, −7)

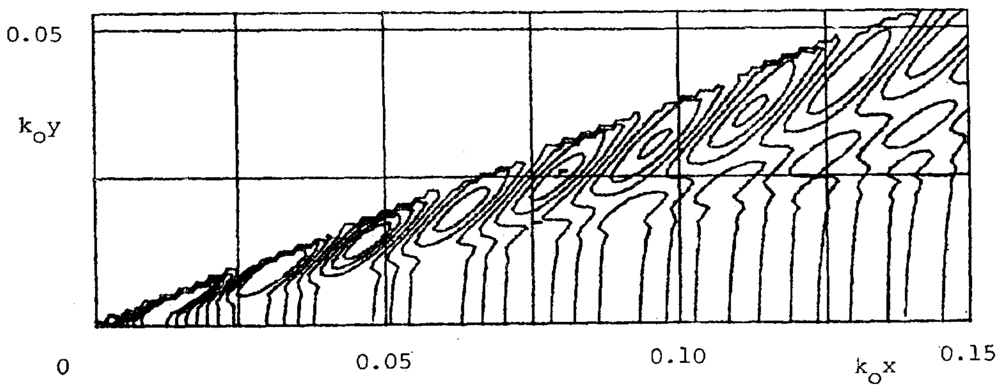

Fig. 10 Contours of surface elevation for $k_0=750$ and $k_0\zeta_0=-0.5$ induced by Havelock source

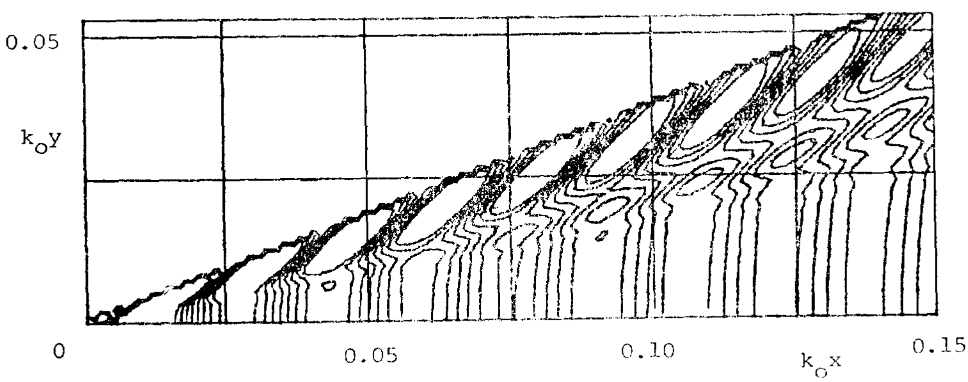

Fig. 11 Contour of sourface elevation for $k_0=500$ and $k_0\zeta_0=-0.5$ induced by Havelock source

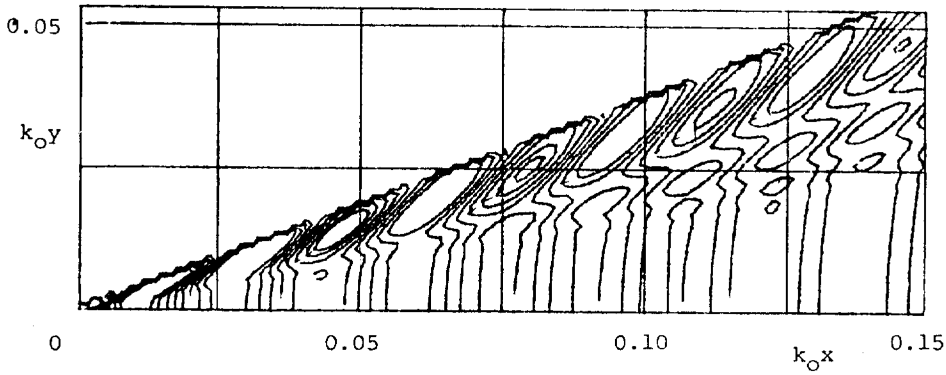

Fig. 12 Contours of surface elevation for $k_0=400$ and $k_0\zeta_0=-0.5$ induced by Havelock source

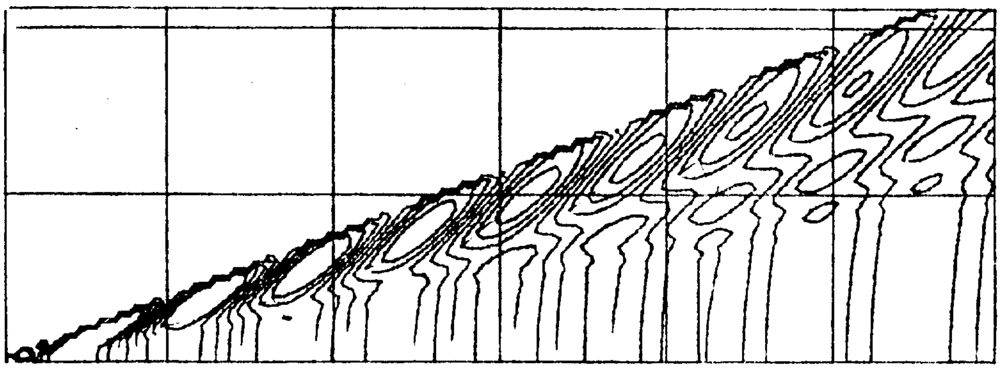

Fig. 13 Contours of surface elevation for $k_0=300$ and $k_0\zeta_0=-0.5$ induced by Havelock source

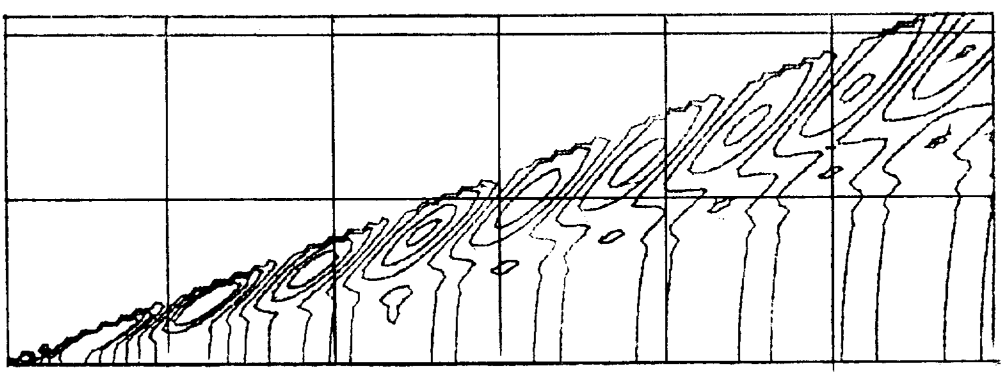

Fig. 14 Contours of surface elevation for $k_0=200$ and $k_0\zeta_0=-0.5$

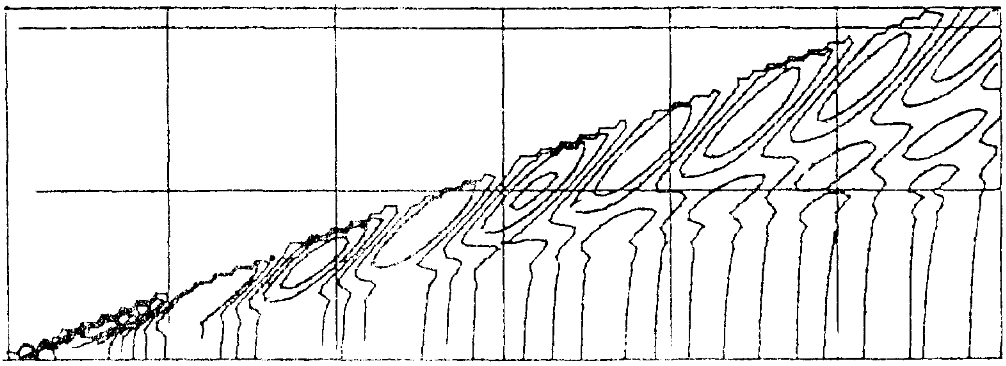

Fig. 15 Contours of surface elevation for $k_0=100$ and $k_0\zeta_0=-0.5$ induced by Havelock source

the free surface. The surface elevation $h(r,\theta)$ is given by $h(x,y)=-(U/g)G_x(x,y,0;\ 0,0,\zeta_0)$ where the Havelock source $G(x,y,0;\ 0,0,\zeta_0)$ is

$$\left.\begin{aligned} G(x,y,0;\ 0,0,\zeta_0)=&-4k_0\int_0^{\pi/2}e^{k_0\zeta_0\sec^2\sigma}\sec^2\sigma\sin(k_0x\sec\sigma)\cos(k_0y\sin\sigma\sec^2\sigma)d\sigma \\ &-\frac{4}{\pi}k_0\int_0^{\pi/2}\sec^2\sigma d\sigma\int_0^{\infty}\frac{e^{k\zeta_0}\cos(kx\cos\sigma)\cos(ky\sin\sigma)dk}{k-k_0\sec^2\sigma} \end{aligned}\right\} \tag{10}$$

We apply the principle of stationary phase to (10) for large k_0 and get the nonuniform expansion of the Kelvin wave

$$\left.\begin{aligned} h(r,\theta)\sim&-2\frac{\sqrt{k_0}}{\sqrt{g}}k_0\left[\frac{2\pi}{k_0r|\psi_{\sigma\sigma}(\theta,\sigma_1)|}\right]^{1/2}e^{k_0\zeta_0\sec^2\sigma_1}\sec^3\sigma_1\cos\left[k_0r\psi(\theta,\sigma_1)+\frac{\pi}{4}\right] \\ &-2\frac{\sqrt{k^0}}{\sqrt{g}}k_0\left[\frac{2\pi}{k_0r|\psi_{\sigma\sigma}(\theta,\sigma_2)|}\right]^{1/2}e^{k_0\zeta_0\sec^2\sigma_2}\sec^3\sigma\cos\left[k_0r\psi(\theta,\sigma_2)-\frac{\pi}{4}\right] \end{aligned}\right\}\text{ for }0<\theta<\theta_0 \tag{11}$$

where $\psi(\theta,\sigma)$, σ_1 and σ_2 were previously defined. Since $\psi_{\sigma\sigma}(\theta,\sigma)=0$ at $\theta=\theta^*$, $h(r,\theta)$ is nonuniform near $\theta=\theta^*$. Hence, similarly as before, $h(r,\theta)$ is written as

$$\left.\begin{aligned} h(r,\theta)\sim&-2\frac{\sqrt{k_0}}{\sqrt{g}}k_0\alpha_1(r,\theta)e^{k_0\zeta_0\sec^2\sigma_1}\sec^3\sigma_1\cos\left[k_0r\psi(\theta,\sigma_1)+\frac{\pi}{4}\right] \\ &-2\frac{\sqrt{k_0}}{\sqrt{g}}k_0\alpha_2(r,\theta)e^{k_0\zeta_0\sec^2\sigma_2}\sec^3\sigma_2\cos\left[k_0r\psi(\theta,\sigma_2)-\frac{\pi}{4}\right] \end{aligned}\right\}\text{ for }\theta_0\leq\theta\leq\theta^* \tag{12}$$

where $\alpha_1(r,\theta)$ and $\alpha_2(r,\theta)$ were already given. The present Kelvin wave induced by the Havelock source alse consists of the transverse and divergent waves which are confined inside the same V-shaped region. We compute $h(r,\theta)$ for the surface elevation similarly as before. The results are given in Figs. 5 through 14.

3. Conclusion

The present treatment near the cusp point seemd to be successful as far as the wave pattern is concerned. But a series of computations must be carried out for different values of θ_0 so that the best value of θ_0 can be determined. The various Kelvin wave patterns in the present paper are extensively discussed in Ref.[2].

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