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# On the Study of System Reliability Analysis of Tension Leg Platforms

by

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# TLP 해양구조물의 시스템 신뢰성 해석에 관한 연구

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## Abstract

In this paper, another method for system reliability analysis, called the extended incremental load method, is introduced. The method is an extension of the conventional incremental load method and has been developed aiming at evaluating the probability of system failure (or system reliability) of continuous structures such as floating offshore structures under the multiple loading condition, more realistically considering the post-ultimate behaviour of failed components and directly using the strength formulae of principle components in a structure with employing the modified safety margin equation proposed herein in the system analysis.

The method has been applied to the Hutton TLP operated in the Hutton field in the North Sea and a certain variant of the design using the TLP Rule Case Committee type improved strength models.

System failure probability and corresponding system reliability indices are derived for a more economical and efficient design. The redundancy characteristics are also addressed. The TLP forms are shown to possess high reserve strength and system safety.

## 요 약

본 논문에서는 시스템 신뢰성 해석을 위한 또 하나의 방법으로써 '확대하중증분법'을 소개하였다. 이 방법은 통상의 하중증분법의 적용범위를 확대한 것인데, 부유식 해양구조물과 같은 연속계 구조물에 대해 복수하중(multiple loading)하의 시스템의 파괴확율(또는 시스템의 신뢰성)을 추정하고, 파괴요소의 후 파괴거동을 보다 실질적으로 고려하며 또한 본 논문에서 제안하는 수정된 안전여부식을 이용하여 구조물내의 주요 부재의 강도공식을 시스템 신뢰성 해석에 직접 이동하기 위한 것이다.

그 방법을 북해의 Hutton 구역에서 가동하는 Hutton TLP와 TLP Rule Case Committee형의 개선된 강도공식을 사용하여 설계한 Hutton TLP의 변형된 형태에 적용하였다.

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보다 경제적이고 효율적인 설계를 위해 시스템의 파괴확율과 그에 대응하는 시스템 신뢰성 지수를 유도하였고, 부정정도(redundancy)의 특성에 대한 언급도 포함하였다. 이로부터 TLP 구조물은 높은 잉여강도(reserve strength)와 시스템 견지에서의 안전성을 갖음을 알 수 있었다.

## Nomenclature

- $B_{ml}$ : loading coefficient of the lth loading of the mth failure mode
- $C_{mk}$ : resistance coefficient of component $k$ or $r_k$ of the mth failure mode
- $L$ : number of loading types acting on a structure
- $P$ : radial pressure
- $P^{(l)}$ : the lth loading acting on a structure
- $R_k$ : resistance of component $k$ or $r_k$
- $X_M$ : strength modelling parameter
- $\underline{X}_M$ : mean bias of strength modelling parameter
- $V_{X_M}$ : strength modelling uncertainty (COV of strength modelling parameter)
- $Z_m$ : safety margin of the mth failure mode
- $Z_m'$ : modified safety margin of the mth failure mode
- $P_{f,sys}$ : probability of system failure
- $\beta_{sys}$ : system reliability index
- $\beta_{path}$ : reliability index of a failure mode (or path)
- $\lambda_T$ : total load factor (reserve strength index of system)
- $\sigma_x$ : axial stress

## 1. Introduction

The development of probability-limit state design has been motivated by a desire to quantify performance of structures and to treat uncertainties in loads, resistances and analysis in a more rational way[1]. Structural reliability theory has been tremendously developed during the last two decadesand has been applied to the practical design of various structures.

Regarding its applications to marine structures, the basic idea was discussed at the International Ship Structures Congress (ISSC) in 1967[2]. The practical application can be found in an early paper by Mansour and Faulkner[3]. Since then many studies have been reported. Most applications are, however, based on component reliability analysis. It has well been appreciated for many years that a more complete estimates of the reliability of a structure must include a structural system reliability analysis. Another important motivation of concerning with the system analysis arised from the fact that the redundancy arising from various reasons[4] may be considered in design state and the system partial safety factor which can be obtained from the system reliability analysis and can reflect the system strength beyond failure of component level could be included in the strength check equation.

In this field, most studies are concerned with discrete structures, such as fixed jacket platforms, and only a few studies have been reported for continuous structures, such as tension leg platforms (TLP) and semi-sumersibles[5~8]. The aim of this paper is to introduce another approximate method for the system reliability analysis for structural systems, discrete and continuous. The metis called herein the extended incremental load method which is an extension of the conventional method porposed by Moses[9] and has been developed to more realistically take into account the effect of post-ultimate behaviour after failure of components on the system strength and consequently, on the system safety level and to directly use the strength models of principle components used in design. These two points are major merits of the present method compared with other methods developed up to the present.

An important task in system reliability analysis is to identify the failure modes. It is not practical to include all possible failure modes even in a simple structure. In practice a small number of failure modes that are expected to significantly contribute to the sytem failure must be considered in evaluating the system reliability. These are referred to as the most important failure modes in this paper. References 7 and 10 well review the state-of-the-art

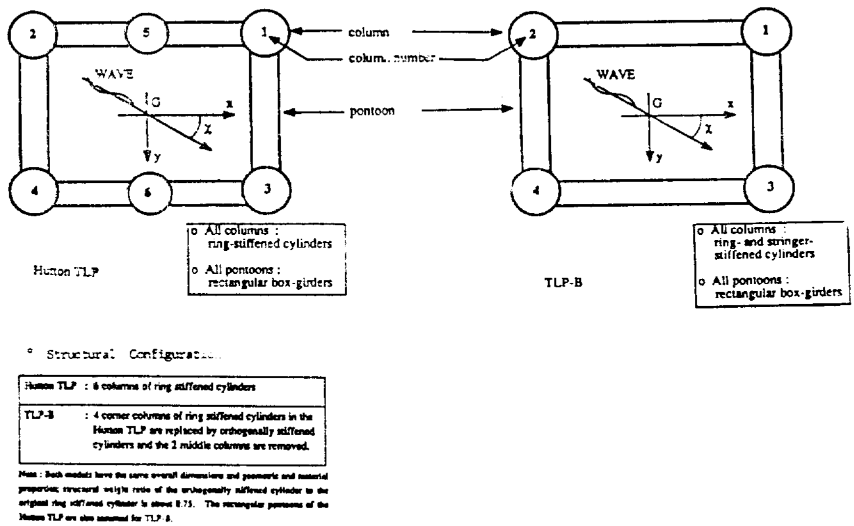



**Fig. 1** TLP Models: $xy$-plane & structural configuration

in the system reliability methods and the proceedures of identifying the important failure modes.

In this study the Hutton TLP, which is the first example of TLP and now is operated in the Hutton field in North Sea of U.K. sector, is chosen as a TLP model and its one variant, TLP-B, is also chosen to compare the system safety level for the structures having different component type, say, ring-stiffened cylinders and stringer-stiffened cylinders. Fig. 1 shows the present TLP models. The system failure probabilities and reliability indices are evaluated for the design load of the Hutton TLP.

Three categories of environmental loadings are considered, that is, static loading due to dead lodd dnd static pressure, quasi-static loading due to current and wind, and hydro-dynamic loading due to wave and motion. The load due to wave drift is not considered. A Morison type approach[11] is employed to evaluate the hydro-dynamic loading. The load effects are obtained by modelling the TLP structure into a finite element type space frame.

To find the important failure modes, the paper use the probabilistic searching technique of which basic idea is from the concept of artificial intelligence. The procedure considers not only probabilistic criteria but also deterministic criteria to search the important failure modes, deterministically as well as probabilistically.

## 2. Modified Safety Margin Equation

When $j$ components, $r_1, \cdots, r_j$, have failed, a general expression of the linear safety margin for the mth failure mode is expressed as Fq.(1)[12]:

$$Z_m=\sum_{k=1}^{j}C_{mk}R_k-\sum_{l=1}^{L}B_{ml}P^{(l)} \tag{1}$$

where these terms are defined in Nomenclature. The first summation term is called the resistance term and the second the loading term. When the failure of $j$th component leads to the collapse of the structure, $C_{mj}=1.0$. In order to directly use the strength formulae the above safety margin equation is modified in the non-dimensional form as follows to consider the effects of the strength modelling parameter in the system reliability analysis[6, 7, 13]. The strength modelling parameter(or error), $X_M$ is usually defined as:

$$X_M=\frac{\text{actual behaviour}}{\text{predicted behaviour}} \tag{2}$$

which represents the subjective uncertainty of the strength model in the reliability analysis[14]. The mean of $X_M$ is referred to as the mean bias, $\underline{X}_M$ and when there is sufficient data, the randomness of

$X_M$ is usually referred as the modelling uncertainty specified by its coefficient of variation($V_{X_M}$). For Good strength models Faulkner et al[15] recommended the statistical requirements related to the strength modelling parameter.

Separating tge resistance term of component $r_j$ in Eq. (3), which is the last failed component and considering that its coefficient is unity:

$$Z_m=R_j+\sum_{k=1}^{j-1}C_{mk}R_k-\sum_{l=1}^{L}B_{ml}P^{(l)}$$
$$=R_j-Q_j \tag{3}$$

where $Q_j$ is the net load effect on component $r_j$ due to the already failed components, $r_1, \cdots, r_{j-1}$ and due to the loading acting on the structure, namely:

$$Q_j=\sum_{l=1}^{L}B_{ml}P^{(l)}-\sum_{k=1}^{j-i}C_{mk}R_k \tag{4}$$

Eq.(3) can be wrtten as Eq.(5) by introducing the strength modelling parameter of component $r_j$, say $X_{M_j}$:

$$Z_m=X_{M_j}R_j-Q_j \tag{5}$$

Dividing both sides of the above equation by $R_j$ and substituting Eq.(4) for $Q_j$ results in the safety margin in the non-dimensional form which has the same physical meaning as Eq.(1):

$$Z_m'=X_{M_j}+\sum_{k=1}^{i-1}C_{mk}\frac{R_k}{R_j}-\sum_{l=1}^{L}B_{ml}\frac{P^{(l)}}{R_j} \tag{6}$$

The term $R_k/R_j$ can be generally regarded as a function of resistance variable vector of component $r_k$ and $r_j$, $\{R\}_k$ and $\{R\}_j$, and $P^{(1)}/R_j$ as a function of resistance variable vector of component $r_j$ and loading variable vector, $\{Q\}_j$ acting on the component. The safety margin equation (1) eventually can be conceptually modified in the non-dimensional form:

$$Z_m'=X_{M_j}+\sum_{k=1}^{i-1}G_k(\{R\}_{k'}\{R\}_j)-\sum_{l=1}^{L}G_l(\{Q\}_l, \{R\}_j) \tag{7}$$

Function $G_k$ represents the contribution of the strength of already failed component. $r_k(k=, j-1)$ on the safety margin and function $G_l$ that of load effects.

Eq.(7) is a feasible way to directly use the strength fomulae for principle members in the system reliability analysis and allows for the uncertainties of loadings and strengths without loss of any physical meaning as Eq.(1). Each function $G_k$ or $G_l$ is treated as a random variable in the analysis and its mean and COV may be obtained easily using the concept of the first-order second moment.

Detail procedures of deriving the safety margin equation and identifying the most important failure modes are found in references 6 and 7.

## 3. Uncertainty Modelling

### 3.1. Design Variables

Uncertainties of design variables are inherent and therefore, objective. They arise from the fabrication procedure of materials and can be characterised by their means, coefficient of variations (COVs) and distribution types. The postulated uncertainties in design variables are listed in Table 1.

**Table 1** Uncertain in design variables

| design variable | COV (%) |
|---|---|
| geometric properties | 4.0 |
| elastic modulus | 4.0 |
| yield stress | 8.0 |

### 3.2. Modelling Uncertainty

3.2.1. Strength Model

In the present system reliability analysis the strength modelling parameter ($X_M$) has been incorporated in the safety margin equation[See Eq.(7)]. The uncertainty of $X_M$ is characterised by its mean bias, $\underline{X}_M$ and COV, $V_{X_M}$, and its distribution type. $\underline{X}_M$ and can vary depending on the form of strength model, which implies that the evaluated system safety level can be much affected by the strength model itself used in the analysis.

For cylindrical component a comparatively simple model, Eq.(8)[16] is used for both TLP models for consistency in the system reliability analysis.

**Table 2** Uncertainty in strength model

| | ring-stiffened cylinder (Hutton) | stringer-stiffened cylinder (TLP-B) |
|---|---|---|
| $\underline{X}_M$ | 0.99 | 0.99 |
| $V_{X_M}$ | 10% | 13% |

$$\left[\frac{P}{P_u}\right]^m+\left[\frac{\sigma_x}{\sigma_{xu}}\right]^n=1 \qquad (8)$$

where $P$ is the radial pressure and $\sigma_x$ is the the total axial stress resulting from axial load, bending moments and pressure. In the above equation $m=2$ and $n=1$ are used. For box-girder of rectangular section the strength model, under combined axial compression and bi-axial bending moments, proposed by Lee[17] is used.

$$\frac{F_x}{F_{xu}}+\left[\left(\frac{M_y}{M_{yu}}\right)^{1.8}+\left(\frac{M_z}{M_{zu}}\right)^{1.8}\right]^2=1.0 \qquad (9)$$

where $F_x$, $M_y$ and $M_z$ are axial compressive force and bi-axial bending moments, and index '$u$' means the ultimate strength. The above equation was derived to best fit the numerical results obtained from the non-linear analysis of rectangular box-girders using a beam-column concept.

The distribution type of $X_M$ is assumed to be log-normal rather than normal as taken by TLP Rule Case Committee[16]. This is based on the fact that among the design variables in the previous setion, yield stress may be supposed to mainly contribute to the strength and hence, on the strength model in use. Table 2 showe the mean biases and COVs of strength models, Eq.(8) and (9).

3.2.2. Loading Model

It is usually expected that the degree of uncertainty with loading model is higher than that with strength. This arises from there being much more unknown factors affecting loads and their effects. The COVs of static and quasi-static load effects are usually imposed to be 10 and 20%, respectively. Referring to the uncertaitainty in dynamic component, when using the extreme value analysis with significant wave heights, the COV is usually taken to be 30% for TLP structures but when evaluating the dynamic loading is based on a single design wave with 100 years return period, the COV will be reduced to be 10%, and two cases may give the similar results. Fig. 2 shows the distributions of extreme value and design load value for dynamic load effects. The distribution type can be modelled to be log-normal, reasonablly. Table 3 summarises the uncertainty for loading model.

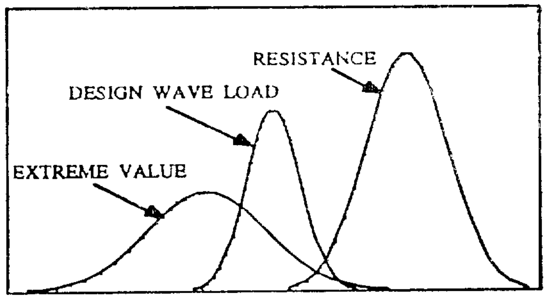



**Fig. 2** Two types of distribution of dynamic load effect

**Table 3** Uncertainty in loading

| load effect | mean bias | COV(%) | distribution type |
|---|---|---|---|
| static | 1.0 | 10.0 | log-normal |
| quasi-static | 1.0 | 20.0 | log-normal |
| dynamic | 1.2*<br>1.0** | 10.0 | log-normal |

* for axial force
** for other dynamic load effects

## 4. System Reliability of TLP Strnctures

System reliability analysis for the two TLP models has been carried out under the same design environmental loading condition (Table 4[18]). The component behaviour is assumed to be ductile and the probability of system failure is evaluated for three wave directions, say $x=0$, 45 and 90 deg. (see

**Table 4** Design environmental loading condition[18]

| | | | |
|---|---|---|---|
| wind | : 1min. mean wind velocity at 10m elevation | | $V_{10}=44.0$m/sec |
| wave | : regular design wave | wave height | $H_w=30.3$m |
| | | wave period | $T_w=14.9\sim18.5$sec |
| current | : 5min. mean velocity at 10m depth | | $V_c=0.85$m/sec |
| mean draft : | | | $T=32.0$m |

**Table** 5 Probability of system failure and system reliability index for the Hutton TLP and TLP-B

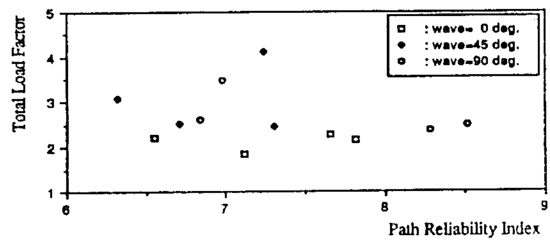
| wave direction / model | 0 deg. | 45 deg. | 90 deg. | average |
|---|---|---|---|---|
| Hutton TLP | $0.294\times10^{-10}(6.55)$ | $0.147\times10^{-9}$ $(6.30)$ | $0.411\times10^{-11}(6.83)$ | $0.602\times10^{-10}(6.44)$ |
| TLP-B | $0.640\times10^{-11}(6.77)$ | $0.510\times10^{-12}(7.13)$ | $0.123\times10^{-12}(7.32)$ | $0.234\times10^{-11}(6.91)$ |

note : number in ( ) is the system reliability index corresponding to the probability of system failure

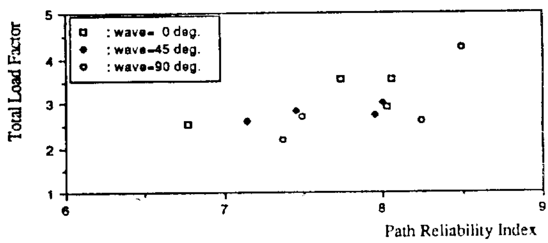
Fig. 1).

Table 5 showsthe probability of system failure, $P_{fsys}$ and corresponding system reliability index, $\beta_{sys}$ for the Hutton TLT and TLP-B. The average value of $P_{fsys}$ is obtained under the assumption that the same probility of each wave direction is given, and the average $\beta_{sys}$ is the value corresponding to the average $P_{fsys}$. From Table 5 it is very interesting to note that average $\beta_{sys}$ of TLP-B is 7.4% greater than that of the Hutton TLP in spite of its less structural material than the Hutton TLP. The higher $\beta_{sys}$ value of TLP-B may be due to that the component on the mid-column of ring-stiffened cylinder in the Hutton TLP has a higher chance of failing than the other components, and hence, the re-distribution of load effects of failed component on the mid-column much affects the reduction of system stiffiness and consequently, the system strength. But, in the case of TLP-B it does not have mid-voloumn and the failure of components on corner columns and pontoons give less reduction of system strength. This might be, that is the re-distribution of load effects due to failed components, the main reason that TLP-B shows a higher system reliability in spite of there being much less structural material (see table in Fig. 1).

When using the incremental load method for system reliability analysis, we can obtain the total load factor, $\lambda_T$ which is the ratio of the ultimate load of a system to the design load and is closely related to the reserve strength of a system. Fig. 3 shows the relation between $\lambda_T$ and the reliability index of failure mode (or path), $\beta_{path}$. Four failure modes which lead to structural collapse have been found for the present TLP models. As can be seen in Fig. 3, the failure mode having a higher $\beta_{path}$ does not always have higher $\lambda_T$, in other words, the deterministically most important failure mode is not identical with the probabilistically most important failure mode. This was also found from the ysystem reliability analysis for discrete structures[6, 19]. The high values of total load factor support the high redundancy of TLP structures.




(1) Hutton TLP




(2) TLP-B

**Fig.** 3 Relation between path reliability index($\beta_{sys}$) and total load factor($\lambda_T$)

## 5. Conclusions

This paper introduces the extended incremental load method as another approximate method for reliability analysis of structural systems, which can be claimed to be suitable for the system reliability analysis of continuous structures as well as discrete structures under the multiple loading condition. The modified safety margin equation (7) opens the way to directly use the strength formulae of principle components in a structure in the system analysis. The method has been applied to TLP structures.

From the results for the Hutton TLP and its variant, TLP-B, TLP-B shows a higher system reliability level in spite of there being much less structural material. This may be due that the redistribution of load effects in survival components due to failed components depends on structural configuration and component type affects the system strength. Since the component behaviour was assumed to be ductile, the actual reliability of TLP structural system should be lower than the evaluated values in this study because the non-ductile behaviour certainly reduces the system stiffness and therefore, system reliability. According to the experimental results for stiffened cylinders, the behaviour of stringer-stiffened cylinders is closer to ductile behaviour than the ring-stiffened cylinders. Considering this, it can be said that stringer-stiffened cylinders are more efficient than ring-stiffened cylinders under the such kind of loading found in TLP structures from the view point of system reliability.

An extension of this study may be to find the system partial safety factor which will be included in safety check equation of component strength through a parametric study.

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