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Hierarchical Feedback Control of Large-Scale Discrete-Time Systems with Coupled States and Inputs

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요 약

상태 및 입력이 상호결합된 대규모 시스템의 최적 추적제어를 위해 Singh의 다층방법을 확장 적용한다. 확장된 다층방법의 정상상태 추적오차와 수렴조건을 해석적으로 유도하며 그 결과 정상상태 추적오차와 수렴속도는 서로 절충되어야 함을 알 수 있다. 또한 성능지수에 공칭입력을 도입함으로써 정상상태 추적오차 및 계산부담면에서 Singh의 방법보다 우수한 새로운 다층방법을 제안한다. 여기서 구한 궤환이득 행렬 및 보상벡타는 모든 초기조건에 대하여 최적이 되므로 on-line 계산량은 매우적다.

Abstract- Singh's multi-level method is extended to the optimal tracking control of a large interconnected dynamical system which has coupled states and coupled inputs. The steady-state tracking error and a convergence condition for the extended multi-level method are derived analytically and the results show that the steady-state tracking error and a convergence rate have to be compromised. Also, a new multi-level method which is advantageous over the Singh's method in steady-state tracking error and computational burden is proposed by introducing nominal inputs into the performance index. The resulting feedback gain matrix and the compensation vector are optimal for all initial conditions so that eventual on-line computation is minimal.

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1. Introduction

A standard centralized optimization technique[1~2] can be used, at least in principle, to control a large interconnected dynamical system, but the high dimensionality of the problem accompanies computational difficulties which are associated with computation time and storage space. To get around these computational difficulties, much work [3~6] has been done on the method of decomposition and coordination. In this method, the large dynamic optimization problem is decomposed into a number of smaller subproblems which can be solved independently and the coordination variables are successively modified to force the optimal soulutions of the independent subproblems to the optimal soution of the overall system.

The main disadvantage that arises in the use of such decomposition and coordination method for the practical control of large interconnected dynamical system is that the control is open-loop in nature so that it is necessary to recalculate it whenever an unknown disturbance changes the initial state of the system. This is undesirable since it ties down a relatively large computer for the calculation and implementation of the control.

Singh at al. [7~8] have proposed a promising hierarchical closed-loop method for the optimal control of large inerconnected dynamical system with coupled states using the interaction prediction method (IPM) [9]. This method is found to be superior to other multi-level methods for a certain class of optimization problems. On the upper-level it has more rapid convergence rate and fewer operations than other coordination rules such as gradient technique [10].

Since some of large interconnected dynamical systems, such as road traffic system [11~12], communication network system [13], have coupled inputs we extend Singh's method to apply more general large interconncted system with coupled states and coupled inputs. Also we derive analytically the steady-state tracking error and a convergence condition for the extended method. The results reveal that unless a particular condition is satisfied the steady-state tracking error always exists and has to be compromised with the convergence condition. Further we propose an efficient hierarchical technique which is advantageous over the Singh's method in steadys-state tracking error and computational burden. The proposed hierarchical technique is based on the transformation of a tracking problem into a regulator problem.

The rest of the paper is divided into three parts. In section 2 the hierarchical optimal solution to the constant target tracking problem of a large interconnected dynamical system is obtained. Aso, its steady-state tracking error and the convergence condition are derived analytically. In section 3 we propose an efficient multi-level technique. In the final section the multi-level methods are applied to a river pollution control example.

2. Soulution to the System with Coupled States and Coupled Inputs

Let us consider the following liner quadratic [LQ] tracking proble of a large interconnected dynamical discrete-time system which is coupled with a number of subsystems.

$$X(k+1)=AX(k)+BU(k)+C, X(0)=X_0 \quad (1)$$

$$J=1/2\sum_{k=1}^{k_f-1}\{[X(k)-X^d]^tQ[X(k)-X^d] + U^t(k)RU(k)\} \quad (2)$$

where A is an $n\times n$ system matrix, B is an $n\times m$ input matrix, C is an $n\times 1$ known constant input vector, X^d is an $n\times 1$ constant desired value of state vector, $Q\geq 0$ is an $n\times n$ diagonal state weighting matrix and $R>0$ is an $m\times m$ diagonal input weighting matrix. It is assumed that (A,B) and (D,A) are stabilizable and detectable pairs, respectively, where $Q=D^tD$.

To overcome computational difficulties associated with the large discrete algebraic Riccati equation and tracking equation the large-scale centralized optimal tracking problem is decomposed into smaller subproblems i-th of which is expressed as follows;

$$x_i(k)=A_ix_i(k)+B_iu_i(k)+c_i+h_i(k), x_i(0)=x_{io} \quad (3)$$

$$h_i(k)=\sum_{j\neq i}^{n}\{L_{ij}\ x_j(k)+M_{ij}\ U_j(k)\} \quad (4)$$

$$j_i=1/2\sum_{k=0}^{k_f-1}\{[x_i(k)-x_i^d]^tQ_i[x_i(k)-x^d{}_i] + u^t{}_i(k)R_iU_i(k)\} \quad (5)$$

where $x_i(k)$ and $u_i(k)$ are an $n_i\times 1$ state vector and an $m_i\times 1$ control vector of i-th subsystem, respectively, $hi(k)$ is an $n_i\times 1$ interaction input vector which comes in from the other sybsystems,

N is the number of interconnected subsystems whcih comprise the overall system, $\sum_{i=1}^{n} n_i = n$ and

$$\sum_{i=1}^{n} m_i = m.$$

The upper-level problem of the hierarchical multi-level technique is essentially updating the coordination vector to force the independent lower-level solutions to the optimal solution of the overall system. For this purpose consider Lagrangian of the decomposed subsystem (3), (4) and (5);

$$\begin{aligned} L = \sum_{i=1}^{n}\sum_{k=0}^{k_f-1} \{&1/2[x_i(k)-x_i^d]^t Q_i[x_i(k)-x_i^d] \\ &+1/2\ u_i^t(k)R_i u_i(k)+\lambda_i^t(k)h_i(k) \\ &-\sum_{j\neq i}^{n}\lambda_j^t(k)[L_{ji}\ x_j\ _i(k)+M_{ij}\ u_i(k)] \\ &+p_i^t(k+1)[-x_i(k+1)+A_i x_i(k) \\ &+B_i u_i(k)+c_i+h_i(k)]\} = \sum_{i=1}^{n} L_i \end{aligned} \tag{6}$$

where λ_i and p_i are an $n_i \times 1$ Lagrange multiplier and costate vector of i-th subsystem, respectively. (6) shows that L is additively separable for given $h_i(k)$ and $\lambda_i(k)$ trajectories. This implies that for any given $h_i(k)$ and $\lambda_i(k)$ trajectories, there are N independent subproblems the i-th subprolem of which is represented by L_i

A necessary condition for updating the $z_i(k)$ and $\lambda_i(k)$ to force the independent subproblm solutions to the optimal soution of the overall problem is given by

$$\frac{\partial L}{\partial h_i(k)} = 0 \text{ and } \frac{\partial L}{\partial \lambda_i(k)} = 0 \tag{7}$$

From this equation two equations are obtained and these equations are used as the upper-level coordination rule from iteration L to $L+1$ as follows;

$$\begin{bmatrix} \lambda_i(k) \\ h_i(k) \end{bmatrix}^{l+1} = \begin{bmatrix} -p_i(k+1) \\ \sum_{j\neq i}^{n}[L_{ij}\ x(k)+M_{ij}\ u_j(k)] \end{bmatrix}^{l} \tag{8}$$

The important point to note is that at the upper-level, it is necessarty to do very little calculations and that the convergence rate is rapid compared to the gradient method.

Now, consider the lower-level problem. The Hamiltonian for the i-th independent subproblem can be written as

$$\begin{aligned} H_i = 1/2&[x_i(k)-x_i^d]^t\ Q_i[x_i(k)-x_i^d(k)] \\ &+1/2u_i^t(k)R_i u_i(k)+\lambda_i^t z_i(k) \\ &-\sum_{j\neq i}^{n}\lambda_j^t(k)[L_{ji}\ x_i(k)+M_{ji}\ u_i(k)] \\ &+p_i^t(k+1)[A_i x_i(k)+B_i u_i(k)+c_i+h_i(k)] \end{aligned} \tag{9}$$

Then the following set of necessary conditions can be obtained.

$$\begin{aligned} x_i(k+1) = A_i x_i(k)+B_i u_i(k)+c_i \\ +h_i(k),\ x_i(0)=x_{io} \end{aligned} \tag{10}$$

$$u_i(k) = Q_i x_i(k) + A_i^{-1}B_i^t p_i(k+1) - R_i^{-1}\sum_{j\neq i}^{n} M_{ji}^t \lambda_j(k) \tag{11}$$

$$\begin{aligned} p_i(k) = Q_i x_i(k)+A_i^t p_i(k+1) \\ -\sum_{j\neq i}^{n} L_{ji}^t\lambda_j(k) - Q_i x_i^d,\ p_i(k_f)=0 \end{aligned} \tag{12}$$

From these equations and the coordination rule optimal trajectories of states and control inputs for a finite interval ($k=0, \cdots, k_f-1$) can be obtained by the following open-loop algorithm 1;

step 1: At lthe upper-level set $L=1$, predict initial values for $h_i(k)=h_i^0(k)$ and $\lambda_i(k)=\lambda_i^0(k)$, $i=1, 2, \cdots, N$ and $k=0, 1\cdots, k_f-1$ and pass them down to the lower-level.

step 2: At the lower-level solve the independent necessary conditions for optimality (10), (11) and (12) for $x_i(k)$, $u_i(k)$ and $p_i(k)$, 1, 2, $\cdots$, N and $k=0, 1, \cdots, k_f-1$, respectively and send them to the upper-level.

step 3: At the upper-level, check for the convergence of (8). i.e., whether their errors are within the pre-determined error bound, ε. If not update $\lambda_i(k)$ and $z_i(k)$ using (8), $i=1, 2, \cdots, N$ and $k=0, 1, \cdots, k_{f-1}$ and set $L=L+1$ and go to step 2.

The following Theorem gives the closed-loop control.

Theorem 1: The optimal tracking control law of the large scale system with coupled states and coupled inputs, (1) for infinite-time case ($k_f \to \infty$) is given by

$$U(k) = GX(k)+d \tag{13}$$

where G and d are a constant feedback gain matrix and a compensation vector, respectively.

proof : As a simple intuitive proof of Theorem 1 we suppose that at the opimum,

$$\lambda_i(k) = -p_i(k+1) \qquad (14\text{-a})$$

$$h_i(k) = \sum_{j \neq 1}^{n} [L_{ij} \ x_j(k) + M_{ij} \ u_j(k)] \qquad (14\text{-b})$$

Substituting (14-a) and (14-b) into the necessary conditions for optimality (10), (11) and (12) we olbtain the following integrated expressions.

$$X(k+1) = \hat{A}X(k) + \hat{B}U(k) + C + \tilde{A}X(k) + \tilde{B}U(k)$$

$$= AX(k) + BU(k) + C \qquad (15)$$

$$U(k) = -R^{-1}[\hat{B}+\tilde{B}]P(k+1)$$

$$= -R^{-1}BP(k+1) \qquad (16)$$

$$P(k) = Q[X(k) - X^d] + [\hat{A}^t + \tilde{A}^t]P(k+1)$$

$$= Q[X(k) - X^d] + A^tP(k+1), \quad P(k_f) = 0 \qquad (17)$$

where $\hat{A}$ and $\hat{B}$ are the block diagonal part of the global system and input matrices A and B, respectively and $\tilde{A}$ and $\tilde{B}$ are the off-block diagonal part of the matrices A and B, respectively.

Since (15), (16) and (17) are the same as the necessary conditions for optimality of standard centralized optimal tracking problem, the optimal control law is given by (13) if k_f approaches infinity. This completes the proof.

Consequently, the procedure to obtatin G and d in (13) can be summarized as follows ;

step 1 : Run the open-loop algorithm 1 with $X(0) = 0$ and let $d = U(0)$.

step 2 : Run the open-loop algorithm n times sucessively for the following initial conditions and obtain $U^i(0)$, $i = 1, 2, \cdots, n$.

$$X^1(0) = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad X^2(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad \cdots,$$

$$X^n(0) = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

step 3 : Let $G = [U^1(0) - d \quad | U^2(0) - d | - - | U^n(0) - d]$

Note that $n+1$ runs of the open-loop algorithm 1 are needed to obtain the feedback gain matrix G and the compensation vector d. However, all this off-line computation is performed independently at subsystem level so that its computation time and storage requiremenmts can be reduced. From now on we call the above procedure to otain G and d the extended method.

Therem 2 : The extended method has the steady-state tracking error given by

$$e_{ss} = \{I_n - A + BR^{-1}B^t[I_n - A^t]^{-1}Q\}^{-1} \{[I_n - A]x^d - C\} \qquad (18)$$

proof : If k_f is large enough for he system to each a steady-state, we obtain the followings form (15), (16) and (17).

$$X_s = AX_s + BU_s + C \qquad (19)$$

$$U_s = -R^{-1}B^tP_s \qquad (20)$$

$$P_s = Q[X_s - X^d] + A^tP_s \qquad (21)$$

where X_s, U_s, P_s are the steady-state state, control and costate vector, respectively, From (19), (20) and (21) we obtain

$$[I_n - A]X_s = -BR^{-1}B^t[I_n - A^t]^{-1}Q[X_s - X^d] + C \qquad (22)$$

Define the steady-state tracking error as

$$e_{ss} = X^d - X_s \qquad (23)$$

Substituting (23) into (22) we obtain (18). This completes the proof.

Theorem 3 : The convergence condition of the algorithm 1 is given by

$$\frac{R}{2} + [\hat{B}^t\hat{T}^*Q\hat{T}\hat{B} - 1/2B^tT^*QTB] > 0 \qquad (24)$$

where $\hat{T}$ is block diagonal part of T and T^* is the adjoint operator of T which is defined by the following linear operator[14] ;

$$y(k) = [Tx](k) \leftrightarrow y(k) = \sum_{j=0}^{k_f-1} \Phi(k, \ j+1)x(k) \qquad (25)$$

where Φ is the state transition matrix of (1).

Remark 1 :

(a) Cohen[15] has derived the convergence condition of IPM for the continuous-time system. It can be easily shown that the condition is identical to that of the discrete-time system. Therefore, the proof of Theorem 3 is omitted.

(b) From Theorem 2, we can see that the steady-state tracking error always exists and depends on Q and R unmless $[I_n-A]X^d-C=0$.

(c) From Theorem 3, the convergence condition (24) will be always satisfied provided that R is large enough. This is because in second term on the left-hand side of (24), R is not present.

(d) From Theorem 2, an increase of $\|Q\|$ or a decrease of $\|R\|$ reduces the steady-state tracking error. But these may violate the convergence condition of Theorem 3. Hence the steady-state tracking error and the convergence condition have to be compromised.

In the following, we propose an efficient multi-level technique which is based on the extended method to reduce the steady-state tracking error

3. Proposed Method

Let us take the performance index to reduce the steady-state tracking error as follows:

$$J^p=1/2\sum_{k=0}^{k_f-1}\{[X(k)-X^d]^t Q[X(k)-X^d]+[U(k)-U^n]^t R[U(k)-U^n]\} \quad (26)$$

where U^n is an $m\times 1$ pre-determined nominal control vector, which will be discussed later.

Define new state and control vectors as

$$Z(k)=X(k)-X^d \quad (27\text{-a})$$

$$V(k)=U(k)-U^n \quad (27\text{-b})$$

Using (27-a) and (27-b) we can tranform the large-scale optimal tracking problem of (1) and (26) into the following regulator problem with constant input.

$$Z(k+1)=AZ(k)+BV(k)+C^p, \quad Z(0)=X(0)-X^d \quad (28)$$

$$J^p=1/2\sum_{k=1}^{k_f-1}\{Z^t(k)QZ(k)+V^tQZ(k)+V^t(k)RV(k)\} \quad (29)$$

where $C^p=[A-I_n]X^d+BU^n+C$.

Decompose the above large scale optimal regulator problem into a number of independent subproblems i-th of which is expressed as follows.

$$Z_i(k+1)=A_iZ_i(k)+B_iv_i(k)+c_i^p+g_i(k),$$

$$z_i(0)-x_i^d \quad (30)$$

$$g_i(k)=\sum_{j\neq i}^{n}[L_{ij}\ z_j(k)+M_{ij}\ v_j(k)] \quad (31)$$

$$J_i^p=1/2\sum_{k=0}^{k_f-1}\{z_i^t(k)Q_iz_i(k)+v_j^T(k)R_iv_i(k)\} \quad (32)$$

where $z_i(k)$ and $v_i(k)$ are an $n_i\times 1$ new state vector and an $m_i\times 1$ new control vector of i-th subsystem, respectively and gi(k) is an ni×i interaction input vector which comes in from the other sybsystems.

Since the procedure to obtain the hierarchical open-loop control is similiar to the extended method in the previous section, we describe the results only.

(i) At the upper-level:

$$\begin{bmatrix}\gamma_i(k)\\ g_i(k)\end{bmatrix}^L=\begin{bmatrix}-q_i(k+1)\\ \sum_{j\neq i}^{n}[L_{ij}\ z_j(k)+M_{ij}\ v_j(k)]\end{bmatrix}^{L+1}$$

where $r_i(k)$ and $q_i(k)$ are an $n_i\times 1$ Lagrange multiplier and costate vector of i-th subsystem, respectively.

(ii) At the lower-level:

$$z_i(k+1)=A_iz_i(k)+B_iv_i(k)+c_i^p+g_i(k),\quad z_i(0)=x_i(0)-x_i^d \quad (34)$$

$$v_i(k)=-R_i^{-1}B_i^t(k+1)q_i(k+1)-R_i^{-1}\sum_{j\neq i}^{n}M_{ji}^t\gamma_j(k) \quad (35)$$

$$q_i(k)=Q_iz_i(k)+A_i^tq_i(k+1)-\sum_{j\neq i}^{n}L_{ji}^t\gamma_j(k),\quad q_i(k_f)=0 \quad (36)$$

Theorem 4: The Optimal control law of the transformed large scale regulator problem for infinmite-time duration is given by

$$V(k)=GZ(k) \quad (37)$$

Proof: Similar to the proof of Theorem 1.

As in the previous section, the procedure to obtain G in (37) can be summarized as follows;

step 1: Solve the open-loop algorithm n times successively for the following initial conditions and obtain $V^i(0)$, i=1, 2, ⋯,n.

$$Z^1(0)=\begin{bmatrix}1\\0\\ \cdot\\ \cdot\\ \cdot\\0\end{bmatrix},\quad Z^2(0)=\begin{bmatrix}0\\1\\0\\ \cdot\\ \cdot\\0\end{bmatrix},\cdots,$$

$$Z^n(0)=\begin{bmatrix}0\\0\\\cdot\\\cdot\\0\\1\end{bmatrix}$$

step 2 : Let $G=[V^1(0)|V^2(0)]\cdots|V^n(0)]$

Theorem 5 : A necessary and sufficient coindition for the optimal tracking with zero steady-state tracking error is

$$BU^n=[I_n-A]X^d-C \qquad (42)$$

and the steady-state optimal tracking control law of (1) and (27) is given by

$$U(k)=GX(k)+d^p \qquad (43)$$

where $d^p=-GX^d+U^n$

Proof : With $BU^n=[I_n-A]X^d-C$ the tracking problem of (1) and (27) can be transformed into the regulator problem of (28) and (29) with $C^p=0$ and from Theorem 4, the feedback gain matrix G which is obtained hierarchically is identical with the standard centralized one. Therefore $[A+BG]$ is an asymptotically stabe matrix [16] if (A, B) and (D, A) are stablizable and detectable pairs, respectively, Hence $X(k)$ and $U(k)$ approach X^d and U^n, respectively from (27-a) and (27-b) as k goes to infinity. Accordingly, from (37) we obtain (43). This completes the proof.

Remark 2 :

(a) The optimal tracking control law (43) is now obtained by the regulator algorithm. Thus it is not necessary to run the open-loop algorithm to obtain the compensation vector d. Morever, the open-loop algorithm saves $(n_i\times n_i)\times(n_i\times 1)$ matrix multiplication $N\times$L times because (36) does not contain Q_ix_i which is contained in (12).

(b) If a vector $[\text{In}-A]X^d-C$ belongs to the column space of B, the nominal control input U^n which is obtained by

$$U^n=[B^tB]^{-1}B^t\{[I_n-A]X^d-C \qquad (44)$$

is the unique solution to (42). In this case, the proposed multi-level method has a zero steady-state tracking error regardless of Q and R.

(c) If a vector $[I_n-A]X^d-C$ does not belong to the column space of B, the nominal control input U^n which is obtained by (44) is the approximate solution to (42).

4. Simulation and Discussions

To illustrate the presented results we consider he river pollution model of river Cam near Cambridge[17] ;

$$x_i(k+1)=A_ix_i(k)+B_iu_i(k)+c_i+h_i(k),\quad i=1,\ 2.$$

where, $A_1=A_2=\begin{bmatrix}0.18 & 0\\-0.25 & 0.27\end{bmatrix}$, $B^1=B_2=\begin{bmatrix}-2\\0\end{bmatrix}$,

$$L_{21}=\begin{bmatrix}0.55 & 0\\0 & 0.55\end{bmatrix},\quad C_1=\begin{bmatrix}4.5\\6.15\end{bmatrix},\quad C_2=\begin{bmatrix}2\\2.65\end{bmatrix},$$

$$x_1(0)=[0\quad 0]^t,\ x_2(0)=[0\quad 1]^t$$

Computer simulations are carried out for the following two cases.

Case Ⅰ. $[I_n-A]X^d-C$ belongs to the column space of B ; $x^d=[4.16\quad 7\quad 5.56\quad 7]^t$

Case Ⅱ. $[I_n-A]X^d-C$ does not belong to the column space of B ; $X^d=[5\quad 7\quad 5\quad 7]^t$

In simulations, k_f is chosen to 30 which is sufficiently long enough for the system to reach a steady-state and ε is chosen to be 10^{-5}.

A summary of the simulation results of both Singh's method and proposed method is given in Table 1.

The simulation results show that the steady-state tracking error of the proposed method is smaller than that of Singh's method in both cases and these results are consistent with Thorem 2 and Theorem 5. Especially the steady-state tracking error of the proposed method in case I is zeor irrespective of Q and R and these weighting matrices affect only the transient response.

5. Conclusion

Sigh's multi-level method is extended to the optimal tracking control of a large interconnected

Table 1 Summary of the simulation results

Method	Weighting Martix		Number of	Steady-State Tracking Error	
	Q	R	Iterations	Case Ⅰ	Case Ⅱ
Singh's Method	$\mathbb{I}_4$	10 $\mathbb{I}_2$	diverge	———	———
	$\mathbb{I}_4$	50 $\mathbb{I}_2$	16	$[-1.13\ .39\ -.36\ -.41]^t$	$[-.34\ .40\ -.89\ .43]^t$
	$\mathbb{I}_4$	100 $\mathbb{I}_2$	12	$[-1.22\ .42\ -.45\ -.47]^t$	$[-.40\ .42\ -.99\ .47]^t$
	$\mathbb{I}_4$	500 $\mathbb{I}_2$	9	$[-1.30\ .45\ -.54\ -.52]^t$	$[-.47\ .45\ -1.09\ .52]^t$
Proposed Method	$\mathbb{I}_4$	10 $\mathbb{I}_2$	diverge	———	———
	$\mathbb{I}_4$	50 $\mathbb{I}_2$	16	0.	$[0.\ .29\ 0.\ .02]^t$
	$\mathbb{I}_4$	100 $\mathbb{I}_2$	12	0.	$[0.\ .29\ 0.\ .02]^t$
	$\mathbb{I}_4$	500 $\mathbb{I}_2$	9	0.	$[0.\ .29\ 0.\ 02]^t$

dynamical system which has coupled states and coupled inputs. The steady-state tracking error and a convergence condition of the extended mothod is derived analytically and the results show that the steady-state tracking error and convergence condition have to be compromised. Also we propose an efficient multi-level technique for the tracking problem which is advantageous over the Singh's method in computational burden and the steady-state traciking error. All the calculations in this method are performed within a hierarchical structure and the resulting controller provides a feedback control which is opitimal for all initial conditions.

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