論 文
39～1～12

수동 Compliance가 능동적 Compliance제어의 안정도에 미치는 영향

A Stability Effect of Passive Compliance on Active Compliance Control

鄭 台 相*
(Tae-Sang Chung)

Abstract- Active compliance is often used in the control of robot manipulators for the implementation of complex tasks such as assembly, multi-finger fine motion, legged-vehicle adaptive control, etc. This technique balances the interactive force between the manipulator tip and its working environment with its position and velocity errors to achieve the operation of a damped spring. This paper investigates the effect of passive compliance on system stability with regard to force feedback implementation for actively compliant motion. Usually it is understood that accurate position control requires a stiff system. However, theoretical examination of control experiments on a legged suspension vehicle suggests that, if the control includes discrete-time force feedback, some passive compliance is necessary at the legs of the vehicle for system stability. This can be an important factor to be considered in manipulator design and control. A theoretical analysis, numerical simulation, and experimental result, confirming the above conclusion, are introduced in this paper.

1. Introduction

When the end effector of a manipulator comes into contact with its working environment, compliant motion is requried to control the end effector position. This is necessary because the manipulator, itself, is not a perfect positioning device and becauses the working environment is not completely known. This is an actively compliant motion allowing the manipulator to adapt to its working environment. The interactive force between the end effector and the environment is measured and used in modifying positional command, usually by a linear gain. A great deal of previous work has been done in force and compliance control of robotic systems, and has been summarized in recent papers.[1-4]

A major research project in legged locomotion, conducted at the Ohio State University, resulted in the development of an experimental prototype, the OSU Hexapod, plus various control schemes and gait selection algorithms for the vehicle.[5, 6] The vehicle was fully equipped with vector force sensors at each leg[7] and, for the control of the vehicle, active compliance[8-10] has been used.

*正 會 員 : Univ. of Kentucky 電氣工學科 助教授・工博
接受日字 : 1989年 9月 21日
1次修正 : 1989年 11月 20日

This technique, which balances forces and position errors to achieve the operation of damped spring in Cartesian coordinates, has been derived from work by Whitney[11] and Hogan[12]. With the algorithm implementing active compliance, it was demonstrated that the OSU Hexapod is capable of walking adaptively over irregular terrain while maintaining body orientation[10]. Fig. 1, is a photohraph of the OSU Hexapod maneuvering over an irregular terrain.

However, with force feedback, the OSU Hexapod experienced unexpected and theoretically inexplicable oscillations.[13] The following factors were considered as possible causes : (1) the force feedback control law itself (2) force interaction of the legs and (3) passive compliance in the system. Previously research has been done for issues (1) and (2) in some degree and the results have been published[14, 15]. Further study on issue (1) is presently under progression [16] for a generalization of the result in[15]. The major concern of this paper is about the issue (3) : the effect of passive compliance on system stability in active compliance control.

It had been believed that one of the reasons for the Hexapod's oscillation, resulting from force gains resonable for a single leg, was due to internal passive compliance in the vehicle, and interaction in force between legs[10]. Thus a digital low-pass filter on the force error had been used to damp out leg interactions in force. A pole of 2.0[rad/sec] was Chosen empirically for the low-pass filter. However, it had not still been understood whether passive compliance might be beneficial or detrimental to the stability of force control for actively compliant motion.

Fig. 1 The Ohio State University Hexapod vehicle maneuvering over an irregular terrain. Note that a spring block was attached at the foot-tip of each leg to decrease vertical stiffness of the vehicle. The spring constant is approximately 41, 900[N/m].

To clarify, experimentally, the confusion about the effect of passive compliance on system stability, the Hexapod body first was stiffened with the attachment of aluminum plates to its frame. However, the effect turned out to be worse, with the system showing more oscillation than before. Thus, as an experimental trial in the opposite direction, a spring block was attached at the foot-tip of each leg to decrease vertical stiffness of the hexapod (see Fig. 1). These blocks are quite soft and thus are major sources of vertical compliance of the vehicle. Experiments with these spring blocks at the foot tips showed increased system stability. The experimental results will be discussed fully in Section Ⅳ.

From. the experiments performed on the OSU Hexapod, it was found that passive compliance bears a significant importance in system stability with regard to force control. The purpose of this paper is to investigate, theoretically, the effect of passive compliance on sytem stablilty with regard to force feedback implementation for actively compliant motion. A theoretical analysis, numerical simulation, and experimental result will be introduced in the following sections : In Section Ⅱ, a model of one dimensional suspension system with passive compliance and active compliance control will be proposed. Based on this model, the effects of passive compliance under continuous control will be studied by a root locus analysis. In Section Ⅲ, the root locus analysis will also be performed but under discrete control. Then the difference of the effects of passive compliance under continuous and discrete controls will be compared. A numerical simulation for discrete control will also be included for visualization of the system response. Section Ⅳ discusses experimental results performed on the OSU Hexapod to confirm the findings in Section II and Ⅲ.

2. Stability Effect of Passive Compliance Under Continuous Control

Since the action and reaction force results from interaction between two contacting objects, the force feedback gain obviously depends on the characteristics of the two objects, in this case their stiffness. In order to study the effects of passive compliance, a one-dimensional suspension model with passive compliance will be proposed in this section. Based on it, the effects of both the continuous control and the discrete control will be compared.

2.1 Model of An Active Suspension System

Fig. 2 is a model of the suspension system, with two legs supporting the mass block of 2M. This model is a reduced one of the OSU Hexapod. Each leg consists of two links with an actuator attached at their joint. Passive compliance sources of the suspension system and the ground are lumped at the contact points, as spring constant K_{SP} and damping coefficient B_P. In this model the velocity and position of the mass block are controlled according to the input command and the force measured at the foot-tip.

It is assumed that both legs of the model in Fig. 2 are controlled indentically and thus the mass block moves only vertically, without interactions between legs. The case of interaction in force between legs was considered in[14]. With this assumption, the passive compliance model can be simplified to a single leg model with mass of M [17] as shown in Fig. 3. This simplification shows only one leg, which is all that is necessary for the purpose of stability analysis.

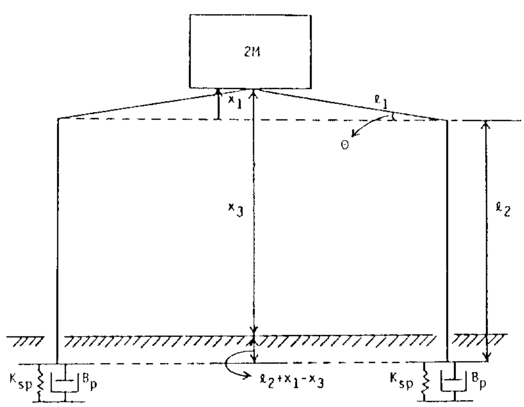

Fig. 2 A model of the vehicle suspension system. Passive compliance parameters of both the suspension system and the supporting ground are lumped at the contact points.

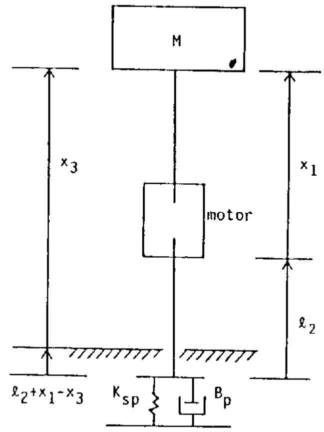

Fig. 3 One-leg model of the vehicle suspension system. Passive compliance parameters of both the suspension system and the supporting ground are lumped at the contact point. The motor block controls the position x_1.

In the system model in Fig. 3, the force measured at the foot-tip is the gravity and acceleration force of the mass block. Thus it can be expressed as

$$f = M(g + \ddot{x}_3) \tag{1}$$

This force is exerted on the spring damper block, with an equal amount of force being reacted upon by the block. Thus the force f can also be expressed as

$$f = K_{SP}[(l_2 + x_1) - x_3] + B_P(\dot{x}_1 - \dot{x}_3) \tag{2}$$

Equating right sides of both (1) and (2) then gives an expression for $\ddot{x}_3$:

$$\ddot{x}_3 = \frac{1}{M}\{K_{SP}[(l_2 + x_1) - x_3] + B_P(\dot{x}_1 - \dot{x}_3)\} - g \tag{3}$$

For completion of the state equations, the actuator model of one joint of the OSU Hexapod is used for realism. Accounting for the linearizing reference wave form and the high gear reduction

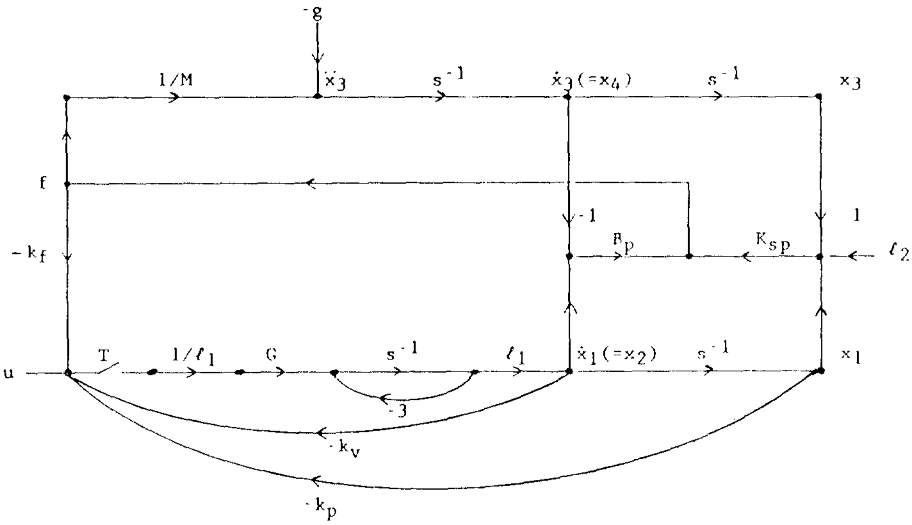

Fig. 4 A signal flow graph of suspension system control of Fig. 3. Linearization for x_1 was applied for θ near zero. Unlabled arrows imply gain of 1. The input $u=k_v\dot{x}_{1d}+k_px_{1d}+k_ff_d$. The sampler is closed in the case of continuous control.

ratio, a simple linear transfer function from input voltage for the power control circuitry to the motor angular position has been identified[18] as

$$\frac{\Theta(s)}{V(s)}=\frac{0.14}{s(s+3)} \tag{4}$$

a formula which has been found to give accurate predictions of actuator performance. In order to combine the actuator model into (3), the term x_1 can be approximated as

$$x_1=l_1\theta \tag{5}$$

by linearizing for θ near zero.

The control law is based on active compliance [9], which is a form of resolved motion rate control.[19]. Thus the control law modifies the desired velocity of the mass block by force and position errors. The commanded velocity becomes

$$\dot{x}_{1c}=\dot{x}_{1d}+k_P(x_{1d}-x_{1a})+k_f(f_d-f_a) \tag{6}$$

where the terms k_p and k_f are position and force gains, respectively. Variables with a subscript d represent the desired setpoints, and those with a represent actual measurements. When the servo loop is closed with the actual velocity, the error signal is computed as

$$V=G_1[k_v(\dot{x}_{1d}-\dot{x}_{1a})+k_P(x_{1d}-x_{1a})+k_f(f_d-f_a)] \tag{7}$$

where G_1 is a compensator gain and k_v is a velocity gain.

By combinig (3) and (5) on the actuator model in (4) and the control law in (7), a signal flow graph is obtained, as shown in Fig. 4, The feedforward gain G is $0.14G_1$. Based on this signal flow graph and the state variables assigned, state equations can then be obtained as follows:

$$\dot{x}_1=x_2 \tag{8}$$

$$\begin{aligned}\dot{x}_2=&-G(K_P+k_fk_{SP})x_1\\&-[G(1+k_fB_P)+3]x_2\\&+k_fK_{SP}x_3+k_fB_Px_4\\&+G[k_ff_d+k_v\dot{x}_{2d}+k_Px_{2d}]\\&+Gk_fK_{sp}l_2,\end{aligned} \tag{9}$$

$$\dot{x}_3=x_4, \tag{10}$$

$$\begin{aligned}\dot{x}_4=&\frac{K_{sp}}{M}x_1+\frac{B_p}{M}x_2-\frac{K_{sp}}{M}x_3\\&-\frac{B_p}{M}x_4+\frac{K_{sp}}{M}l_2-g\end{aligned} \tag{11}$$

2.2. Root Locus Analysis for Continuous Control

The 4×4 system matrix is formulated by the

Table 1 Parameter Values for Computer Simulation of A One-Leg Suspension System

body mass	M	640Kg
gravitational acceleration	g	$9.8 m/sec^2$
position error gains	k_p	$1.17 sec^{-1}$
velocity error gains	k_v	1.0
force error gains	k_f	8.35×10^{-4}(m/sec)/N
compensator gain	G	7.924 volts/(rad/sec)
passive spring constants	K_{sp}	varing, N/m
passive damping coefficients	B_p	870 and 3,670N/(m/sec)
smapling time	T	varying, sec

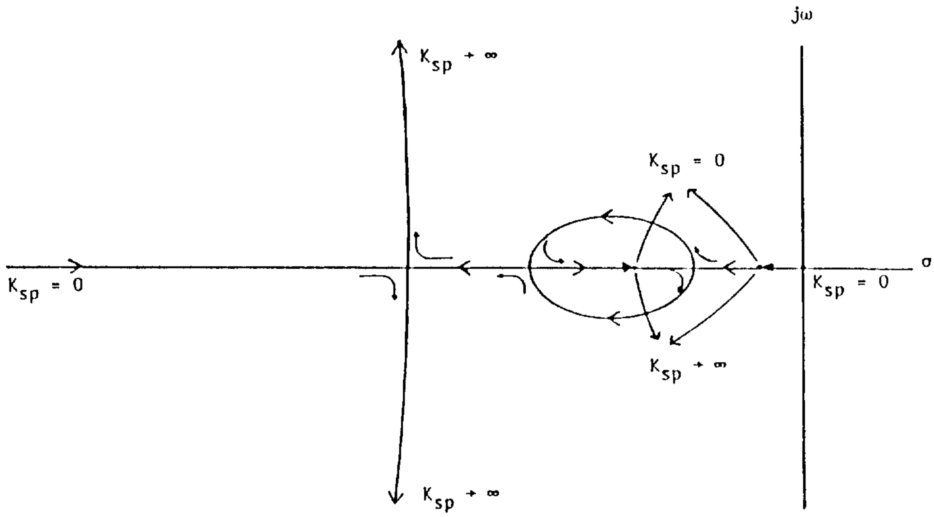

Fig. 5 Loci of eigenvalues of the system matrix for the case that the passive damping coefficient B_p is greater than the active one k_v/k_f. The loci were drawn by varying the passive spring constant K_{sp}.

coefficients of state variables in the right sides of (8) through (11). The eigenvalues of this matrix can then be obtained numerically, with various conditions of passive compliance, in order to observe the effect of passive compliance on system stability. The loci of eigenvalues are drawn in Fig. 5, varying the passive spring constant K_{sp} and setting the passive damping coefficient B_p at a value greater than the active one, k_v/k_f. The parameters used were tabulated in Table 1. The loci in Fig. 6 represent the case where the passive damping coefficient is less than the active one. If the passive spring constant is increased (the system becomes stiffer), two loci branch away in both directions parallel to the imaginary axis, meaning that the system responses have higher frequency oscillations but with the same damping envelop. In all cases, however, the loci of eigenvalues are in the left half of the s-plane. Since the real parts of eigenvalues are negative, the system will be stable for all values of spring constant K_{sp}, if continuous control is performed.

An effect of the damping coefficient to the system response can be pointed out by comparing Fig. 5 and Fig. 6. Notice that decreasing the pasive damping coefficient B_p pushes the two root loci which are parallel to the imaginary axis toward the imaginary axis. Thus, decreasing the passive damping coefficient also decreases the damping of the oscillatory response corresponding to the non-

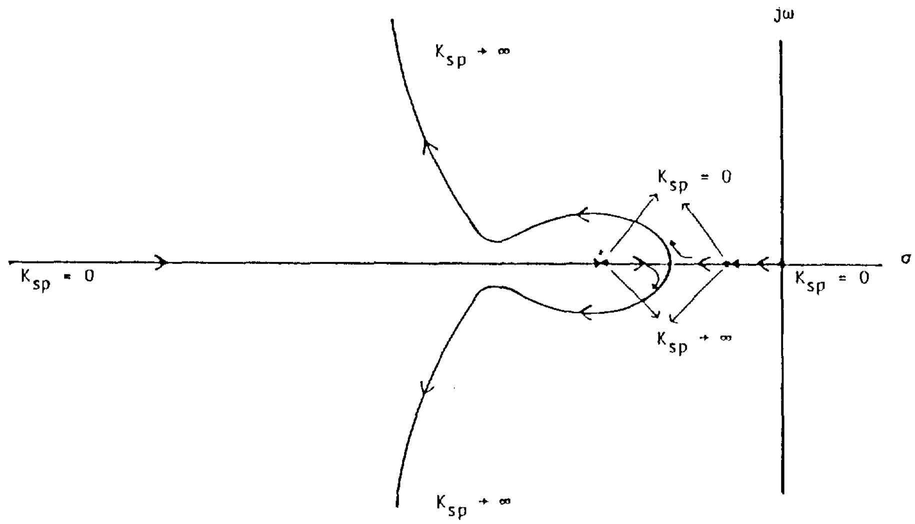

Fig. 6 Loci of eignevalues of the system matrix for the case of $B_p < k_v/k_f$. The loci were drawn by varying the passive spring constant K_{sp}.

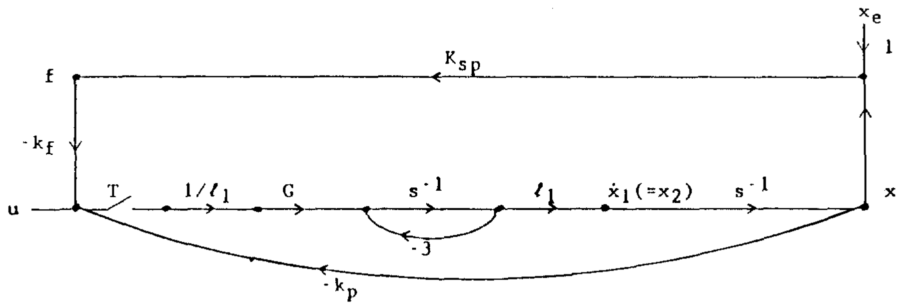

Fig. 7 A signal flow graph of a reduced model of the suspension system in Fig.3. The sampler is closed in the case of continuous control.

dominant complex conjugate pole pair. However the frequency of the oscillation is affected only by the passive spring constant.

In order to understand more simply the effects of passive compliance on system stability in case of continuous control and to compare it to that in case of discrete control, a gross simplification is made, reducing the system order. Assumptions for this are as follows:

(a) x_3=constnat
(b) $B_P=0$
(c) $k_v=0$

Condition (a) implies that the mass block is fixed relative to the supporting ground. This is for the case that the manipulator contacts its working environment with its base being fixed on the ground. Although the condition (b) is the extreme case on B_p, it is not the direct cause of the system instability. This is because that the damping of the system response does not become zero by this condition, since there is also electrical damping factor in the system such as the motor back emf. This condition only helps the stability effect of the passive spring constant be observed easily. Assumption (c) is only for simplicity. The following analysis would also be valid without assumptions (b) and (c); the only difference being that the related equations would be much more complex.

Fig. 7 is a signal flow graph for the reduced model. The ruduced-order state equations are then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -G(k_p+k_fK_{sp}) & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ G(u+k_fK_{sp}x_e) \end{bmatrix} \quad (12)$$

where $x_e = x_3 - l_2 =$ constant. The system matrix A is obtained as

$$A = \begin{bmatrix} 0 & 1 \\ -G(k_p+k_fK_{sp}) & -3 \end{bmatrix} \quad (13)$$

Thus the characteristic equation is defined from this matrix as

$$\det(sI-A) = s^2+3s+G(k_p+k_fK_{sp}) = 0 \quad (14)$$

Eigenvalues are the roots of (14), and they expressed as

$$s = \frac{-3 \pm \sqrt{9-4G(k_p+k_fK_{sp})}}{2} \quad (15)$$

If the sign of the discriminant or the term inside the square root in (15) is positive, i.e.

$$9 > 9-4G(k_p+k_fK_{sp}) > 0, \quad (16)$$

then the system has two real negatve poles. In this case, the system is stable without any oscillation. If the discriminant becomes negative with a change in any one of G, k_p, k_f, and K_{sp}, then the system has poles forming a complex conjugate pair. In this case the response is a damped oscillation with the damping envelope of $e^{-3 \cdot t/2}$. The radian frequency of the oscillation is

$$w = \sqrt{4G(k_p+k_fK_{sp})-9} \quad (17)$$

If the spring constant K_{sp} is increased or the spring is made stiffer with all the other parameters fixed, the frequency of oscillation is increased, accordingly, by (17). However, the damping term is not changed ; thus the system is still stable, if continuous control is performed. The reduced model coinciedes with its original in terms of its stability in passive compliances.

3. Stability Effect of Passive Compliance Under Discrete Control

It was found that making the system stiffer causes higher frequency oscillation on the system response. However, since the damping term is not affected much, the oscillation will die out. Thus the system is stable, under continuous force feedback control, even with the variations in passive compliance. In contrast to this result, experiments performed on the OSU Hexapod under discrete control showed that the system stability is affected by passive compliance significantly. In regard to discrete control, Whitney[1, 20] pointed out that higher force feedback gains can be used if the environment is more compliant, since less force would build up over a fixed control time period. In order to verify, analytically, the experimental results, the state equations for continuous control will be modified for discrete control, and a similar stability analysis will be performed.

3.1 Discrete Model of an Active Suspension System

The state variables fed back through the digital computaion loop being regarded as constant input for a whole interval between two control points, the following hybrid state equations result from equations (8) through (11) :

$$\dot{x}_1(t) = x_2(t), \quad (18)$$

$$\begin{aligned} \dot{x}_2(t) = & -3x_2(t) - G(k_p+k_fK_{sp})x_1(kT) \\ & - G(k_v+k_fB_p)x_2(kT) \\ & + k_fK_{sp}x_3(kT) + k_fB_px_4(kT) \\ & + G[k_ff_d + k_v\dot{x}_2d + k_px_{2d}] + Gk_fK_{sp}l_2, \end{aligned} \quad (19)$$

$$\dot{x}_3(t) = x_4(t), \quad (20)$$

$$x_4 = \frac{K_{sp}}{M}x_1(t) + \frac{B_p}{M}x_2(t) - \frac{K_{sp}}{M}x_3(t) - \frac{B_p}{M}x_4(t) + \frac{K_{sp}}{M}l_2 - g \quad (21)$$

where $kT \le t < (k+1)T$, and f_d, $\dot{x}_{2d}$, and x_{2d} are for $t = kT$.

Pure discrete state equations are obtained through discretization of the above hybrid state equations. If the above hybrid state equations. If the about hybrid state equations are in the form

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (22)$$

with

$$u(t) = u(kT), \text{ for } kT \le t < (k+1)T, \quad (23)$$

then their solution is expressed[21] as

$$x[(k+1)T]=e^{AT}x(kT)+\int_0^T e^{As}Bds(kT) \quad (24)$$

or

$$x[(k+1)T]=Gx(kT)+Hu(kT), \quad (25)$$

with

$$G=e^{AT} \quad (26)$$

and

$$H=\int_0^T e^{As}Bd_s. \quad (27)$$

In this case, the control $u(kT)$ can be computed as

$$u(kT)=Cx(kT)+Dv(kT) \quad (28)$$

with the matrices C and D for the control law and the variable $v(kT)$ for external input. Thus the closed-loop discrete state equation can be represented as

$$x[(k+1)T]=[G+HC]x(kT)+HDv(kT) \quad (29)$$

If there is a one-step computation delay in the feedback loop, the control $u(kT)$ is computed based on the external input $v[(k-1)T]$ and the state measurement $x[(k-1)T]$:

$$u(kT)=Cx[(k-1)T]+Dv[(k-1)T]. \quad (30)$$

Substituting (30) into (25) gives

$$x[(k+1)T]=Gx(kT)+HCx[(k-1)T] + HDv[(k-1)T]; \quad (31)$$

thus the state equation can be augmented as

$$y[(k+1)T]=\begin{bmatrix} G & HC \\ I & 0 \end{bmatrix}y(kT)+\begin{bmatrix} HD \\ 0 \end{bmatrix}z(kT) \quad (32)$$

with substitutions of

$$y_i(kT)=x_i(kT), \quad (33)$$

$$y_{i+n}(kT)=x_i[(k-1)T], \quad (34)$$

and

$$z(kT)=v[(k-1)T], \text{ for } i=1\cdots n \quad (35)$$

3.2 Root Locus Analysis

The stability of the discrete system can be tested by determining whether the eigenvalues of the system matrix are within a unit circle on the z-plane. As shown in (32), the order of the system was doubled with a one-step computation delay in the feedback loop. Computation delay is typical in computing-bound control of complex mechanical systems like multi-legged robot vehicles. However, for theoretical interest and simplicity of the analysis, it is assumed here that there is no computation delay, and the system matrix in (29), $[G+HC]$, is used in the following root locus analysis. The effect of computation delay will be included in the simulation to follow.

The loci of eigenvalues of the discrete system matrix in (29) are drawn in Fig. 8, varying the sampling time T. Keeping T very small, which almost amounts to continuous control, all the eigenvalues are within the unit circle. As T is increased from zero, some loci move toward the unit circle boundary. When the sampling time T is 0.1[sec], an eigenvalue reaches the unit circle boundary. A further increase in T causes the

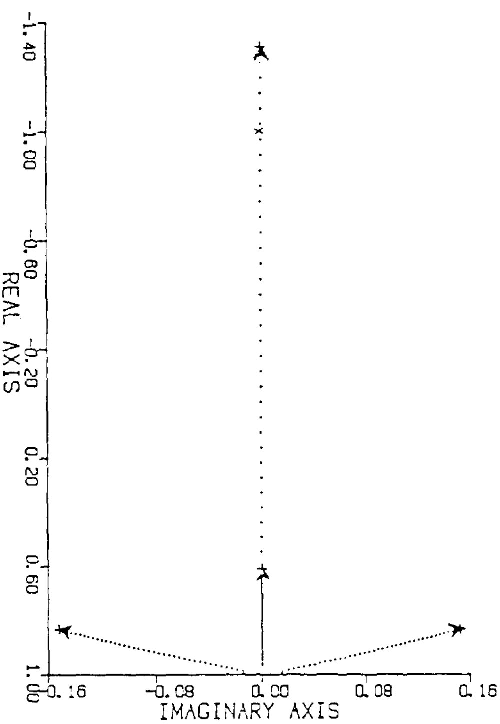

Fig. 8 Loci of eigenvalues of the system matrix for the discrete control system. The loci were drawn by varying the sampling time T. As T increases some eigenvalues cross over the unit circle boundary. resulting in an unstable system.

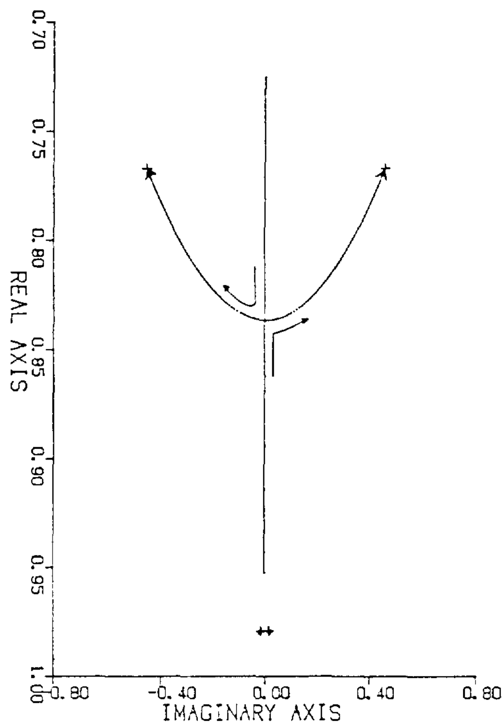

Fig. 9 Loci of eigenvalues of the system matrix for the discrete control system. The loci were drawn by varying the passive spring constant K_{sp}. As K_{sp} increases, some eigenvalues cross over the unit circle boundary, resulting in an unstable system.

eigenvalue to cross over the unit circle as expected, resulting the system instatility.

Fig. 9 is the loci of eigenvalues with spring constant K_{sp}, only, varing. As K_{sp} is increased, a pair of loci move away from the real axis. This implies that the frequency of damped oscilation is accordingly increasing as is the case with continuous control. A further increase in K_{sp} makes some loci cross over the unit circle, making the system unstable, which is not the case with continous control. The critical passive spring constant was $K_{sp}=68{,}000[N/m]$ with the discrete control frequency of 50Hz.

A similar stability test can be accomplished analytically on a reduced model of discrete control, as was done in the case of the reduced model implementing continuous control. For this, the sampler in Fig. 7 samples with a non-zero sampling and control time T. When the control $u(t)$ is constnat between 0 and T, When the control $u(t)$ is constant between 0 and T, $x_2(t)$ is given by

$$x_2(t)=u(0)\frac{1-e^{-3t}}{3}+x_2(0)e^{-3t}, \tag{36}$$

and then $x_1(t)$ is computed as

$$\begin{aligned} x_1(t)&=\int_0^t x_2(t)\,dt+x_1(0)\\ &=u(0)\frac{3t+e^{-3t}-1}{9}\\ &\quad+x_2(0)\frac{1-e^{-3t}}{3}+x_1(0). \end{aligned} \tag{37}$$

Now substituting $t=T$ in (36) and (37) gives

$$\begin{aligned} x_1(T)&=u(0)\frac{3T-1+e^{-3T}}{9}\\ &\quad+x_2(0)\frac{1-e^{-3T}}{3}+x_1(0), \end{aligned} \tag{38}$$

$$x_2(T)=u(0)\frac{1-e^{-3T}}{3}+x_2(0)e^{-3T}. \tag{39}$$

When the sampling time T is very small, an approximation of

$$e^{-3T}=1-3T \tag{40}$$

can be valid. Applying this approximation into (38) and (39) gives

$$x_1(T)=x_1(0)+Tx_2(0), \tag{41}$$

$$x_2(T)=(1-3T)x_2(0)+Tu(0). \tag{42}$$

These two equations are generalized without any difficulty, with time index k, as follows:

$$x_1[(k+1)T]=x_1(kT)+Tx_2(kT), \tag{43}$$

$$x_2[(k+1)T]=(1-3T)x_2(kT)+Tu(kT). \tag{44}$$

The control $u(kT)$ is computed from system input and state feedback, by the designed control law. Thus state feedback may be delayed, by computation, in real operation. However, if computation delay is included, it doubles the system order, making it hard to handle analytically. Therefore, for simplicity of analysis, it is assumed here that there is no computation delay. The control $u(kT)$ then is expressed as

$$\begin{aligned} u(kT)=G[&v(kT)-(k_p+k_fK_{sp})x_1(kT)\\ &+k_fK_{sp}x_e] \end{aligned} \tag{45}$$

By substituting (45) into (44), final discrete state equations are obtained:

$$x_1[(k+1)T]=x_1(kT)+Tx_2(kT), \qquad (46)$$

$$x_2[(k+1)T]=-GT(k_p+k_fK_{sp})x_1(kT) +(1-3T)x_2(kT) +GT[v(kT)+k_fK_{sp}x_e] \qquad (47)$$

The system matrix for state equations of (46) and (47) is defined as

$$A=\begin{bmatrix} 1 & T \\ -GT(k_p+k_fK_{sp}) & 1-3T \end{bmatrix} \qquad (48)$$

The characteristic equation is then expressed as

$$\det(zI-A)=z^2-(2-3T)z+(1-3T) + T^2G(k_p+k_fK_{sp})=0 \qquad (49)$$

The eigenvalues are the roots of (49), and are expressed as

$$z=\frac{(2-3T)\pm\sqrt{(2-3T)^2-4[(1-3T)+T^2G(k_p+k_fK_{sp})]}}{2} \qquad (50)$$

In order for the system response to be stable and non-oscillatory, the discriminant(the inside of the square root in (50)) should be positive and the eigenvalues in (50) should be in $0<z<1$. These conditions are expressed as follows:

$$(2-3T)^2-4[(1-3T)+T^2G(k_p+k_fK_{sp})]>0, \qquad (51)$$

$$0<z<1. \qquad (52)$$

If any one of T, G, k_f, k_p, and K_{sp} is increased from a certain value satisfying the condition expressed in both (51) and (52) with remaining parameters, the conditions of (51) and (52) are violated. However the system is still stable (with oscillation) as long as the eigenvalues are within the unit circle in z-plane. If the parameters listed are increased further over a certain range, the eigenvalues are driven out of the unit circle, resulting in an unstable system. This was not true with the continuous control case. Notice that in any case decreasing the control period T makes the system stable, as in the case of the continous control system.

It is worthwhile to notice that passive spring constant K_{sp} affects the system stability. Actually, the product of the force gain k_f and the spring constant K_{sp} acts as a single parameter. This implies that higher force feedback gain be used if the system itself, or the environment, is more compliant: or if K_{sp} becomes smaller. If the system is very stiff (K_{sp} is very large), then the force feedback gain should be decreased so as to stabilize the system. However this is not compatible with actively compliant motion. Although the discussion here does not give any quantitative suggestions, it clearly explains why some passive compliance is necessary for stability in controlling contact force by discrete control. Since a manipulator usually is hightly non-linear and its order is high, it may be reasonable to adjust its passive compliance by empirical menas.

3.3 Numerical Simulation of the Sampled-Data System

Based on the hybrid state equations (18) through (21), a numerical simulation was performed with a step input. Although the vehicle will not experience a step input in normal walking, its responses to it will still reveal how system stability is affected by passive compliance. The differential equations were numerically integrated by the Runge-Kutta fourth order technique.[22] The frequency of numerical integration(reciprocal of integration interval) was chosen to be an integer multiple of the control frequency so that no integration interval crossed a control point. Parameters tested with each simulation were sampling time T and passive spring constant K_{sp}.

Fig. 10 shows simulation results for various sampling times. with other parameters fixed. It clearly shows that, with the sampling time decreased, oscillation of the response is reduced as expected. The step response becomes unstable with the control frequency less that 15[Hz]. However, the analytic test showed that the minimum control frequency is 10[Hz] when no computation delay is assumed. Thus, it be concluded that the computation delay causes a detrimental effect on the stability.

Fig. 11 is another simulation result, varying spring constant K_{sp} only. The control frequency is fixed around 50Hz, which is a practical value used for the OSU Hexapod control. As the spring

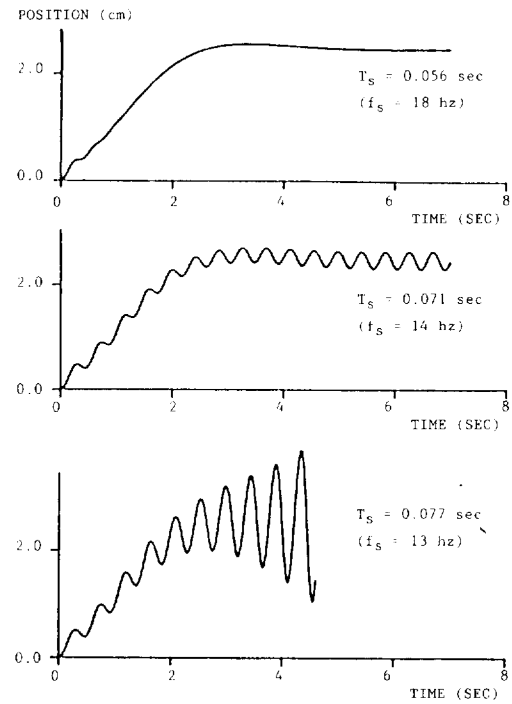

Fig. 10 Force responses of the model suspension system under discrete control. Simulation was performed with the passive spring constant being fixed at 69, 900[N/m]. As the sampling time increases, the response begins oscillation. The unstable oscillation begins with a control ferquency of approximately 14Hz.

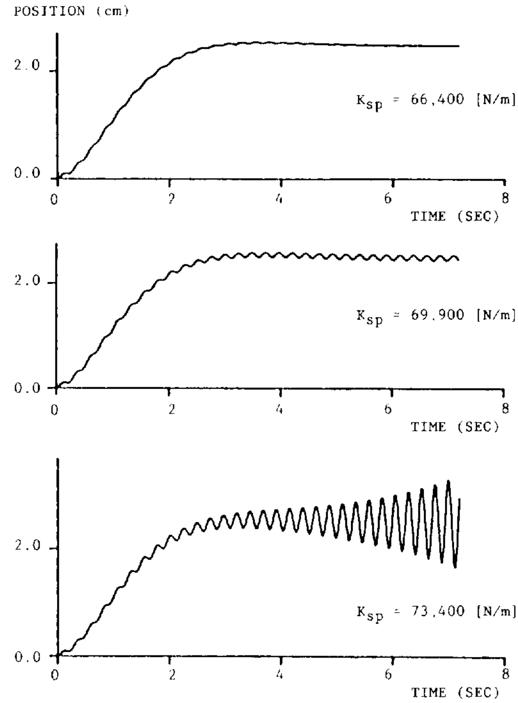

Fig. 11 Force responses of the model suspension system under discrete control. Simulation was performed with the control frequency of 50Hz. As the passive spring constant increases, the response begins oscillation. The unstable oscillation begins with the spring constant of approximately 69,900[N/m].

constant K_{sp} is increased, the response enters into oscillation. This result corresponds to the stability analysis given in this section. The simulation shows that the maximum passive stiffness is around 69,900[N/m] for the system stability, which agrees very well with the result of the root locus analysis given in this section.

4. Experimental Results

It was determined theoretically that some passive compliance is necessary for system stability in implementing force control by digital computer. In this section, the result of the experiment performed on the OSU Hexapod, will be presented as confirmation of the validity of the stability analysis based on the passive spring constant and force feedback gain.

For a practical evaluation of the control algorithm for actively compliant motion, experiments were performed with the Hexapod vehicle walking forward with a leg duty factor of 2/3. Under these circumstances, it was judged, the control system would be activated by a dynamically changing input, with almost all the possible postures of the legs occurring in walking. The force setpoints were generated based on the static equilibrium condition of the vehicle. Generally, the system of equations for force constraints are underspecified if the vehicle is supported by more than three legs[7]. Thus there is an infinite number of solutions. Among many solution methods based on different optimization criteria[14, 23], the pseudoinverse technique which gives the minimum norm solution is used for

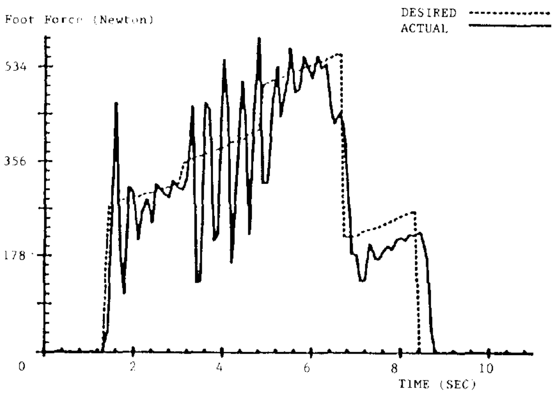

Fig. 12 Foot force tracking of leg 1 of the OSU Hexapod using active compliance. The vehicle was walked without spring blocks on each leg. The control frequency was 50Hz. The foot force setpoint was generated as the paseudoinverse solution of the force constraint equation. The force response shows undamped large amplitude oscillation.

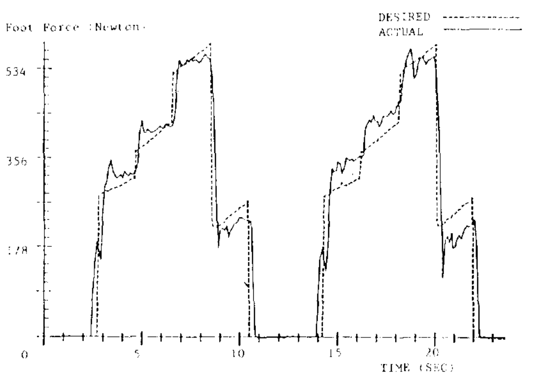

Fig. 13 Foot force tracking of leg 1 of the OSU Hexapod using active compliance control. The vehicle was walked with spring blocks on each leg. The spring constant was approximately 41,900[N/m]. The control frequency was 50Hz. Comparing with Fig. 12, no significant large-amplitude oscillation is observed.

experiments in this section.

It had been belived that the Hexapod's oscillation with force feedback control, resulting from force gains reasonable for a single leg, was due to internal passive compliance in the vehicle, and interaction in force between legs[10]. To

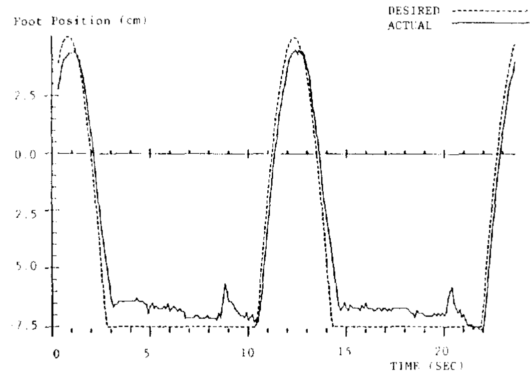

Fig. 14 Foot position tracking which corresponds to the foot force tracking of Fig. 13. Notice that position errors in this figure were introduced by active compliance to counteract the force errors observed in Fig. 13.

clarify, experimentally, the confusion about the effect of passive compliance on system stability, the Hexapod body first was stiffened with the attachment of aluminum plates to its frame. However, the effect turned out to be worse, with the system showing more oscillation than before. Fig. 12 shows the force responses of the experiment with the Hexapod body being stiffened. As plotted in Fig. 12, the force response with the stiffer system shows undamped large-amplitude oscillations. The oscillations were so strong that considerable impact was applied to the system, causing the vehicle to vibrate as a whole.

In order to decrease vertical stiffness of the Hexapod and to eliminate a undesirble vibration experienced with the stiffened system, a spring bock was deliberately attached at the foot-tip of each leg(see Fig. 1). These blocks are quite soft and thus are major sources of vertical compliance of the vehicle. The spring constant of these bloks is approximately 41,900[N/m], allowing about 1.65[cm] compression when half of the vehicle mass is loaded on one leg. As expected, experiments with these spring blocks at the foot tips showed increased system stability. Fig. 13 is the force tracking of the experiment and Fig. 14 is the plot of the corresponding position tracking. It can be observed that large discontinuities in force setpoints existed whenever the support pattern was changed. As plotted, the force response of the passively more compliant system does not show undamped large-amplitude oscillations anymore and it tracks the setpoint smoothly.

Although, in general, the actual force tracked the desired force quite well, even with larger discontinuities in force setpoints, there exists a small approximately constnat gap as shown in Fig. 13. The force gap resulted from using an inaccurate total vehicle weight and an inaccurate center of gravity in formulating force constraint equations. Even with these inaccuracies, the gap in force was compensated by allowing position errors, as shown in Fig. 14. Thus the existence of errors can be interpreted as homogeneous components in stable states of the control system in terms of position and force.

Another discrepancy in force tracking is apparent in Fig. 13. Note that the actual force became non-zero before the commanded force when the legs alternated their phases between support and transfer. This was caused by the trajectory of desired force having been generated according to the leg kinematic cycle phase, not according to the ground contact of the leg. It can be observed from Fig. 14 that the force discrepancy was compensated by position errors, as was programmed by the active compliance algorithm. With this capability the vehicle can negotiate with or accommodate itself to irregular terrains.

The comparison of two experiments with different passive compliances really confims the validity of the stability analysis given in previous sections. Generally passive compliance is beneficial to the stability of force control for actively compliant motion, and more specifically some passive compliance is necessary to make the system stable when force feedback control is implemented by a digital computer. It should be noted, however, that position control is less accurate with a more compliant system. The system stability can still be maintained by reducing force feedback gain instead of making the system more compliant. However this makes the system actively stiffer. Thus a criterion must be determined for optimal setting of the passive compliance for trading between position accuracy and system strability with force control.

5. Conclusions

The purpose of this paper was primarily to investigate the effect of passive compliance on system stability with regard to force feedback implementation of actively compliant motion. In order to illustrate the problem a model of an active suspension system and a continous controller was presented. Based on this model, the effect of passive compliance was investigated by deriving eigenvalues of the system matrix. It was found that increasing the spring constant of passive compliance or the force feedback gain also increases the frequency of oscillation. However the damping envelope is not affected. Thus system stability is maintained with a change of passive compliance or force feedback gain.

In order to determine whether the above was true with discrete control, a time-sampled model of a control system, based on the model of a continuous control system, was derived. Both analysis and simulation were performed on this model in order to investigatie system stability. In contrast to the case of continuous control, it was found that the system stability is closely related to the product of force feedback gain, the passive spring constant, and the square of the sampling time. When this product term increases, the system becomes unstable. In any case, the system can be made stable by decreasing the sampling time, which makes it act more nearly as a continuous system. This result also has been shown by experiments on the OSU Hexapod. Another noticeable point is that the computational delay causes a detrimental effect on the system stability. When there is a computational delay, the system must be controlled with a somewhat higher frequency than for the case of no computation delay.

In summary, it can be concluded that in implementing actively compliant motion by force feedback with discrete control, although the system needs to be stiff for an accurate position control, some passive compliance is necessarh for stabilization of the system. This factor should be considered in future manipulator design if compliance motion is intended for the robot manipulator. Quantitative guidelines are not yet well established, but should be dependent on the force feedback gain and the desired accuracy of position control. Also further research is suggested for a technique which will dynamically estmate the total effective passive compliance in the manipulator and its working environment and adjust the force feedback gain according to the estimation, in order not to cause stability problem but still to achieve a maximal active compliance effect.

REFERENCES

[1] D.E. Whiteny, "Historical perspective and state of the art in robot force control," in *Proc. 1985 IEEE Int. Conf. Robotics and Automation,* pp. 262-268.

[2] M.T. Mason, "Compliant motion," *Robot Motion,* ed. M. Brady, J.M. Hollerbach, et al. MIT Press, Cambridge, Mass, 1982, pp. 305-322.

[3] J.K. Salisbury and J.J. Craig, "Articulated hands : Force control and kinematic issues," *Int. J. of Robotics Res.,* vol. 1, no. 1. pp. 4-17, 1982.

[4] H. Kazerooni, T.B. Sheridan, and R.K. Houpt, "Robust compliant motion for manipulators, Part 1 : The fundamental concepts of compliant motion," *IEEE J. Robotics and Automation,* vol. RA-2, no. 2, pp. 83-92, June 1986.

[5] R.B. McGhee, "Vehicular legged locomotion," *Advanced in automation and robotics,* ed. by. G.N. Saridis, Jai Press, Inc., Greenwich, Cong., 1985.

[6] D.E. Orin, "Supervisory control of a multi-legged robot," International Journal of Robotics Research, vol. 1, no. 1, pp. 79~91, Spring 1982.

[7] D.R. Pugh, An *autopilot for a terrain-adaptive hexapod vehicle,* M.S. thesis, The Ohio State University, Columbus, Ohio, September, 1982.

[8] J.K. Salisbury. "Active stiffness control of a manipulator in Cartesian coordinates," *IEEE* Conf. Decision and Control, Albuquerque, New Mexico, November, 1980.

[9] C.A. Klein and R.L. Briggs, "Use of active compliance in the control of legged vehicles," *IEEE Trans. Syst., Man, Cybern.,* vol. SMC-10, no. 7, pp. 393-400, 1980.

[10] C.A. Klein, K.W. Olson, and D.R. Pugh, "Use of force and attitude sensors for locomotion of a legged vehicle over irregular terrain," *Int. J. Robotics Res.,* vol. 2, no. 2, pp. 3-17, Summer 1983.

[11] D.E. Whitney, "Force feedback control of manipulator fine motion," *ASME J. Dyn. Syst. Meas., Contr.,* vol. 99, pp.91-97, June 1977.

[12] N. Hogan, "Mechanical impedance control in assistive devices and manipulators," in *Proc. 1980 JACC,* San Francisco, CA., 1980.

[13] T.S. Chung, *Force and compliance control for rough-terrain locomotion by multi-legged robot vehicle,* Ph. D dissertation, The Ohio State University, Columbus, OH, 1985.

[14] C.A. Klein and T.S. Chung, "Force interaction and allocation for the legs of a walking vehicle," *IEEE J. Robotics and Automation,* vol. RA-3, no. 6, pp. 546-555, December 1987.

[15] T.S. Chung, "An inherent stability problem in Cartesian compliance and an alternative structure of compliance control *IEEE J. Robotics and Automation,* accepted for publication subject to revision, July, 1989.

[16] T.S. Chung, *A Generalization of the inherent stability problem in Cartesian compliance and an alternative structure of compliance control,* Research Initiation Proposal to National Science Foundation, Grant No. IRI-8910994, July, 1989.

[17] R.B. McGhee, E. Nakano, N. Koyashi, and H. Adachi, "An approach to computer coordination of motion for energy-efficient walking machine," *Bulletine of Mechanical Engineering Laboratory,* Agency of Industrial Science and Technology, MITI, Ibaraki 305, Japan, 1985.

[18] J.R. Buckett, *Design of an on-board electronic joint control system for a hexapod vehicle,* M.S. Thesis, The Ohio State University, Columbus, Ohio, March, 1977.

[19] D.E. Whitney, "Resolved motion rate control of manipulators and human prostheses," *IEEE Trans. Man-Mach. Syst.*, vol. MMS-10, no. 2, pp. 47-53, June, 1969.

[20] D.E. Whitney, "Force feedback control of manipulator fine motions," *Proc. 1976 Joint Automatic Control Conf.*, San Francisco, 1976.

[21] Chan, S.P., S.Y. Chan, and S.G. Chan, *Analysis of linear networks and systems : A matrix-oriented approach with computer application,* Addison-Welsley Publishing Company, Reading Massachusetts, 1972.

[22] A. Ralston, "Runge-Kutta methods with minimum error bounds," *Mathematics of Computation,* Vol. 16, pp. 431-437, 1962.

[23] D.E.Orin, and S.Y.Oh, "Control of force distribution in robotic mechanisms containing closed kinematic chains," *ASME Journal of Dynamic Systems, Measurement, and Control,* vol. 103, pp. 134-141, June, 1981.