醫工學會誌 : 第11巻, 第1號, 1990
J. of KOSOMBE : Vol. 11, No. 1

研 究 論 文
90－ 1 －03

LQG/LTR 방법에 대한 연구

서 병 설

A Study on the LQG/LTR Method

Byung Suhl Suh

要 旨

본 연구에서 LQG/LTR 방법의 우측면 영점에서의 문제점을 위해 새로운 근사방법이 제시되었다. 이론적 분석과 시뮬레이숀 결과는 Stein 방법보다 개선되어질 수 있음을 보였다.

Abstract

In this paper a new approximation method is presented for the nonminimum phase problem of LQG/LTR. Theoretical analysis and simulation results show that it can be better approximated than Stein's method.

1. Introduction

LQG(Linear Quadratic Gaussian) design method has been considered to be successful in such industrial applications as aircraft, submarine, chemical plants, etc. However, it was not known until 1977 if LQG optimal regulators had any guaranteed robustness properties.

In 1978, Doyle [1] showed a counter example that LQG design resulted in very small gain and phase margins. In other words, it is proven that arbitrary LQG regulators do not have any guaranteed robustness. As a remedy, LQG-LTR(linear quadratic gaussian-loop transfer recovery) method was invented by Doyle and Stein(2). This method was the outcome of possible ways of improving the robustness properties of LQG regulators. The technical ideas needed to prove the LQG-LTR result were known previously by Kwakernaak (3). Unfortunately, this method also, like other theories, has an important theoretical limitations that it is applicable to the only systems which have minimum phase zeros.

That is, it is not applicable to the nonminimum phase systems. Recently, Stein and Athans(4) have suggested three options to improve the limitations. The first one is that there is a possibility to improve it with a Poisson integral constraint associated with nonminimum phase zeros and sensitivity function. The second one is a generalization of LQG-LTR process by replacing the Kalman fil-

〈접수 : 1990년 2월 10일〉

Dept. of Electronic Communication Engineering Han Yang University Seoul Korea

(本 研究는 1989年度 文教部 學術研究 助成費에 依하여 研究되었음)

ter design with arbitrary choice of filter gains. The last one is to approximate nonminimum phase systems with minimum phase design models.

In the first option, it can be commented that the integral constraint and interpretations for the minimum phase is, at monent, limited to SISO (single input single output) systems.

And also, even though the constraint for MIMO (multiinput-multioutput) will be available, it is unknown how much the constraint can be effective for solving the nonminimum phase problem. In the second one, the replacemnt of Kalman filter gain with an arbitrary one does not any more guarantee such important Kalman filter "built-in" properties as performance, nominal stability, robustness in the LQG-LTR method. In a sense, the meaning of LQG-LTR will be lost. The last one called "Stein method" is a simple, clever approximation method. This one also, like other methods, serious theoretical limitations which can be well effective in the low frequency range due to the multiplicative error resulted from the approximation. The error tends to grow as the frequency increase and become a considerable amount in the high frequency range.

In this research a new LQG-LTR method for nonminimum phase plants will be studied to improve the Stein's approximation error except the very low frequency range, by a new approximation technique.

2. Stein's Approximation Method

Stein's method basically results from a trial to define the design plant model (DPL) in the LQG/LTR procedure [4] to be one of the best approximate minimum phase models of a given nonminimum phase plant by temporarily adding a suitable multiplicative error.

Technically, the method is to collect all nonminimum phase zeros into all pass factors, i.e.,

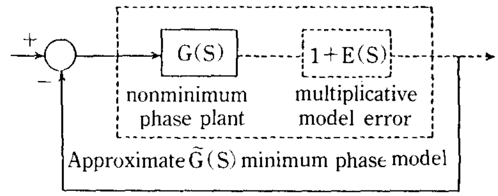

Fig. 1 The process of obtaining an approximate minimum phase model by the multiplicative error

$$G(S) = Gm(S)\ Bz(S) \tag{2.1}$$

where G(S) is nonminimum phase transfer function matrix and Gm(S) is minimum phase transfer function matrix. And Bz(S) is all pass factor,

$$Bz(S)\ Bz(S)^H = I \text{ for all } S \tag{2.2}$$

The nonminimum phase plant G(S) is possible to be represented with a multiplicative error and is then approximated by the minimum phase model G(S), i.e.,

$$G(S) = Gm(S)\ [I + E(S)] \cong Gm(S) \tag{2.3}$$

, which give rise to the multiplicative error E(S). For a single zero at S= +Z, G(S) and E(S) will be

$$G(S) = Gm(S)\ \frac{S-Z}{S+Z} \tag{2.4}$$

$$E(S) = \frac{2S}{S+Z} \tag{2.5}$$

The magnitude of error E(s) grows with the increase of frequency and is limited to 2 as $S \to \infty$

And so, Stein's method will have such a theoretical limitation that cannot be applicable in the high frequency.

3. New Approximation Method

A new approximation method is proposed below to improve the Stein's method. This method is to approximate a given nonminimum phase plant with a minimum phase model by temporarily adding an additive error, i.e.,

G(S)	+	E(S)	=	$\tilde{G}$(S)	(3.1)
a given nonminimum phase plant		an additive error		an approximate minimum phase model	

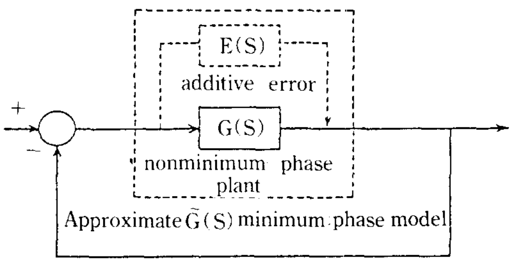

Fig. 2 The process of obtaining an approximate minimum phase model by an additive error

The best approximation depends on the magnitude of E(S) generated by the choice of $\tilde{G}$(S). In order to achieve that E(S) is as small as possible over all the frequency ranges, the following procedures are suggested.

Step. 1 Let the error function E(S) have the following form :

$$E(S)=\frac{S(b_{n-2}S^{n-2}+b_{n-3}S^{n-3}+\cdots+b_1S+b_0)}{a_nS^n+a_{n-1}S^{n-1}+\cdots+a_0} \quad n=1,2,\cdots \quad (3.2)$$

Here, the order n of E(S) is decided by the one of the given plant function G(S).

Step. 2. Choose minimum phase zeros arbitrarily for the numerator of $\tilde{G}$(S). The denominator of $\tilde{G}$(S) may be decided as the common denominator of G(S) and E(S) for the convenience of calculation. And the number of minimum phase zeros of G(S) are decided according to the order of numerator generated after reducing the common denominator of them.

Step. 3 If the minimum phase zeros of $\tilde{G}$(S) is chosen, the unknown coefficients of E(S) and $\tilde{G}$(S) will be completely decided by the relationship of (3.1)

Thus, $\tilde{G}$(S) will be a minimum phase model transfer function matrix obtained by the present approximation method.

It is noted that magnitude of the error function E(S) is reduced as the frequency increase or decreases. And also, the one in the mid-frequency range can be controlled to a certain degree by the choice of minimum phase zeros of $\tilde{G}$(S) Hence this new approximation can have an advantage over Stein's one.

To clarify the procedure presented above, consider a nonminimum phase plant with

$$G(S)=\frac{-S+10}{S(S+1)} \quad (3.3)$$

Step. 1 Since the order of G(S) is 2, G(S) will take the following form :

$$E(S)=\frac{b_0S}{a_2S^2+a_1S+a_0} \quad (3.4)$$

Step. 2 Referring to the procedure of step. 2 can be chosen in the form :

$$\tilde{G}(S)=\frac{(S+1)^3}{S(S+1)\ (a_2S^2+a_1S+a_0)} \quad (3.5)$$

Step. 3 By the relationship of (3.1) the coefficients of E(S) and $\tilde{G}$(S) can be completely decided.

$$E(S)=\frac{\frac{80}{11}S}{\frac{69}{11}S^2+40S+100} \quad (3.6)$$

and

$$\tilde{G}(S)=\frac{\frac{11}{69}S^3+\frac{110}{23}S^2+\frac{1100}{23}S+\frac{11000}{69}}{S^4+\frac{509}{69}S^3+\frac{1540}{69}S^2+\frac{1100}{69}S} \quad (3.7)$$

4. Simulation

In this section the linear time-invariant SISO and MIMO systems will be considered to demonstrate the numerical simulations of the proposed new approximatiom method. And it will be then compared with Stein's method. It is clearly shown in the previous section that the magnitude of error function E(S) in the both low and high frequency can be reduced significantly by the presented method while Stein's one cannot in the high frequency. And so, the magnitude only in the middle frequency range ($10^{-3}\sim 10^{3}$ rad/sec) will be concerned for simulations.

Example 1 (SISO)

Given $G(S)=\frac{-S+10}{S(S+1)}$

By the procedures presented in the section 3 the following transfer function of the approximate

minimum phase zeros	transfer functions of approximate minimum phase models
(S+1)	$\frac{100}{21}S^3+\frac{300}{7}S^2+\frac{300}{7}S^3+\frac{100}{21}\ /\ S^4+\frac{52}{21}S^3+\frac{41}{21}S^2+\frac{10}{21}S$
(S+10)	$\frac{11}{69}S^3+\frac{110}{23}S^2+\frac{1100}{23}S+\frac{11000}{69}\ /\ S^4+\frac{509}{69}S^3+\frac{1540}{69}S^2+\frac{1100}{69}S$
(S+100)	$\frac{11}{13299}S^3+\frac{3300}{13299}S^2+\frac{33000}{13299}S+\frac{3,300,000}{13299}\ /\ S^4+\frac{1542299}{13299}S^3+\frac{1243000}{13299}S^2+\frac{110000}{13299}S$

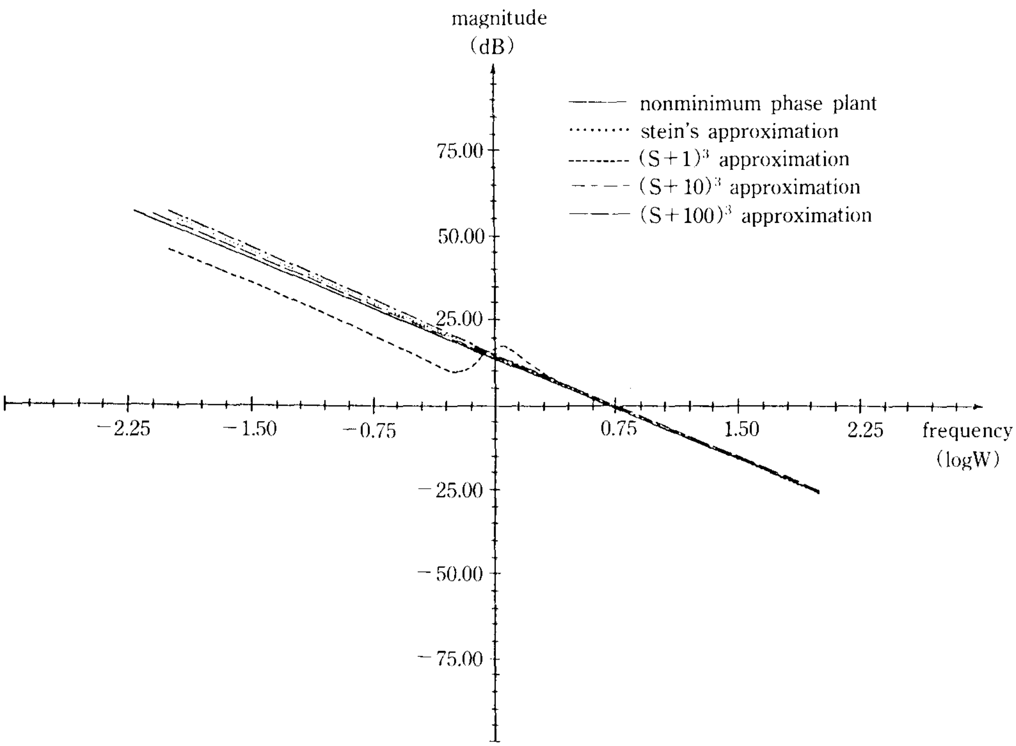

Fig. 3. Singular value plots of limiting filter loops for an SISO example.

minimum phase models are obtained in according to the choice of minimum phase zeros of $\tilde{G}(S)$.

Simulation results in the Fig. 3 show that the presented method can be better approximated to the limiting filter loop in the LQG/LTR procedure (5) by putting the minimum phase zeros far into the left half planne. This is actually due to the decrease of the magnitude of E(S). It is clearly noted that the degree of approximation depends upon the choice of minimum phase zeros and the presented method can be better approximated in the case of $(S+100)^3$ than Stein's one.

Example 2 (MIMO)

$$\text{Given } G(S)=\begin{bmatrix} \frac{S-1}{S^2+2S+1} & \frac{1}{S+1} \\ 0 & \frac{1}{S-1} \end{bmatrix}$$

Simulation results in the Fig. 4 also show that $(S+100)^3$, the minimum phase zeros left far in the left half plane, can be better approximated than $(S+10)^3$.

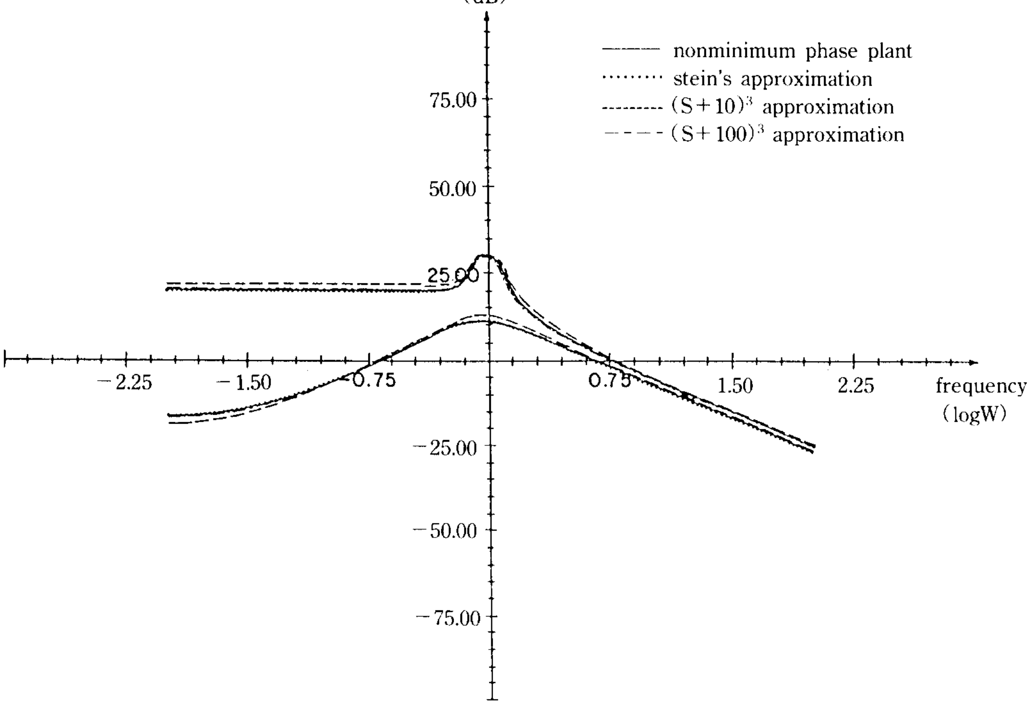

Fig. 4 Singular value plots of limiting filter loops for an MIMO example.

5. Conclusion

In this paper a new approximation method is presented to improve Stein's one. Theoretical analysis and simulation results show that the significant improvement is achieved and the better approximation can be also made by a suitable choice of minimum phase zeros.

The initial choice of minimum phase zeros are arbitrarily decided and the better choice of them are based on the trial and error. And the theoretical and optimal procedures are not suggested. The important future research needing additional work is how to make the optimal procedure theoretically for the best approximation.

References

1. J. C. Doyle, "Guaranteed Margins for LQG Regulators," IEEE Trans. on Auto Control, Vol. AC-23, August 1978, pp. 756-757.
2. H. Kwakernaak, "Optimal Low-Sensitivity Linear Feedback Systems," Automatica, Vol. 5, May 1969.
3. J. C. Doyle and G. Stein, "Multivariable Feedback Design : Concepts for a Classical/Modern Synthesis," IEEE Trans. on Auto. Control, Vol. AC-26, No. 1, February 1981, pp. 4-16.
4. G. Stein and H. Athans, "LQG/LTR Procedure for Multivariable Feedback," IEEE Trans. on Auto Control, Vol AC-32, No. 2, February 1987, pp. 105-114.