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ELECTRODYNAMIC JET FORMATION

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ABSTRACT

The original axisymmetric, stationary electrodynamic model of the central engine in an active galactic nucleus proposed by Macdonald and Thorne consists of a supermassive black hole with magnetic field lines that pass through the region just outside the event horizon of the black hole. Each magnetic field line rotates with a constant angular velocity which will exceed the speed of light at large radii. Even though the field lines are purely mathematical entities this condition sets a stringent physical constraint on the motion of the magnetic field lines and the particles on them. In this paper we will show that we can remove this auxiliary constraint in our model by allowing nonstationary processes. As a result the magnetic field lines can be twisted and wound up in a region lying outside of the quasi-stationary magnetosphere of the black hole. We conclude that astrophysical jets are formed in that region due to the twisted and wound magnetic field lines powered by the Blandford-Znajek process and the other driving forces.

I. INTRODUCTION

An axisymmetric, stationary model of the central engine in an active galactic nucleus (AGN) has been investigated as the setting for the Blandford-Znajek process (Blandford and Znajek 1977), which consists of the supermassive black hole surrounded by a magnetized accretion disk. The model was reformulated and extended by Thorne and Macdonald (1982), Macdonald and Thorne (1982, hereafter MT), Thorne, Price, and Macdonald (1986), Thorne (1986), and Blandford (1987) in "3 + 1" -spacetime formalism, which allowed them to set out the astrophysical equations in forms familiar to theoretical astrophysicists.

In this model each magnetic fiedl line must rotate with constant angular velocity. The angular velocity of the field line, Ω^F, must be constant along the line (MT, eq [5.4]),

$$\boldsymbol{B}^P \cdot \nabla \Omega^F = 0, \tag{1.1}$$

where B^P is the poloidal component of the magnetic field. Equation (1.1) expresses Ferraro's law of isorotation (Ferraro 1937). Therefore, the velocities of the field lines will exceed the speed of light far from the center. Even though the field lines are purely mathematical entities this condition sets a stringent physical constraint on the motion of the magnetic field lines and the particles on them.

We have extended this axisymmetric, stationary model to include the possibility of time evolution in the previous papers (Park and Vishniac 1989a, hereafter paper I: Park and Vishniac 1989b, hereafter paper II). The main point was to add the secular effects of mass accretion to the original axisymmetric, stationary model. Our model was summarized in Figure 1 in Park and Vishniac (1988). In this paper we investigate the motion of the magnetic field lines in our axisymmetric, nonstationary model and find that Ferraro's law does not apply in the region lying outside of the magnetosphere of the black hole. We therefore remove the auxiliary constraint in our nonstationary model and examine the consequences.

Following papers I and II, we will summarize the equations of axisymmetric, nonstationary electrodynamics in §II. The breakdown of Ferraro's law will be demonstrated in §III. Finally, we will discuss the astrophysical implications of our results and our conclusions in §IV. Throughout this paper we define our units such that $c = G = 1$, and the central black hole is assumed to be a Kerr black hole which possesses the total mass M, the angular momentum, J, and the angular momentum density $a(= J/M)$.

II. AXISYMMETRIC, NONSTATIONARY ELECTRODYNAMICS

In this section we will review the axisymmetric, nonstationary electrodynamics. Axisymmetric, nonstationary conditions can be represented as (I, eq. [3.1]; II, eq. [2.1]),

$$\boldsymbol{m}\cdot\nabla f = 0; \; L_m \boldsymbol{f} = 0 \tag{2.1a}$$

and

$$\frac{\partial f}{\partial t} = \dot{f} \neq 0; \; \frac{\partial \boldsymbol{f}}{\partial t} = \dot{\boldsymbol{f}} \neq 0, \tag{2.1b}$$

where, as in papers I and II, **m** is a Killing vector of the axisymmety, $\boldsymbol{L}$ means the Lie derivative, and f and **f** are any scalar and vetor, respectively.

To describe the sphericaly symmetirc spacetime we use the spherical coordinate system (r, θ, φ) whose unit vectors are expressed as $\boldsymbol{e}_{\hat{r}}$ $\boldsymbol{e}_{\hat{\theta}}$, and $\boldsymbol{e}_{\hat{\varphi}}$, respectively ($\boldsymbol{e}_{\hat{r}} \times \boldsymbol{e}_{\hat{\theta}} = \boldsymbol{e}_{\hat{\varphi}}$). We also use the cylindrical coordinate system ($\boldsymbol{R}$, φ, z) with the unit vectors $\boldsymbol{e}_{\hat{R}}$, $\boldsymbol{e}_{\hat{\varphi}}$, and $\boldsymbol{e}_{\hat{z}}$ ($\boldsymbol{e}_{\hat{R}} \times \boldsymbol{e}_{\hat{\varphi}} = \boldsymbol{e}_{\hat{z}}$) to describe the axisymmetry of the accretion disk.

Throughout this paper **m** has the same magnitude as ω, the separation between the symmetric axis of the black hole and a Zero-Augular-Momentum-Observer (ZAMO; see Bardeen 1973, and

MT),

$$\tilde{\omega} = \frac{\Sigma}{\rho} \sin \theta, \tag{2.2a}$$

where

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \tag{2.2b}$$

$$\Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta, \tag{2.2c}$$

and

$$\Delta = r^2 + a^2 - 2Mr. \tag{2.2d}$$

1. General Case

Let ∂A be an **m**-lop, A be any surface bounded by ∂A but not intersecting the event horizon of the black hole, and $d\boldsymbol{\Sigma}$ be the normal vector on an infinitesimal area on A. Then we can define the total electric current passing downward through A, $I(\mathbf{x}, t)$, the total magnetic flux passing upward through A, $\Psi(\mathbf{x}, t)$, and the total electric flux passing upward through A, $\Phi(x, t)$, as(I, eq. [3.2]; II, eq. [2.3]),

$$I(\mathbf{x}, t) = -\int_A \alpha \boldsymbol{j} \cdot d\boldsymbol{\Sigma}, \tag{2.3a}$$

$$\Psi(\mathbf{x}, t) = \int_A \boldsymbol{B} \cdot d\boldsymbol{\Sigma}, \tag{2.3b}$$

and

$$\Phi(\mathbf{x}, t) = \int_A \boldsymbol{E} \cdot d\boldsymbol{\Sigma}, \tag{2.3c}$$

where **j** is the current vector, and α is the lapse function of the ZAMO. The value of α is given by

$$\alpha = \frac{\rho}{\Sigma} \sqrt{\Delta} \tag{2.4}$$

In this case the Maxwell equation for the ZAMO at the given point around the central black hole are given by the condition (2.1) as (see I, eqs. [2.9])

$$\nabla \cdot \boldsymbol{E} = 4\pi \rho_e, \tag{2.5a}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{2.5b}$$

$$\nabla \times (\alpha E) = -\{\dot{B} - (B\cdot\nabla\omega)m\}, \tag{2.5c}$$

and

$$\nabla \times (\alpha B) = \dot{E} - (E\cdot\nabla\omega)m + 4\pi\alpha j, \tag{2.5d}$$

where

$$\omega = \frac{2aMr}{\Sigma^2} \tag{2.5e}$$

is the ZAMO's angular velocity. In this case the electromagnetic fields are given by (I, eqs. [3.3], [3.5], and [3.6]; II, eq. [2.5])

$$E^T = -\frac{2}{\alpha\,\varpi}\left(\frac{\dot{\Psi}}{4\pi}\right)e_{\hat{\varphi}}, \tag{2.6a}$$

$$E^P = E - E^T, \tag{2.6b}$$

$$B^T = -\frac{2}{\alpha\,\varpi}\left(I - \frac{\dot{\Phi}}{4\pi}\right)e_{\hat{\varphi}}, \tag{2.6c}$$

and

$$B^P = -\frac{e_{\hat{\varphi}} \times \nabla\Psi}{2\pi\,\varpi} \tag{2.6d}$$

where T, P denote the toroidal and poloidal components respectively.

2. Degenerate Case

In this subsection we will discuss degenerate, axisymmetric, nonstationary electrodynamics. In this case we have

$$E\cdot B = 0 \tag{2.7}$$

Equation (2.7) corresponds to the requirement that there are no unbalanced forces causing the particles to accelerate along the magnetic field lines. This should be a reasonable approximation when considering the extremely diffuse plasma in the magnetospher.

By the same arguments in papers I and II, we define the physical velocity of the magnetic field limes relative to the ZAMO (I, eq. [4.2]; II, eq. [3.2]),

$$v^F = -\frac{\omega - \Omega^F}{\alpha}\,m + v\,(x,\ t)_{inf}\,e_{\hat{R}} \tag{2.8a}$$

such that

$$\boldsymbol{E}^P = -(\boldsymbol{v}^F)^T \times \boldsymbol{B}^P - (\boldsymbol{v}^F)^P \times \boldsymbol{B}^T \tag{2.8b}$$

and

$$\boldsymbol{E}^T = -(\boldsymbol{v}^F)^P \times \boldsymbol{B}^P, \tag{2.8c}$$

where $v_{\rm inf}$ is the radial infall velocity of the magnetic field lines. Notice that $v_{\rm inf}$ is the entire reason why our model is nonstationary.

Now, from equation (2.8b), we can get the poloidal component of the electric field (I, eq. [4.3]; II, eq. [3.3])

$$\boldsymbol{E}^P = \frac{\omega - \Omega^F}{2\pi\alpha} \nabla\Psi + \frac{2v_{\rm inf}}{\alpha\omega}(I - \frac{\dot{\Phi}}{4\pi})\boldsymbol{e}_{\hat{z}}, \tag{2.9}$$

which was not specified in equation (2.6b).

III. THE BREAKDOWN OF FERRARO'S LAW

Armed with the axisymmetric, nonstationary electrodynamic equations in §II we will investigate the electrodynamics of the region lying outside of the magnetosphere of the black hole. In papers I and II we stated that the magnetosphere of the black hole itself will be quasi-stationary on timescales longer than the horizon timescale because the strength of the magnetic field is constrained to be (I, eq. [4.5])

$$B_{\perp 4} \sim \sqrt{\frac{\dot{m}}{M_8}}, \tag{3.1}$$

where $B_{\perp 4}$ is the strength of the magnetic field in the unit of 10^4 gauss, $\dot{m}$ is the ratio of the mass accretionrate $\dot{M}_+$ to the Eddington accretion rate $\dot{M}_E$, and M_8 is M in the unit of $10^8\ M_\odot$. This follows from the requirement that the black hole magnetic field be just powerful enough to affect the flow of the accreting matter. This requirement is reasonable if the disk system is capable of generating magnetic fields through dynamo action, an assumption which seems necessary to explain the large magnetic fields needed to power the nonthermal radiation from AGN. A weaker magnetic field would grow through the accretion of flux from the disk. A stronger field would tend to cause the accretion of canceling flux from the disk.

The region lying outside of the magnetospohere of the black hole, however, will be directly affected by the accreting magnetic field lines. Throughtout the remainder of this paper we wil assume that the degenerate condition (2.7) is satisfied in that region which lies far enough ($\hat{r} > a$) from the central black hole to have an essentially Newtonian structure with

$$\alpha \rightarrow 1, \tag{3.2a}$$

$$\tilde{\omega} \rightarrow \hat{R} = R, \tag{3.2b}$$

and

$$\omega \rightarrow 0 \tag{3.2c}$$

The last condition means that ZAMOs are just distant observers in this case. We also set as

$$\boldsymbol{v}^F = v^{\hat{\varphi}} + v^{\hat{R}} = v^{\hat{\varphi}} \boldsymbol{e}_{\hat{\varphi}} + v^{\hat{R}} \boldsymbol{e}_{\hat{R}}, \tag{3.3}$$

where $\mathbf{v}^{\hat{R}}$ and $\mathbf{v}^{\hat{\varphi}}$ are the radial and angular components, respectively. This means

$$v_{\text{inf}} = v^{\hat{R}} \tag{3.4}$$

in the region under consideration

Therefore, the electromagnetic fields in that region are given by

$$\boldsymbol{E}^T = -\frac{2}{R}\left(\frac{\dot{\Psi}}{4\pi}\right)\boldsymbol{e}_{\hat{\varphi}}, \tag{3.5a}$$

$$\boldsymbol{E}^P = -\frac{\Omega^F}{2\pi}\nabla\Psi + \frac{2v^{\hat{R}}}{R}\left(I - \frac{\dot{\Phi}}{4\pi}\right)\boldsymbol{e}_{\hat{z}} \tag{3.5b}$$

$$\boldsymbol{B}^T = -\frac{2}{R}\left(I - \frac{\dot{\Phi}}{4\pi}\right)\boldsymbol{e}_{\hat{\varphi}}, \tag{3.5c}$$

and

$$\boldsymbol{B}^P = -\frac{\boldsymbol{e}_{\hat{\varphi}} \times \nabla\Psi}{2\pi R}. \tag{3.5d}$$

By the definitions (2.3) we have (see MT, eqs. [4.9] and [4.10a])

$$\nabla I = -2\pi R\boldsymbol{e}_{\hat{\varphi}} \times \boldsymbol{j}^P \tag{3.6a}$$

and

$$\nabla\Psi = 2\pi R\boldsymbol{e}_{\hat{\varphi}} \times \boldsymbol{B}^P \tag{3.6b}$$

in the Newtonian limit. We also have

$$\dot{\Psi} = -2\pi R e^{\hat{\varphi}} = -2\pi R v^{\hat{R}} B^{\hat{z}} \tag{3.7a}$$

and

$$I - \frac{\dot{\Phi}}{4\pi} = -\frac{1}{2} R B^{\hat{\varphi}}. \tag{3.7b}$$

We get

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}} \tag{3.8}$$

from one of the Maxwell equations (eq. [2.5c]). Substituting equations (3.5), (3.6), and (3.7) into eqution (3.8) and dropping $(\upsilon^{\hat{R}})^2$-order terms, we get

$$\begin{aligned}\nabla \times \boldsymbol{E} = {} & B^{\hat{z}} \frac{\partial \upsilon^{\hat{R}}}{\partial R} \boldsymbol{e}_{\hat{z}} + \upsilon^{\hat{R}} \, \nabla \hat{B}^{z} \times \boldsymbol{e}_{\hat{\varphi}} - R(\boldsymbol{B}^{P} \cdot \nabla \Omega^{F}) \boldsymbol{e}_{\hat{\varphi}} \\ & + B^{\hat{\varphi}} \left(\frac{\partial \upsilon^{\hat{R}}}{\partial R} - \frac{\upsilon^{\hat{R}}}{R} \right) \boldsymbol{e}_{\hat{\varphi}}.\end{aligned} \tag{3.9}$$

Splitting equation (3.8) into toroidal, poloidal components we get the relation

$$\boldsymbol{B}^{P} \cdot \nabla \Omega^{F} = \frac{B^{\hat{\varphi}}}{R} \left(\frac{\partial \upsilon^{\hat{R}}}{\partial R} - \frac{\upsilon^{\hat{R}}}{R} + \frac{(\dot{B})^{\hat{\varphi}}}{B^{\hat{\varphi}}} \right) \tag{3.10}$$

from the toroidal component. The right-hand side of equation (3.10) clearly vanishes in the stationary case (see eq. [1.1]).

Equation (3.10) shows that Ferraro's law of isorotation naturally breaks down in our nonstationary model and the magnetic field lines can be twisted and wound up in the region under consideration. Therefore, the field lines will not move at speed of light even in the most distant regions in our model. The "acceleration region"(see MT) in our nonstationary model seems to lie much closer to the hole than that in the original stationary model.

IV. CONCLUSIONS

In our axisymmetric, nonstationary model of the central engine in an AGN the magnetic field lines are dragged toward the symmetry axis of the accretion disk, which is also the rotational axis of the black hole. Therefore, the magnetic field lines will be twisted and wound up around the this axis. Several hypothesis have been proposed to explain the origin of astrophysical jets (see e.g., Begelman et al. 1984; Rees 1986). Shibata and Uchida (1984, and references therein) proposed a mechanism in which both the acceleration and the collimination of the astrophysical jets are caused by the action of twisted and wound magnetic field lines. In this model, however, the driving force is just the action of the local field on the bulk current, $\mathbf{j} \times \mathbf{B}$.

If this mechanism operates in the acceleration region of our model, where the power extracted by the Blandford-Znajek process is deposited, it seems likely that this provides a natural explanation for the energetic collimation and the helical motion of the observed jets. As MT have pointed out, much of the power extracted by the Blandford-Znajek process is used to create charged particle pairs and the rest will accelerate the particles. Therefore, we conclude that astrophysical jets are formed just outside the quasi-stationary magnetosphere of the black hole due to the twisted and wound magnetic field lines powered by the Blandford-Znajek process and mediated by the bulk current forces acting on the plasma.

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